Homogeneous isotropic turbulence: Particles, color is ω_z

Duane Rosenberg, Everest wall, OakRidge, 2015

Stably stratified rotating turbulence
Part II

Annick Pouquet[1,2]

Corentin Herbert[3], Raffaele Marino[4*], Pablo Mininni[5], Cecilia Rorai[6*] & Duane Rosenberg[7]

1: LASP; 2: NCAR; 3: Weizmann; 4: Berkeley; 5: U. Buenos Aires; 6: Nordita; 7: OakRidge

NSF/XSEDE - ASC090050 & TG-PHY100029 and Yellowstone (ASD/NCAR); INCITE/DOE - DE-AC05-00OR22725 * *NSF/CMG 1025183*

What's different in rotating &/or stratified turbulence?

- *Examples of rotating stratified flows and Boussinesq eqs.*
- *Resolving characteristic scales, taking into account all parameters*
- *Direct and inverse cascades in homogeneous isotropic turbulence*
- *Bi-directional constant-flux energy cascades & oceanic mixing*

- Development of large vertical velocity in stratified flows
- Bolgiano-Obukhov scaling and the role of potential energy
- *Role of helicity (velocity-vorticity correlations)*

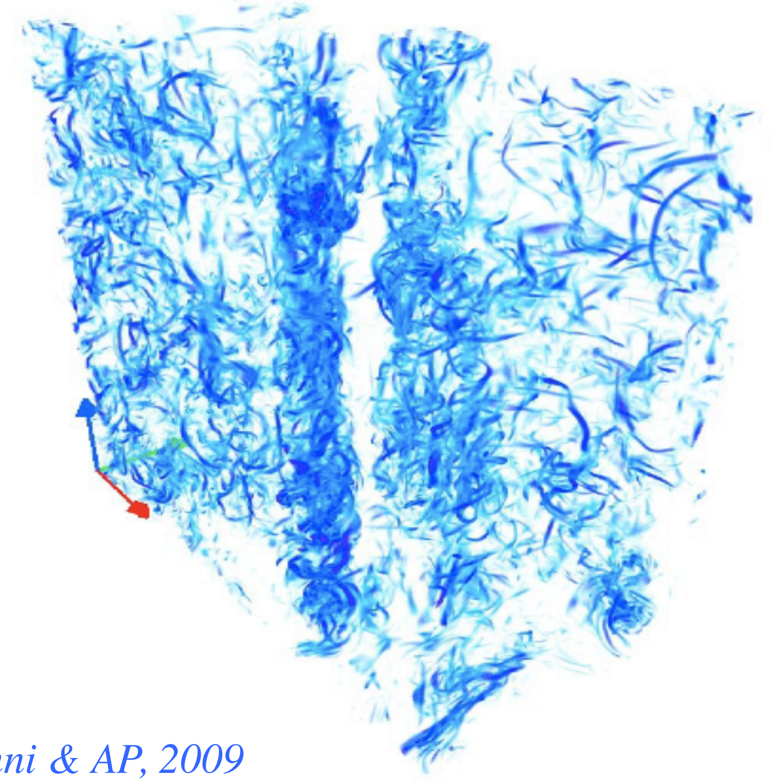
Kolmogorov:

"I soon understood that there was little hope of developing a pure, closed theory*, and because of absence of such a theory the investigation must be based on hypotheses obtained on processing experimental data."

