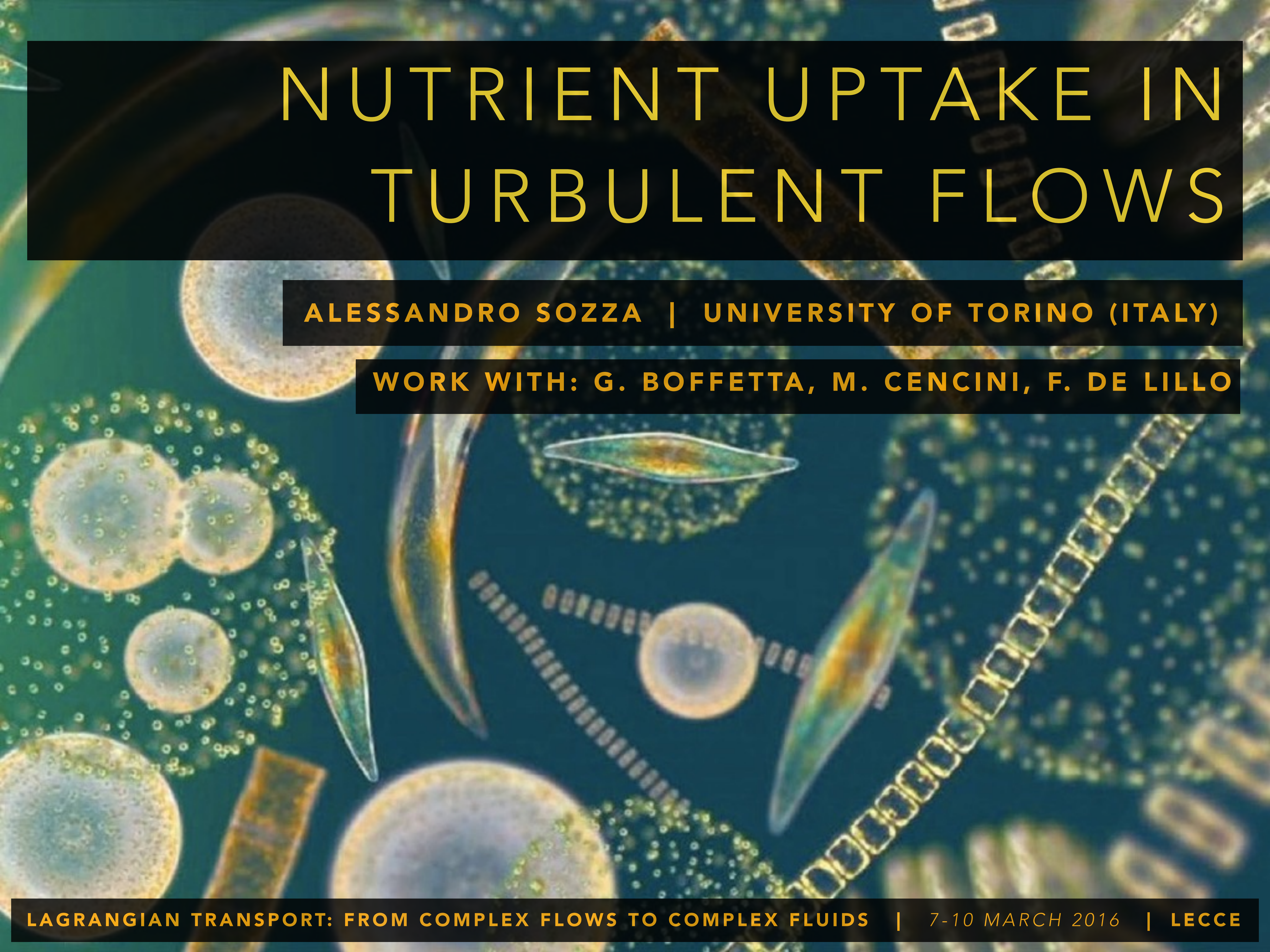


NUTRIENT UPTAKE IN TURBULENT FLOWS

ALESSANDRO SOZZA | UNIVERSITY OF TORINO (ITALY)

WORK WITH: G. BOFFETTA, M. CENCINI, F. DE LILLO

LAGRANGIAN TRANSPORT: FROM COMPLEX FLOWS TO COMPLEX FLUIDS | *7-10 MARCH 2016* | LECCE

General Motivation

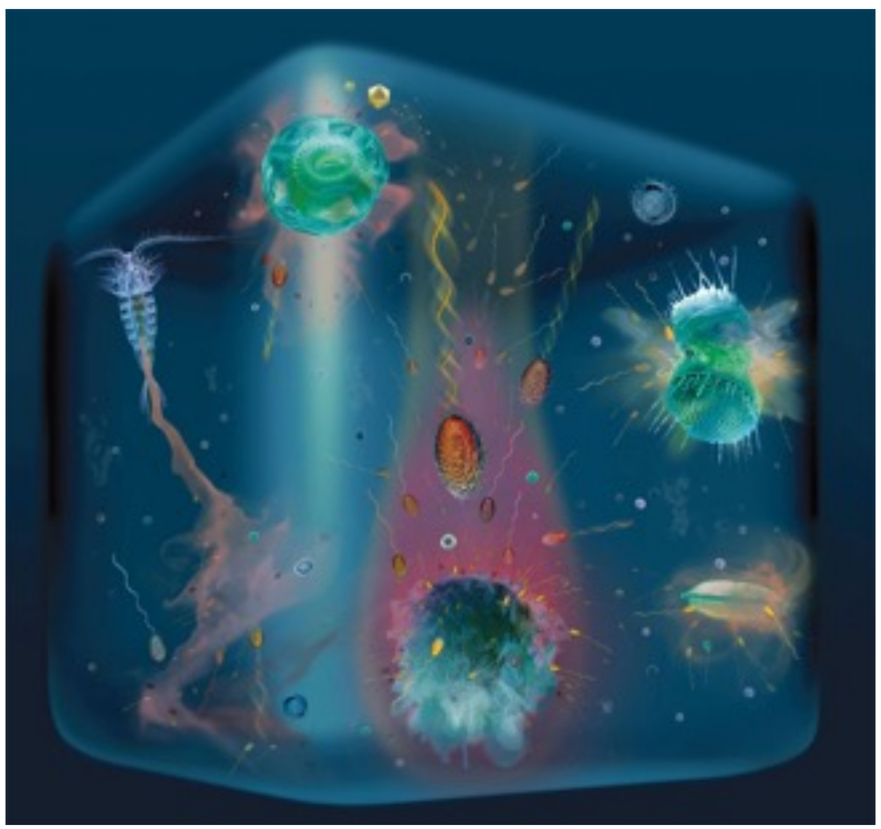
Life at Low Reynolds

Transport of dissolved nutrient
to the cell via osmosis

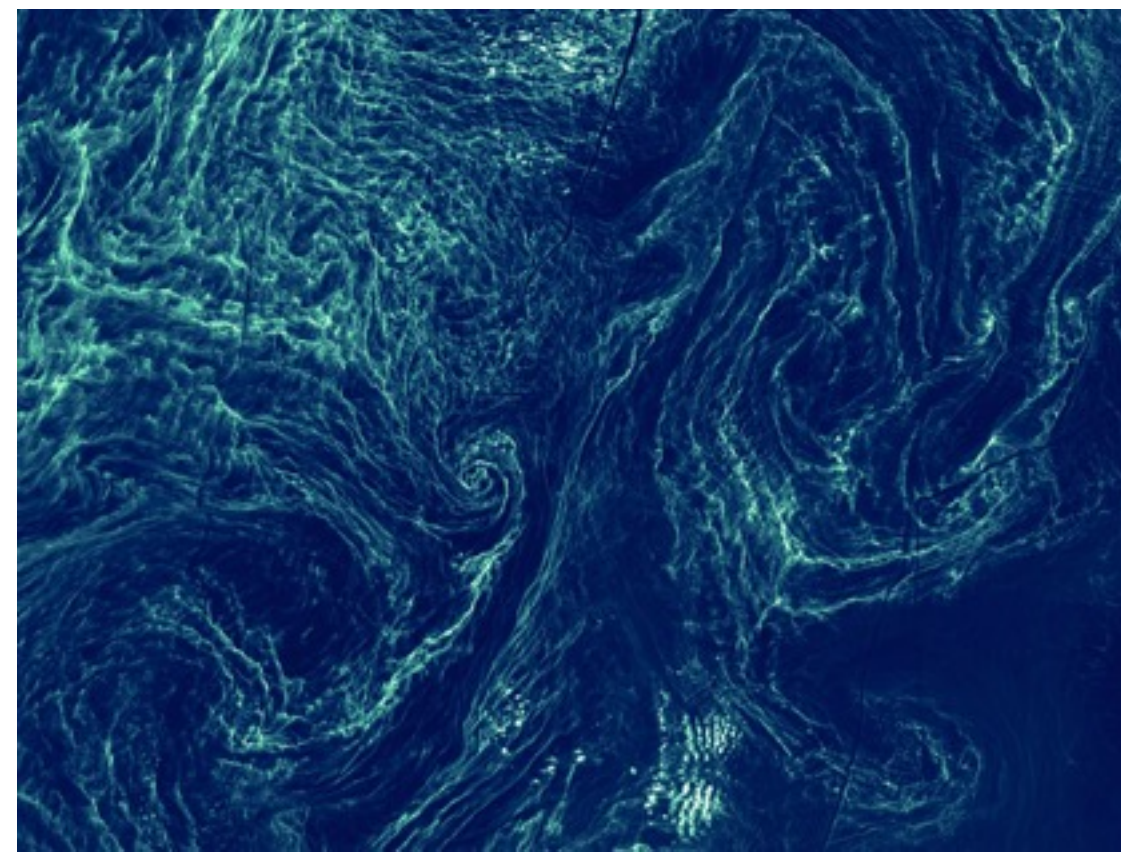
Uptake is limited by diffusion.

