Clustering and Turbophoresis in a Shear Flow without Walls

Filippo De Lillo	University of Turin, Italy
Massimo Cencini	CNR, Italy
Stefano Musacchio	CNRS, Lab. J.A. Dieudonné, France
Guido Boffetta	University of Turin, Italy

cost
Flowing Matter

Lecce
7-10 March 2016

cnrs dépasser les frontières

Université Nice Sophia Antipolis

Outline of the talk

Turbophoresis:	Brief introduction
Our numerics:	Kolmogorov flow Inertial particles Results of numerical simulations
Models:	Fokker-Planck eq. for particle position/velocity Large inertia case Weak inertia case Stochastic models

Turbophoresis vs. fractal clustering

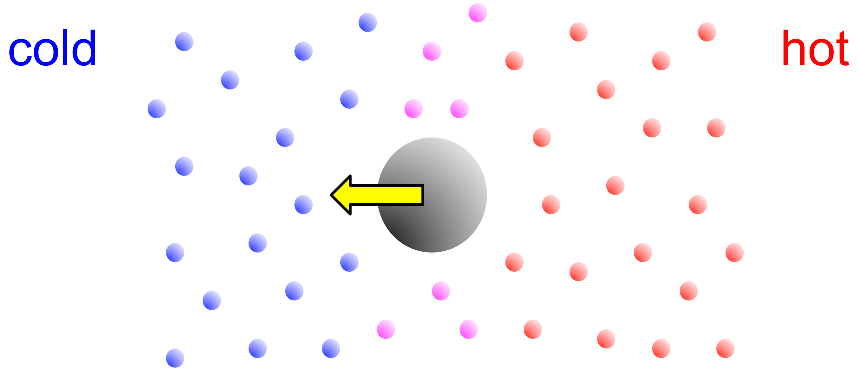
F. De Lillo M. Cencini, SM, G. Boffetta
"Clustering and Turbophoresis in a Shear Flow without Walls"
Phys. Fluids 28, 035104 (2016)

Thermophoresis & Turbophoresis

Thermophoresis: Brownian particle in a gas with gradients of temperature

Diffusivity
(Sutherland, Einstein 1905)

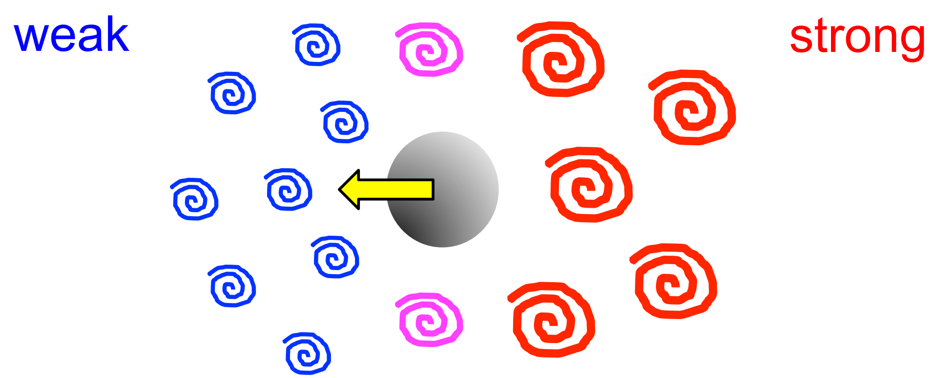
$$D = \frac{k_B T}{6\pi a \mu}$$



Turbophoresis: Inertial particle in a flow with gradients of turbulent intensity

Turbulent diffusivity

$$D_{turb} \sim \tau_c \langle (v')^2 \rangle$$

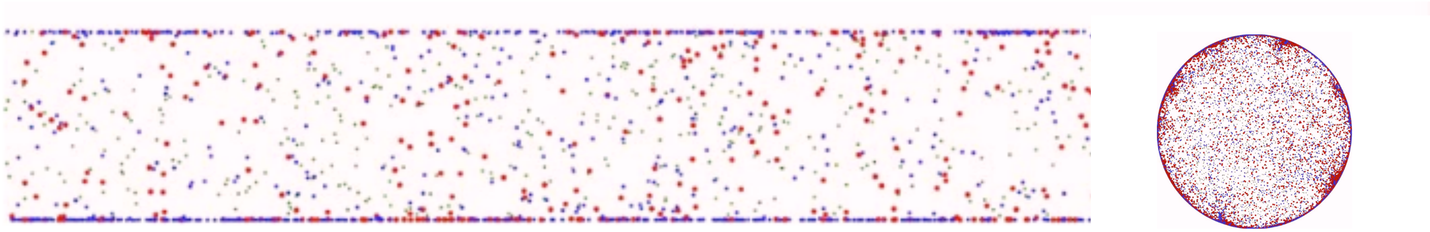


Turbophoresis

Drift of particles down gradients of turbulent intensity

Caporaloni, M., Tampieri, F., Trombetti, F. and Vittori, O. (1975) J. Atmos. Sci. 32, 565
Reeks, M.W. (1983) J. Aerosol Sci. 14, 729-739