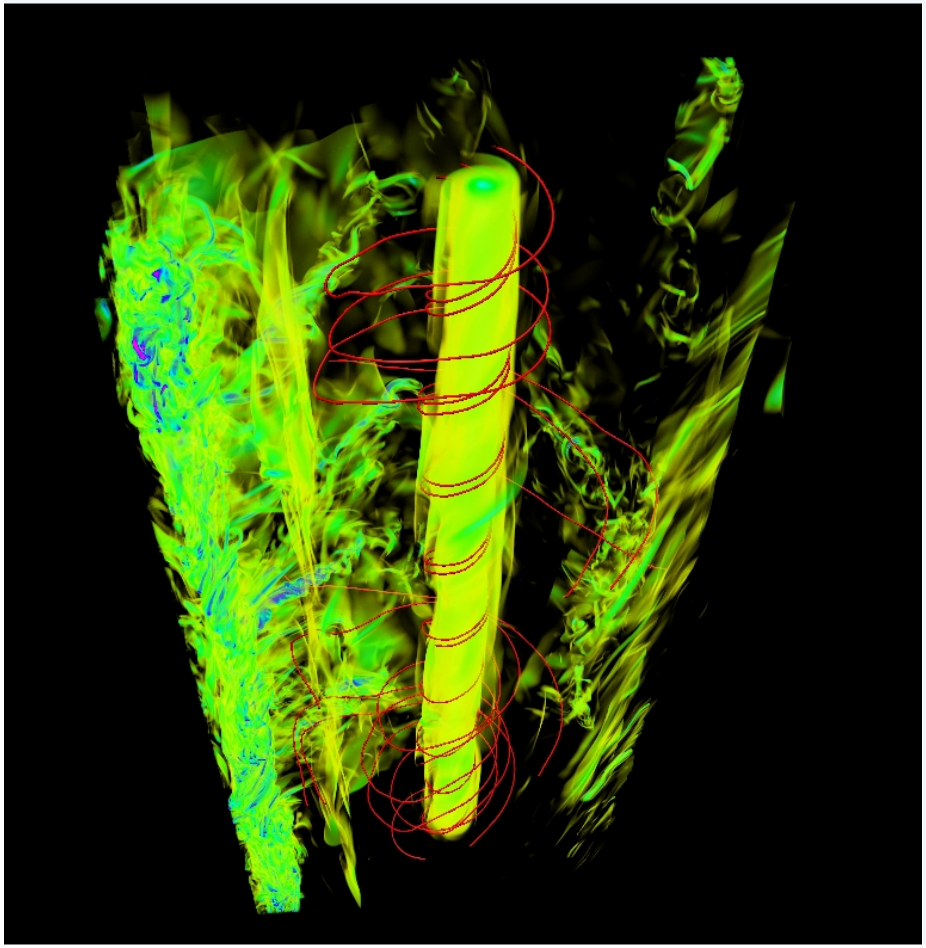
** of turbulence*

Rotation, no stratification, vorticity

Taylor-Green non-helical forcing,
k_F=4, 512^3 grid, Ro=0.35 →



Mininni & AP, 2009

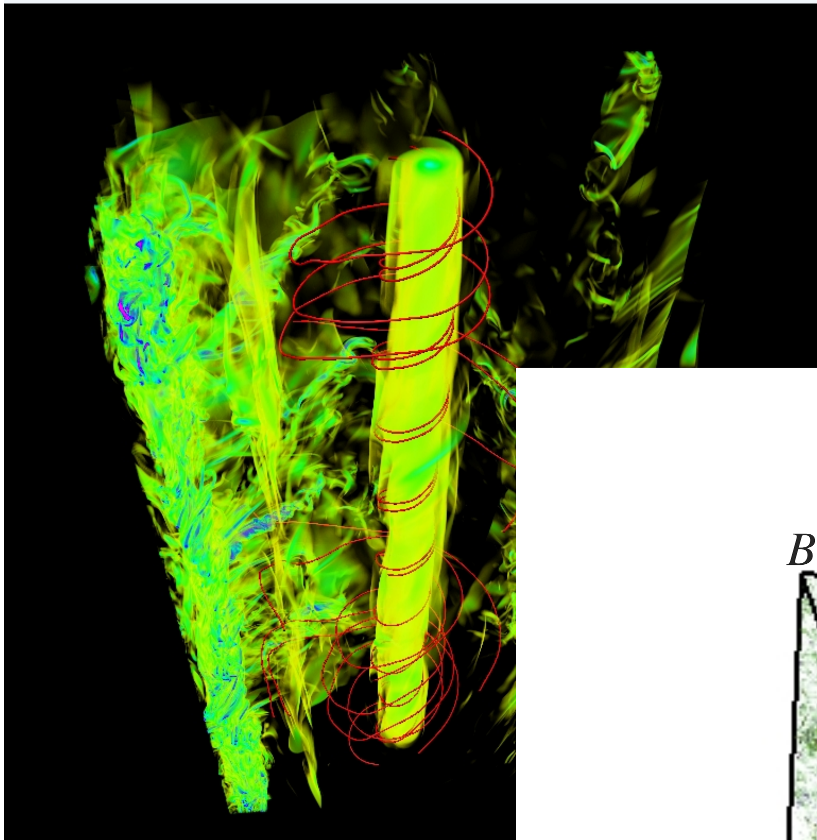


