# A computational continuum model of poroelastic beds

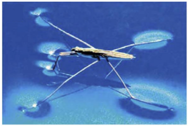
## (Limitations and possibilities)

Uğis Lācis[1], Shervin Bagheri[1] and Sudhakar Yogaraj[1]

[1] *Linné FLOW Centre, KTH Mechanics*

# Motivation and objective

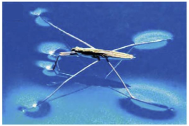
## Inspiration from nature

# Motivation and objective

## Inspiration from nature


All of these animals move in the surrounding medium (fluid) and interact with it.



# Motivation and objective

Inspiration from nature

**There are different surface textures[1] used by motile animals**

[1] Liu, K. & Lei, J., Bio-inspired design of multiscale structures for function integration, *Nano Today* **6(2)**, 155-175 (2011).

# Motivation and objective

Inspiration from nature

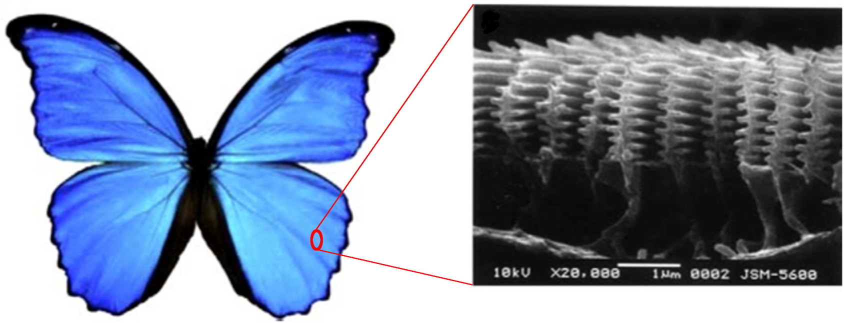
**There are different surface textures[1] used by motile animals**

[1] Liu, K. & Lei, J., Bio-inspired design of multiscale structures for function integration, *Nano Today* **6(2)**, 155-175 (2011).

# Motivation and objective

## Inspiration from nature

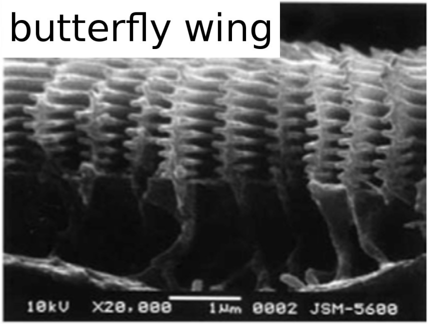
**There are different surface textures[1] used by motile animals**




[1] Liu, K. & Lei, J., Bio-inspired design of multiscale structures for function integration, *Nano Today* **6(2)**, 155-175 (2011).

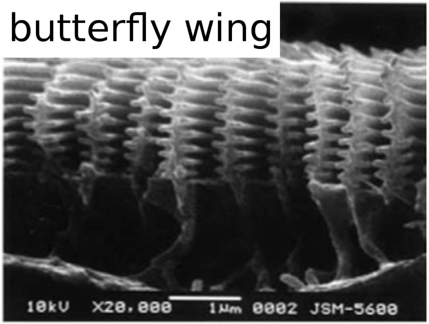
# Motivation and objective

Main aim of this work



# Motivation and objective

## Main aim of this work



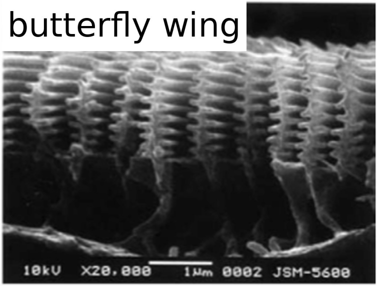

**Understand how the porous and elastic multi-scale surface texture interact with freely moving fluid**

# Outline

# Outline

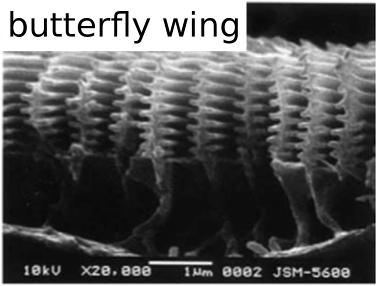
# Illustration and methods

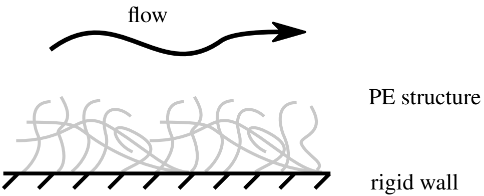
Poroelastic bed

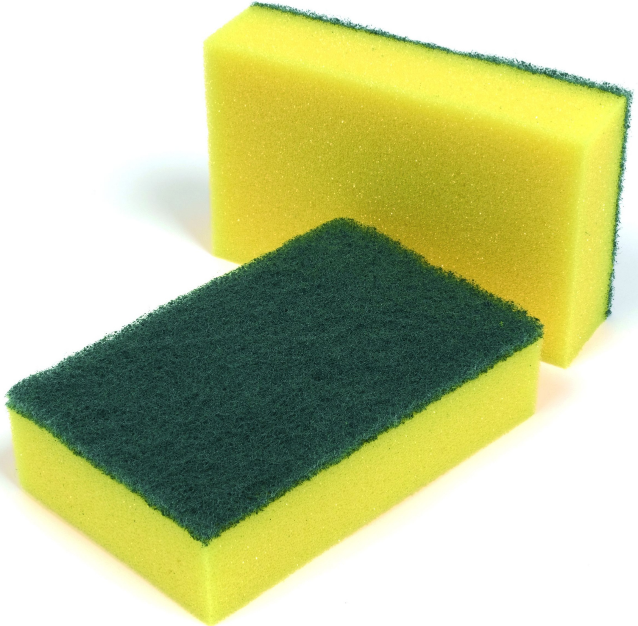


# Illustration and methods

## Poroelastic bed

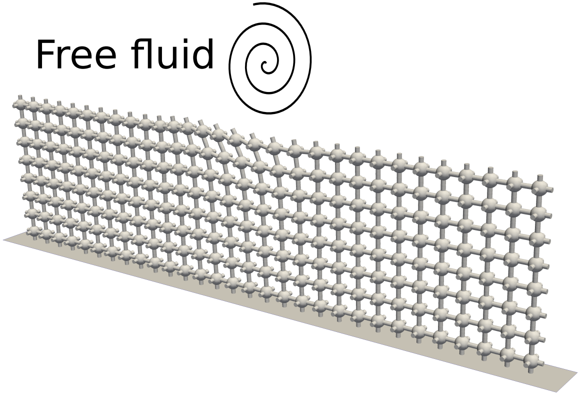




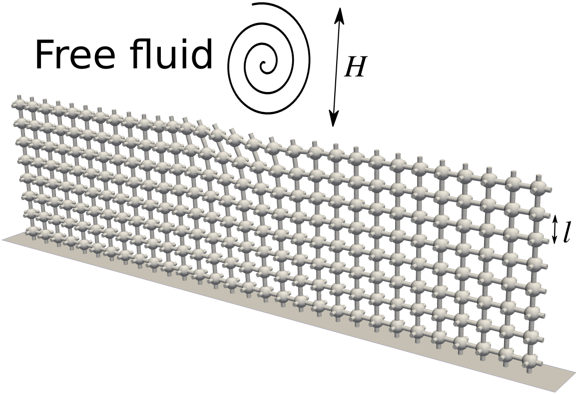
# Illustration and methods

Poroelastic bed



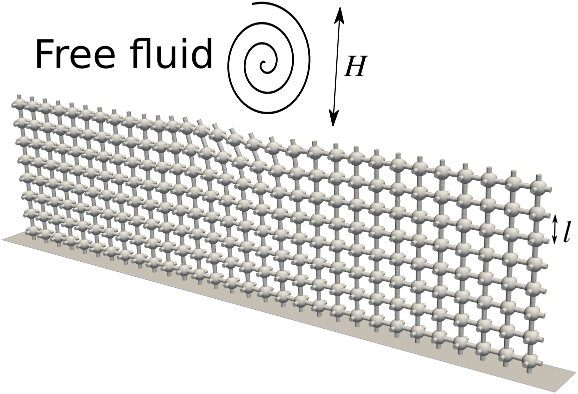
# Illustration and methods

Poroelastic bed



# Illustration and methods

Poroelastic bed




- System is inherently multi-scale, $\epsilon = l/H \lesssim 0.1$

# Illustration and methods

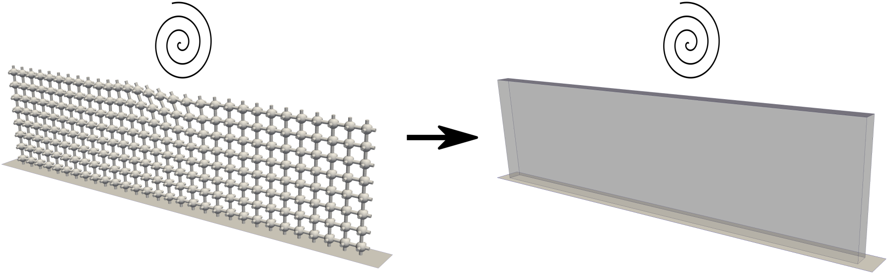
## Method overview

Direct numerical simulations are often not feasible due to extreme computational requirements

