Nonlinear response of a driven tracer particle in simple fluid models

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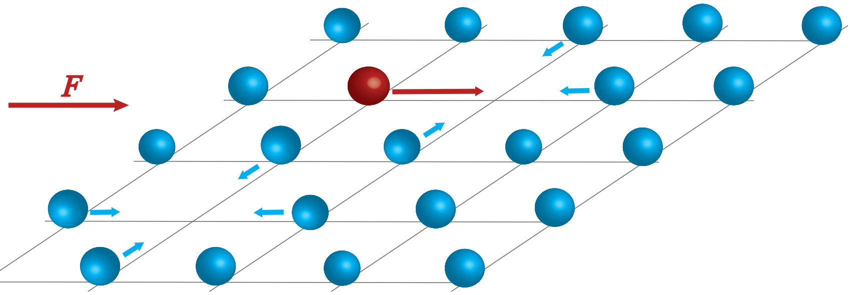
Two simple fluid models

- Driven tracer particle in a lattice gas
  - Analytical microscopic theory
  - Negative differential mobility



In collaboaration with O. Bénichou, P. Illien, G. Oshanin, R. Voituirez UPMC, Paris

Phys. Rev. Lett. **113**, 268002 (2014), Phys. Rev. E **93**, 032128 (2016)

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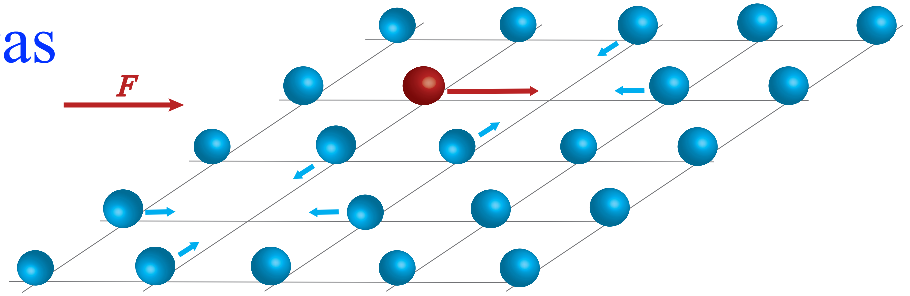

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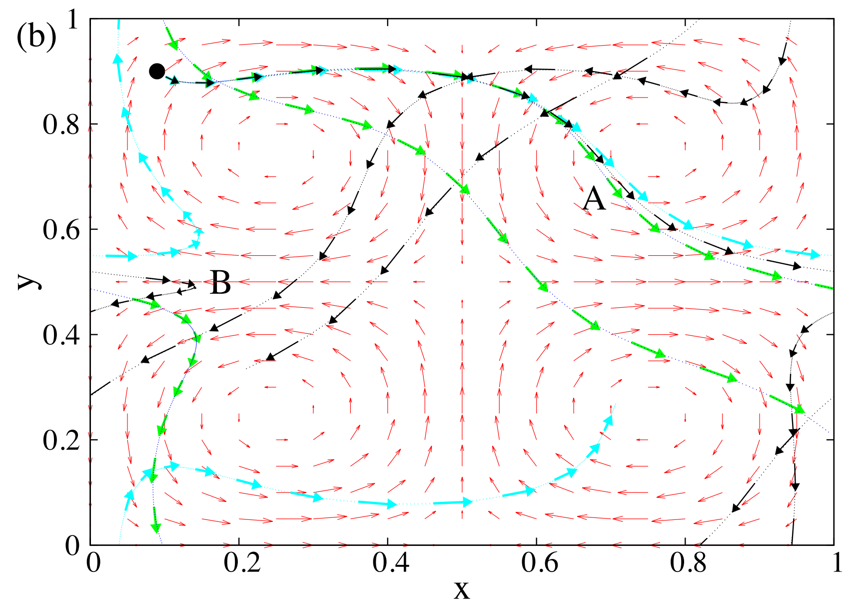
- Driven inertial particle in a steady flow
  - Nonlinear dynamics
  - Absolute negative mobility

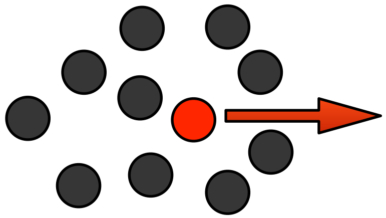
With F. Cecconi, A. Puglisi, A. Vulpiani
ISC-CNR, Univ. Sapienza, Roma

Phys. Rev. Lett. **117**, 174501 (2016)



Active microrheology and negative differential mobility

Tracer particle (TP) driven by
an external force F in a host medium



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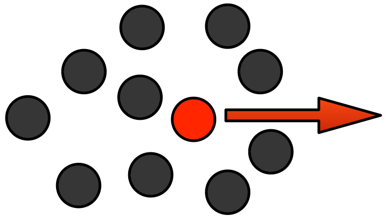