← ABC forcing, zoom

k_F=7, 1536^3 grid
Re=5100, Ro=0.06

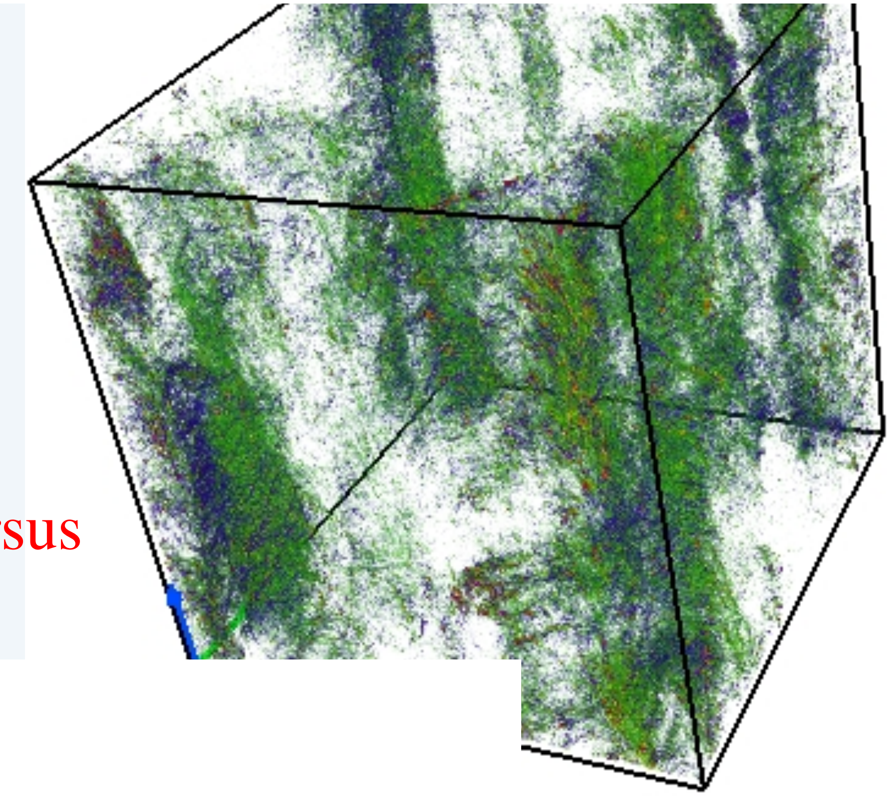
1536^3 grid, k_F=7,
Re=5100,
Ro=0.06

Helical ABC forcing
Mininni & AP, 2010



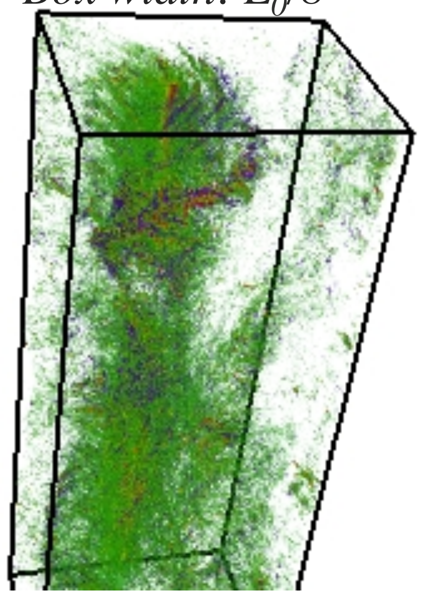
$L_\Omega \sim l_{min}$