# Illustration and methods

## Method overview

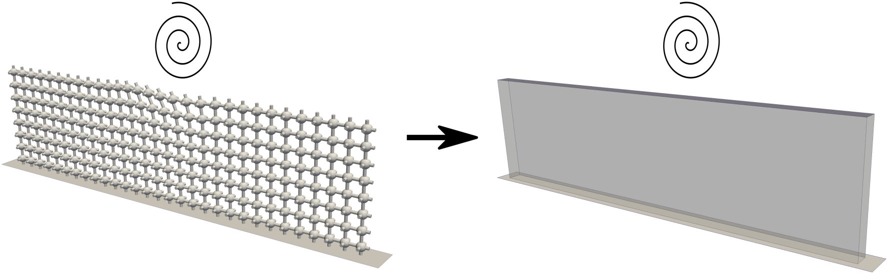
Direct numerical simulations are often not feasible due to extreme computational requirements



# Illustration and methods

## Method overview

Direct numerical simulations are often not feasible due to extreme computational requirements

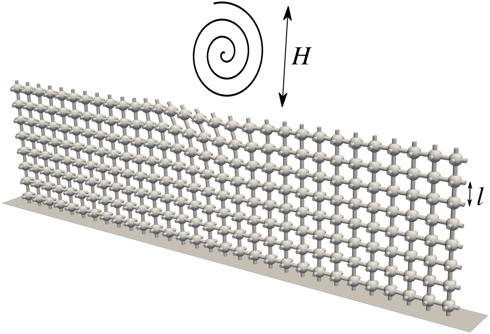


**How to do this mathematically without free parameters?**

# Illustration and methods

## Scale separation and estimates

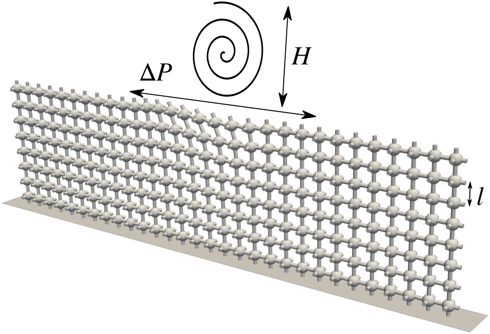
Scale estimates are important for physical understanding of problem and to later apply mathematical framework



# Illustration and methods

## Scale separation and estimates

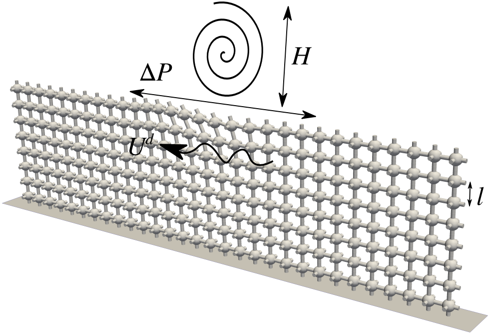
Scale estimates are important for physical understanding of problem and to later apply mathematical framework



# Illustration and methods

## Scale separation and estimates

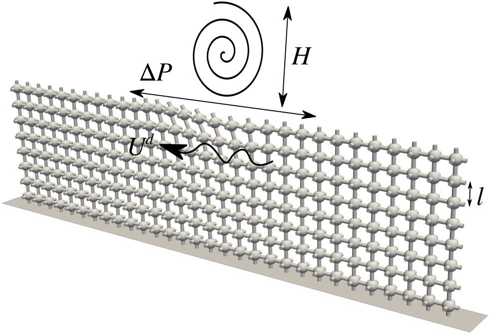
Scale estimates are important for physical understanding of problem and to later apply mathematical framework



# Illustration and methods

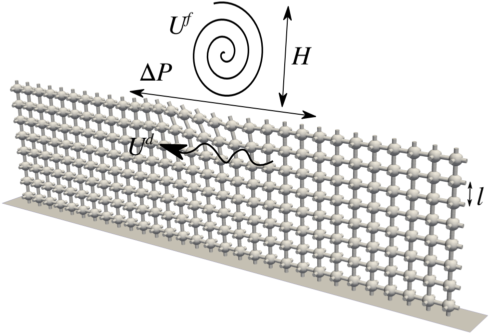
## Scale separation and estimates

Scale estimates are important for physical understanding of problem and to later apply mathematical framework




$$U^d \sim \frac{l^2 \Delta P}{\mu H}$$

Scale estimates are important for physical understanding of problem and to later apply mathematical framework

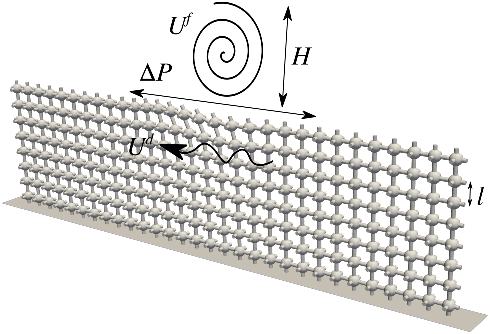



$U^d \sim \frac{l^2 \Delta P}{\mu H}$

# Illustration and methods

## Scale separation and estimates

Scale estimates are important for physical understanding of problem and to later apply mathematical framework

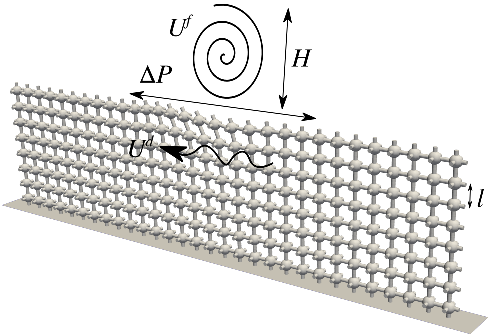



$$U^d \sim \frac{l^2 \Delta P}{\mu H}; \quad U^d \sim \left(\frac{l}{H}\right)^2 U^f$$

# Illustration and methods

## Scale separation and estimates

Scale estimates are important for physical understanding of problem and to later apply mathematical framework




$U^d \sim \frac{l^2 \Delta P}{\mu H}$; $U^d \sim \left(\frac{l}{H}\right)^2 U^f$; shear $\sim \frac{U^f}{H}$;

# Illustration and methods

## Multi-scale expansion

- Scale separation parameter $\epsilon = l/H \lesssim 0.1$

# Illustration and methods

## Multi-scale expansion

- Scale separation parameter $\epsilon = l/H \lesssim 0.1$
- Slow (micro) coordinate $x_1 = x$
- Fast (macro) coordinate $x_0 = \epsilon x$

# Illustration and methods

## Multi-scale expansion

- Scale separation parameter $\epsilon = l/H \lesssim 0.1$
- Slow (micro) coordinate $x_1 = x$
- Fast (macro) coordinate $x_0 = \epsilon x$

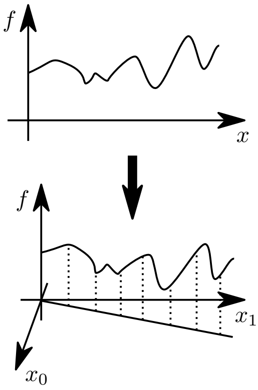
- Introduce function in double dimensions
  $f\left(x\right) \;\rightarrow\; f\left(x_0, x_1\right)$

# Illustration and methods

## Multi-scale expansion

- Scale separation parameter $\epsilon = l/H \lesssim 0.1$
- Slow (micro) coordinate $x_1 = x$
- Fast (macro) coordinate $x_0 = \epsilon x$

- Introduce function in double dimensions
  $f\,(x) \;\rightarrow\; f\,(x_0, x_1)$



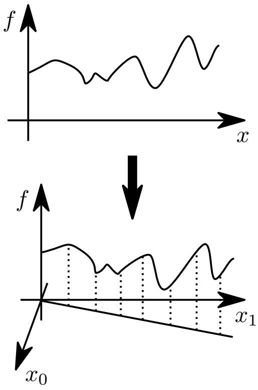
# Illustration and methods



- Scale separation parameter $\epsilon = l/H \lesssim 0.1$
- Slow (micro) coordinate $x_1 = x$
- Fast (macro) coordinate $x_0 = \epsilon x$

- Introduce function in double dimensions
  $f\left(x\right) \;\rightarrow\; f\left(x_0, x_1\right)$

- Expansion in both coordinate and amplitude
  - Derivative $\left(\right)_{,i} = \left(\right)_{,i_1} + \epsilon \left(\right)_{,i_0}$
  - Variable $f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \ldots$



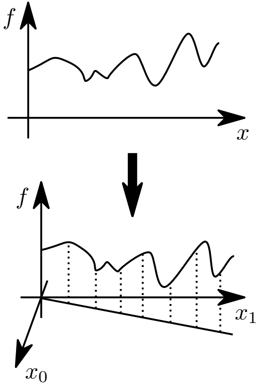
# Illustration and methods

## Multi-scale expansion

- Scale separation parameter $\epsilon = l/H \lesssim 0.1$
- Slow (micro) coordinate $x_1 = x$
- Fast (macro) coordinate $x_0 = \epsilon x$

- Introduce function in double dimensions
  $f\left(x\right) \;\rightarrow\; f\left(x_0, x_1\right)$

- Expansion in both coordinate and amplitude
  - Derivative $\left(\right)_{,i} = \left(\right)_{,i_1} + \epsilon \left(\right)_{,i_0}$
  - Variable $f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \ldots$

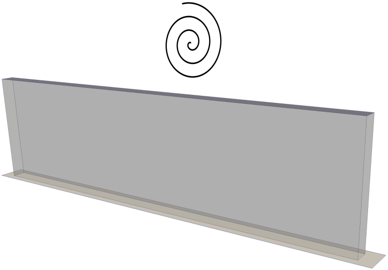



- Group terms at different orders, solve, homogenize, ...

