FloMat 2015
Roma – Istituto degli Studi Romani – 24-27 March 2015

Cages and anomalous diffusion in vibrated dense granular media

Andrea Puglisi
in collaboration with A Gnoli, C. Scalliet, H. Touchette and A Vulpiani

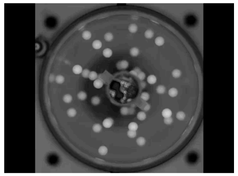
Consiglio Nazionale delle Ricerche

institute for complex systems

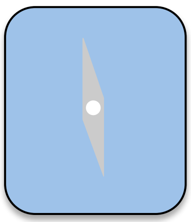
SAPIENZA Università di Roma

The experiment

Recent history of the experiment



- dilute granular medium + symmetric intruder



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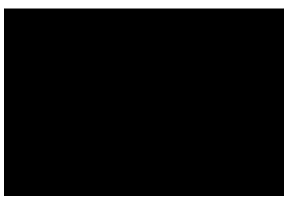
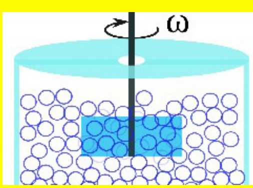


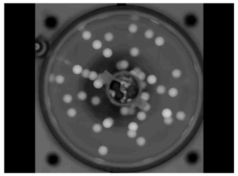
- moderately dense granular medium + perturbed intruder



- **high density** granular medium + symmetric intruder

TODAY

Brief digression on the dilute case



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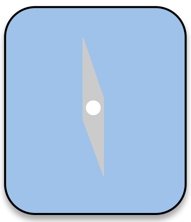

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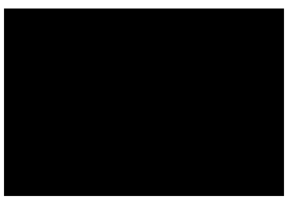

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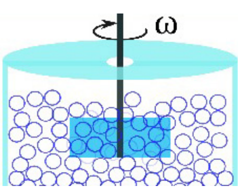
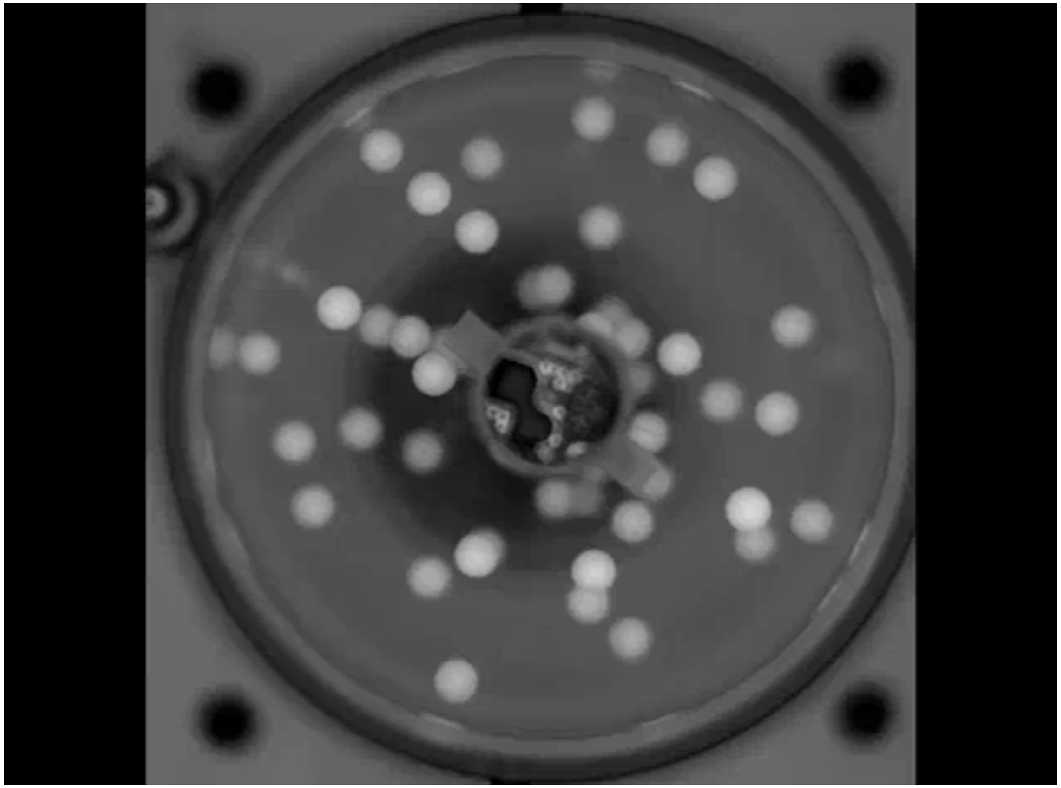