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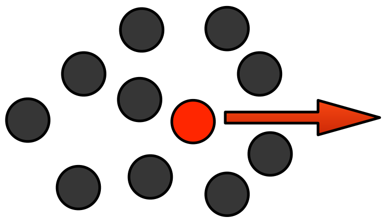

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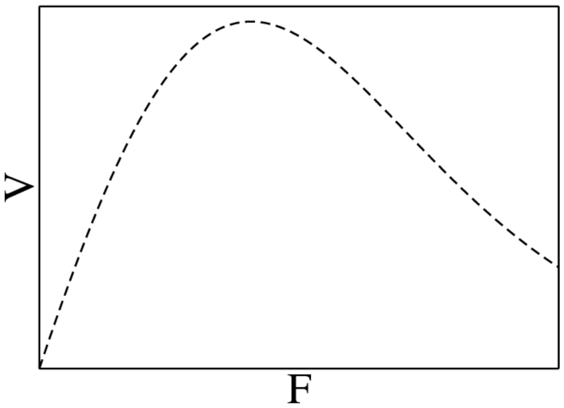
Nonlinear response regime: increasing the applied force can reduce the probe's drift velocity in the force direction $\mu(F) \leq 0$



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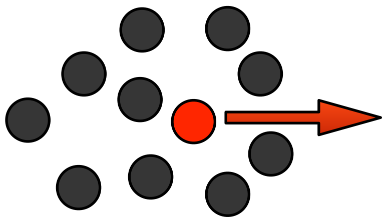

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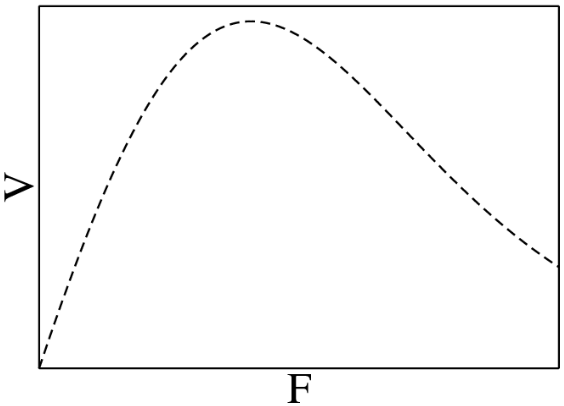
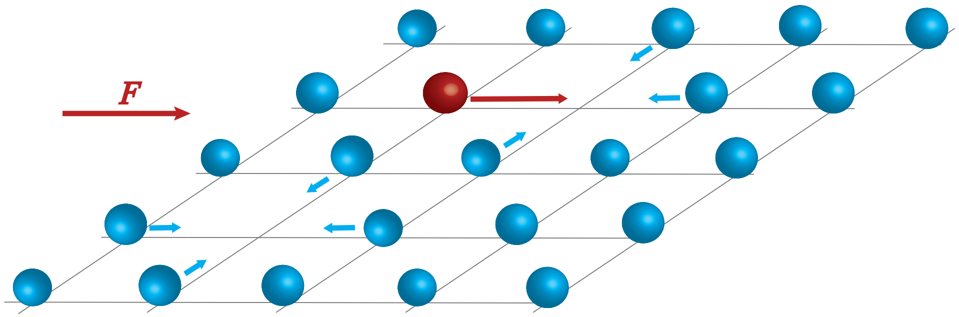


"Getting more from pushing less" (Zia et al. Am. J. Phys. 2002)

Driven tracer in a hard-core lattice gas

(N-1) hard-core particles,
symmetric exclusion process,
average waiting time τ^*



Tracer driven by a force F
asymmetric exclusion process,
average waiting time τ

Density $\rho = \dfrac{N}{V}$

Tracer jump probabilities, local detailed balance $p_\nu = \dfrac{e^{(\beta/2)\boldsymbol{F}\cdot\boldsymbol{e}_\nu}}{\sum_\mu e^{(\beta/2)\boldsymbol{F}\cdot\boldsymbol{e}_\mu}}$

$\nu = \pm 1, \ldots, \pm d \quad \boldsymbol{F} = F\boldsymbol{e}_1$

$\tau^*/\tau = \infty$ Lorentz gas

(at low density Leitmann & Franosch PRL (2013))

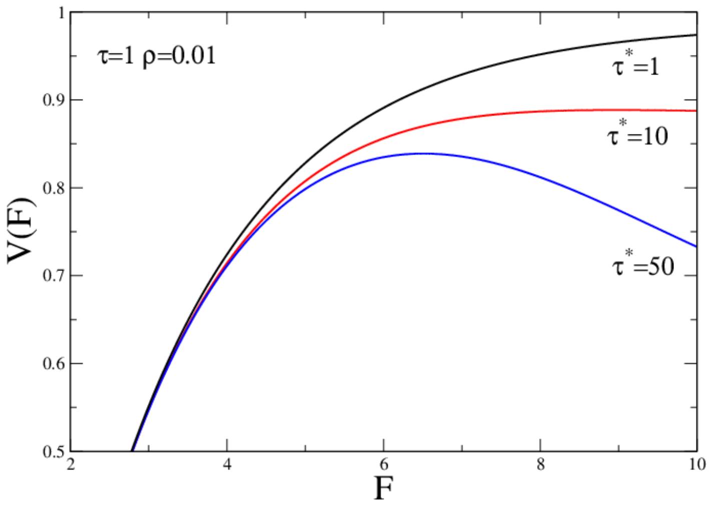
Force-velocity relation?