In a turbulent world

Turbulence generates **inhomogeneous** distribution of nutrients, oxygen…



J.R. Taylor, R. Stocker, Science (2012)

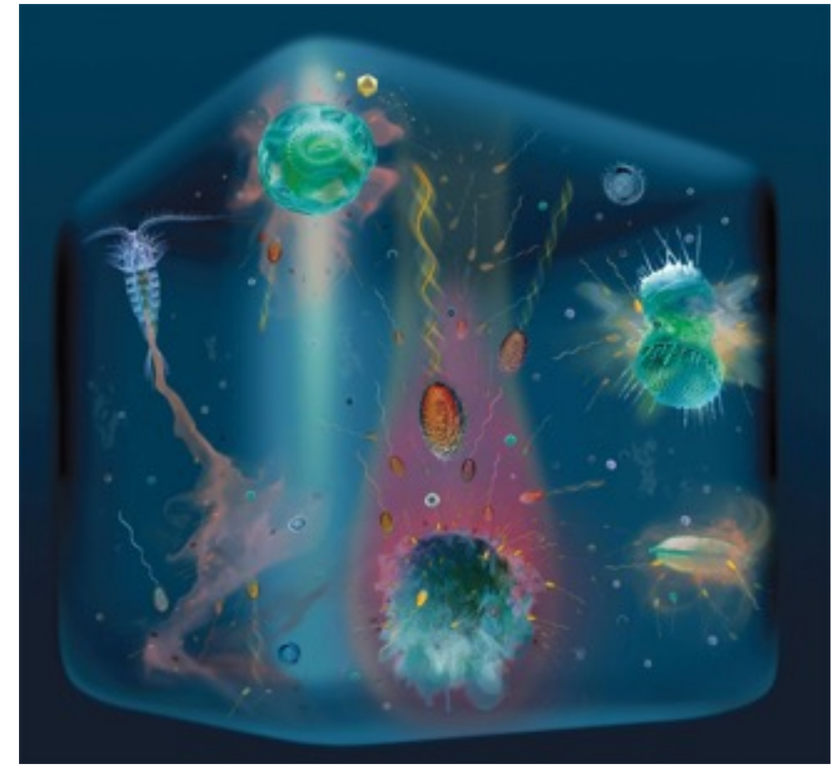


Phytoplankton bloom in the Baltic Sea. (ESA)

What are the strategies for an efficient feeding?

How turbulence affects nutrient uptake?

Scale For Marine Microbes



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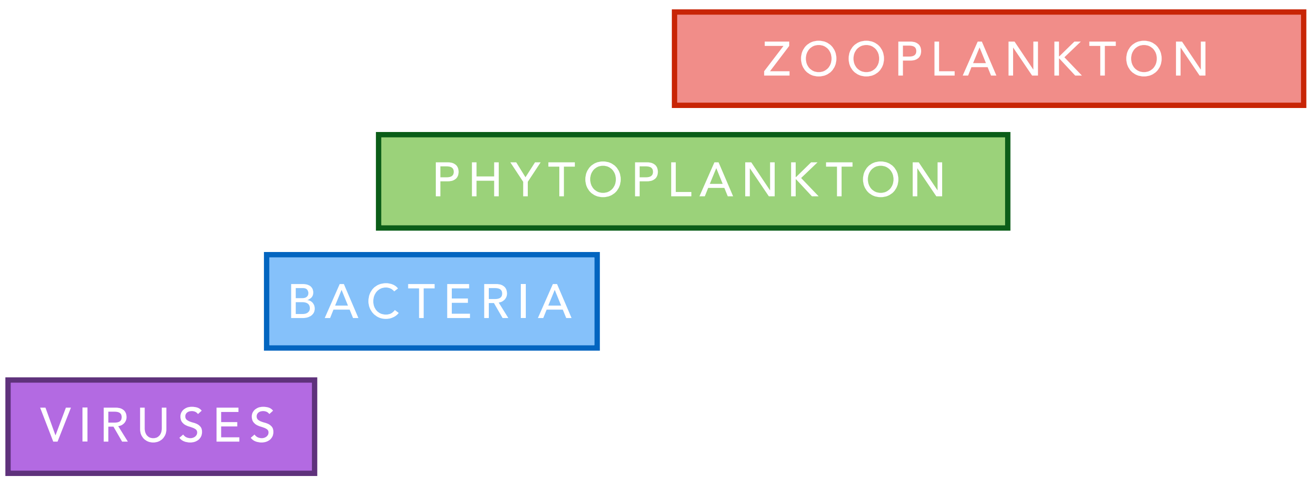
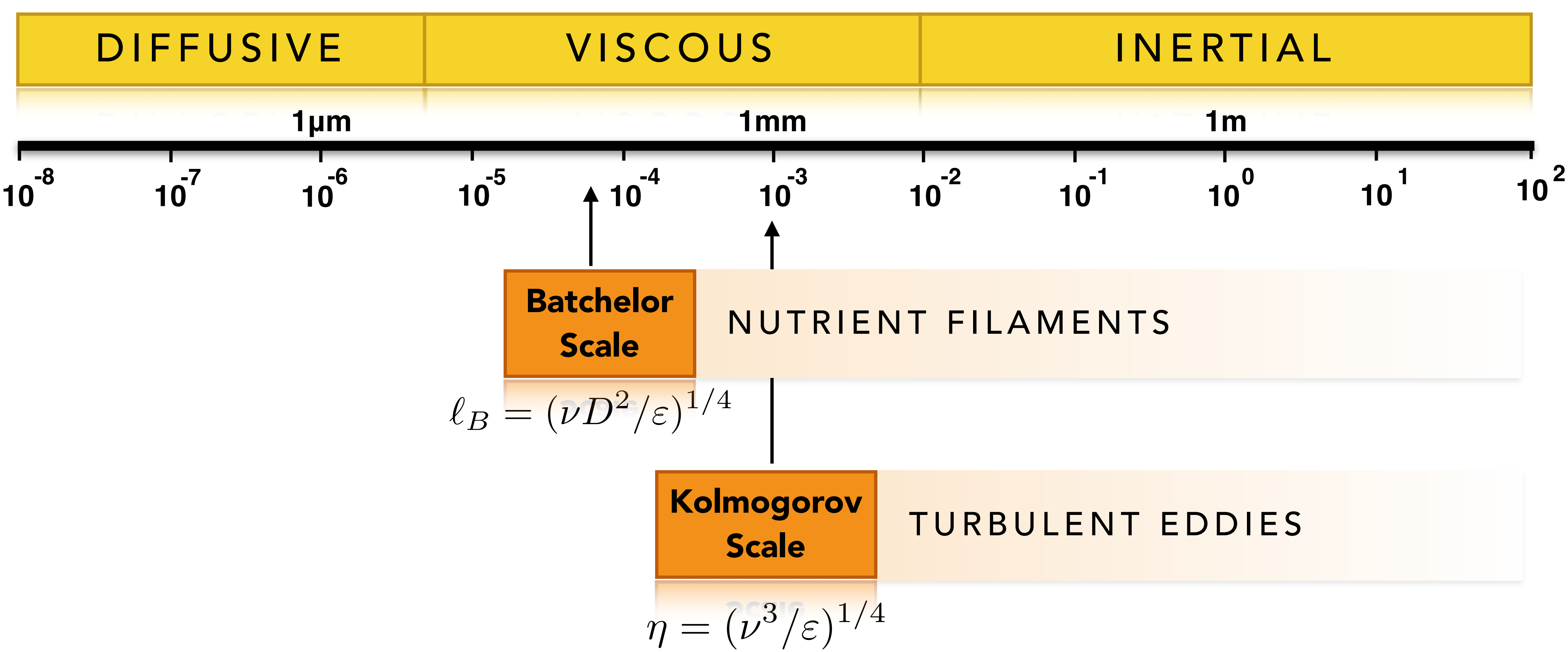




Life at Low Reynolds...

Consider an absorbing sphere in a homogeneous nutrient field

$$\boxed{\partial_t \varrho = D \triangle \varrho}$$

Boundary Conditions:
(Dirichlet BC)

$$\begin{cases} \varrho(R) = 0 & \text{(Perfect Absorber)} \\ \lim_{r \to \infty} \varrho(r) = \varrho_\infty \end{cases}$$

Flux per unit area:
(Fick's Law)

$$J_r = -D \frac{\partial \varrho}{\partial r}$$

Uptake:

$$\kappa = - \int_s \boldsymbol{J} \cdot \hat{n} \, dS$$

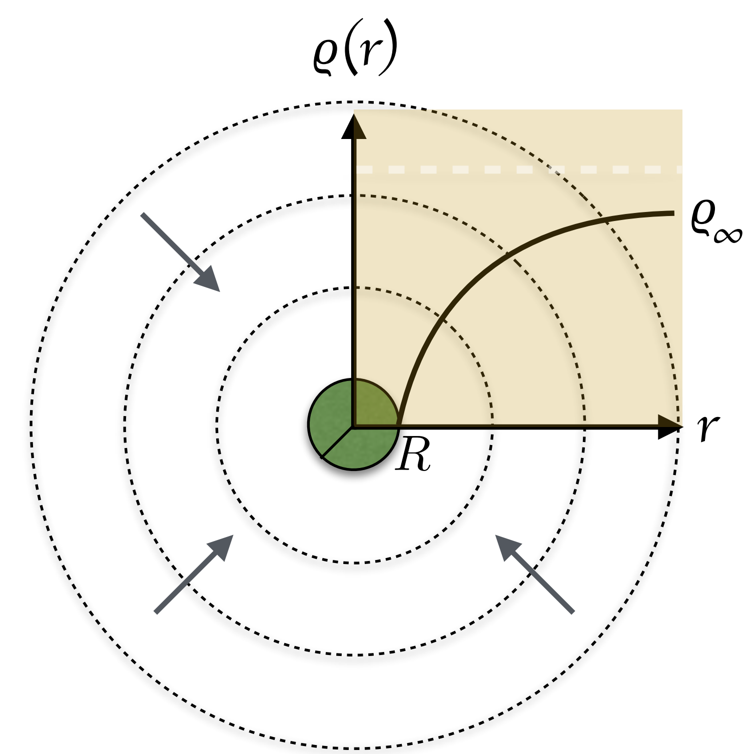
Smoluchowski rate

$$\kappa_s = 4\pi D R \varrho_\infty$$

(Diffusion-Controlled Processes)