In wall-bounded flows: Preferential accumulation at the wall



F. Picano, G. Sardina, and C.M. Casciola, (2009) Phys. Fluids 21, 093305

Turbophoresis does not require the presence of walls!

Kolmogorov flow

Navier-Stokes eq. with sinusoidal shear force

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{F}(z) \qquad \boldsymbol{F}(z) = [F_0 \cos(z/L), 0, 0]$$

Laminar regime $\quad Re = \frac{UL}{\nu} < \sqrt{2}$

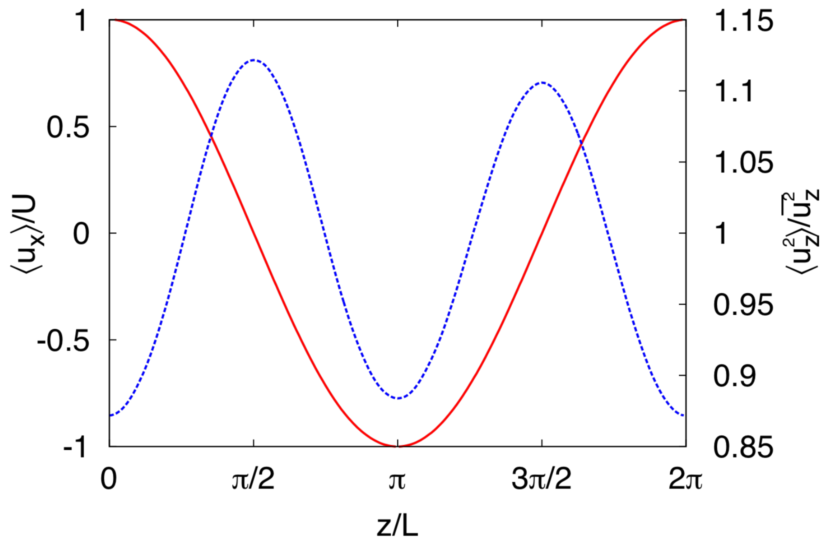
$$\boldsymbol{u} = [U \cos(z/L), 0, 0] \qquad U = L^2 F_0 / \nu$$

Turbulent regime

$$\langle \boldsymbol{u} \rangle = [U \cos(z/L), 0, 0]$$

$$\langle u_z^2 \rangle \propto U^2 (1 - b \cos(2z/L))$$

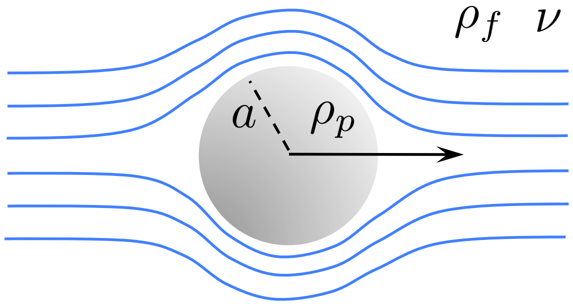
SM, G. Boffetta, Phys. Rev. E 89, 023004 (2014)



Inertial particles

Gatignol Maxey-Riley eq. for small heavy particle:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{v} \\ \dot{\boldsymbol{v}} &= -\frac{1}{\tau}[\boldsymbol{v} - \boldsymbol{u}(\boldsymbol{x},t)] \end{aligned}$$