Argument for NDM at low density

Strong external force $\epsilon = 2e^{-\beta F/2} \ll 1$

$$V(F) = \frac{1-\epsilon}{\tau + 4\rho(1-\epsilon)\frac{\tau^*}{3+4\epsilon\tau^*/\tau}}$$

Criterion for NDM $\tau^*/\tau \gtrsim \rho^{-1/2}$



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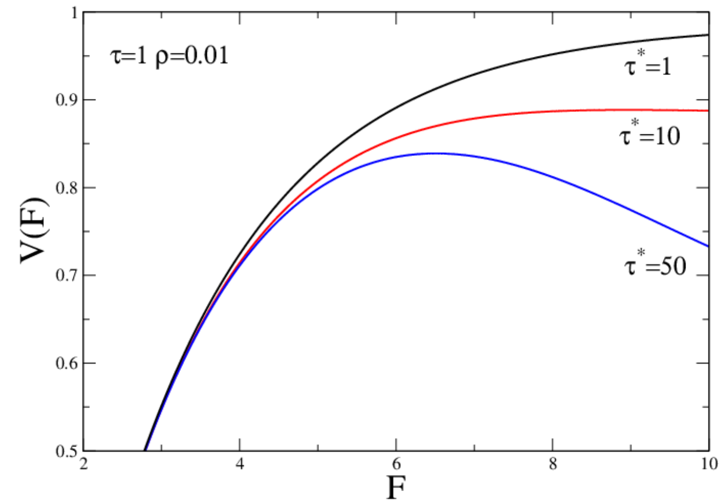

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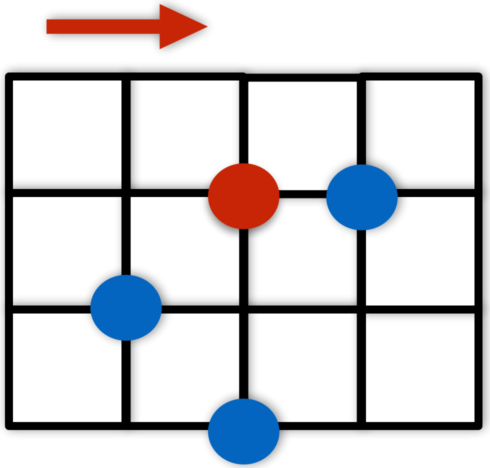
Physical mechanism: a large force

→ reduces the flight time between two consecutive encounters with bath particles;

→ increases the escape time from traps created by surrounding obstacles



For τ^* large enough ("slow" obstacles), traps are sufficiently long lived to slow down the TP when F is increased → NDM

Analytical computation of the tracer velocity

Master Equation for $P(\boldsymbol{R}_{TP}, \eta; t)$

$\boldsymbol{R}_{TP}$ tracer position

η obstacle configuration

Tracer velocity $V(F) \equiv \dfrac{d\langle \boldsymbol{R}_{TP} \cdot \boldsymbol{e}_1 \rangle}{dt} = \dfrac{1}{2d\tau^*}(A_1 - A_{-1})$

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Decoupling approximation

$$\langle \eta(\boldsymbol{R}_{TP} + \boldsymbol{\lambda})\eta(\boldsymbol{R}_{TP} + \boldsymbol{e}_\nu)\rangle \approx \langle \eta(\boldsymbol{R}_{TP} + \boldsymbol{\lambda})\rangle\langle \eta(\boldsymbol{R}_{TP} + \boldsymbol{e}_\nu)\rangle$$