Stationary Solution:

$$\varrho(r) = \varrho_\infty \left(1 - \frac{R}{r} \right)$$



Time-dependent Solution

$$\boxed{\partial_t \varrho = D \triangle \varrho}$$

Transforming in Laplace $\hat{\varrho}(r,s) = \int_0^\infty e^{-st} \varrho(r,t)\, dt \longrightarrow s\hat{\varrho} - \varrho_\infty = D \triangle \hat{\varrho}$

General solution is in the form: $\hat{\varrho}(r,s) = \dfrac{\varrho_\infty}{s} + A \dfrac{e^{-Kr}}{r} + B \dfrac{e^{Kr}}{r}$ $\quad K = \sqrt{s/D}$

Imposing BCs: $\hat{\varrho}(r,s) = \dfrac{\varrho_\infty}{s} \left(1 - \dfrac{R}{r} e^{-K(r-R)} \right)$

Inverse transforming $\varrho(r,t) = \varrho_\infty - \dfrac{R}{r} \varrho_\infty \left[1 + \mathrm{Erf}\left(\dfrac{R-r}{2\sqrt{Dt}} \right) \right]$

(Carslaw and Jaeger, 1959)
(Osborn, 1996)

UPTAKE $\begin{cases} J_r = -D \dfrac{\partial \varrho}{\partial r} \\ \kappa = -\displaystyle\int_s \boldsymbol{J} \cdot \hat{n}\, dS \end{cases} \longrightarrow \boxed{\kappa(t) = 4\pi D R \varrho_\infty \left(1 + \dfrac{R}{\sqrt{\pi D t}} \right)}$

Optimal Cell Size

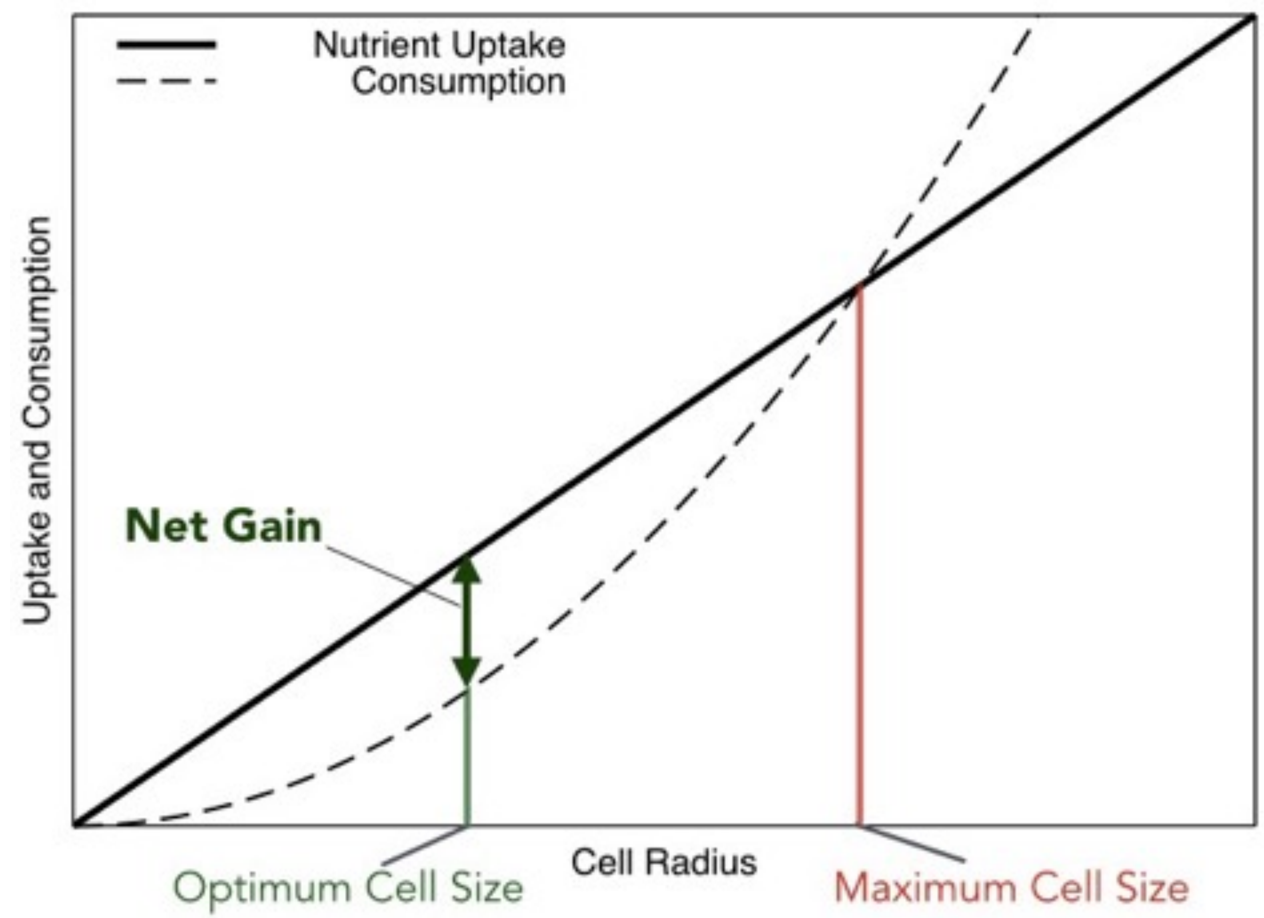
Diffusion is a limiting factor in cell growth

Cell size is the compromise between **uptake** by diffusion and **metabolism**

Uptake: $\kappa \propto R$

Metabolism: $M \propto R^{\gamma}$

From T. Kiørboe

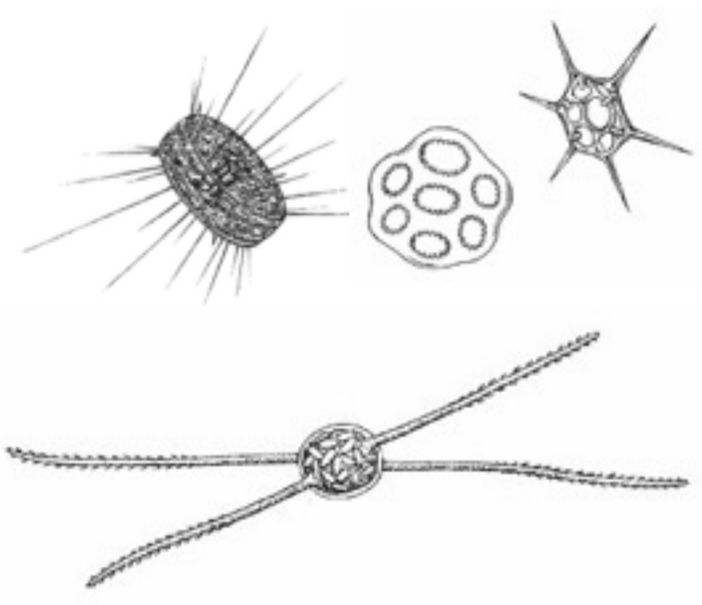


Physical properties of the fluid shape the cell biology.

Life at Low Reynolds...