# Outline

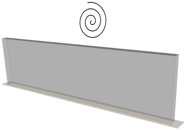
# Governing equations

## Poroelastic bed



# Governing equations

## Poroelastic bed



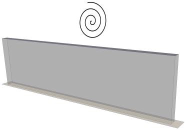
- Free fluid, Navier-Stokes:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \boldsymbol{\nabla})\,\vec{u} = -\boldsymbol{\nabla} p + \frac{1}{Re}\nabla^2 \vec{u}$$

$$\boldsymbol{\nabla} \cdot \vec{u} = 0$$

# Governing equations

## Poroelastic bed



- Free fluid, Navier-Stokes:

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \boldsymbol{\nabla}\right) \vec{u} = -\boldsymbol{\nabla} p + \frac{1}{Re} \nabla^2 \vec{u}$$

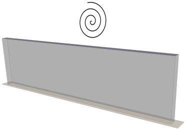
$$\boldsymbol{\nabla} \cdot \vec{u} = 0$$

- Poroelastic material, linear elasticity:

$$\rho \frac{\partial^2 \vec{v}}{\partial t^2} = \nabla \cdot \left[ \mathbf{C} \quad : \frac{1}{2} \left\{ \boldsymbol{\nabla} \vec{v} + \left( \boldsymbol{\nabla} \vec{v} \right)^T \right\} \right]$$

# Governing equations

## Poroelastic bed



- Free fluid, Navier-Stokes:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \boldsymbol{\nabla})\, \vec{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathit{Re}} \nabla^2 \vec{u}$$

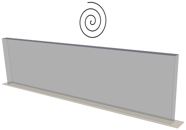
$$\boldsymbol{\nabla} \cdot \vec{u} = 0$$

- Poroelastic material, linear elasticity:

$$(1-\theta)\rho \frac{\partial^2 \vec{v}}{\partial t^2} = \nabla \cdot \left[ \mathbf{C}^{\mathrm{ef}} : \frac{1}{2} \left\{ \boldsymbol{\nabla} \vec{v} + (\boldsymbol{\nabla} \vec{v})^T \right\} \right]$$

# Governing equations

## Poroelastic bed



- Free fluid, Navier-Stokes:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \boldsymbol{\nabla})\, \vec{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathit{Re}} \nabla^2 \vec{u}$$

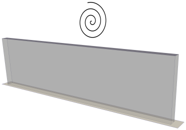
$$\boldsymbol{\nabla} \cdot \vec{u} = 0$$

- Poroelastic material, linear elasticity:

$$(1-\theta)\rho \frac{\partial^2 \vec{v}}{\partial t^2} = \nabla \cdot \left[ \mathbf{C}^{\mathrm{ef}} : \frac{1}{2} \left\{ \boldsymbol{\nabla} \vec{v} + (\boldsymbol{\nabla} \vec{v})^T \right\} \right] - \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} p^-$$

# Governing equations

## Poroelastic bed



- Free fluid, Navier-Stokes:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \boldsymbol{\nabla})\,\vec{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathit{Re}} \nabla^2 \vec{u}$$

$$\boldsymbol{\nabla} \cdot \vec{u} = 0$$

- Poroelastic material, linear elasticity:

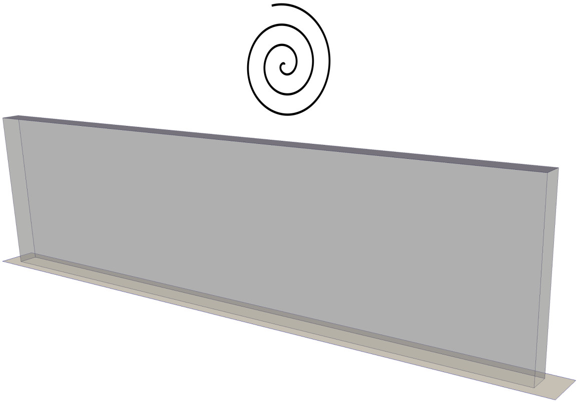
$$(1-\theta)\rho \frac{\partial^2 \vec{v}}{\partial t^2} = \nabla \cdot \left[ \mathbf{C}^{\mathrm{ef}} : \frac{1}{2} \left\{ \boldsymbol{\nabla} \vec{v} + (\boldsymbol{\nabla} \vec{v})^T \right\} \right] - \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} p^-$$

Additional information in book[2]

[2] Mei, C. C. & Vernescu, B., Homogenization methods for multiscale mechanics, *World scientific* (2010)

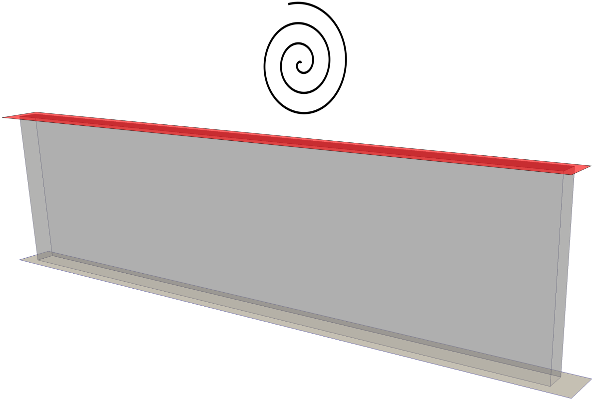
# Governing equations

Interface conditions



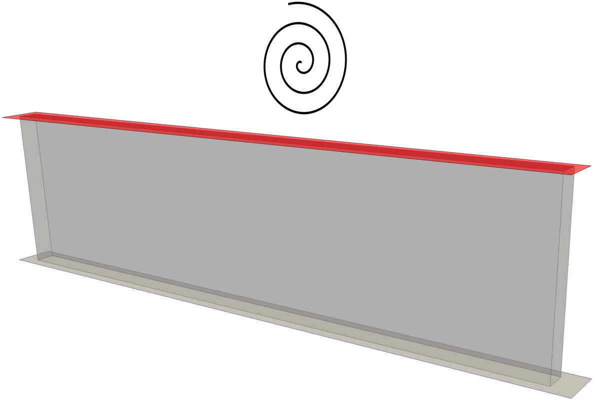
# Governing equations

Interface conditions



# Governing equations

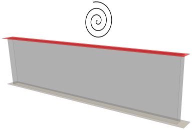
## Interface conditions



**Interface treatment is non-trivial**

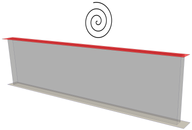
# Governing equations

Interface conditions, velocity



# Governing equations

Interface conditions, velocity

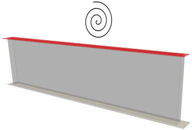


Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

Interface conditions, velocity



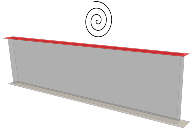
- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^{+}$, $p = P + p^{+}$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^{+}$, $p = P + p^{+}$

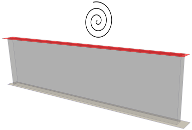
$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right) \vec{U} = -\boldsymbol{\nabla} P + \frac{1}{Re} \nabla^2 \vec{U}$$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

## Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^+$, $p = P + p^+$

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right) \vec{U} = -\boldsymbol{\nabla} P + \frac{1}{Re} \nabla^2 \vec{U}$$

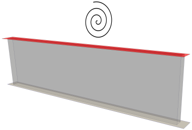
$$\vec{U} = 0 \qquad \text{on interface (red)}$$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

## Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^{+}$, $p = P + p^{+}$

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right) \vec{U} = -\boldsymbol{\nabla} P + \frac{1}{Re} \nabla^2 \vec{U}$$

$$\vec{U} = 0 \qquad \text{on interface (red)}$$

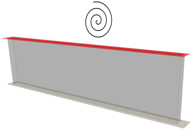
- Perturbations driven mostly by shear stress:

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

## Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^{+}$, $p = P + p^{+}$

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right) \vec{U} = -\boldsymbol{\nabla} P + \frac{1}{\mathit{Re}} \nabla^2 \vec{U}$$

$$\vec{U} = 0 \qquad \text{on interface (red)}$$

- Perturbations driven mostly by shear stress:

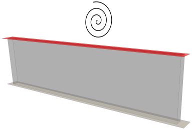
$$\boldsymbol{\Sigma}^{U} \cdot \hat{n}$$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

## Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^+$, $p = P + p^+$

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right)\vec{U} = -\boldsymbol{\nabla} P + \frac{1}{Re}\nabla^2 \vec{U}$$

$$\vec{U} = 0 \qquad \text{on interface (red)}$$

- Perturbations driven mostly by shear stress:

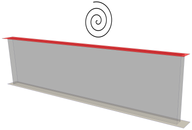
$$\boldsymbol{\Sigma}^{U} \cdot \hat{n} + \boldsymbol{\Sigma}^{u^+} \cdot \hat{n}$$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

## Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^{+}$, $p = P + p^{+}$

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right) \vec{U} = -\boldsymbol{\nabla} P + \frac{1}{Re} \nabla^2 \vec{U}$$

$$\vec{U} = 0 \qquad \text{on interface (red)}$$

- Perturbations driven mostly by shear stress:

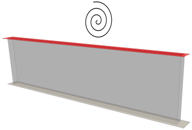
$$\boldsymbol{\Sigma}^{U} \cdot \hat{n} + \boldsymbol{\Sigma}^{u^{+}} \cdot \hat{n} = \boldsymbol{\Sigma}^{u^{-}} \cdot \hat{n}$$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

Interface conditions, velocity



- Decomposing fast flow: $\vec{u} = \vec{U} + \vec{u}^{+}$, $p = P + p^{+}$

$$\frac{\partial \vec{U}}{\partial t} + \left(\vec{U} \cdot \boldsymbol{\nabla}\right)\vec{U} = -\boldsymbol{\nabla} P + \frac{1}{Re}\nabla^2 \vec{U}$$

$$\vec{U} = 0 \qquad \text{on interface (red)}$$

- Perturbations driven mostly by shear stress:

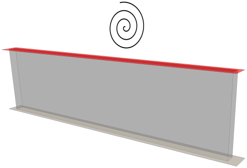
$$\boldsymbol{\Sigma}^{U} \cdot \hat{n} + \boldsymbol{\Sigma}^{u^{+}} \cdot \hat{n} = \boldsymbol{\Sigma}^{u^{-}} \cdot \hat{n}$$

Additional information in publication[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

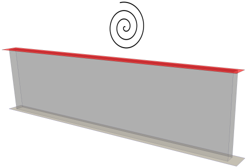
Interface conditions, velocity



[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

Interface conditions, velocity

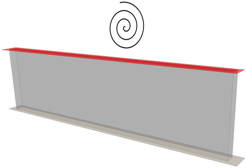


$$\vec{u} = \qquad -\frac{\mathbf{K}}{\mu}\cdot\boldsymbol{\nabla}p^{-} + \mathbf{L} : \left[\boldsymbol{\nabla}\vec{u} + \left(\boldsymbol{\nabla}\vec{u}\right)^{T}\right]$$

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

Interface conditions, velocity



$$\vec{u} = \quad -\frac{\mathbf{K}}{\mu}\cdot\boldsymbol{\nabla} p^{-} + \mathbf{L} : \left[\boldsymbol{\nabla}\vec{u} + (\boldsymbol{\nabla}\vec{u})^{T}\right]$$

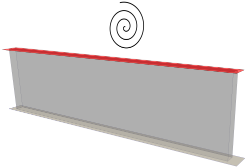
Generalized BJ[4] condition **without** any **empirical** parameters[3]

[4] Beavers, G. S. & Joseph, D. D., Boundary conditions at a naturally permeable wall, *J. Fluid Mech.* **30(01)**, 197-207 (1967)

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

Interface conditions, velocity



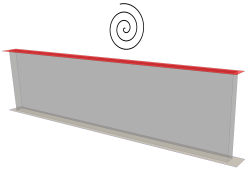
$$\vec{u} = \partial_t \vec{v} - \frac{\mathbf{K}}{\mu} \cdot \boldsymbol{\nabla} p^- + \mathbf{L} : \left[ \boldsymbol{\nabla} \vec{u} + (\boldsymbol{\nabla} \vec{u})^T \right]$$

Generalized BJ condition **without** any **empirical** parameters[3]

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

# Governing equations

## Interface conditions, velocity



$$\vec{u} = {\color{red}\partial_t \vec{v}} - \frac{\mathbf{K}}{\mu} \cdot \boldsymbol{\nabla} p^- + \mathbf{L} : \left[ \boldsymbol{\nabla} \vec{u} + (\boldsymbol{\nabla} \vec{u})^T \right]$$

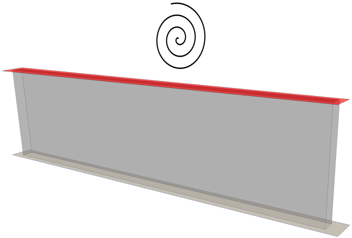
$\boxed{\text{Generalized BJ condition \textbf{without} any \textbf{empirical} parameters}}$ [3]

- Permeability $\mathbf{K} \sim l^2$ and slip length $\mathbf{L} \sim l$, disappears as $l \to 0$

[3] Lācis, U. & Bagheri, S., A framework for computing effective boundary conditions at the interface between free fluid and a porous medium, *J. Fluid Mech.* **812**, 866-889 (2017)

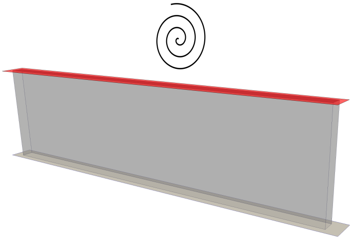
# Governing equations

Interface conditions, stress



# Governing equations

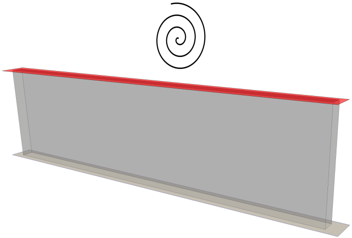
Interface conditions, stress



- Natural way to match free fluid and poroelastic material

# Governing equations

## Interface conditions, stress



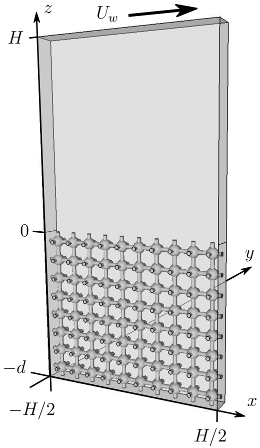
- Natural way to match free fluid and poroelastic material
- We propose to use the total stress continuity

$$\left[\mathbf{C}^{\mathrm{ef}} : \frac{1}{2}\left(\boldsymbol{\nabla}\vec{v} + (\boldsymbol{\nabla}\vec{v})^T\right) - \boldsymbol{\alpha} p^-\right] \cdot \hat{n} = \left[-p\boldsymbol{\delta} + \frac{1}{Re}\left(\boldsymbol{\nabla}\vec{u} + (\boldsymbol{\nabla}\vec{u})^T\right)\right] \cdot \hat{n}$$

# Outline

# Response to flow vortex

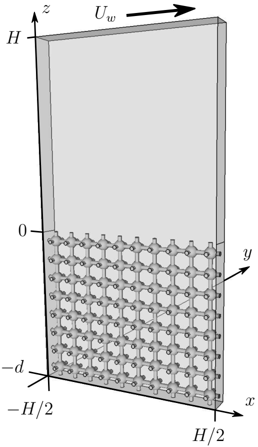
Flow field[5]

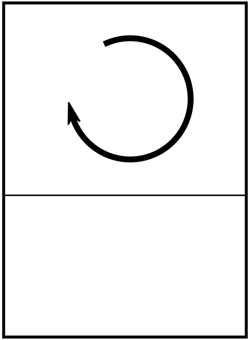



[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]

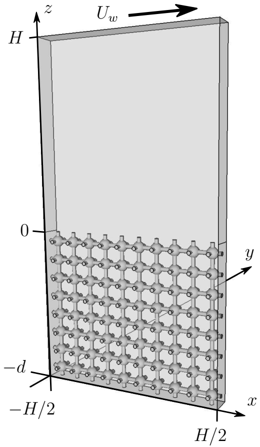


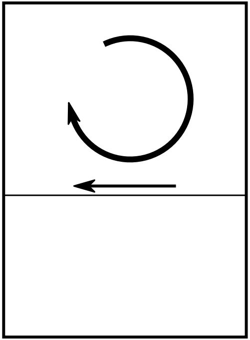



[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]

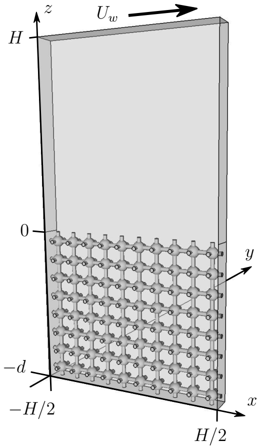


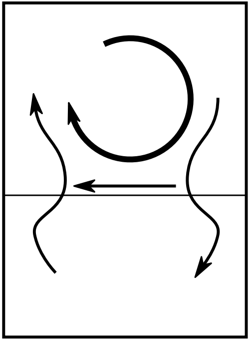



[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]

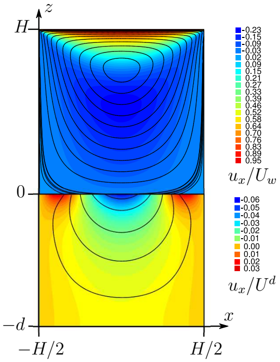


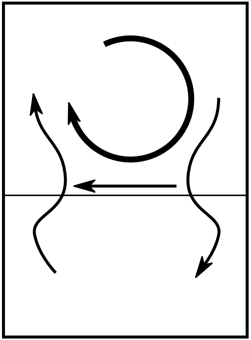



[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]

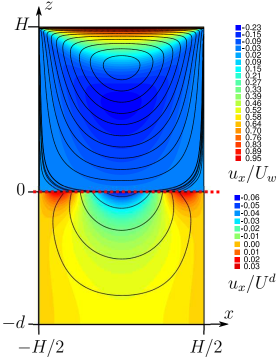


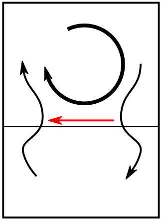



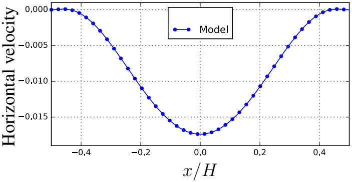
[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]