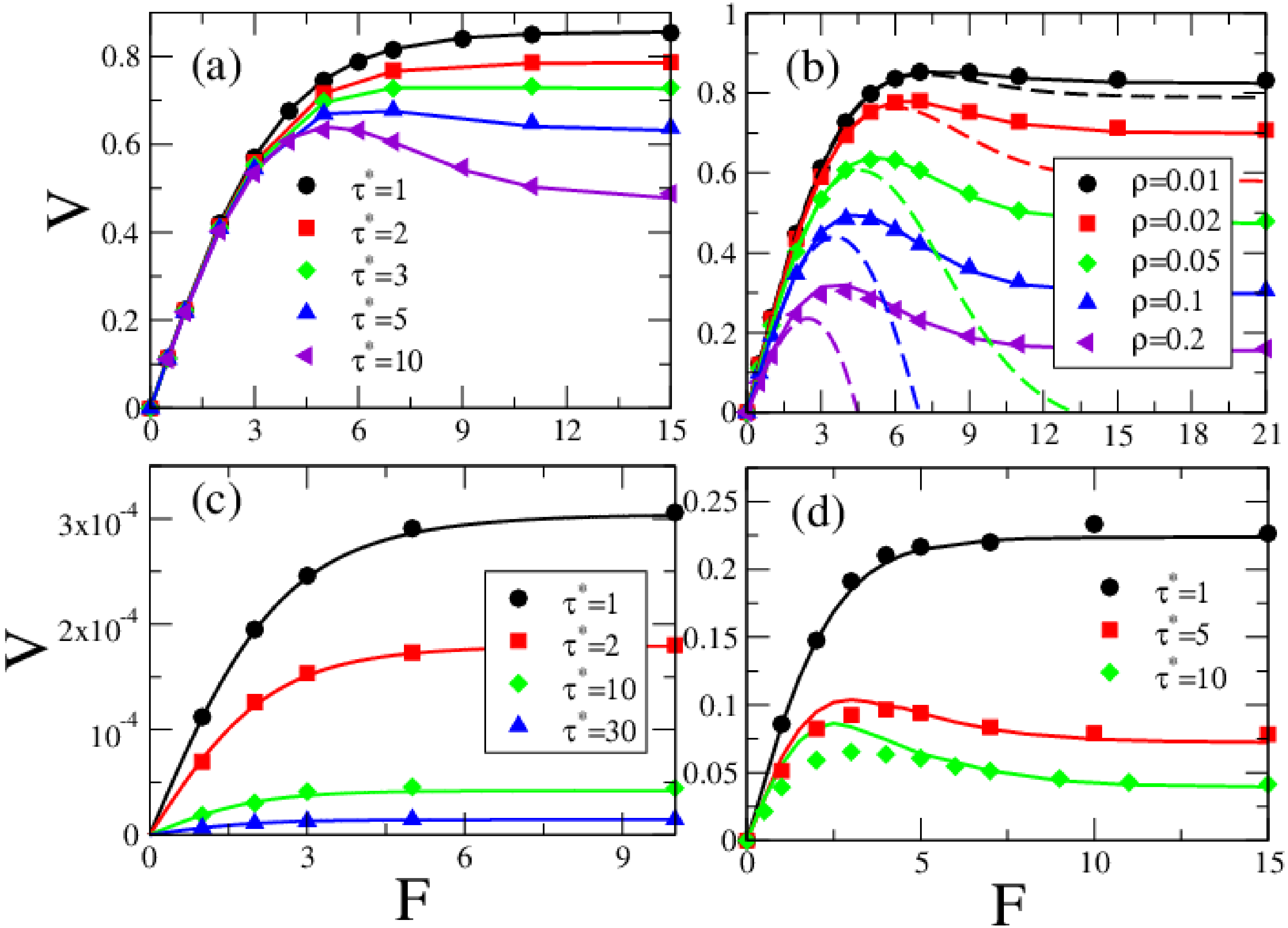
for $\boldsymbol{\lambda} \neq \boldsymbol{e}_\nu$

Nonlinear system of equations

$$A_\nu = 1 + \frac{2d\tau^*}{\tau} p_\nu \left[1 - \rho - \rho(A_1 - A_{-1}) \frac{\det C_\nu}{\det C} \right]$$