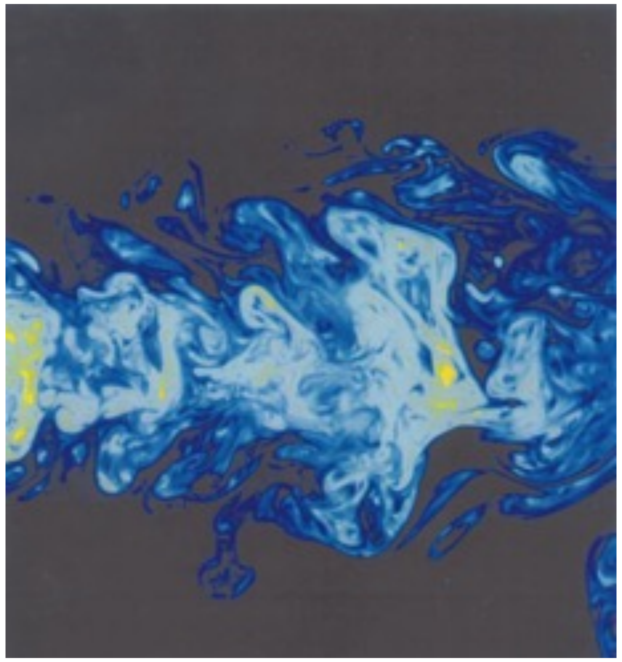
Strategies: Shape, Motility & Flow

Shape as response to nutrient limitation

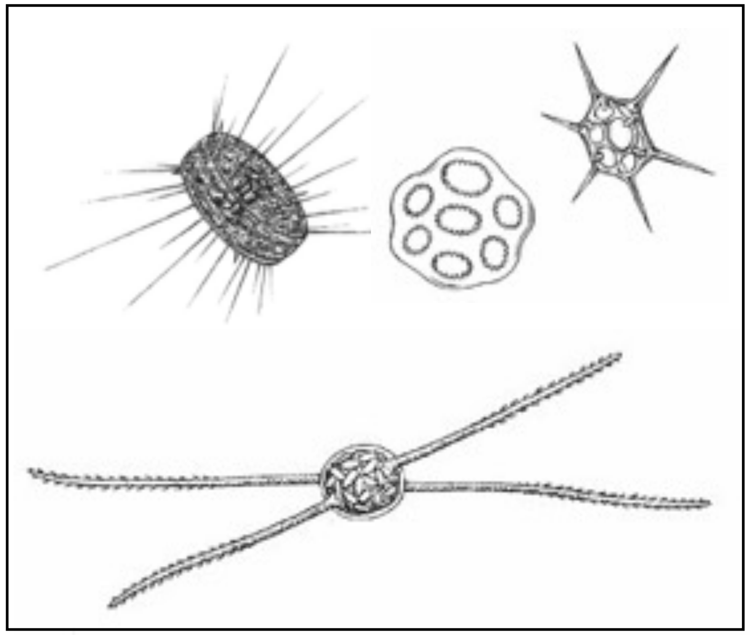


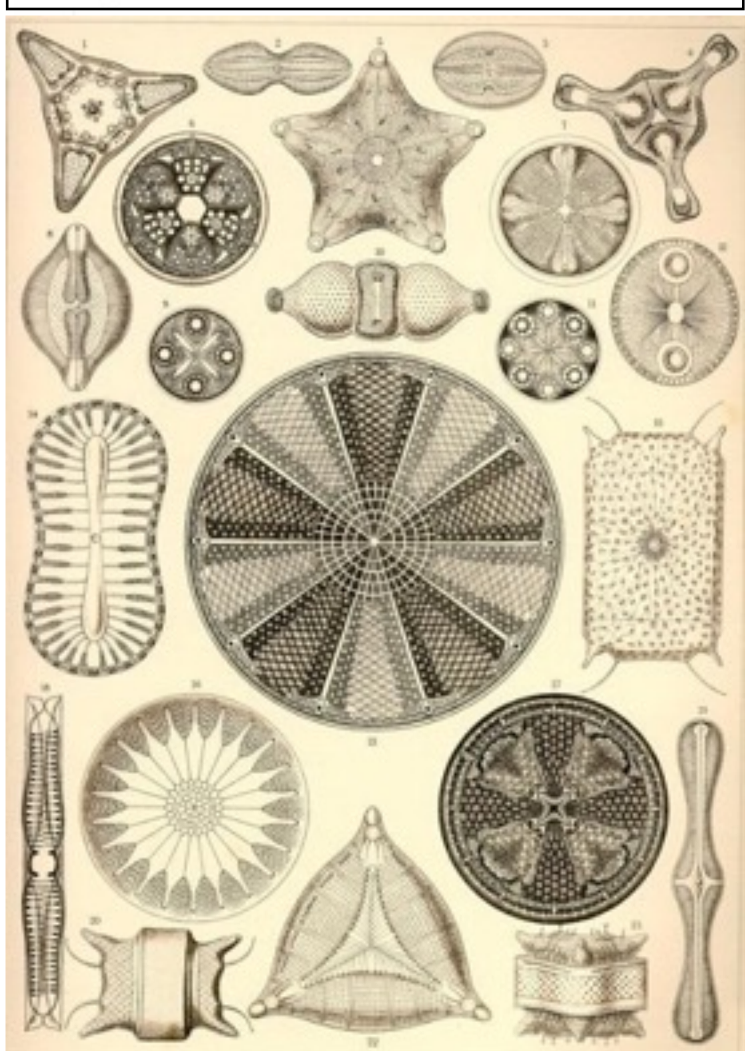
Motility to reach regions of fresh nutrient

Turbulent flow may create complex nutrient landscape



Strategies: Effect of Shape



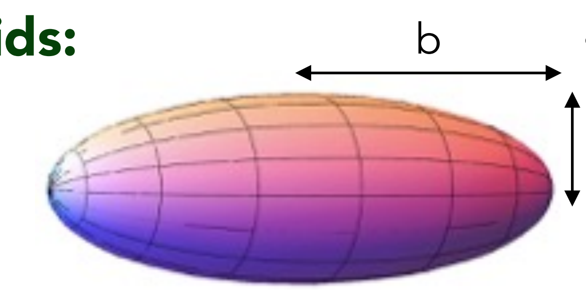


The cell shape has a modest effect on uptake:

Some bacteria appear to exploit this effect by becoming elongated in nutrient-deprived conditions (Steinberger et al. 2002).

Prolate ellipsoids have marginally greater uptake than spheres of equal volume (Clift et al. 2005).

Smoluchowski rate for prolate spheroids:



$$\kappa_s = 4\pi D a \frac{\sqrt{E^2 - 1}}{\log(E + \sqrt{E^2 - 1})}$$

$$E = \frac{b}{a} > 1$$

Cell may obtain a benefit of 4 to 20%.

Strategies: Effect of Motility

Peclet Number: $Pe = \frac{V_s R^2}{D}$

Swimming in a homogeneous environment may enhance uptake.

Sherwood Number: $Sh \equiv \frac{\kappa}{\kappa_s}$ $\frac{\text{Uptake by advection}}{\text{Diffusive Uptake}}$

$$\left. \begin{array}{l} Sh \sim 1 + \frac{1}{2}Pe + Pe^2 \log(Pe) \\ Sh \sim 0.62 Pe^{1/3} \end{array} \right| \begin{array}{l} Pe \ll 1 \\ Pe \gg 1 \end{array} \Bigg|$$

Frankel & Acrivos *(1968)*

But it implies an **energetic cost!**

If resources are **heterogeneous** motility can confer an important fitness advantage.

Competition between motility and nutrients mixing may be important and depends on the peculiar details of the species.

Turbulence may enhances uptake?

Shear Flow (Mean field approximation of turbulence)

Shear Rate: $\gamma = \left(\frac{\varepsilon}{\nu}\right)^{1/2}$ Peclet Number: $Pe = \frac{\gamma R^2}{D}$

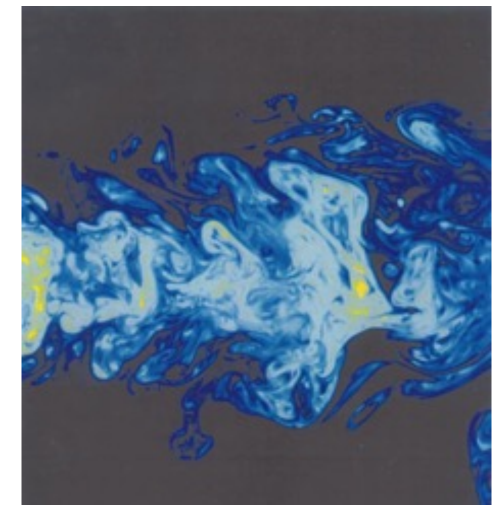
Karp-Boss, Boss, Jumars (1996)

Sherwood Number: $Sh \equiv \frac{\kappa}{\kappa_s}$ $\frac{\text{Uptake by advection}}{\text{Diffusive Uptake}}$

$Sh \simeq 1 + 0.36\, Pe^{1/2}$	$Pe \ll 1$	***Frankel & Acrivos** (1968)*
$Sh \simeq 0.9\, Pe^{1/3}$	$Pe \gg 1$	***Batchelor** (1979)*

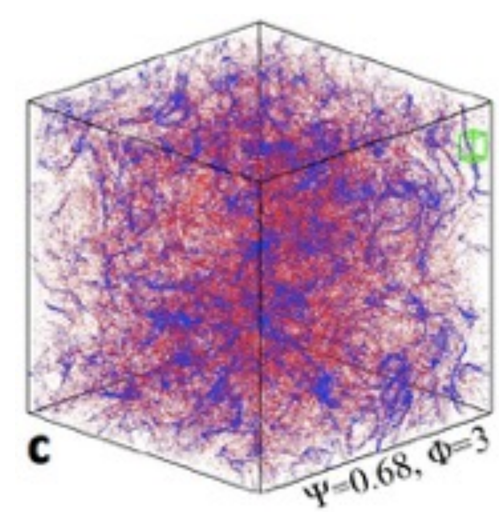
Enhancement of uptake by bacteria is negligible (0.3%) while becomes important for larger phytoplankton species (20-30%)

Turbulent flow may create complex nutrient landscape



Preferential concentration

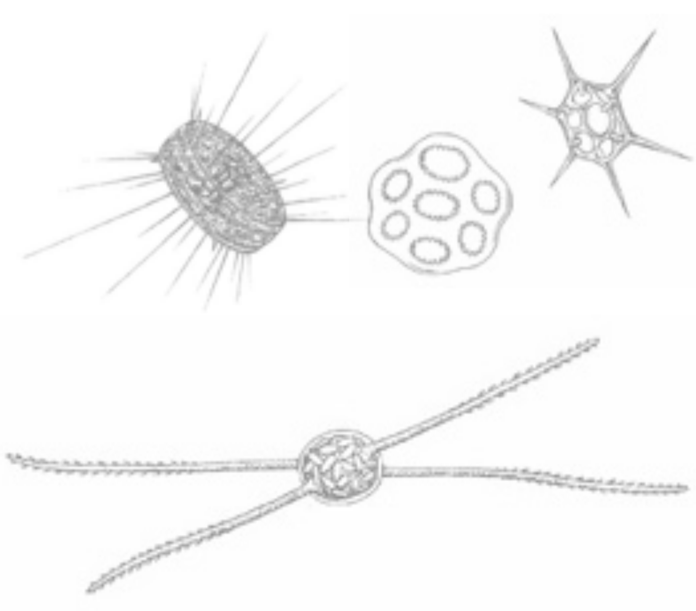
(eg. gyrotactic model)



Life at Low Reynolds…

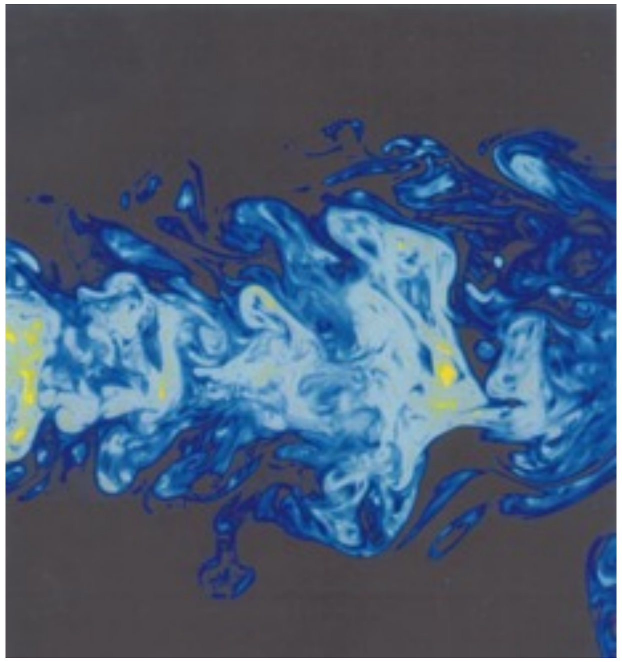
Strategies: Shape, Motility & Flow

Shape as response to nutrient limitation



Motility to reach regions of fresh nutrient

Turbulent flow may create complex nutrient landscape



How turbulence affects nutrient uptake?