versus



$L_\Omega \sim 30\ l_{min}$

Box width: $L_0/8$



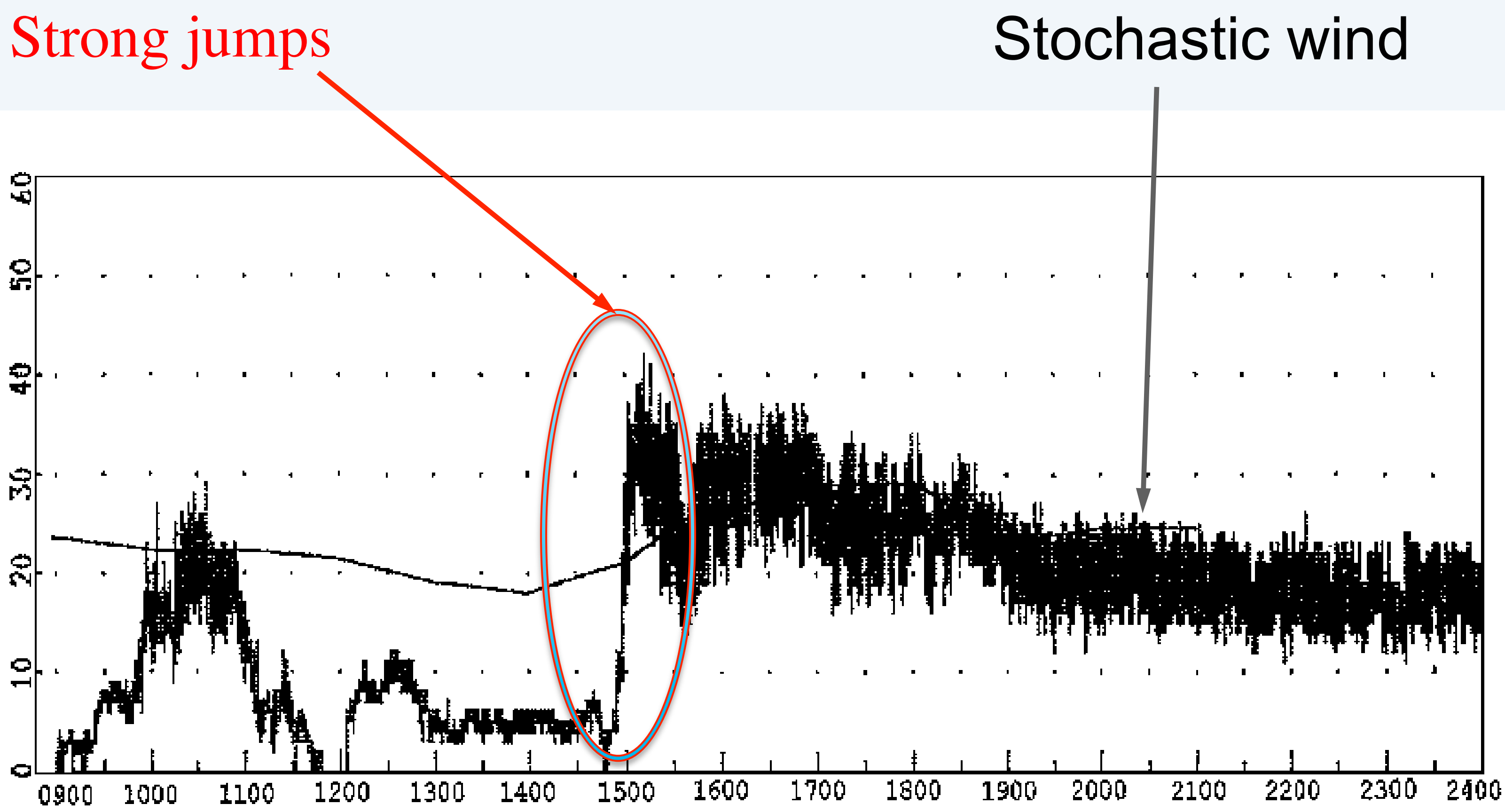
3072^3 grid,
k_F=4,
Re~ 24000,
Ro~0.07

Mininni et al., 2012

The emergence of strong velocity fields

Rorai et al. 2014

``Intermittency''