Solution for $V(F)$ for arbitrary values of the parameters

Comparison with Monte Carlo numerical simulations



$d = 2, \tau = 1$

(a) $\rho = 0.05$

(b) $\tau^* = 10$

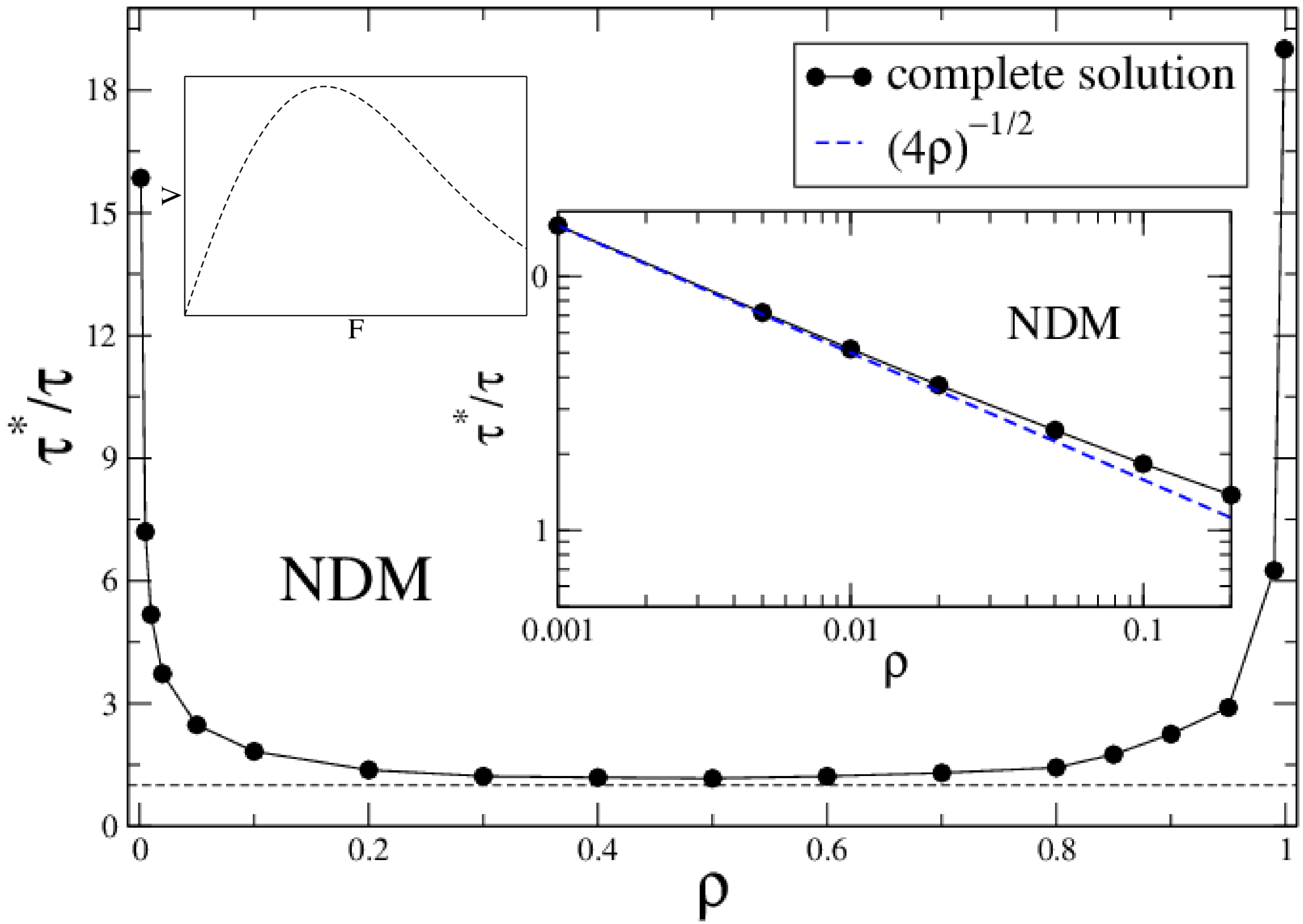
(c) $\rho = 0.999$

(d) $\rho = 0.5$

Good agreement in a wide range of parameters

Criterion for negative differential mobility

Parameter space: time scales τ^*/τ and density ρ



Physical mechanism: coupling between density and time scales ratio

Nonlinear response of a driven inertial tracer

Transport properties of particles of non-negligible mass in fluids

Inertial tracer in a steady cellular flow

Thermal noise

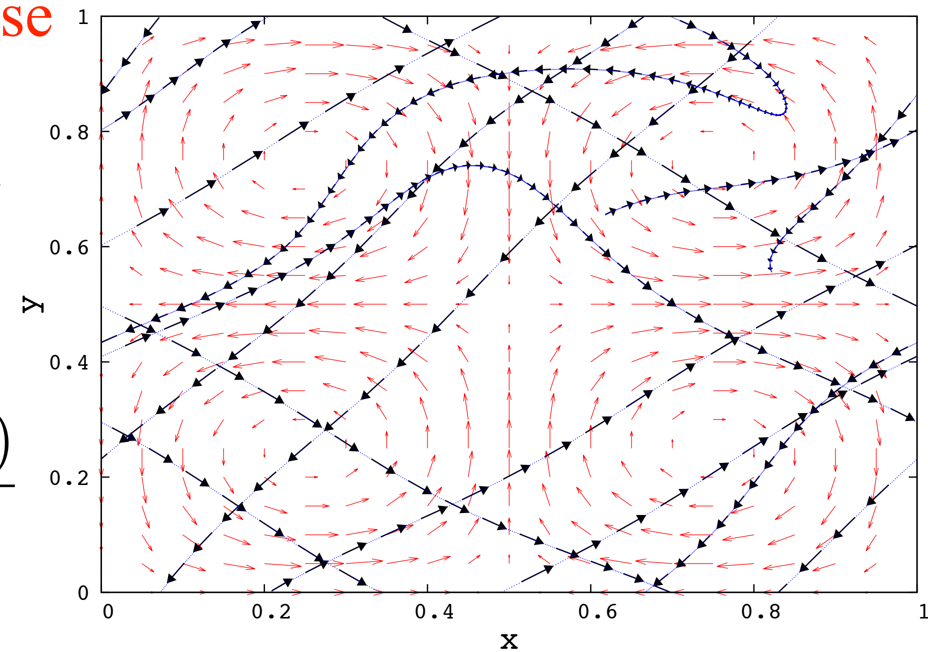
$$\dot{x} = v_x, \qquad \dot{y} = v_y$$