- high density granular medium + symmetric intruder

Brownian motion in a granular gas
(with dry friction)

Gnoli, Puglisi, Touchette, Europhys. Lett. 102, 14002 (2013)



Diffusion limit $\frac{m}{M} \to 0$

The granular noise becomes Ornstein-Uhlenbeck

$$\dot{\omega} \approx -\gamma\omega + \sqrt{\Gamma_g}\eta$$

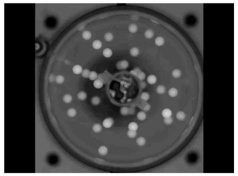
$$\gamma = \gamma_a + \gamma_g$$

$$\gamma_g = (1+\alpha)\sqrt{\frac{2}{\pi}}\lambda^{-1}\frac{m}{M}v_0\langle g^2\rangle_{surf}$$

$$\langle \eta \rangle = 0$$
$$\langle \eta(t)\eta(t')\rangle = \delta(t-t')$$

$$\Gamma_g = (1+\alpha)\gamma_g\frac{m}{I}v_0^2$$

Recent history of the experiment



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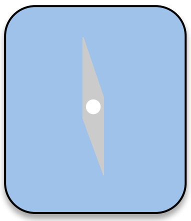

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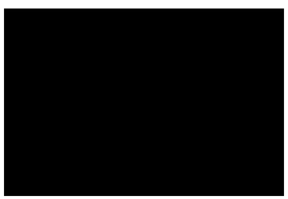

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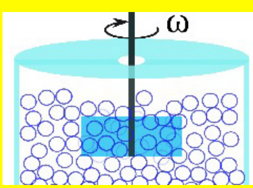
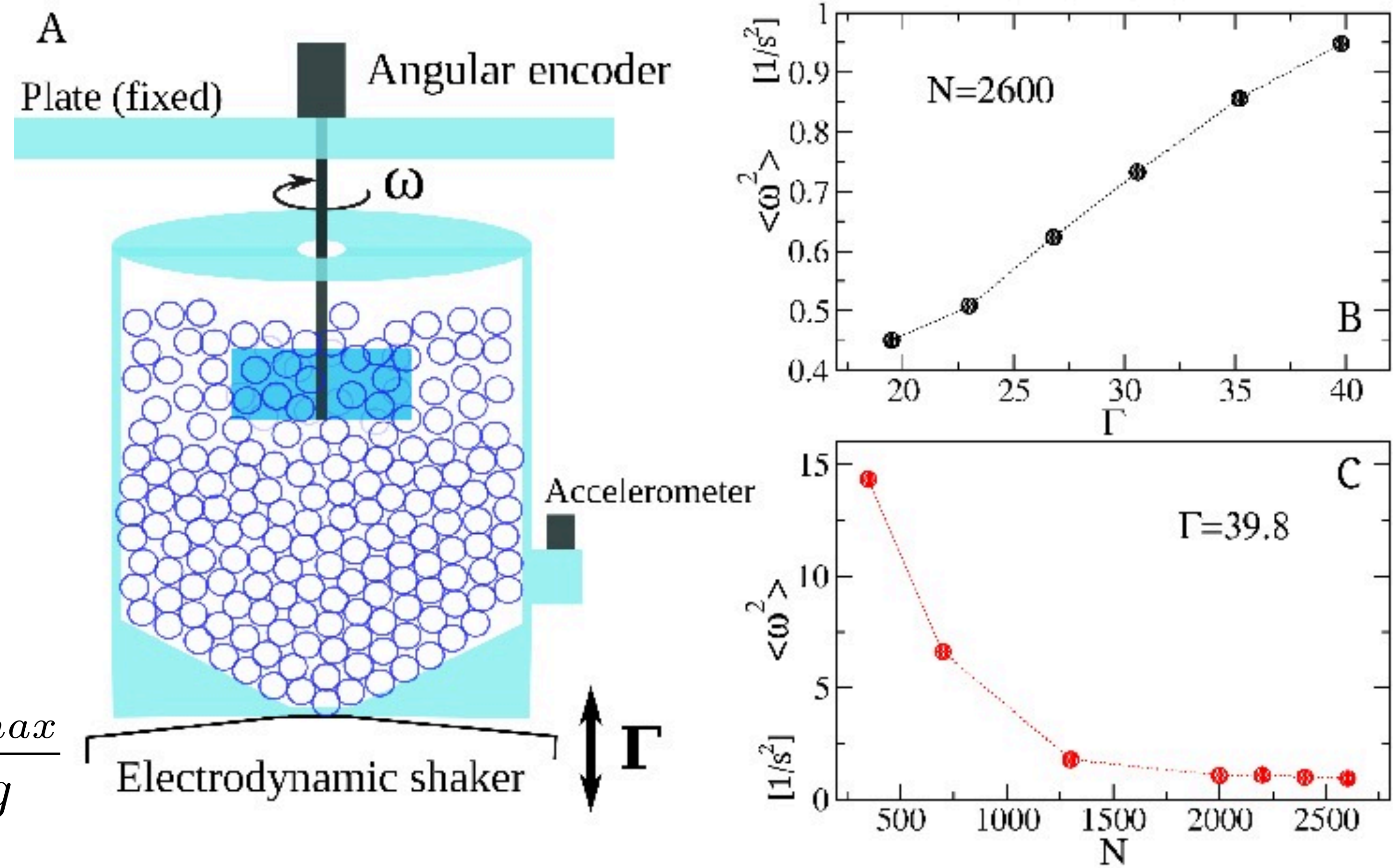