Stokes time $\tau = \dfrac{2a^2\rho_p}{9\nu\rho_f}$

Small-scale fractal clustering

Stokes number $St = \tau/\tau_\eta \qquad \tau_\eta = (\nu/\varepsilon)^{1/2}$

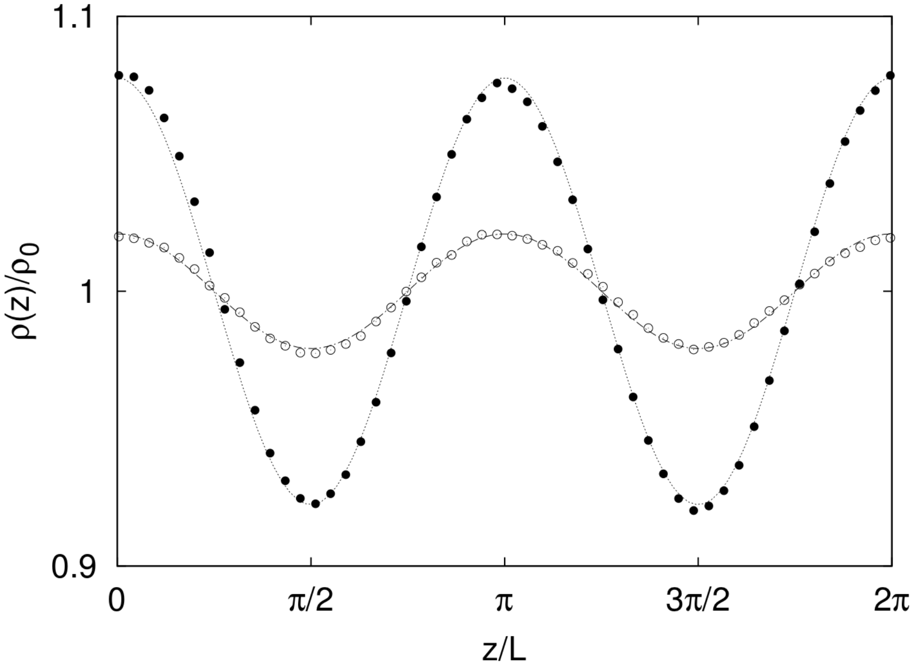
$$T/\tau_\eta \sim Re^{1/2}$$

Large-scale turbophoretic clustering

Inertia parameter $S = \tau/T \qquad T = E/\varepsilon$

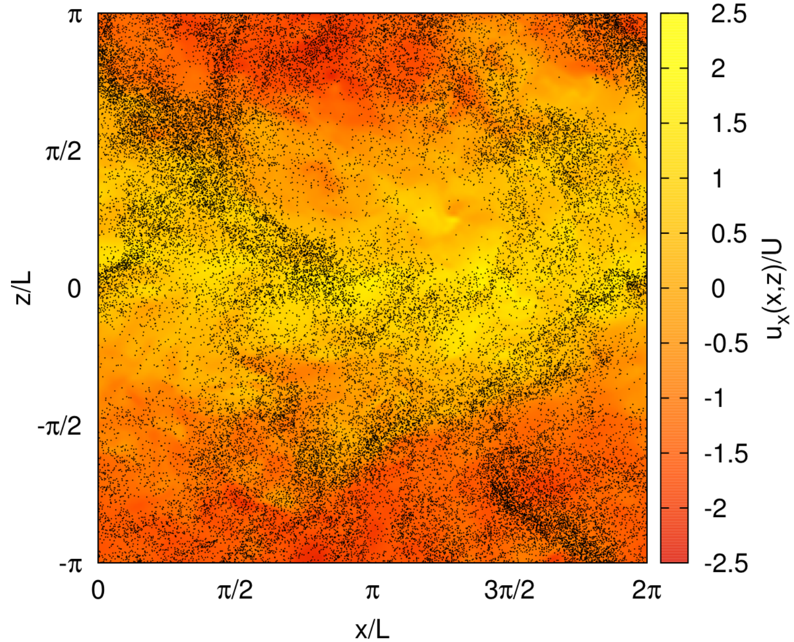
Particle density profiles

$$\rho(z) = \rho_0(1 + a(S)\cos(2z/L))$$



$S = 2.6 \times 10^{-3}$ (empty circles)

$S = 7.9 \times 10^{-2}$ (filled circles)



Vertical cut of istantaneous particle distribution (black dots) and streamwise fluid velocity