$$\dot{v}_x = -\frac{1}{\tau}(v_x - U_x) + F + \sqrt{2D_0}\xi_x$$

$$\dot{v}_y = -\frac{1}{\tau}(v_y - U_y) + \sqrt{2D_0}\xi_y$$

$$U_x = \frac{\partial \psi(x,y)}{\partial y}, \qquad U_y = -\frac{\partial \psi(x,y)}{\partial x}$$

Divergenceless velocity field



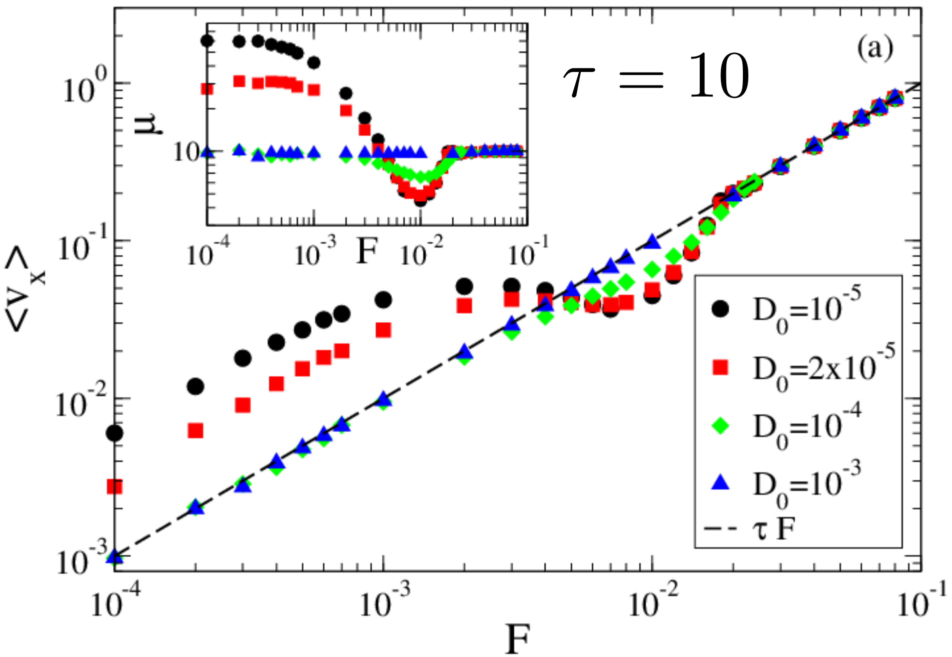
Stream function $\psi(x,y) = \frac{LU_0}{2\pi} \sin\left(\frac{2\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right)$

Parameters: $U_0 = 1,\ L = 1,\ \tau^* = L/U_0 = 1$

τ Stokes time

Negative differential and absolute mobility

Force-velocity relation



Mobility $\mu = \langle v_x \rangle / F$

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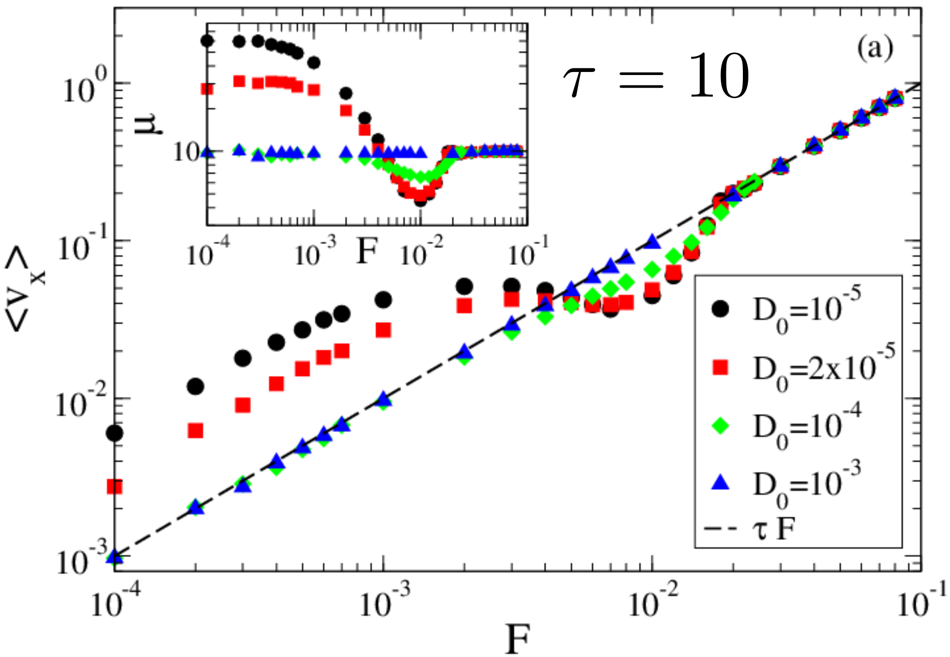