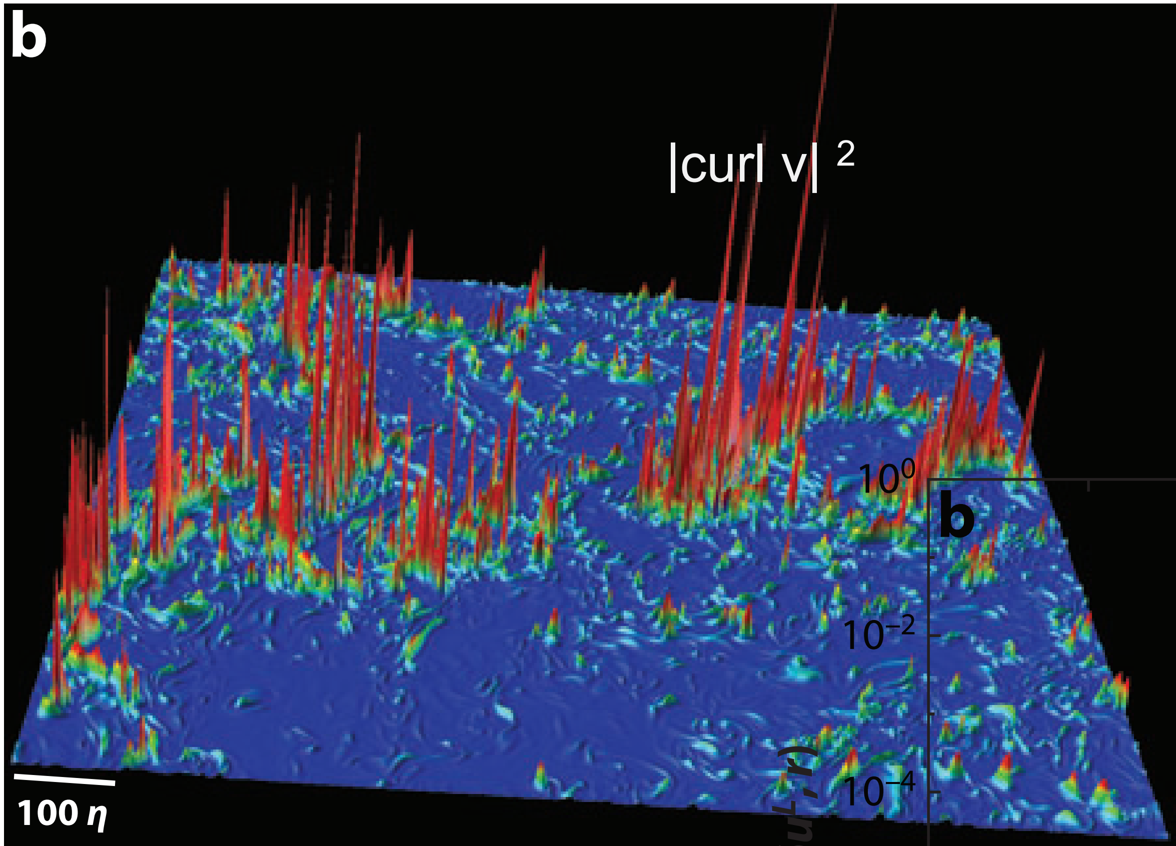


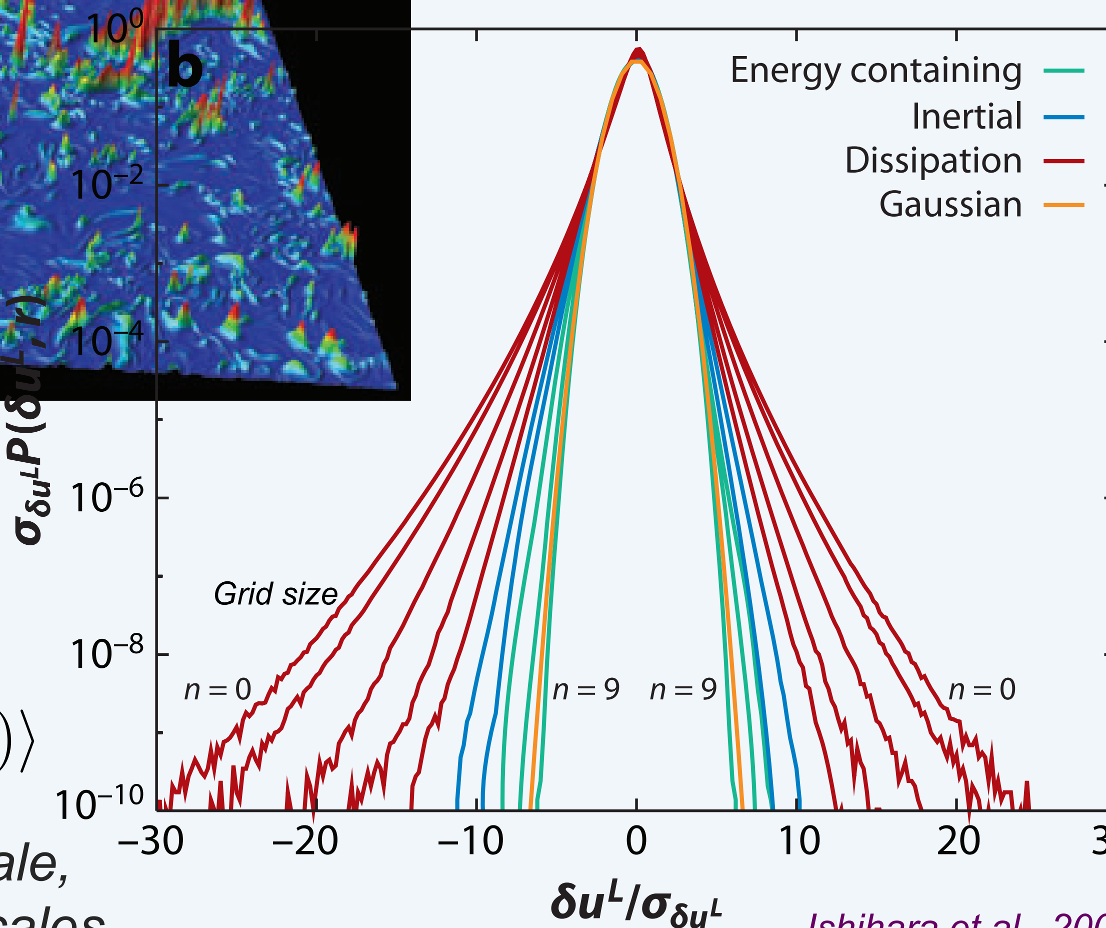
igure 6. Anemograph trace for Bellambi Point on 26 December 1996 (wind speed in knots), taken from Batt and Leslie (1998), Fig. 7.

Intermittency which manifests itself as heavy tails in Prob. Distrib. Fns.