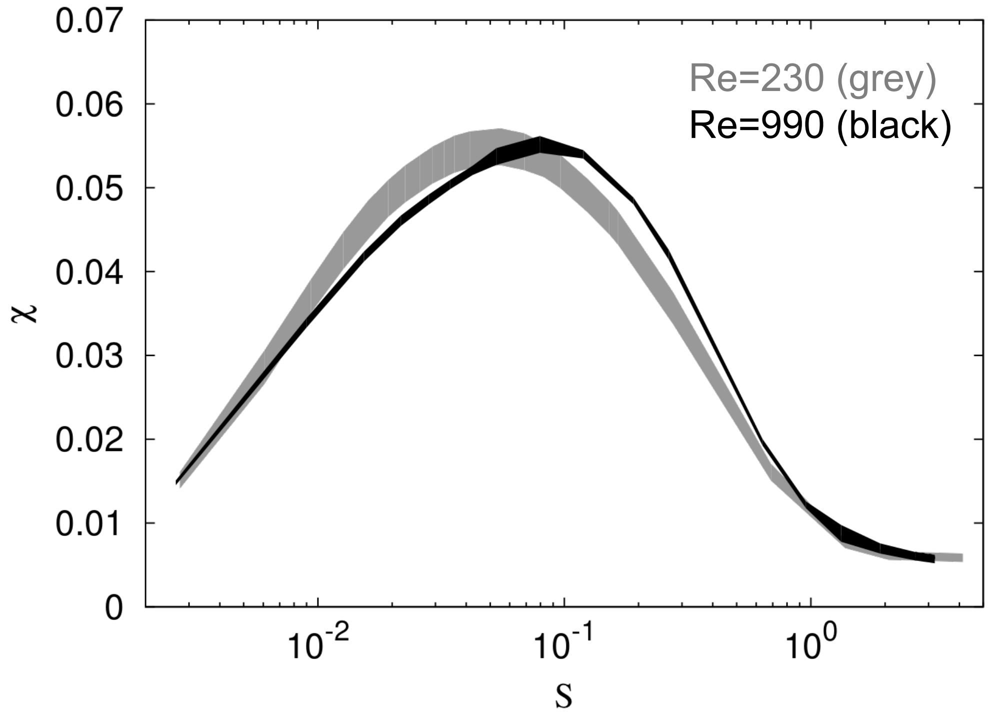
Particles accumulate in the regions of weak turbulence intensity (weak mean shear)

Turbophoresis

Rms relative deviations from uniform distribution

$$\chi = \left[\frac{1}{L_z} \int_0^{L_z} \left(1 - \frac{\rho(z)}{\rho_0} \right)^2 dz \right]^{1/2}$$

$$\rho(z) = \rho_0 (1 + a(S) \cos(2z/L))$$
$$\chi(S) = a(S)/\sqrt{2}$$



Maximum of turbophoresis for $S = \tau/T \simeq 10^{-1}$

1D Model for particle density profile

1) Gaussian random fluid velocity

Fokker-Planck eq. for $P(z,v)$

$$\frac{\partial P}{\partial t} = -v\partial_z P + \frac{1}{\tau}\partial_v(vP) + \frac{\kappa(z)}{\tau^2}\partial_z^2 P$$

Eddy diffusivity: $\kappa(z) \propto \langle u_z^2 \rangle$

2) Fast relaxation of particle velocity distribution

Eq. for $\rho(z) = \int dv P(z,v)$

$$\partial_t \rho(z) = \partial_z J(z)$$

Flux (non-Fick relation): $J(z) = \partial_z[\kappa(z)\rho]$

Steady fluxless solution: $\rho(z) \propto \kappa^{-1}(z) \propto \langle u_z^2 \rangle^{-1}$

C. Lopez and U. M. B. Marconi, Phys. Rev. E 75, 021101 (2007)
S. Belan, I. Fouxon, and G. Falkovich, Phys. Rev. Lett. 112, 234502 (2014)

Prediction for particle density profile

Particle density profile: $\rho(z) \propto \kappa^{-1}(z) \propto \langle u_z^2 \rangle^{-1}$

Kolmogorov flow: $\langle u_z^2 \rangle \propto U^2(1 - b\cos(2z/L))$

$\rho(z) = \rho_0(1 + a(S)\cos(2z/L))$ $a(S) = b$ Wrong!

Large inertia (S >> 1): Particle velocity < fluid velocity
Amplitude of spatial modulation decreases with inertia
a(S) decreases with S for S >>1

Weak inertia (S << 1): Only fast turbulent eddies ($\tau_\ell < \tau$)
contribute to the spatial dependent eddy diffusivity
a(S) increases with S for S << 1

Maximum of turbophoresis for $\tau \simeq T = E/\varepsilon$

Prediction for large inertia

Local diffusivity proportional to the particle “temperature field”