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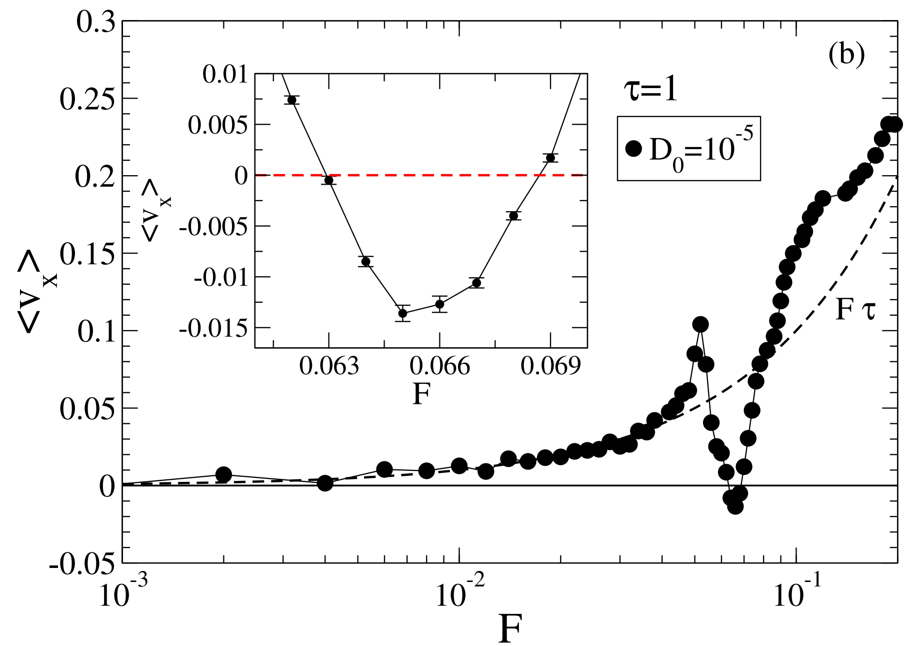


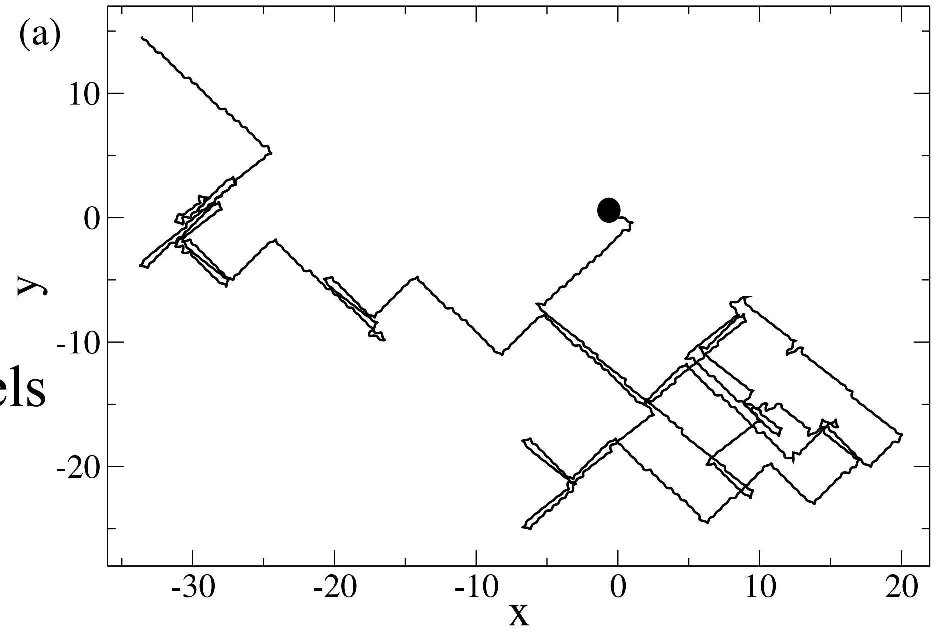
Absolute negative mobility occurs for some values of the driving force

Possible physical mechanism

Typical trajectory for $D_0 = 0$

The motion is realized along preferential “channels”