→ *Problem for e.g. wind farms*

Turbulence at 4096^3 resolution
$R_\lambda \sim 1200$





Velocity differences $\delta u(l)$
on distances $l \sim 2^n \Delta x$

$$\delta u_x(\ell) = \langle u_x(x+\ell) - u_x(x) \rangle$$

They are Gaussian at large scale,
and with heavy tails at small scales

Ishihara et al., 2009

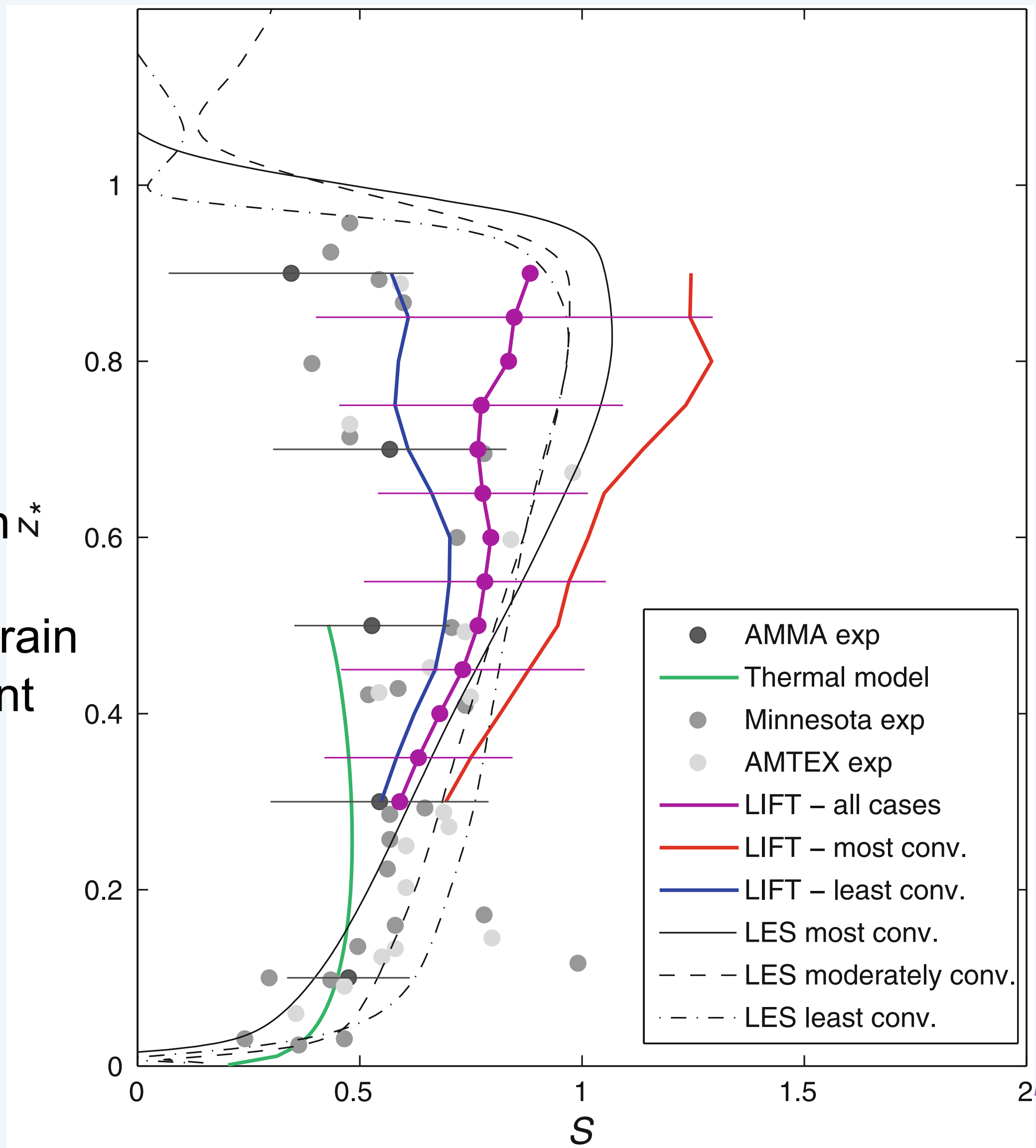
Skewness of vertical velocity in the convective planetary boundary layer