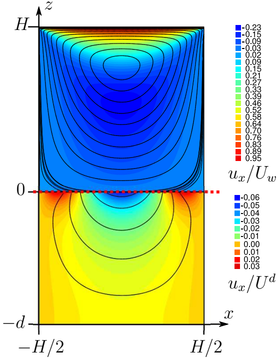



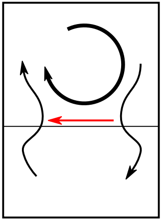


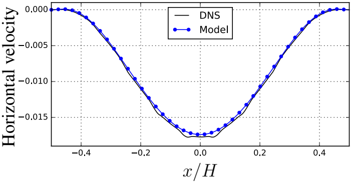

[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]



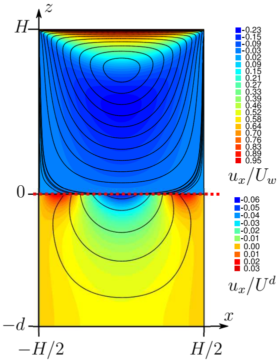



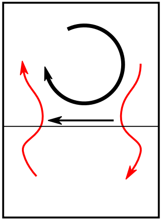


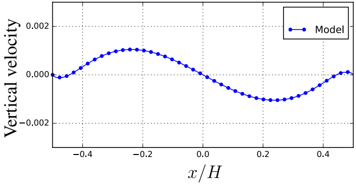

[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]



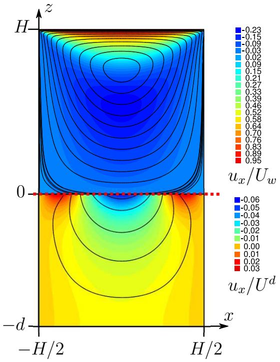



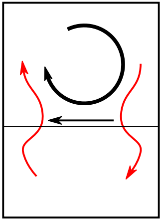


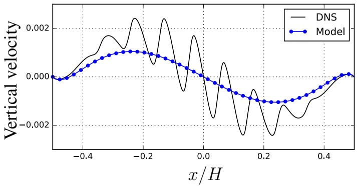

[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Flow field[5]



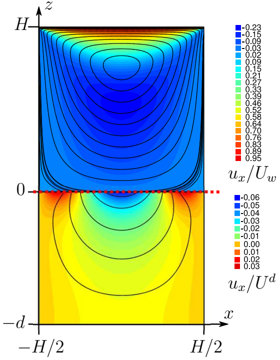


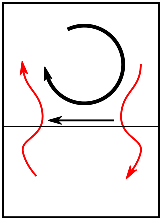

[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

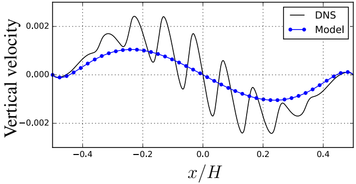
# Response to flow vortex

Flow field[5]

No empirical fitting parameters



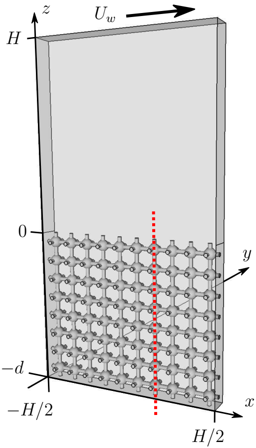



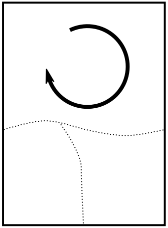



[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[5]

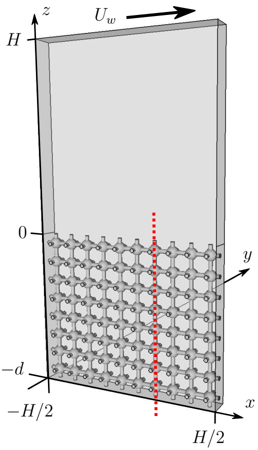


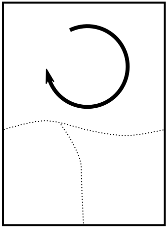



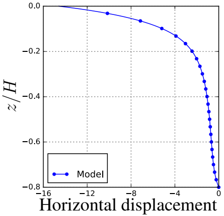
[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[5]



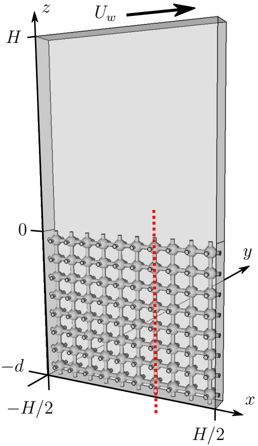



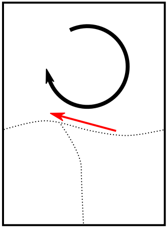


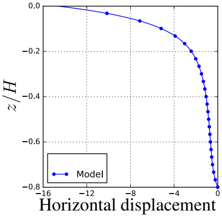

[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[5]



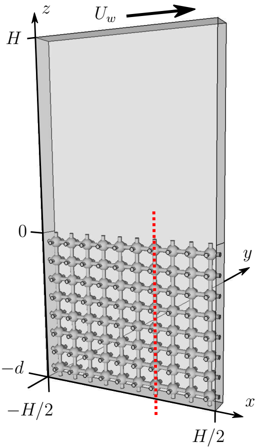



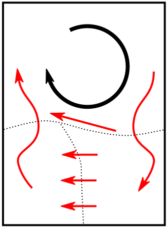


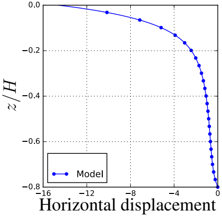

[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

## Displacement field[5]



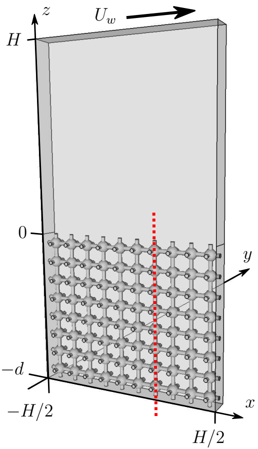



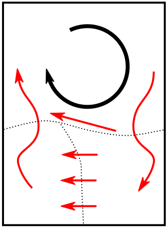


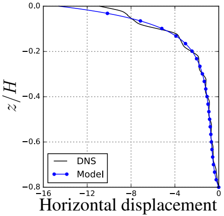

[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[5]



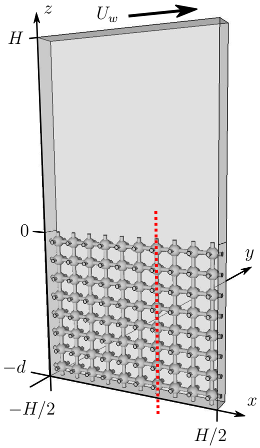



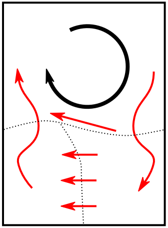


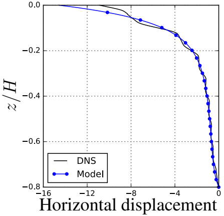

[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[5]



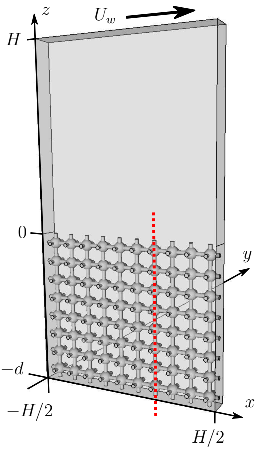


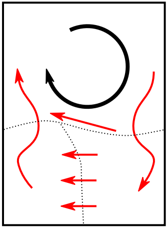

**Fully non-empirical model**

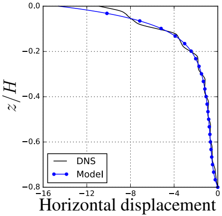
[5] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[5]






Fully non-empirical model

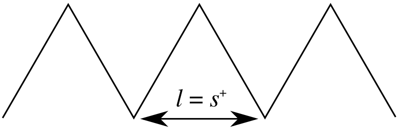
> 10 times less d.o.f.

[5]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Outline

# Limitations and applicability

## What we cannot capture

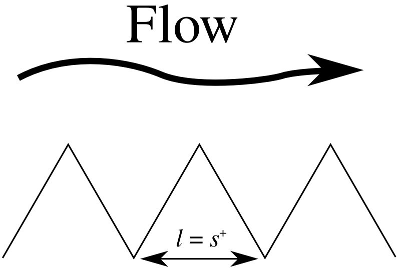



- Illustration through breakdown of riblet drag reduction[6]

[6]Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

What we cannot capture

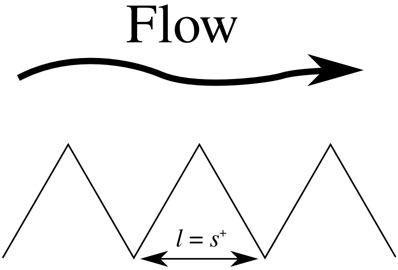



- Illustration through breakdown of riblet drag reduction[6]

[6] Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

What we cannot capture



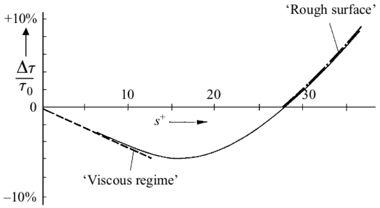



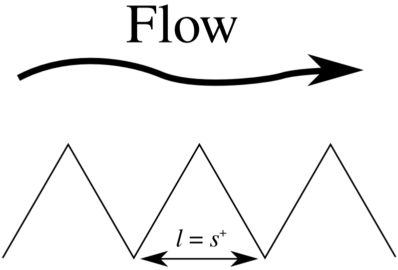

FIGURE 15. General structure of a drag reduction curve.