Numerical Approach

Pseudo-Spectral Method + IBM

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nu \triangle \boldsymbol{u} + \boldsymbol{f}$$

NAVIER-STOKES (DNS)

$$\partial_t \varrho + \boldsymbol{u} \cdot \nabla \varrho = D \triangle \varrho - \beta \varrho \sum_i^N \delta(\boldsymbol{x} - \boldsymbol{\xi}) + \boldsymbol{f}_\varrho$$

NUTRIENT FIELD

$\left\{ \begin{array}{l} \boldsymbol{\xi} \text{ : particle position} \\ \beta \text{ : rate of absorption} \end{array} \right.$

SINK TERM

FORCING BY UPWELLING

Differently from the analytical problem:

PERIODIC BOX (Lost Infinity)

NON-STATIONARY PROCESS (Scalar in decay)

POINTLIKE PARTICLES (No surface integral)

Uptake: $$\kappa(t) = \int_{\Omega_p} d^3\boldsymbol{x}\, \beta \varrho(\boldsymbol{x}, t) f(\boldsymbol{x} - \boldsymbol{\xi})$$

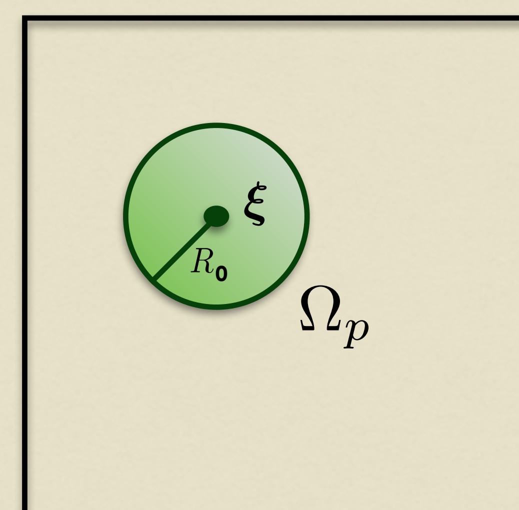
The uptake of the particle is obtained
by integrating the instantaneous sink term.

IMMERSED BOUNDARY

The sink term describes
the diffusive halo around the cell with

$$f(\boldsymbol{x}) \neq 0 \quad \text{for} \quad \boldsymbol{x} \subset \Omega_p$$

centered on the particle position.



Numerical Approach

STAGNANT CONFIGURATION $\boldsymbol{u} = 0$

$$\partial_t \varrho = D \triangle \varrho - \beta \rho \sum_i^N \delta(\boldsymbol{x} - \boldsymbol{\xi})$$

NUTRIENT FIELD

$\boldsymbol{\xi}$: particle position

β : rate of absorption

DECAY OF THE NUTRIENT FIELD IN PRESENCE OF SINKS

(Non-stationary process)

Uptake: $$\kappa(t) = \int_{\Omega_p} d^3\boldsymbol{x}\, \beta \varrho(\boldsymbol{x}, t) f(\boldsymbol{x} - \boldsymbol{\xi})$$

The uptake of the particle is obtained
by integrating the instantaneous sink term.

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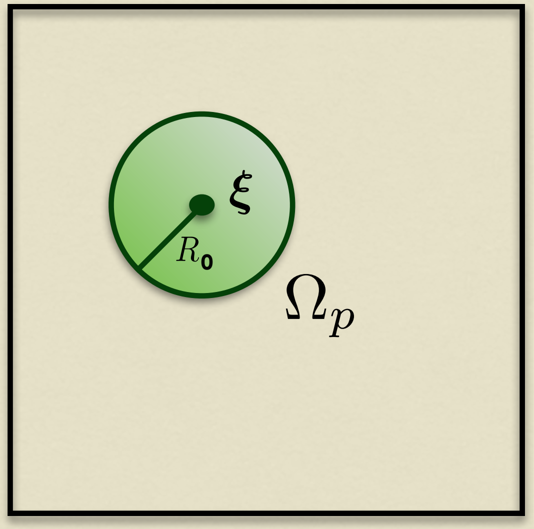
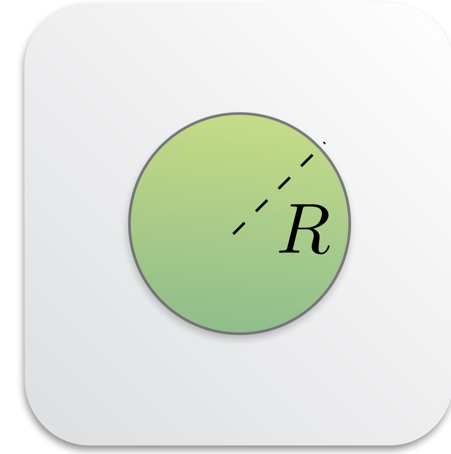


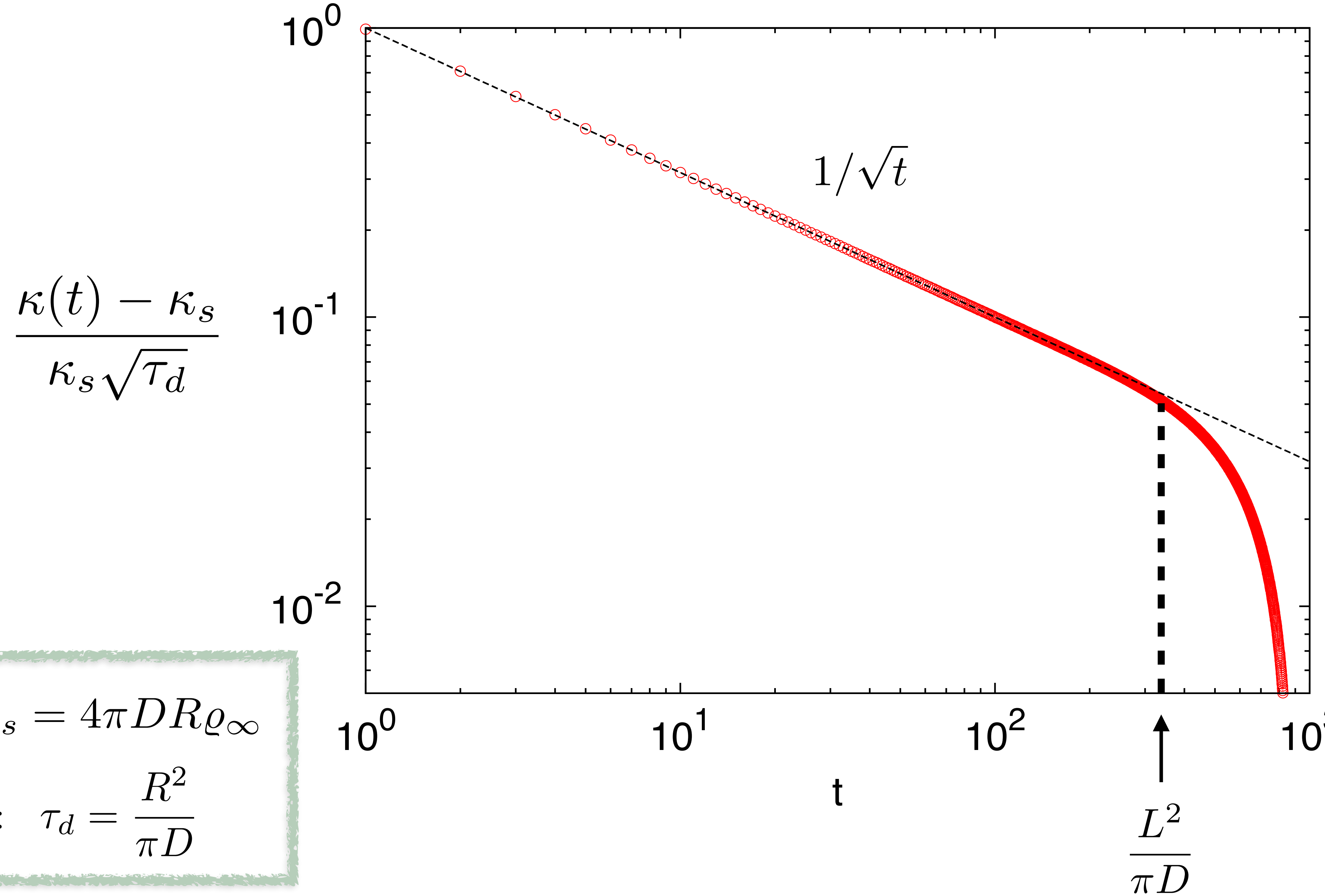
Uptake

Single Cell



Time-Dependent Solution:
***Osborn** (1996)*

$$\kappa(t) = 4\pi D R \varrho_\infty \left(1 + \frac{R}{\sqrt{\pi D t}}\right)$$