Z normalized by boundary layer depth

LIFT: Lidar In Flat Terrain
* Aircraft measurement

Δx=30m, Δt=1s

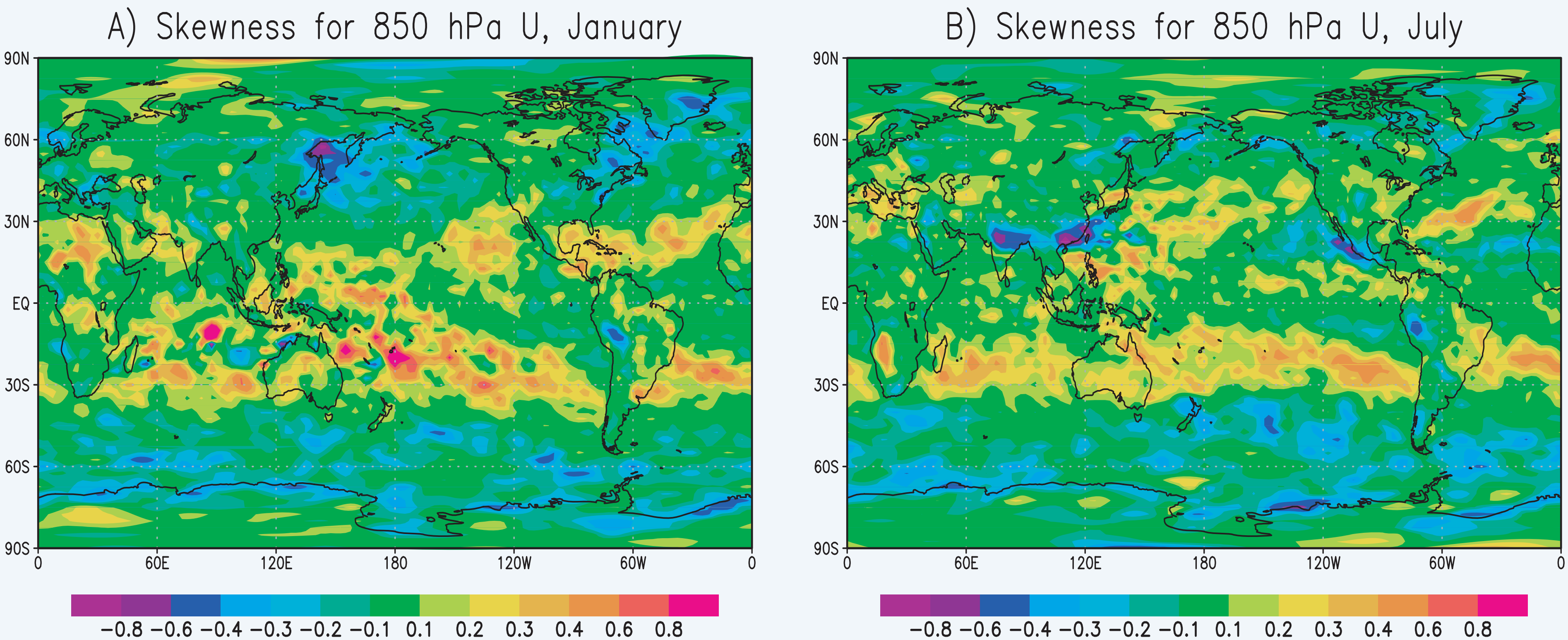
LES on 512^3 points



Lenschow et al., 2012

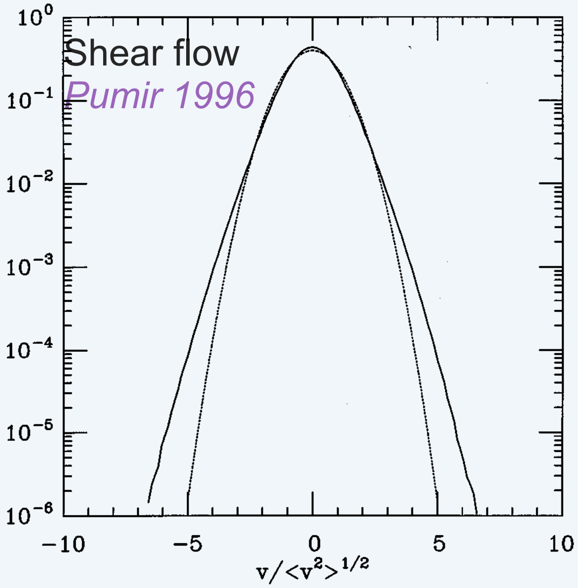
Troposphere

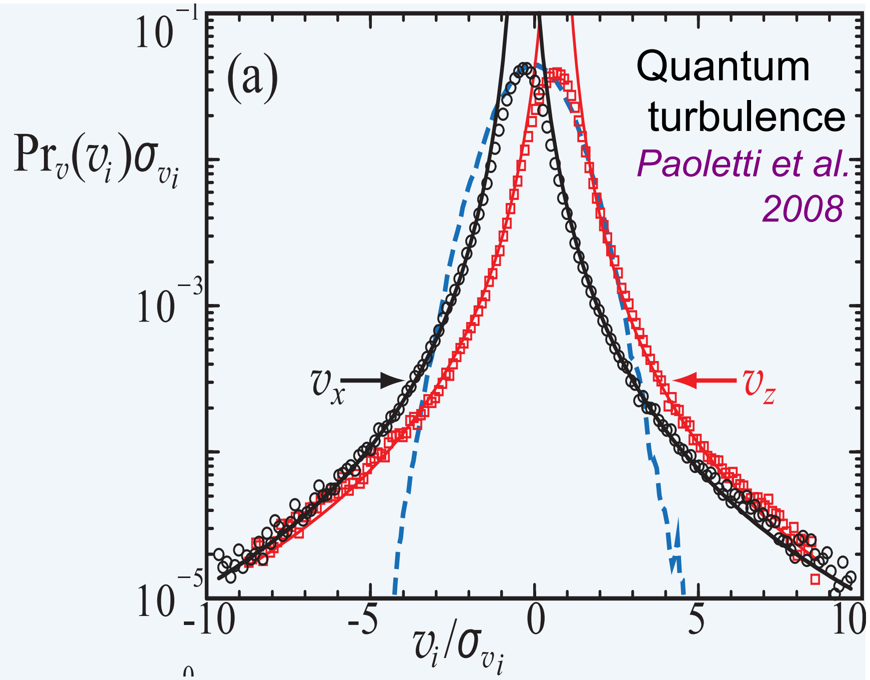
27 years *(Y>1976)* daily sampled, 5 vertical levels, Δx ~ 250km (code: 40km)