- Illustration through breakdown of riblet drag reduction[6]

[6] Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

What we cannot capture



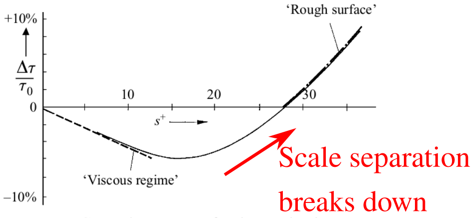



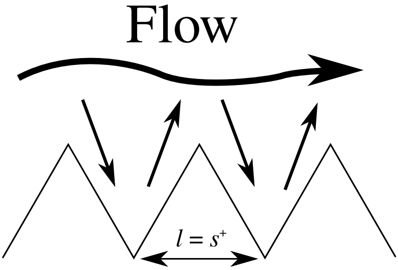

FIGURE 15. General structure of a drag reduction curve.

- Illustration through breakdown of riblet drag reduction[6]

[6] Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

What we cannot capture



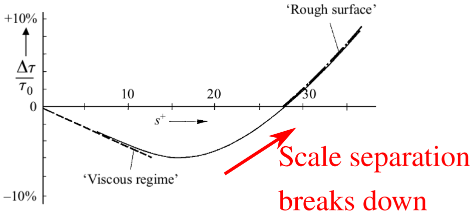



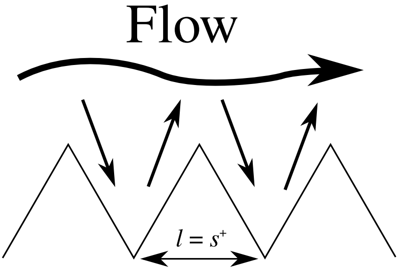

FIGURE 15. General structure of a drag reduction curve.

- Illustration through breakdown of riblet drag reduction[6]

[6] Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

What we cannot capture



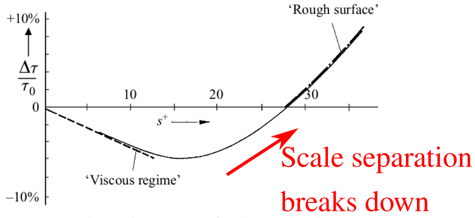




FIGURE 15. General structure of a drag reduction curve.

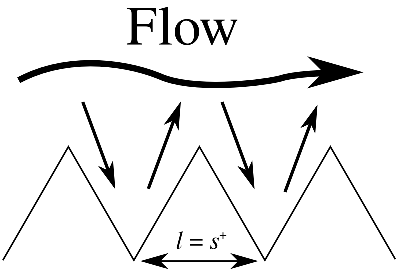
- Illustration through breakdown of riblet drag reduction[6]
- Current model is limited to $\mathbf{L} = const$

$$\vec{u} = \mathbf{L} : \left[\nabla\vec{u} + (\nabla\vec{u})^T\right]$$

[6] Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

What we cannot capture



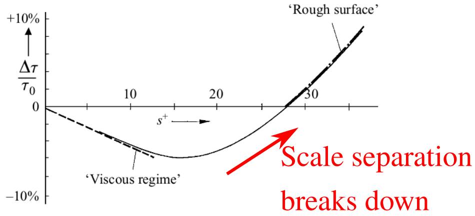




FIGURE 15. General structure of a drag reduction curve.

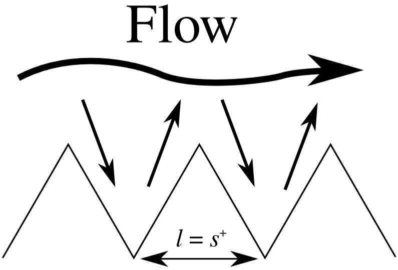
- Illustration through breakdown of riblet drag reduction[6]
- Current model is limited to $\mathbf{L} = const$, could have $\mathbf{L} = f\,(shear, ...)$

$$\vec{u} = \mathbf{L} : \left[\nabla\vec{u} + (\nabla\vec{u})^T\right]$$

[6]Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

## What we cannot capture



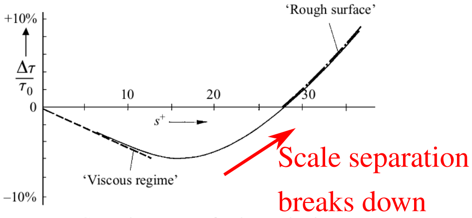




FIGURE 15. General structure of a drag reduction curve.

- Illustration through breakdown of riblet drag reduction[6]
- Current model is limited to $\mathbf{L} = const$, could have $\mathbf{L} = f\,(shear, ...)$

$$\vec{u} = \mathbf{L} : \left[\nabla\vec{u} + (\nabla\vec{u})^T\right] + ...$$

[6]Bechert, D. W., et al., Experiments on drag-reducing surfaces and their optimization with an adjustable geometry, *J. Fluid Mech.* **338**, 59-87 (1997)

# Limitations and applicability

## What we can describe

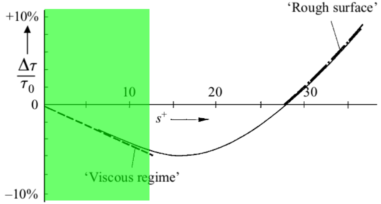



FIGURE 15. General structure of a drag reduction curve.

- Riblet drag reduction linear part[7]

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

What we can describe

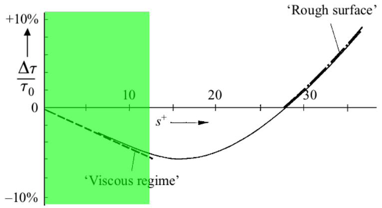
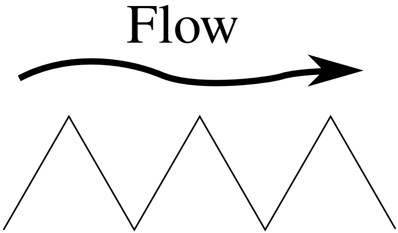



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

## What we can describe

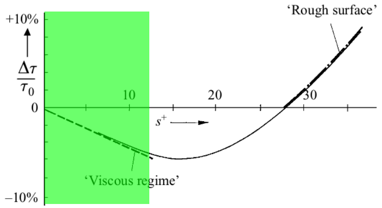
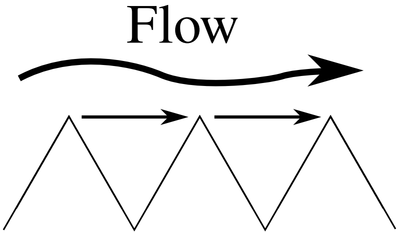



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

## What we can describe

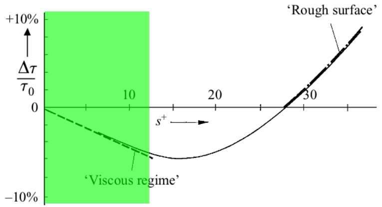
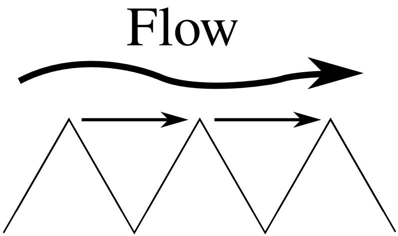



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]
- Current model can add – (anisotropic) porosity

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

## What we can describe

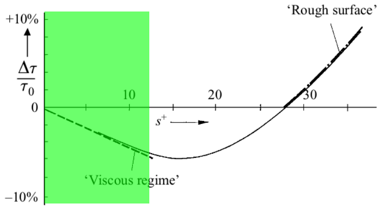
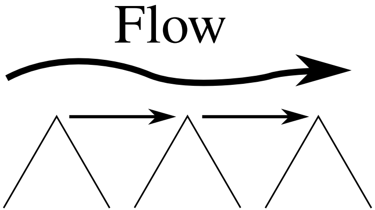



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]
- Current model can add – (anisotropic) porosity

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

## What we can describe

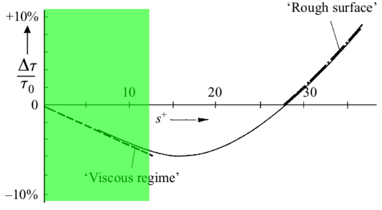
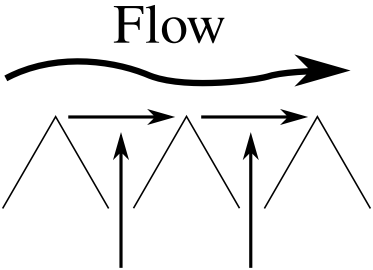



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]
- Current model can add – (anisotropic) porosity

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

## What we can describe

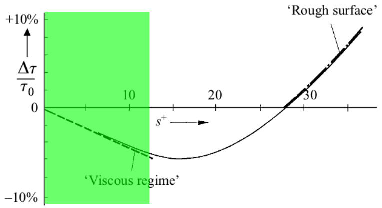
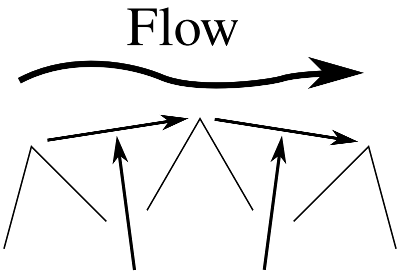



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]
- Current model can add – (anisotropic) porosity, (anisotropic) elasticity

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Limitations and applicability

## What we can describe

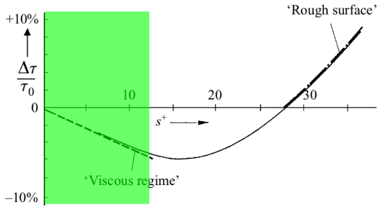
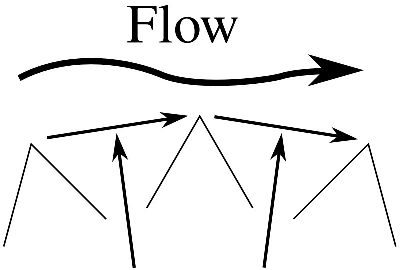



FIGURE 15. General structure of a drag reduction curve.