Smoluchowski rate: $\kappa_s = 4\pi D R \varrho_\infty$

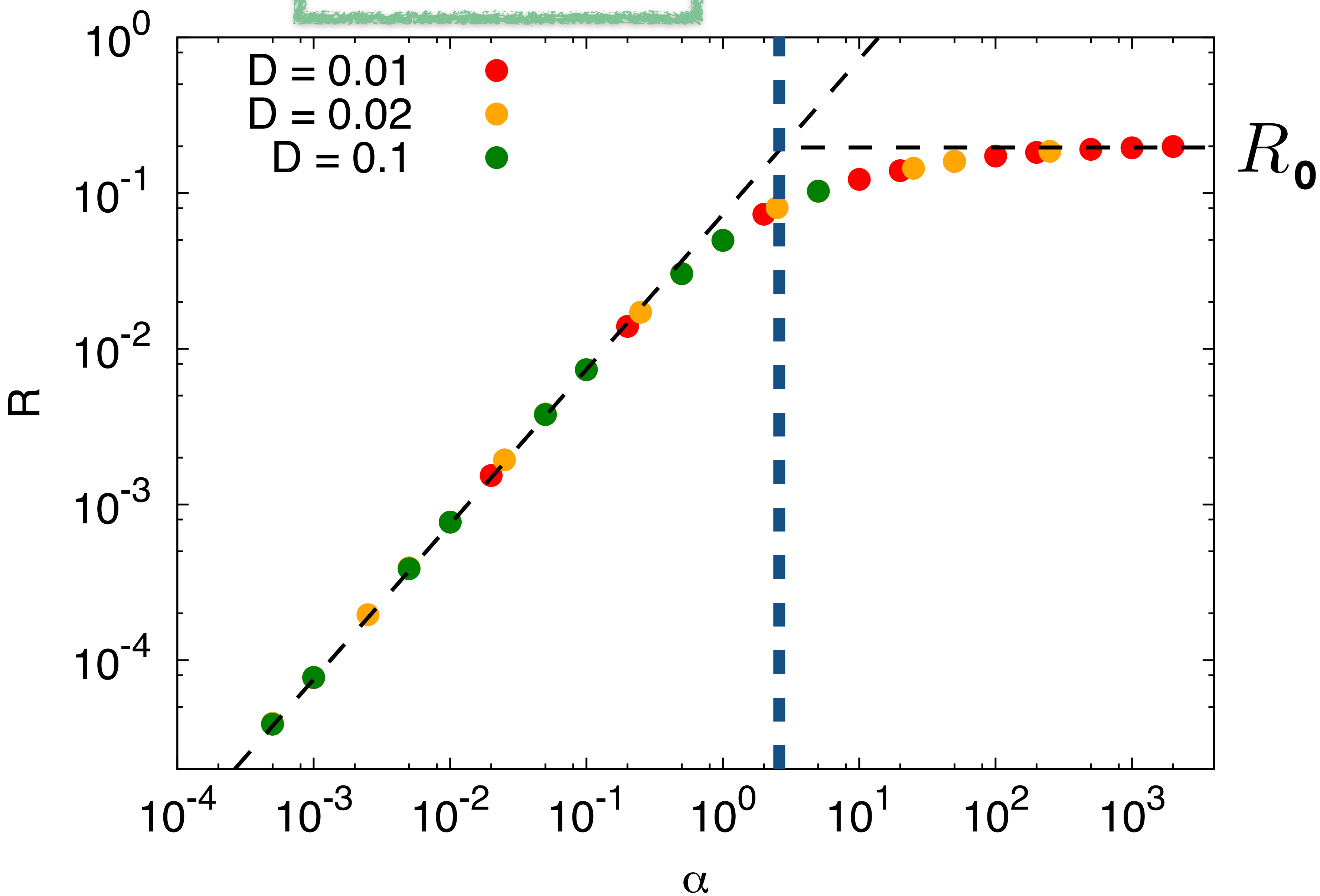
Diffusive Timescale: $\tau_d = \frac{R^2}{\pi D}$

Effective Radius

$$\beta = \kappa_s = 4\pi D R \rho_\infty$$

By defining $\beta = \alpha D$

$$\boxed{R = \frac{\alpha}{4\pi\rho_\infty}}$$



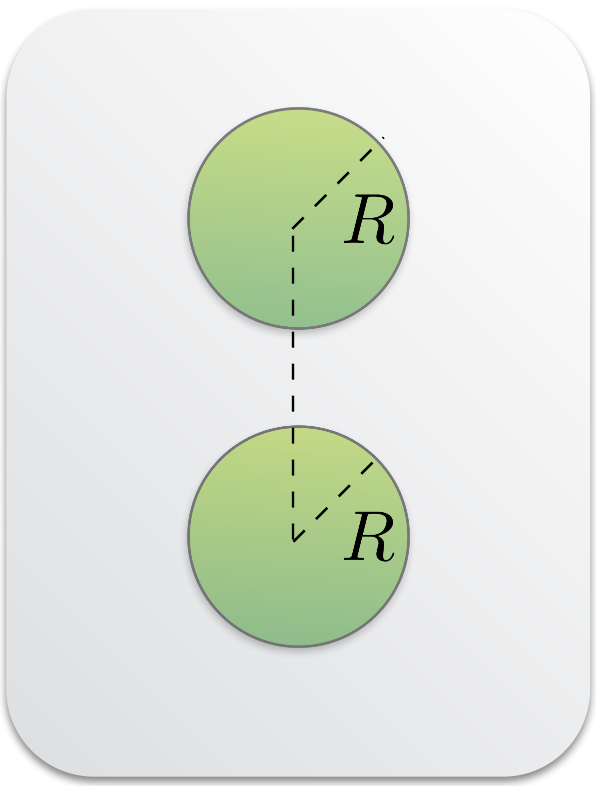
Effective radius is controlled by α.

For α>1, radius is fixed by the size of the discretize delta.

β determines the stiffness of the sink.

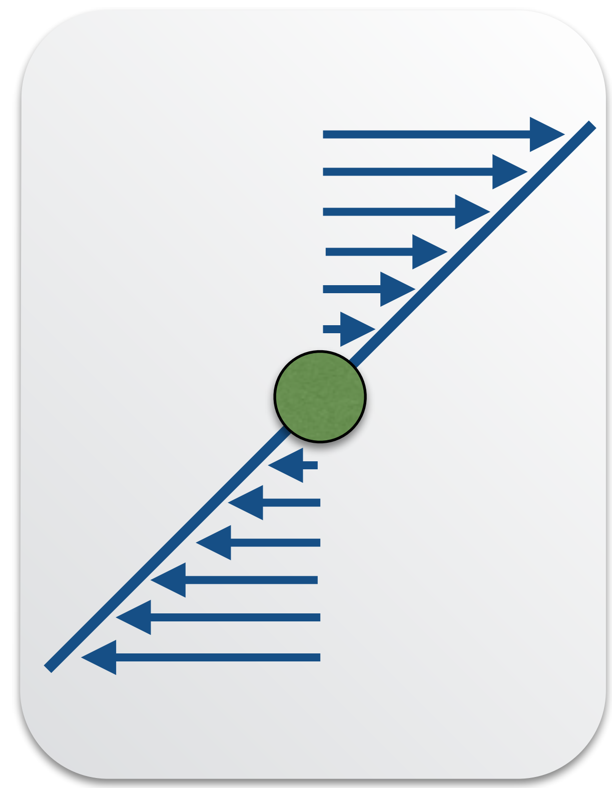
Two Simple Cases

Two Cells



Diffusive Interactions / Competition

Linear Shear

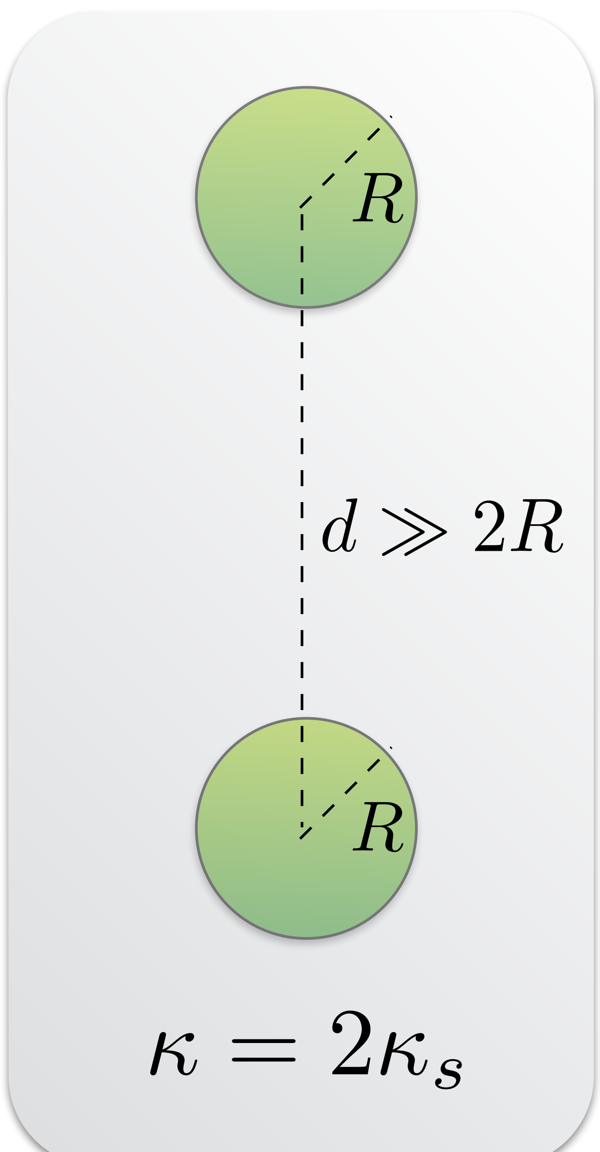


Uptake Enhancement

Diffusive Interactions

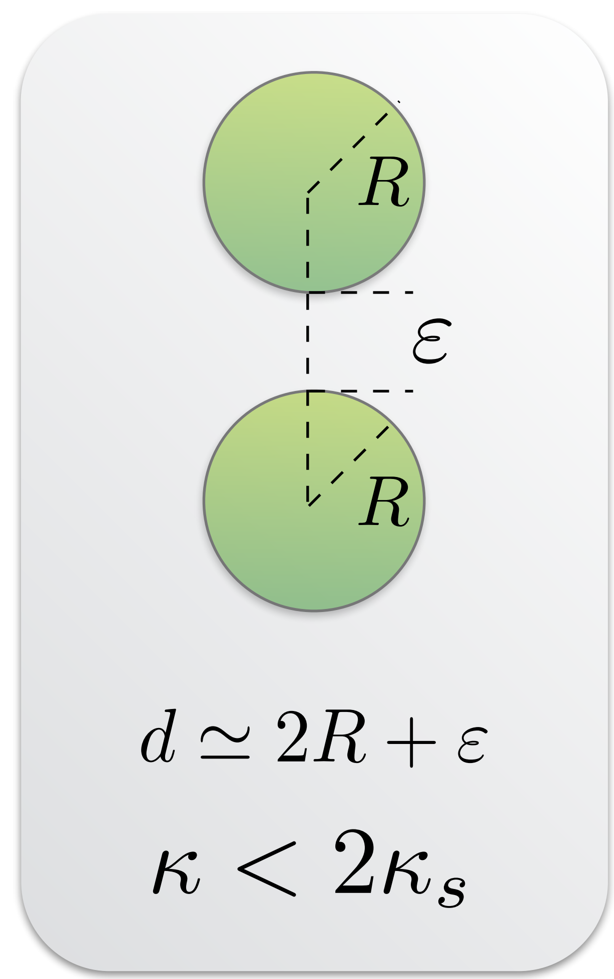
Non-interacting Spheres

Total uptake is
the algebraic sum
of contributions



Nutrient Shielding

Each sphere
shields a part of flux
from the other.