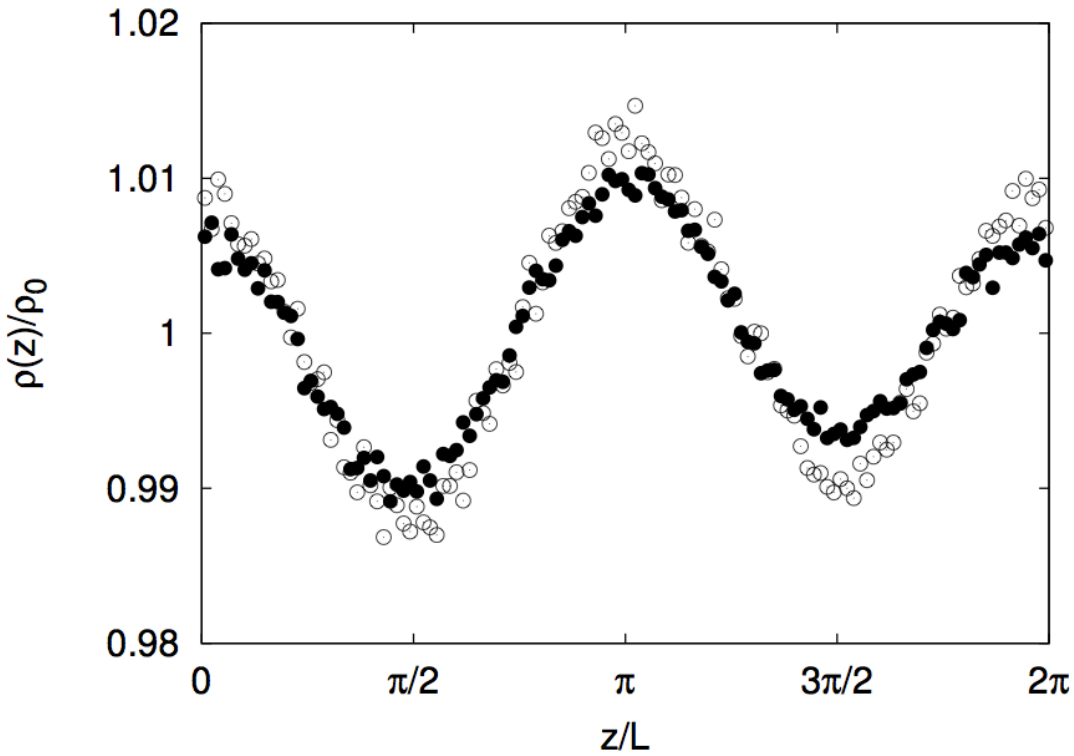
M. Caporaloni, F. Tampieri, F. Trombetti, and O. Vittori, J. Atmos. Sci. 32, 565–568 (1975)

$$\kappa(z) \sim \langle v_z^2(z) \rangle$$

Particle density profile

$$\rho(z) \propto \langle v_z^2 \rangle^{-1}$$

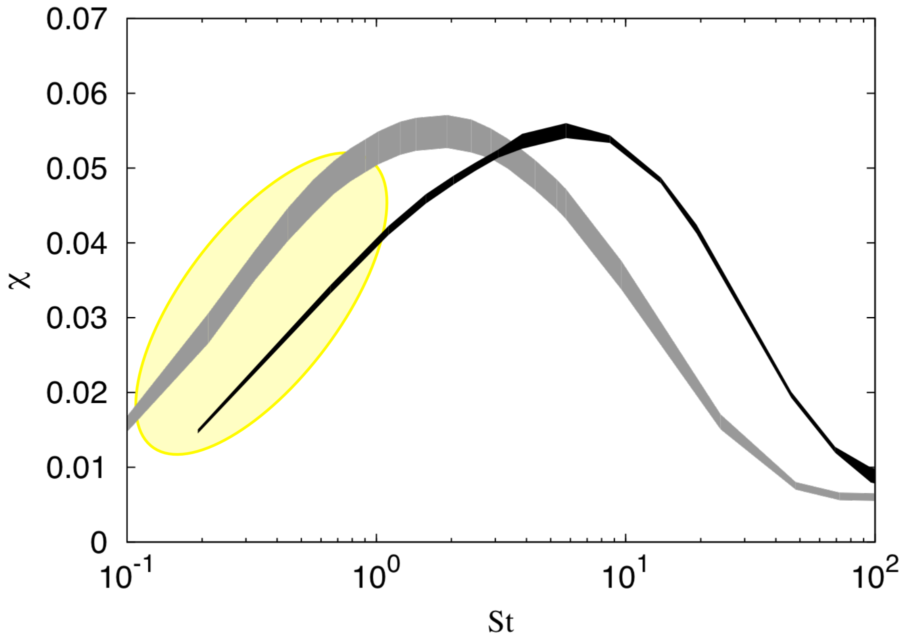
- ● $\rho(z)/\rho_0$ (Re=230, S=4.1)
- ○ $\langle v_z^2 \rangle^{-1} L_z / \int_0^{L_z} \langle v_z^2 \rangle^{-1} dz$