- **high density** granular medium + symmetric intruder

TODAY

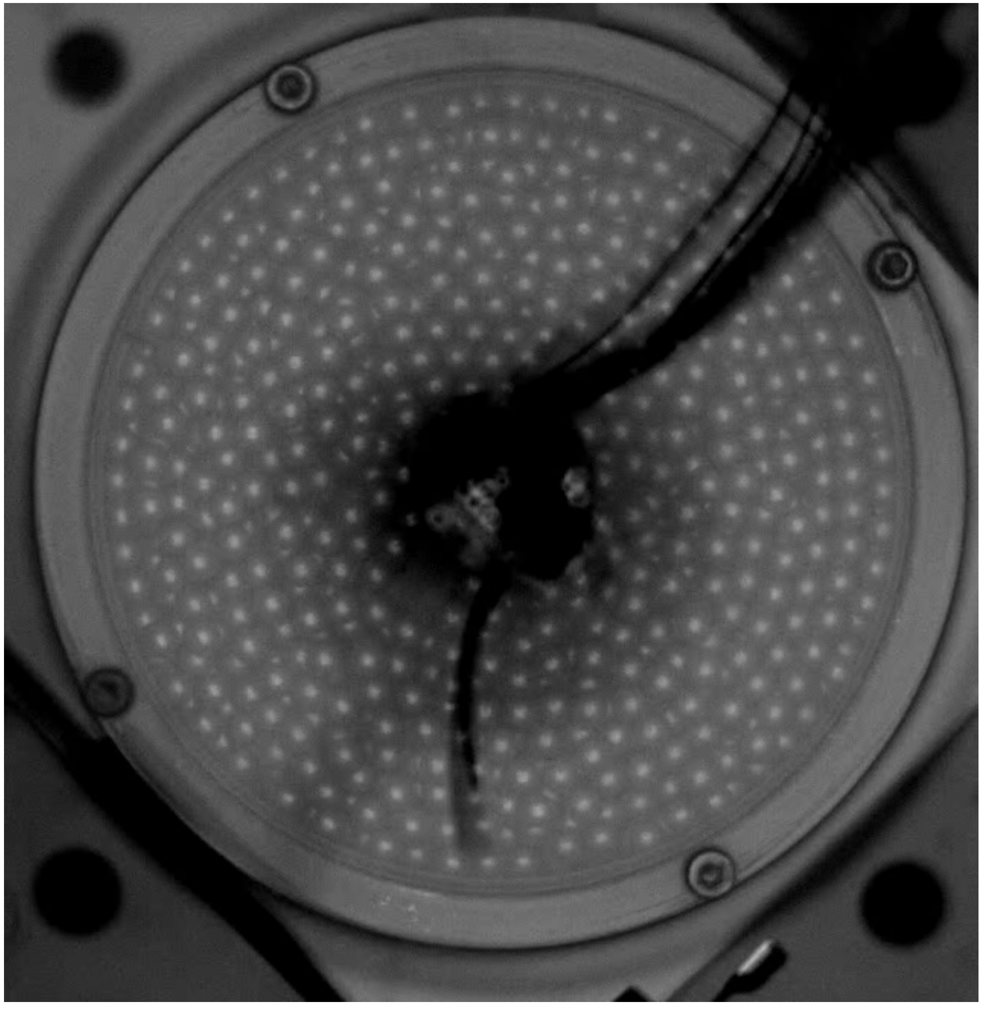
Approaching "jamming"