Both inertia and noise activate random transitions between the channels



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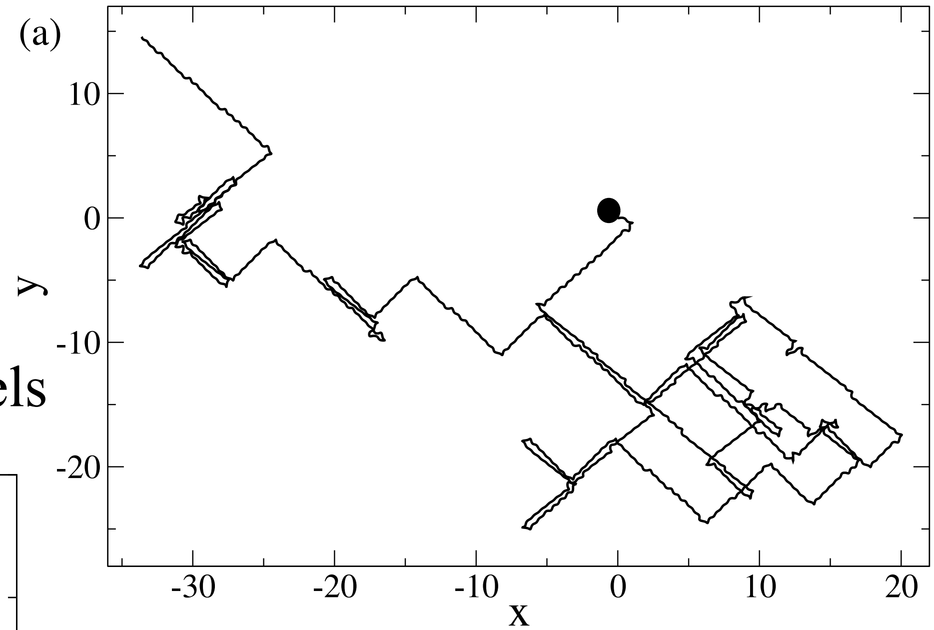

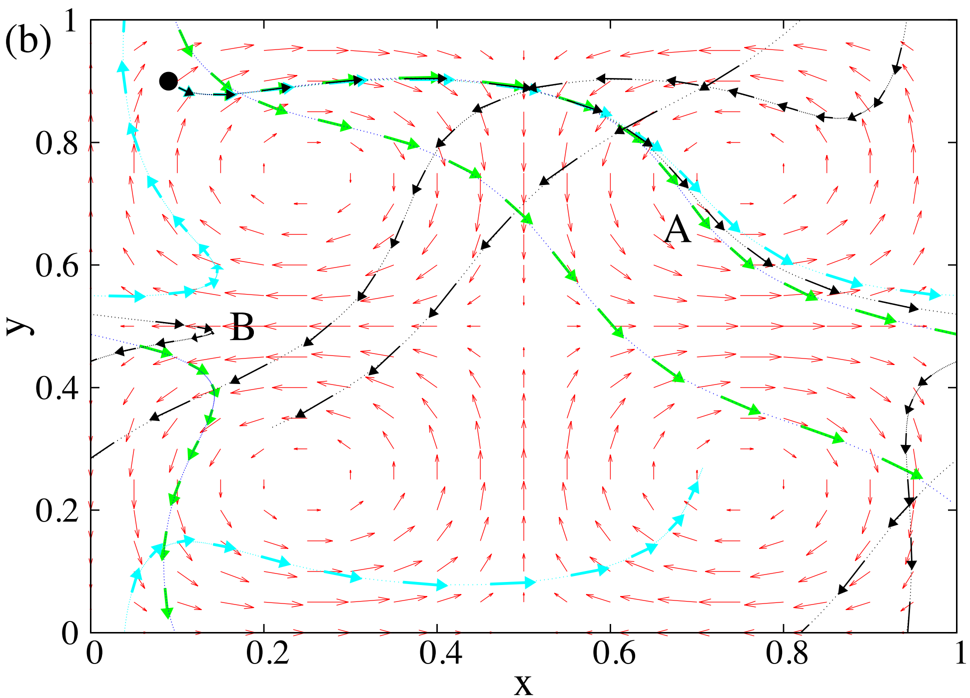

F=0.065

The tracer is pushed from region A (downstream channel) to region B (upstream channel)

F=0.04

F=0.09

With a smaller or larger force, the particle avoids the adverse region B and continues its run along downstream channels

Conclusions and perspectives

- Nonlinear response reveals anomalous behaviors (e.g. NDM and ANM)
  - Analytical results for a lattice gas model
  - Numerical investigation for inertial particles in steady laminar flows

» Interaction of many driven tracers in the system?

» Possible analytical approaches for steady laminar flows?

References:

Microscopic theory for negative differential mobility in crowded environments
Bénichou, Illien, Oshanin, Sarracino, Voituriez, Phys. Rev. Lett. **113**, 268002 (2014)

Nonlinear response and emerging nonequilibrium microstructures in confined crowded environments
Bénichou, Illien, Oshanin, Sarracino, Voituriez, Phys. Rev. E **93**, 032128 (2016)

Nonlinear response of inertial tracers in steady laminar flows
Sarracino, Cecconi, Puglisi, Vulpiani, Phys. Rev. Lett. **117**, 174501 (2016)