- Riblet drag reduction linear part[7]
- Current model can add – (anisotropic) porosity, (anisotropic) elasticity
- Examples in nature – mostly anisotropic

[7] Luchini, P., Manzo, F., & Pozzi, A., Resistance of a grooved surface to parallel flow and cross-flow, *J. Fluid Mech.* **228**, 87-109 (1991)

# Outline

**Model of flow above poro-elastic material with no empirical parameters**

# Conclusions and outlook

Conclusions

**Model of flow above poro-elastic material with no empirical parameters**

- Model equations and boundary conditions have been developed
- Model simulation has been validated with fully resolved simulation
- Poroelastic material response to free flow vortex has been explained

# Conclusions and outlook

Conclusions

**Model of flow above poro-elastic material with no empirical parameters**

- Model equations and boundary conditions have been developed
- Model simulation has been validated with fully resolved simulation
- Poroelastic material response to free flow vortex has been explained

**Limitations and applicability**

- Model can not capture micro-scale variations
- Model is valid for linear regime in drag reduction curve

# Conclusions and outlook

Conclusions

**Model of flow above poro-elastic material with no empirical parameters**

- Model equations and boundary conditions have been developed
- Model simulation has been validated with fully resolved simulation
- Poroelastic material response to free flow vortex has been explained

**Limitations and applicability**

- Model can not capture micro-scale variations
- Model is valid for linear regime in drag reduction curve

**All model codes (FreeFEM++) available in Github**

# Conclusions and outlook



## Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic

[8] Yogaraj, S., ongoing work with NEK5000

# Conclusions and outlook

Outlook

## Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic

## Extending the model

- Complement the model with two-phase flow

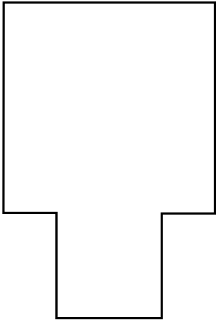
[8] Yogaraj, S., ongoing work with NEK5000

## Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic

## Extending the model

- Complement the model with two-phase flow



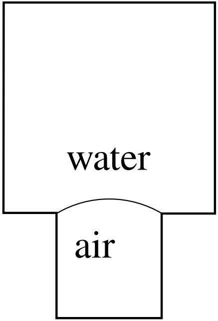
[8] Yogaraj, S., ongoing work with NEK5000

# Conclusions and outlook

Outlook

## Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic




## Extending the model

- Complement the model with two-phase flow

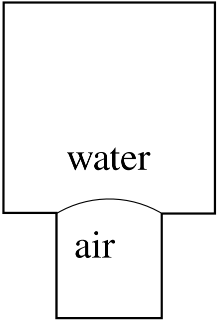
[8]Yogaraj, S., ongoing work with NEK5000

# Conclusions and outlook

Outlook

### Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic



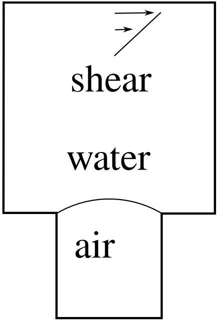

### Extending the model

- Complement the model with two-phase flow
- Investigate response to shear, pressure, ...

[8] Yogaraj, S., ongoing work with NEK5000

## Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic




## Extending the model

- Complement the model with two-phase flow
- Investigate response to shear, pressure, ...

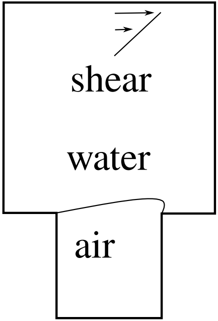
[8] Yogaraj, S., ongoing work with NEK5000

# Conclusions and outlook

Outlook

## Applying the developed model

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic




## Extending the model

- Complement the model with two-phase flow
- Investigate response to shear, pressure, ...

[8] Yogaraj, S., ongoing work with NEK5000

# Conclusions and outlook

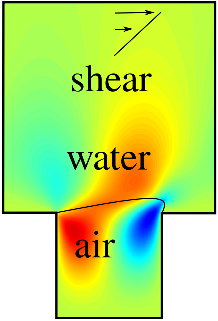
Outlook

**Applying the developed model**

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic

**Extending the model**

- Complement the model with two-phase flow
- Investigate response to shear, pressure, ...



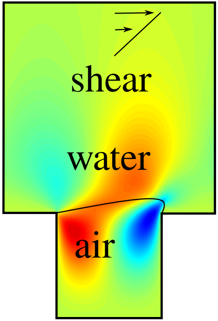

[8] Yogaraj, S., ongoing work with NEK5000

# Conclusions and outlook

Outlook

**Applying the developed model**

- Adding porosity to riblets[8]
- Testing anisotropic poroelastic models
- Most examples from nature – anisotropic



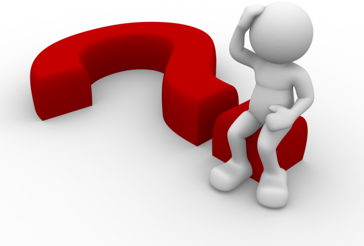

**Extending the model**

- Complement the model with two-phase flow
- Investigate response to shear, pressure, ...
- Understand stability from system energy

[8] Yogaraj, S., ongoing work with NEK5000

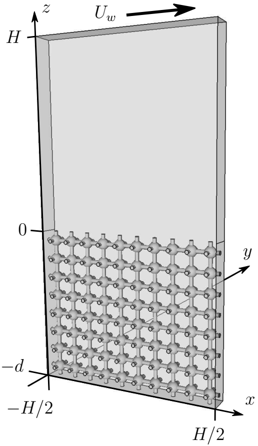
Thank you for your attention!

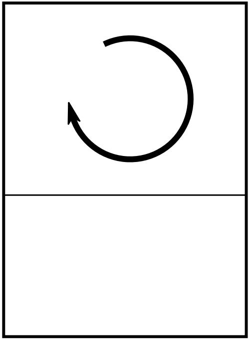
Questions and discussion



# Response to flow vortex

Displacement field[8]

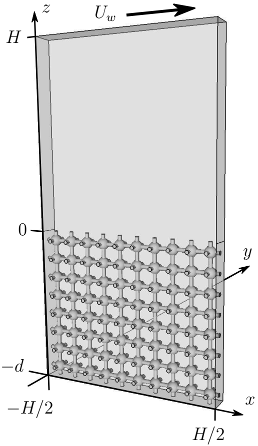


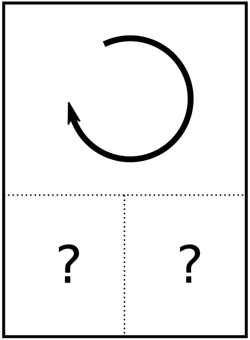



[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]

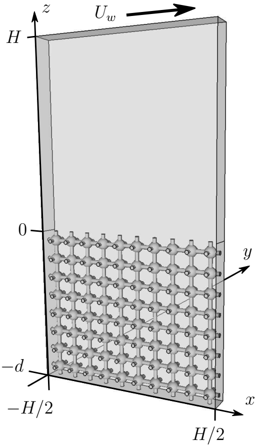


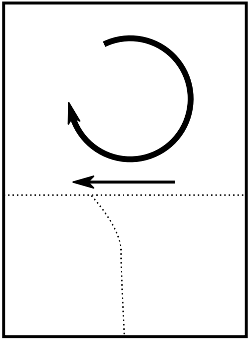




[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]

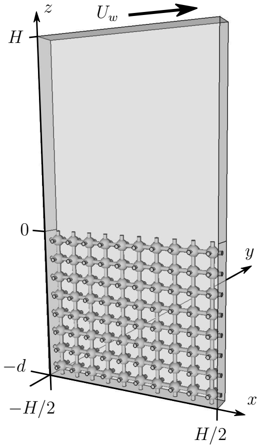


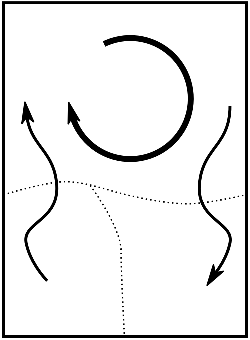



[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]

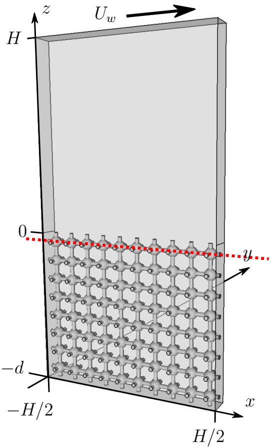


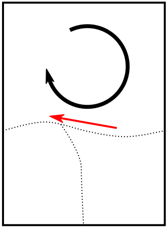



[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]