Weak inertia: compressibility

Inhomogeneities are present
also for weak inertia

$$St = \tau/\tau_\eta < 1$$



Re=230 (grey)
Re=990 (black)

First order expansion for weak inertia

$$\boldsymbol{v} = \boldsymbol{u} - \tau(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}) + o(\tau)$$

E. Balkovsky, G. Falkovich, and A. Fouxon,
Phys. Rev. Lett. 86, 2790 (2001)

Particle velocity field is compressible

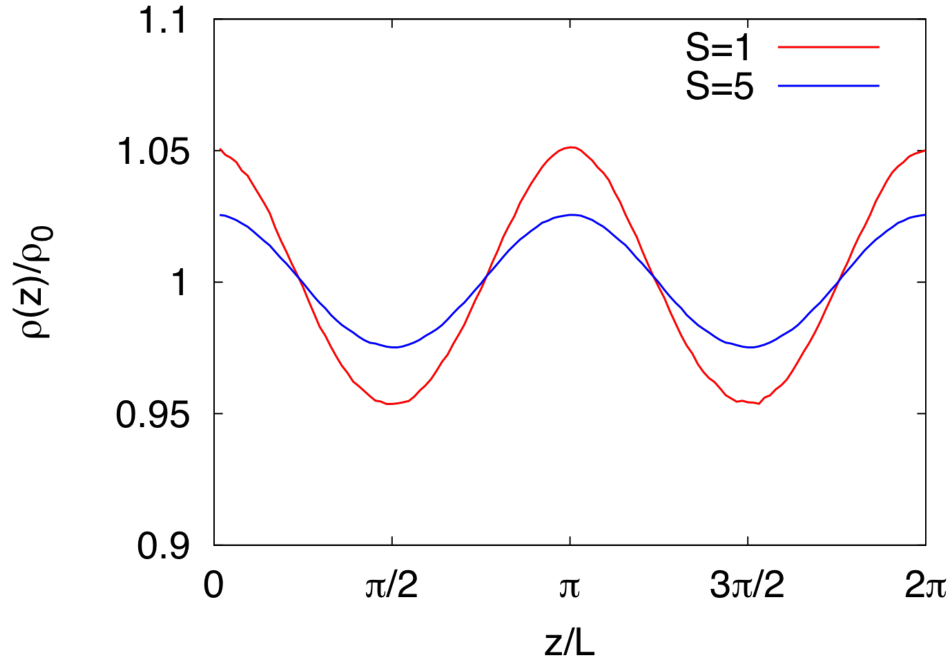
$$\langle \nabla \cdot \boldsymbol{v} \rangle = -\tau \langle \cdot \nabla (\boldsymbol{u} \cdot \nabla \boldsymbol{u}) \rangle = -\tau \partial_z^2 \langle u_z^2 \rangle$$