Scalliet, Gnoli, Puglisi, Vulpiani, under review

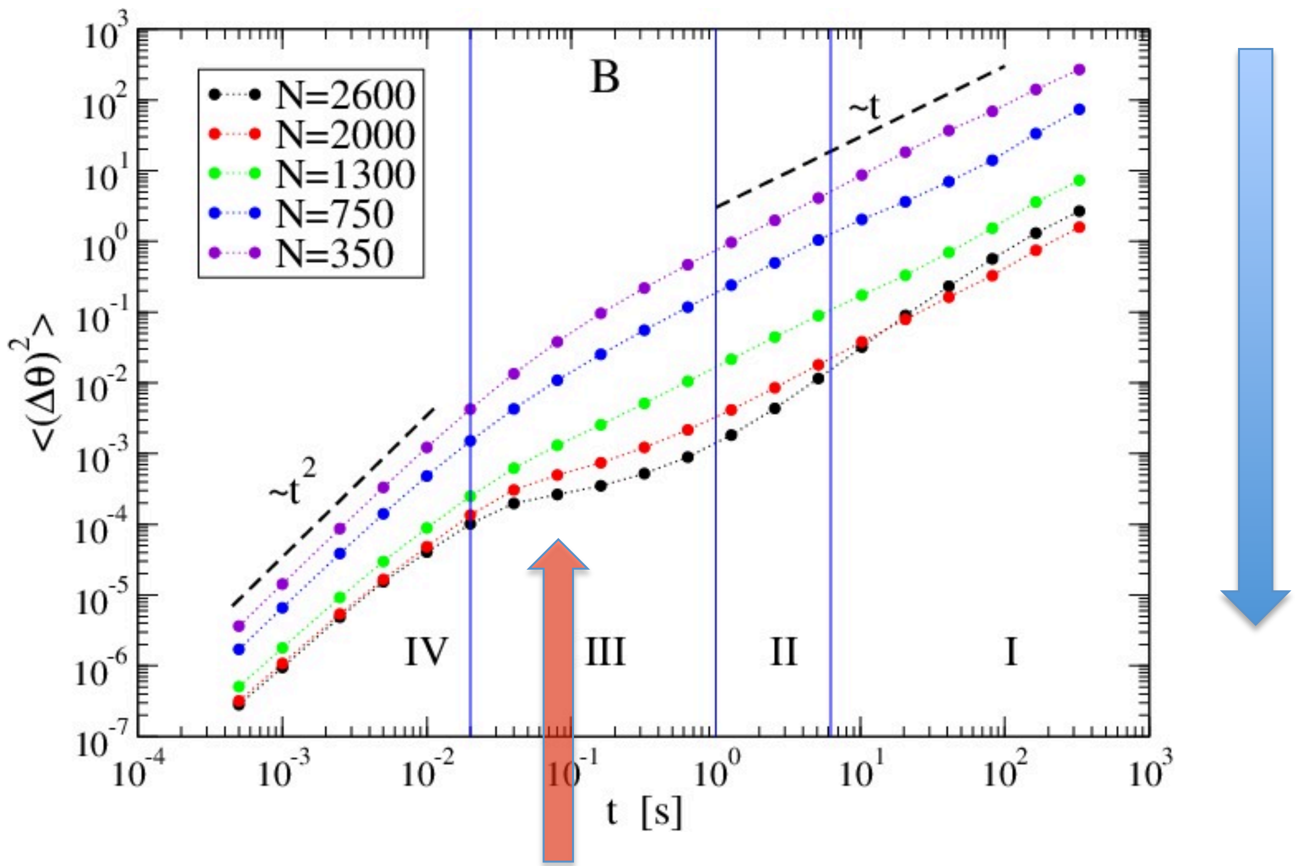


$$\Gamma = \frac{a_{max}}{g}$$

More dense, less hot



Mean square displacement: from gas to dense liquid



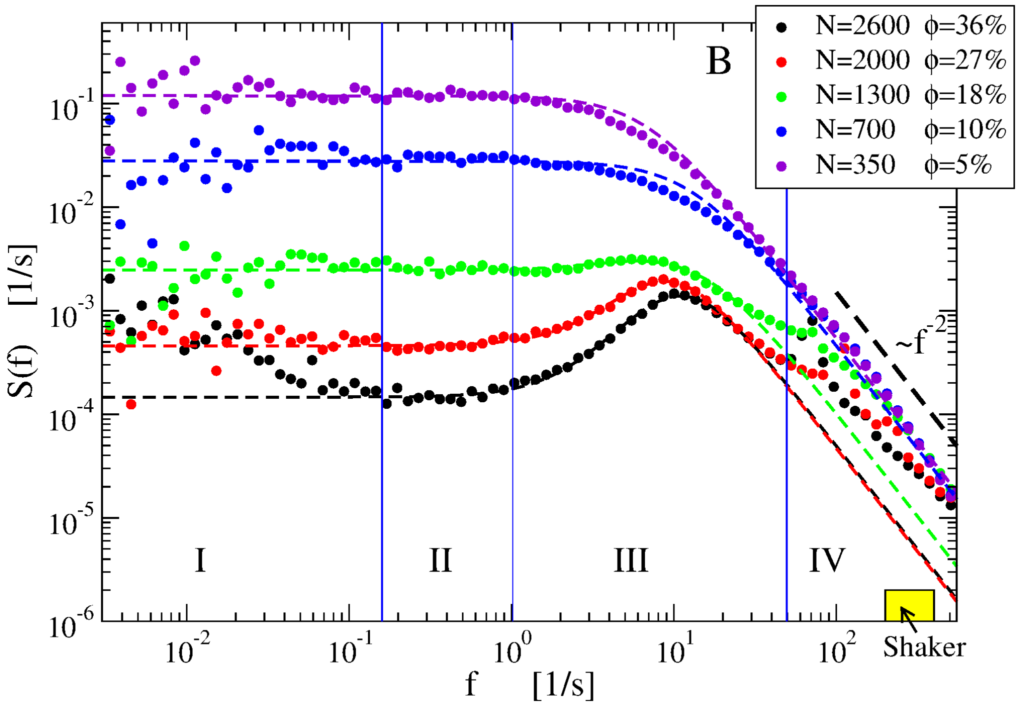
Increasing density

Caging appears at high density

Power spectrum of angular velocity

$$S(f) = \frac{1}{2\pi t_{TOT}} \left| \int_0^{t_{TOT}} \omega(t) e^{i(2\pi f)t} dt \right|^2 \qquad \lim_{t\to\infty} \langle \Delta\theta^2(t) \rangle / t \sim 2\pi S(f \to 0)$$