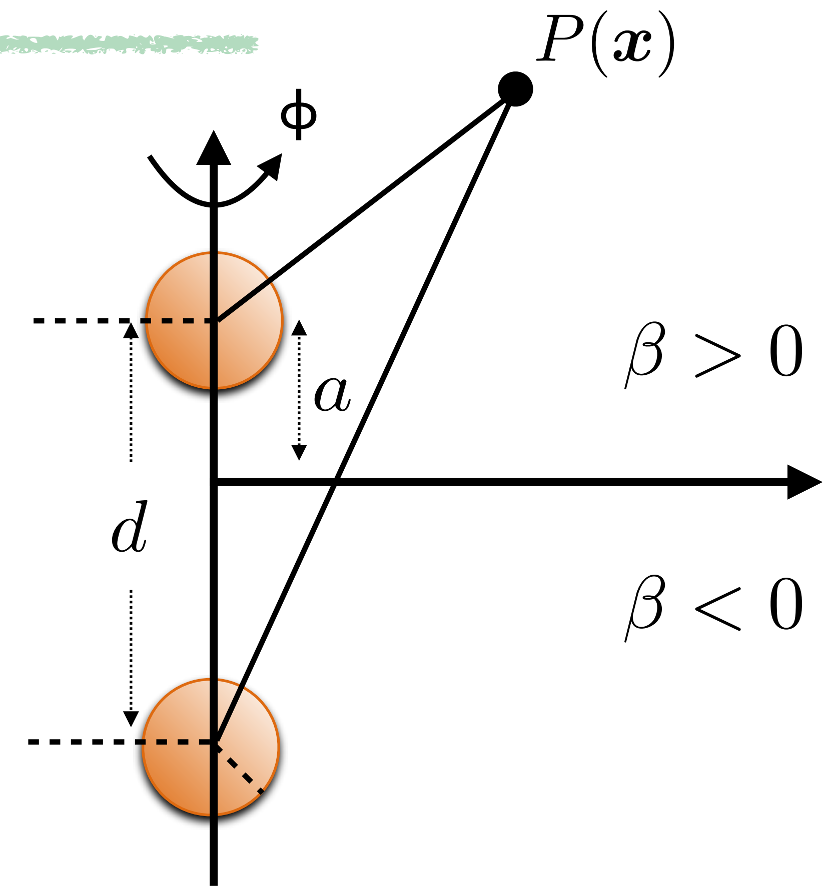


Diffusive Interactions

Solve the Laplace equation in bispherical coordinates.

$$\begin{cases} x = a\dfrac{\sin\alpha\cos\phi}{\cosh\beta - \cos\alpha} \\ y = a\dfrac{\sin\alpha\sin\phi}{\cosh\beta - \cos\alpha} \\ z = a\dfrac{\sinh\beta}{\cosh\beta - \cos\alpha} \end{cases}$$

Morse, Feshbach (1953)



⋮

Solution is expressed in terms of a multipole expansion:

$$\kappa = 2\kappa_s\sqrt{\chi^2 - 1}\sum_{n=0}^{\infty}\frac{2}{1 + (\chi + \sqrt{\chi^2 - 1})^{2n+1}}$$

R. Samson, J.M. Deutch, J. Chem. Phys. (1977)

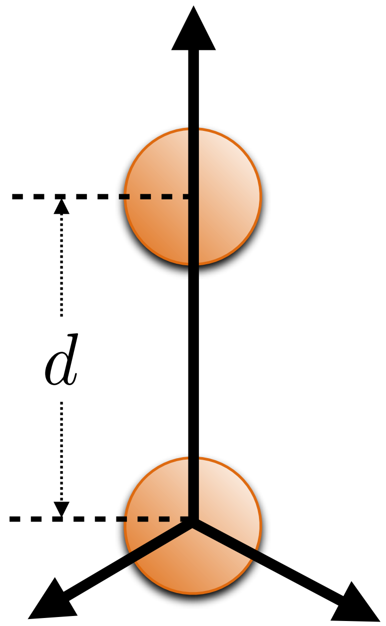
$$\chi = d/2R$$

At the first order approximation:

$$\boxed{\frac{\kappa}{2\kappa_s} = \frac{d}{R + d}}$$

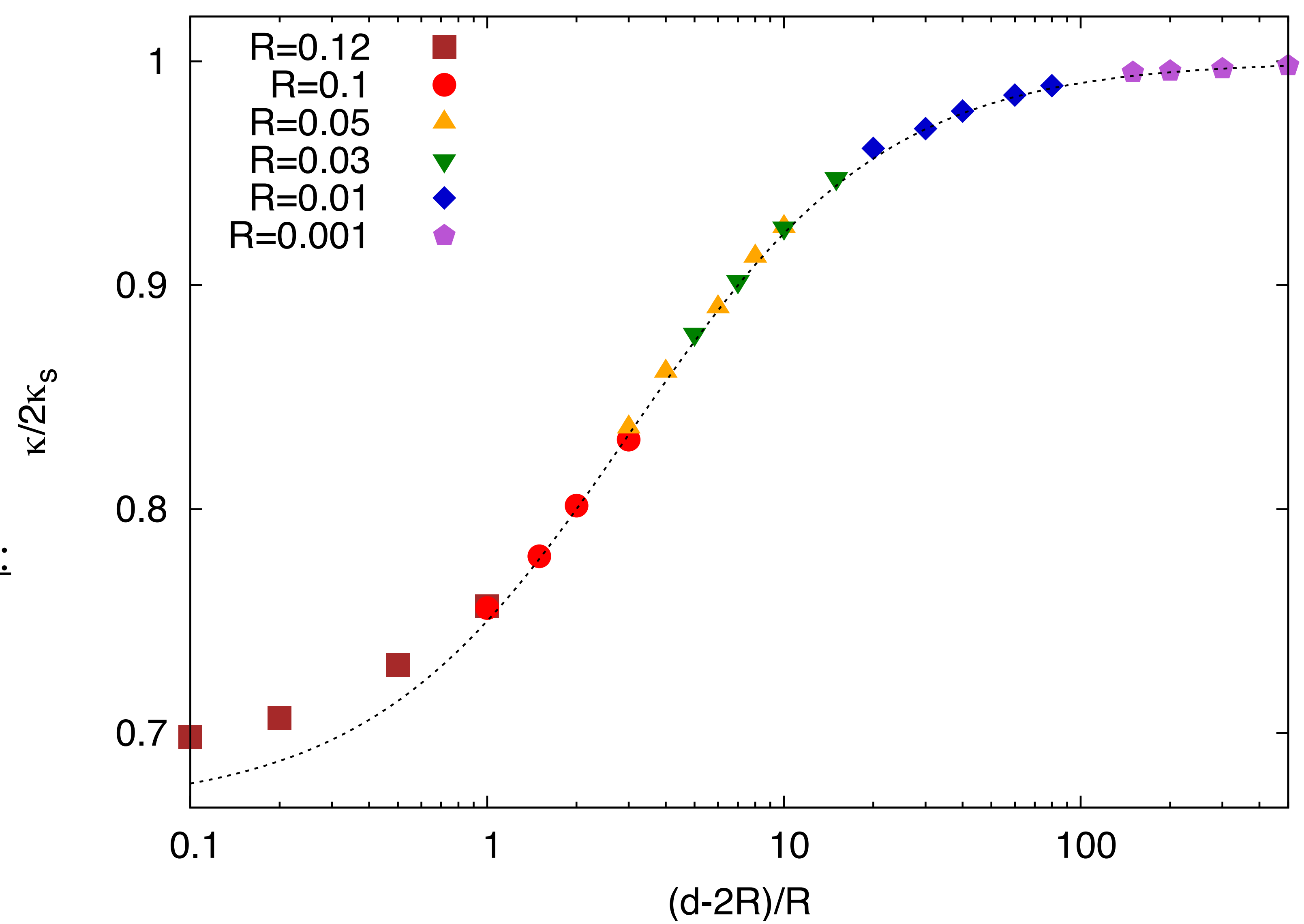
Competition effects are long range interactions.