Negative mean divergence in the minima of $\langle u_z^2 \rangle$

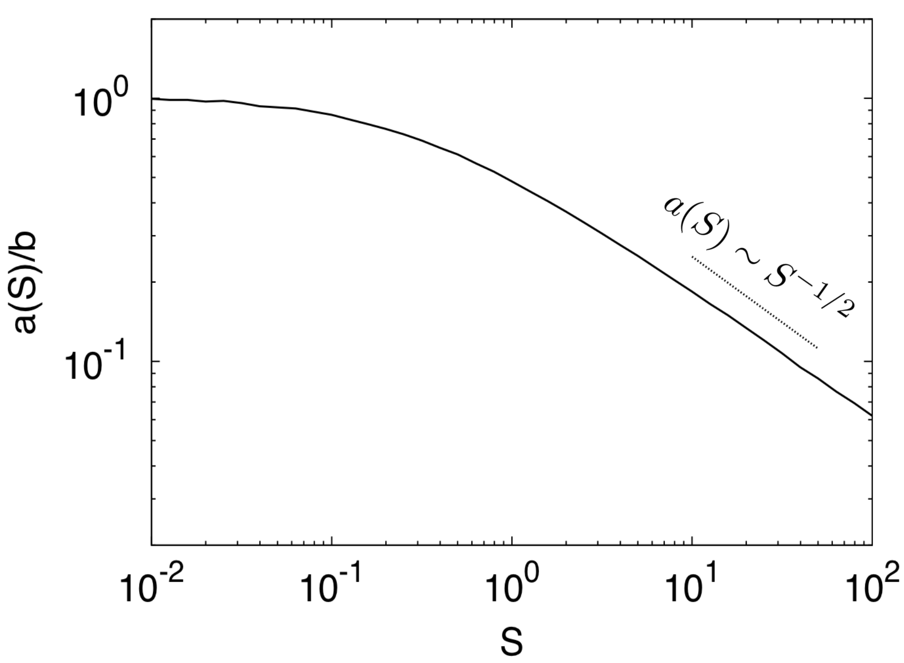
Stochastic model for large inertia

$$\frac{dz}{dt} = v \qquad \frac{dv}{dt} = -\frac{v}{\tau} + \frac{1}{\tau}\sqrt{2\kappa(z)}\eta \qquad \kappa(z) = \kappa_0(1 - b\cos(2z/L))$$

Particle density profile (b=0.1)



Amplitude a(S)

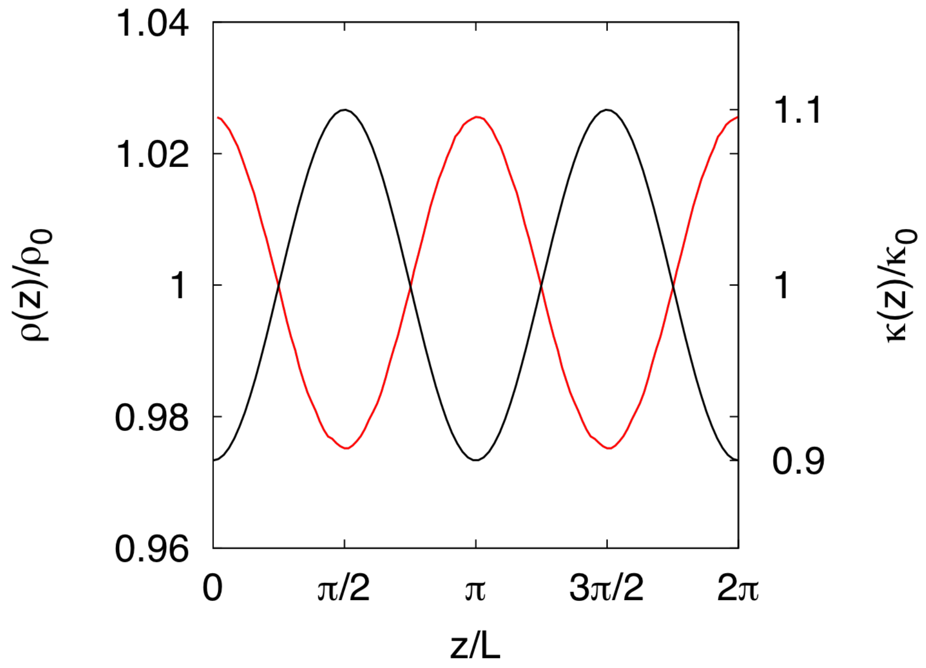


$$\rho(z) = \rho_0(1 + a(S)\cos(2z/L)) \qquad S = \frac{\tau\kappa_0}{L^2}$$

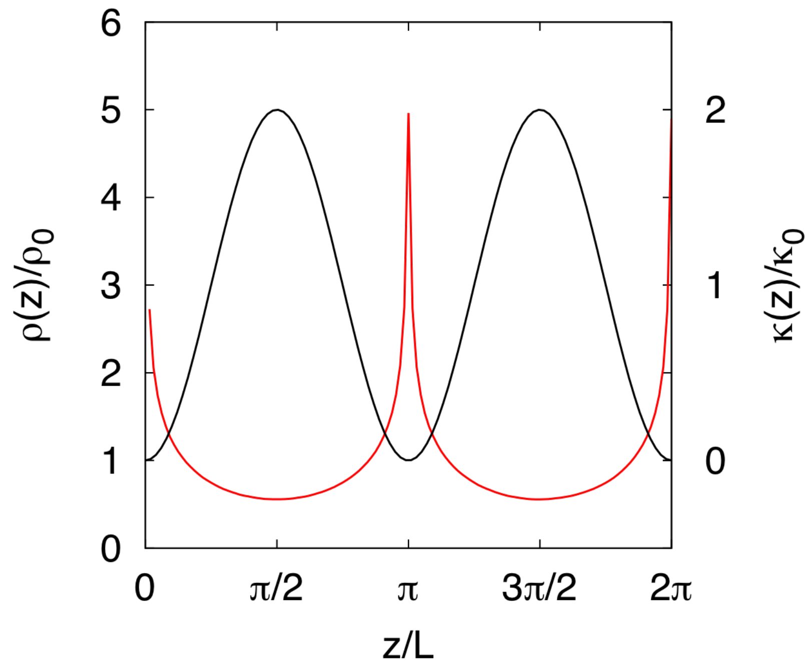
Stochastic model

$$\frac{dz}{dt} = v \qquad \frac{dv}{dt} = -\frac{v}{\tau} + \frac{1}{\tau}\sqrt{2\kappa(z)}\eta \qquad \kappa(z) = \kappa_0(1 - b\cos(2z/L))$$