In the gas phase: $S(f) = \frac{T}{\pi\gamma} / [1 + (2\pi I f/\gamma)^2]$



Increasing density

A simple model for caging

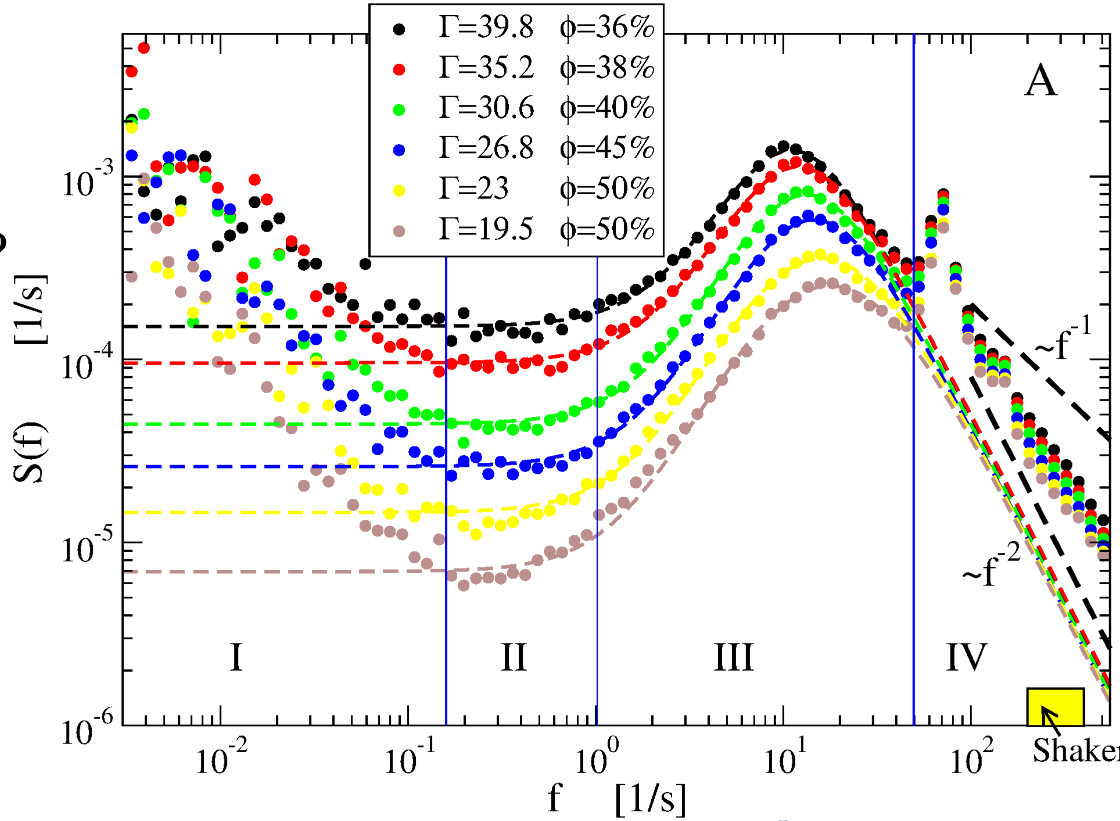
Motion in a diffusing harmonic trap

$$\begin{aligned}
\dot{\theta}(t) &= \omega(t) \\
I\dot{\omega}(t) &= -\gamma\omega(t) - K[\theta(t) - \theta_0(t)] + \sqrt{2\gamma T}\xi(t) \\
\dot{\theta}_0(t) &= \sqrt{2D_0}\xi'(t)
\end{aligned}$$

$$S(f) = \frac{1}{\pi}\frac{D_0K^2 + \gamma T(2\pi f)^2}{\gamma^2(2\pi f)^2 + [K - I\ (2\pi f)^2]^2}$$

Decreasing "temperature"