Diffusive Interactions



Define $x \equiv \dfrac{d-2R}{R}$

$$\frac{\kappa}{2\kappa_s} = \frac{d}{R+d} \longrightarrow \boxed{\frac{\kappa}{2\kappa_s} = \frac{x+2}{x+3}}$$



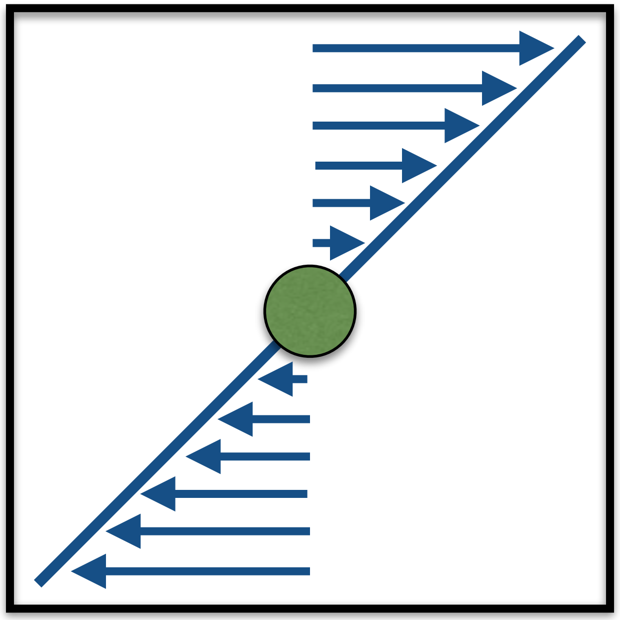
Interactions are not well resolved for:

1. distances below grid size
2. distances larger than half-box

Linear Shear Flow

$$\partial_t \varrho + \boldsymbol{u} \cdot \nabla \varrho = D \triangle \varrho$$

$$\boldsymbol{u} = (\gamma z, 0, 0)$$

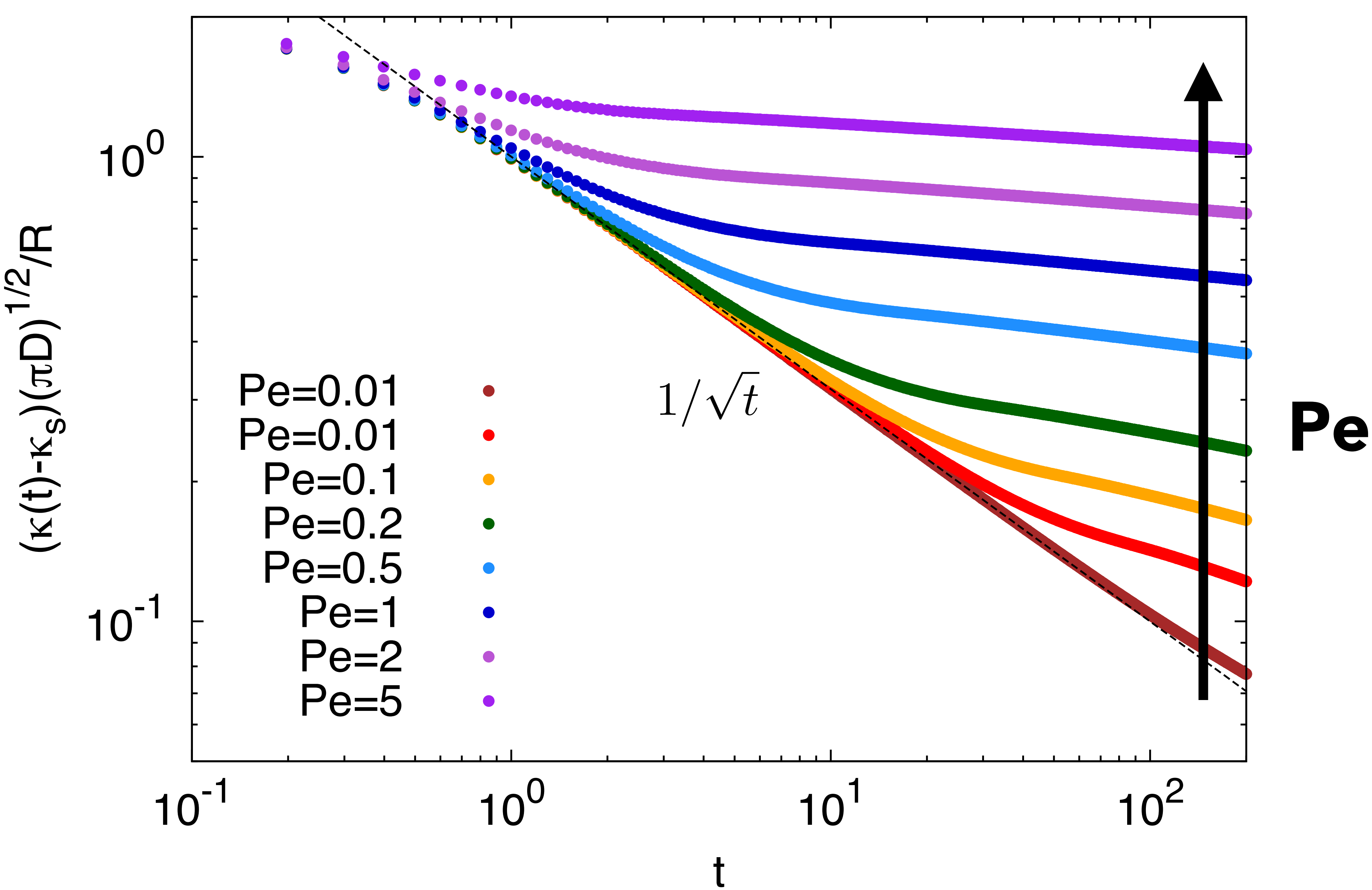


$$Pe = \frac{\gamma R^2}{D}$$

Linear Shear Flow

Uptake Enhancement

$$\kappa(t) = \int_{\Omega_p} d^3\boldsymbol{x}\, \beta\varrho(\boldsymbol{x},t) f(\boldsymbol{x}-\boldsymbol{\xi})$$



Linear Shear Flow

Low Pe Approximation

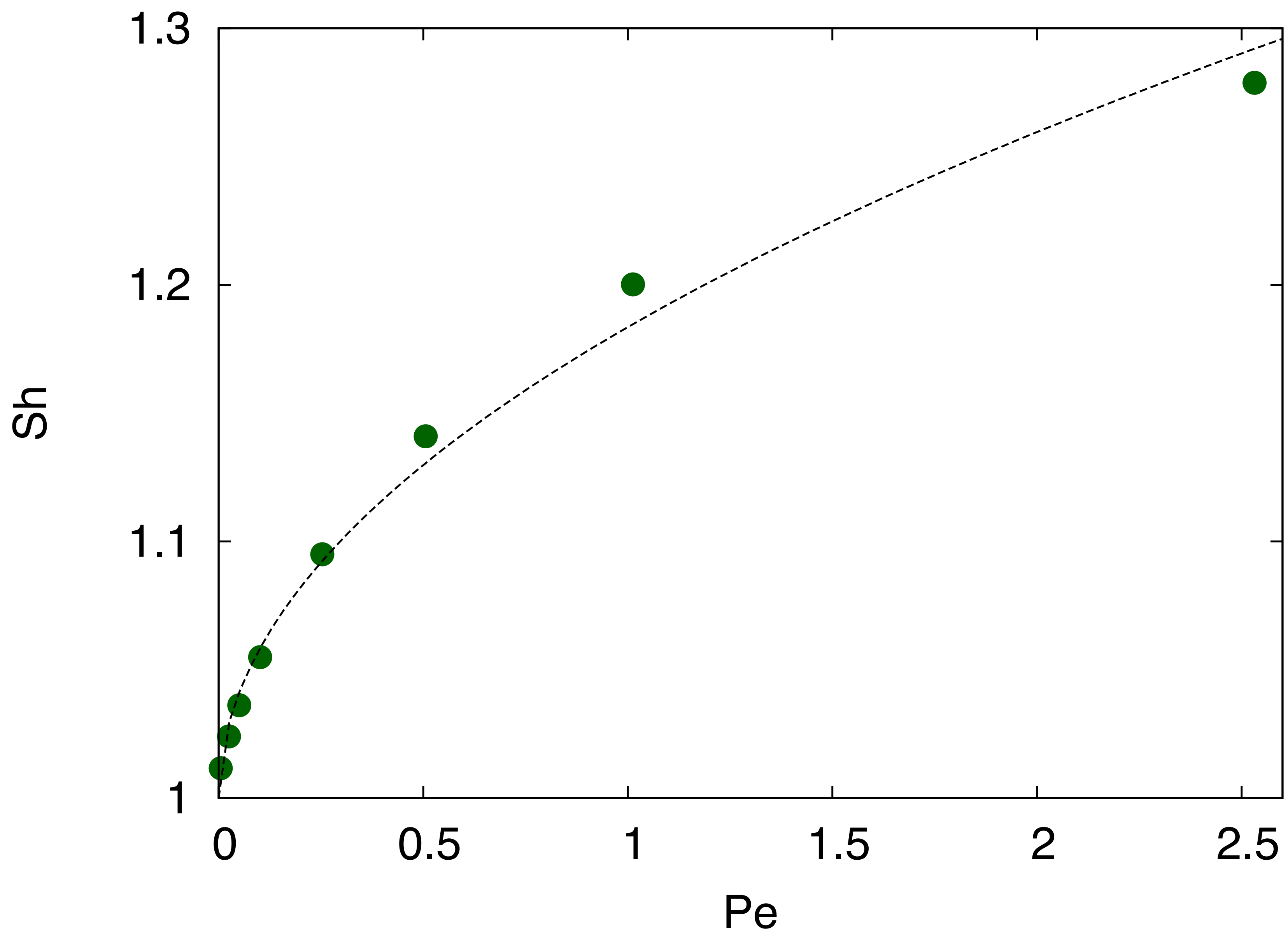
Sherwood Number:

$$Sh \equiv \frac{\kappa}{\kappa_s}$$

$$\frac{\text{Uptake in presence of the flow}}{\text{Diffusive Uptake}}$$

$$\boxed{Sh \simeq 1 + 0.36\, Pe^{1/2}}$$

Frankel & Acrivos POF (1968)
Batchelor JFM (1979)



Future Perspectives

Preliminary results can be used as benchmark for further investigation in:

1. Turbulent flows
1. Passive Particles
2. Gyrotactic Model

2. Diffusion-Controlled Processes in complex geometries
1. N-Body Systems (Random/Fractal)
2. Complex Shapes

Thank You!