Kolmogorow flow (b=0.1)



Wall-bounded flow (b=1)



red line = particle density profile
black line = noise amplitude

Localization transition for S ~ 1

M. Wilkinson and B. Mehlig, PRE 68, 040101 (2003)
S. Belan, I. Fouxon and G. Falkovich PRL 112, 234502 (2014)

Turbophoresis vs. Fractal clustering

Turbophoresis: Large-scale inhomogeneities
Induced by local variations of turbulence intensity

Fractal clustering: Small-scale inhomogeneities
Due to dissipative and chaotic particle dynamics

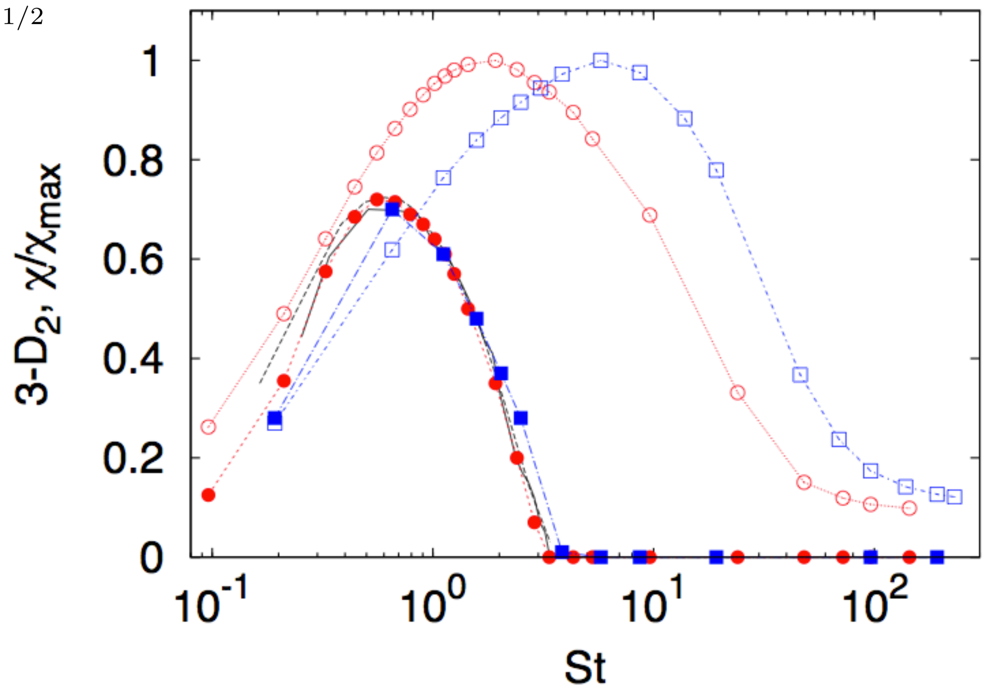
Correlation dimension D_2 $\quad P_2(r) \sim r^{D_2}$

○ Turbophoresis $\chi = \left[\frac{1}{L_z} \int_0^{L_z} \left(1 - \frac{\rho(z)}{\rho_0} \right)^2 dz \right]^{1/2}$

● Codimension $3 - D_2$

Re=230 (red circles)
Re=990 (blue squares)

$$\frac{St_{turbo}}{St_{fract}} \sim Re^{1/2}$$



Conclusions

Two mechanisms for particle clustering in shear flow without walls

Turbophoresis: Large-scale inhomogeneities
Induced by local variations of turbulence intensity
Maximum at $\tau \simeq T = E/\varepsilon$

Fractal clustering: Small-scale inhomogeneities
Due to dissipative and chaotic particle dynamics
Maximum at $\tau \simeq \tau_\eta = (\nu/\varepsilon)^{1/2}$

F. De Lillo M. Cencini, SM, G. Boffetta
"Clustering and Turbophoresis in a Shear Flow without Walls"
Phys. Fluids 28, 035104 (2016)

Thank you!