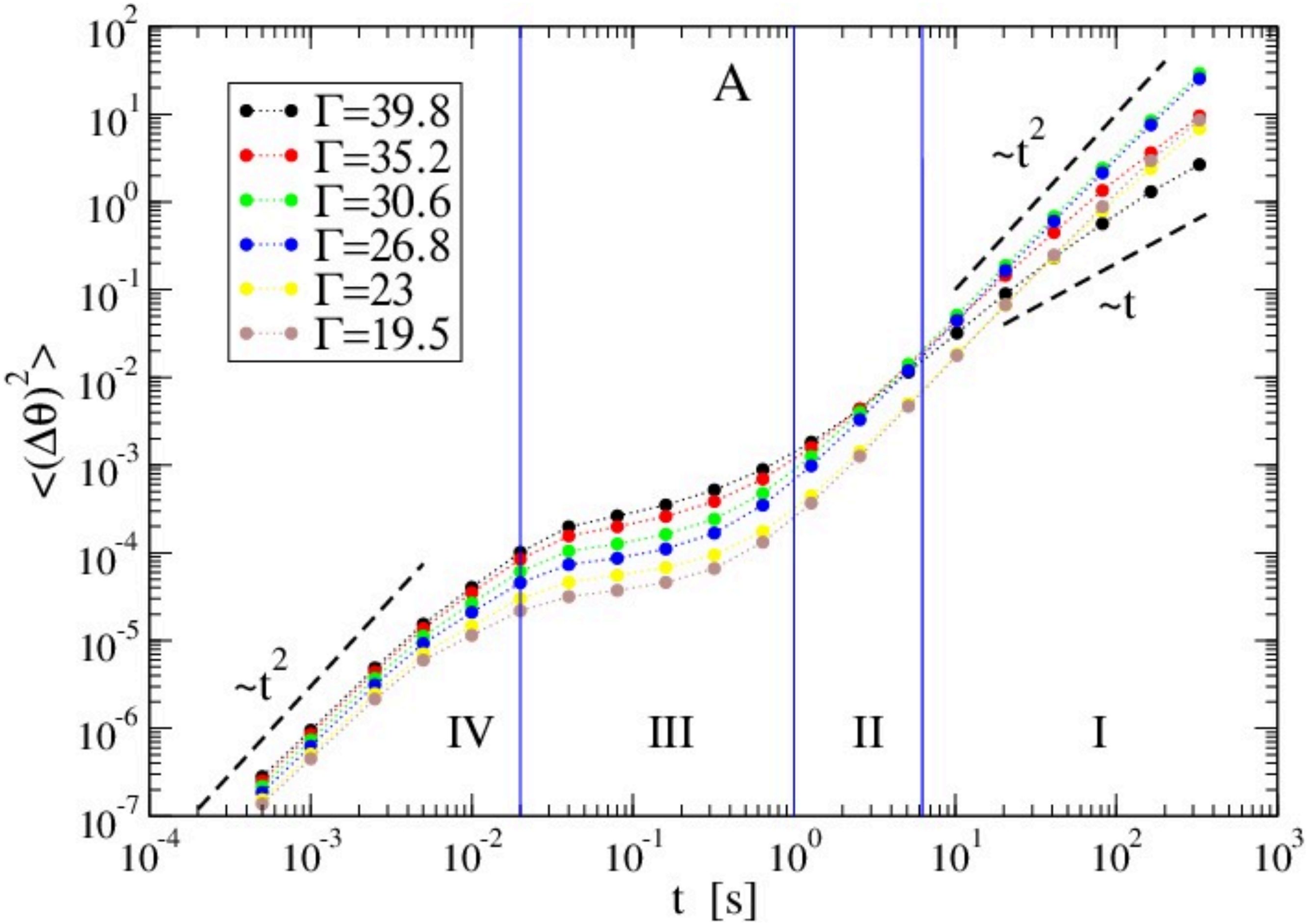
$$\lim_{f \to 0} S(f) \sim \infty ?$$



$$\sim f^{-1}$$

Caging

Superdiffusion at low “temperature”



Modelling super-diffusion?