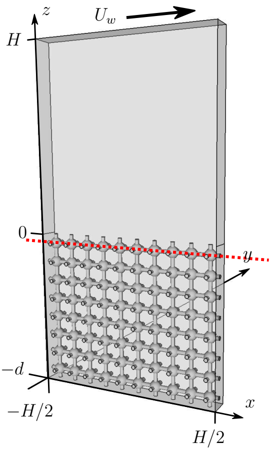


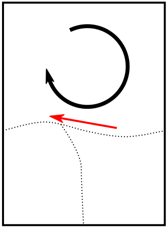



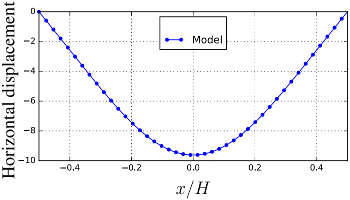
[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]



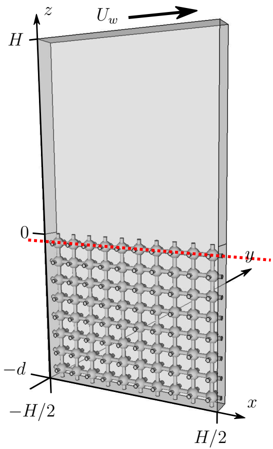



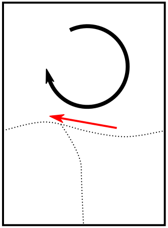


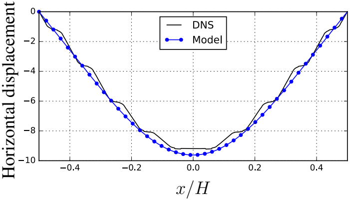

[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]



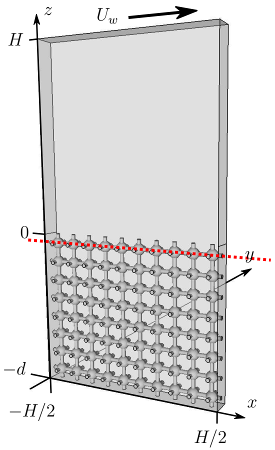



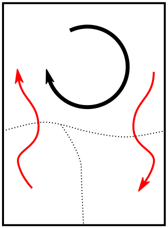



[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]

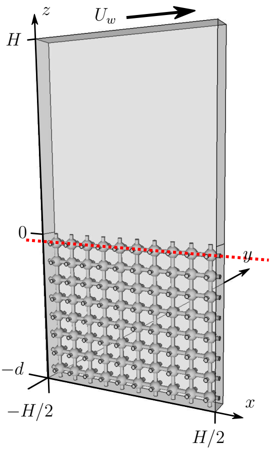


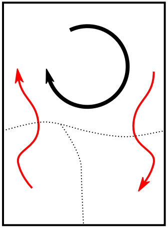



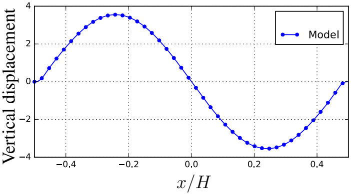
[8]Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]



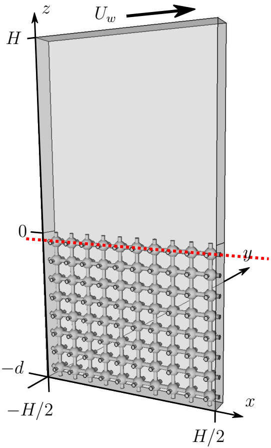



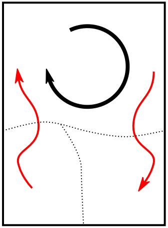


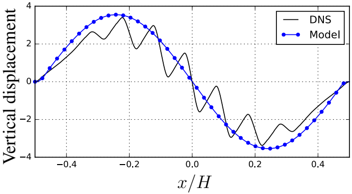

[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Response to flow vortex

Displacement field[8]

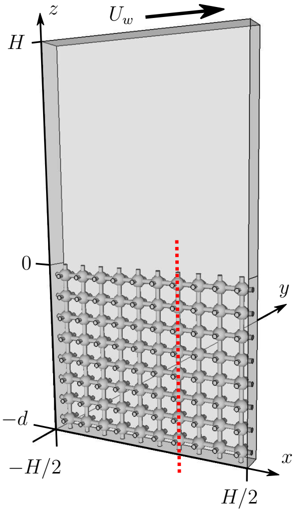


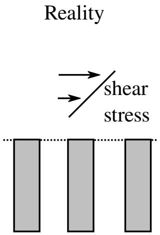



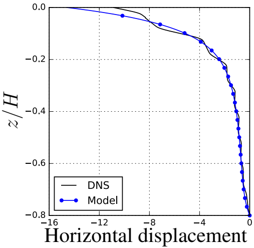



[8] Lācis, U., Zampogna, G. A. & Bagheri, S., A computational continuum model of poroelastic beds, *Proc. R. Soc. A.* **473**, 20160932 (2017)

# Results, displacement field



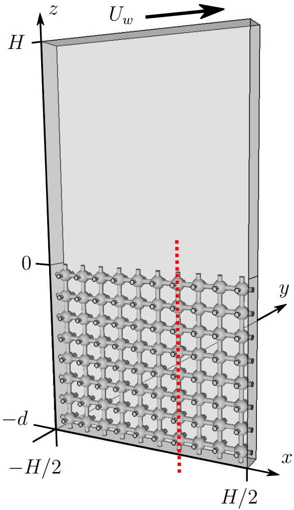



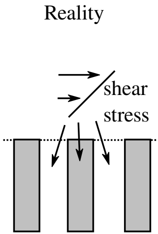



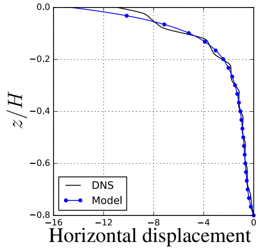

Fully non-empirical model

$10^4$ times faster

# Results, displacement field



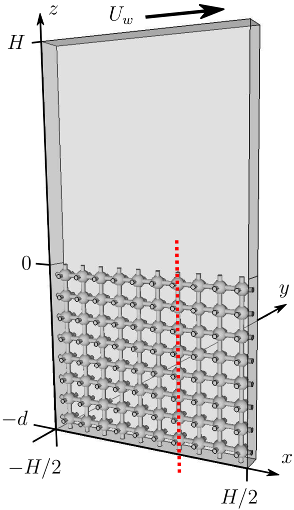



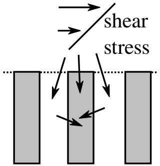

Fully non-empirical model

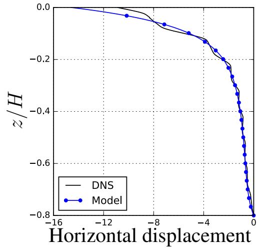
$10^4$ times faster

# Results, displacement field




Reality

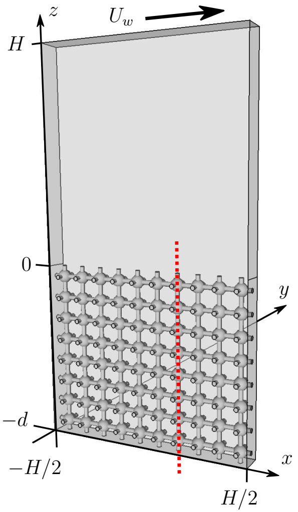


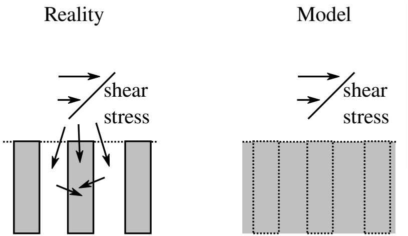



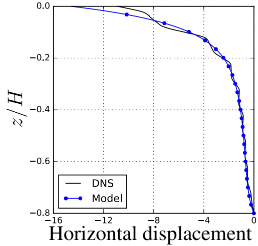

Fully non-empirical model

$10^4$ times faster

# Results, displacement field



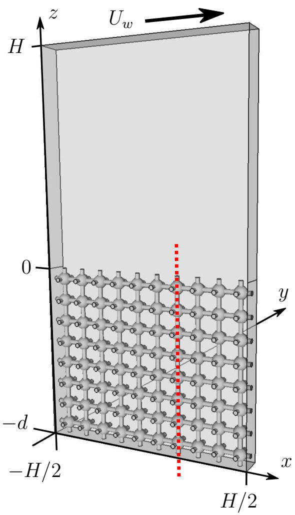



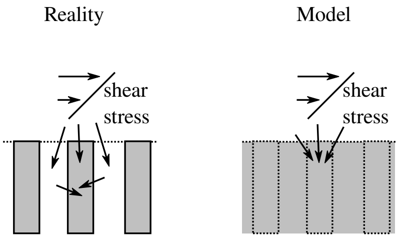



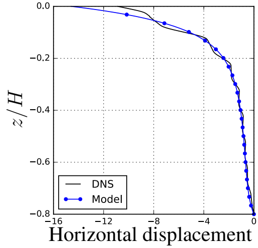

Fully non-empirical model

$10^4$ times faster

# Results, displacement field




Fully non-empirical model

$10^4$ times faster