A model for long timescales

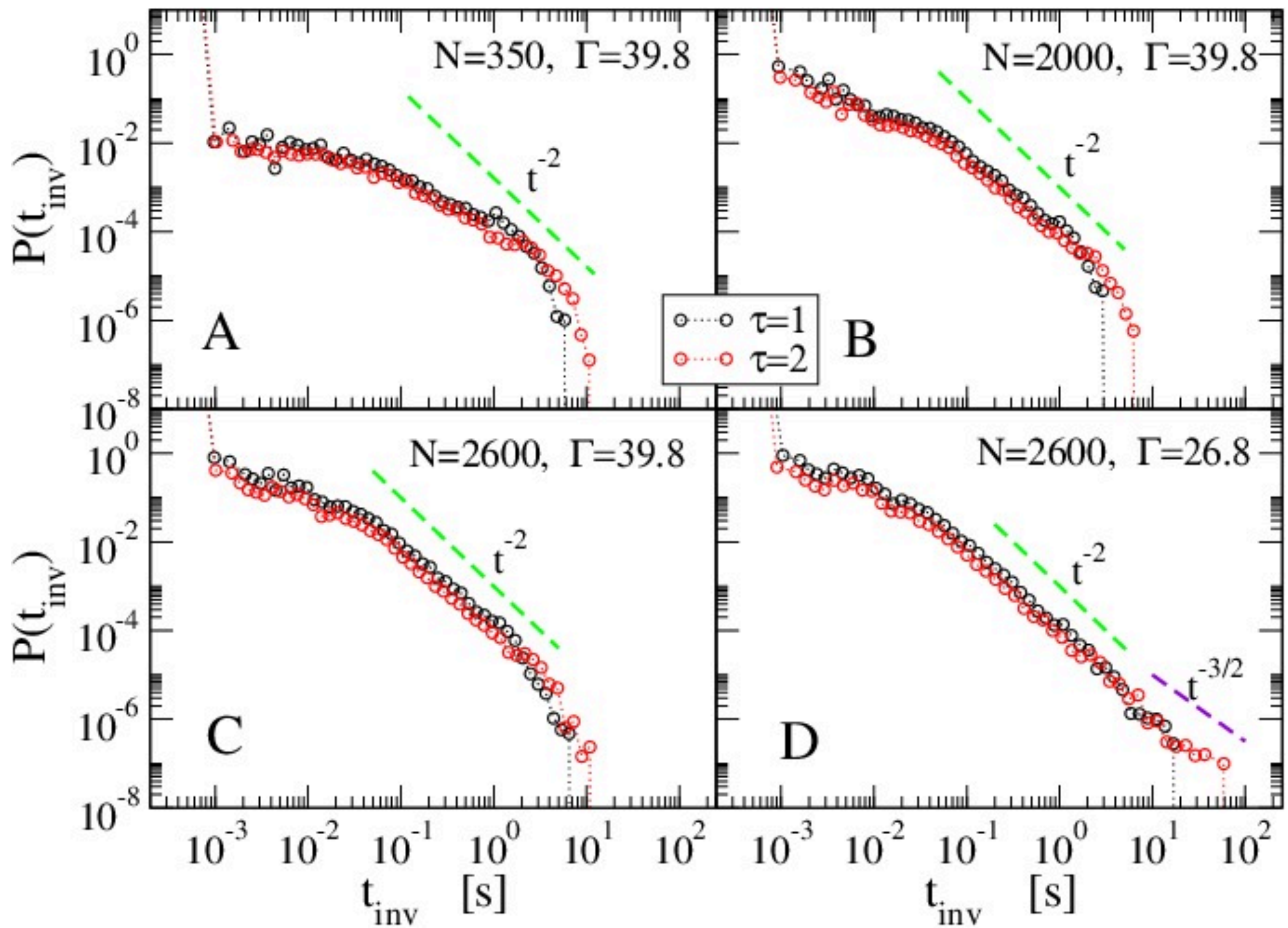
$$\omega(t) = \omega_s(t) + \omega_f(t)$$

Continuous time random walk for $\omega_s(t)$
changing signs at random intervals of time t_{inv}

$$P(t_{inv} = x) \sim \begin{cases} h(x) & x \in [0, t^*) \\ x^{-g} & x \in [t^*, t_{max}] \\ 0 & x > t_{max}. \end{cases}$$

$$\langle [\Delta\theta(t)]^2 \rangle \sim \begin{cases} t & g > 3 \;\; or \; t_{max} < \infty \\ t^{4-g} & 2 < g < 3 \\ t^2 & 1 < g < 2 \end{cases}$$

Long times



N=350, Γ=39.8
N=2000, Γ=39.8
N=2600, Γ=39.8
N=2600, Γ=26.8
A
B
C
D
τ=1
τ=2
t^-2
t^-3/2
P(t_inv)
t_inv [s]

Conclusions

- A simple experiment: a tracer in a fluidized granular medium
- The strongly shaken dilute regime is consistent with a Ornstein-Uhlenbeck stochastic process
- At high density new phenomena emerge:
    - Caging
    - $\sim f^{-1}$ high frequency tails of the spectrum
    - Superdiffusion at very low "temperatures"