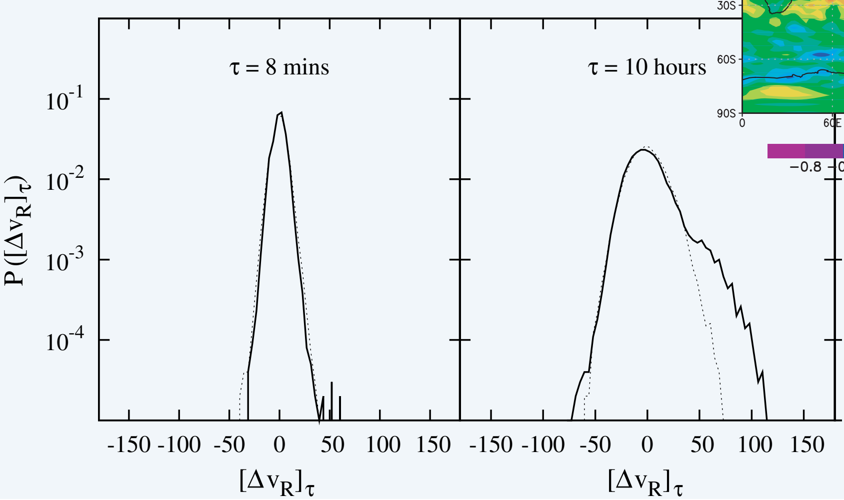
Skewness of temperature (ERA40 data)

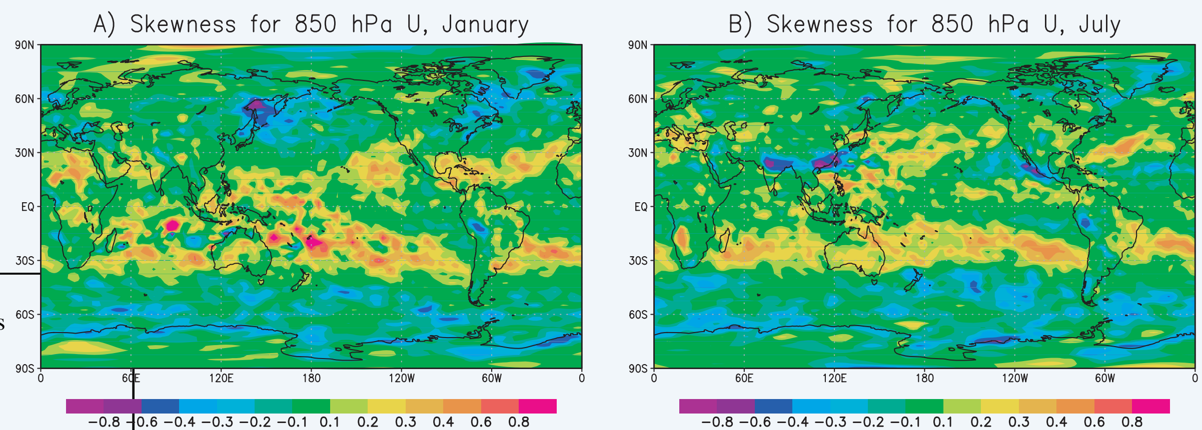
Petoukhov et al., 2008





δv_r: Solar Wind shear layers
Marino et al., 2012





Skewness of temperature (ERA40 data)
Petoukhov et al., 2008

How do **waves** alter the dynamics?

Stable Boussinesq stratification → *gravity waves*

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} = -\nabla P - Nbe_z \quad + \mathrm{F}$$

$$\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b = Nw\ ,$$

$$\nabla\cdot\mathbf{u} = 0\ .$$

$T_{dissipation} / T_{nonlinear}$ — $Re = U_0 L_0/\nu$ — *Reynolds number*

$T_{wave} / T_{nonlinear}$ — $Fr = U_0/[L_0 N]$ — *Froude number*

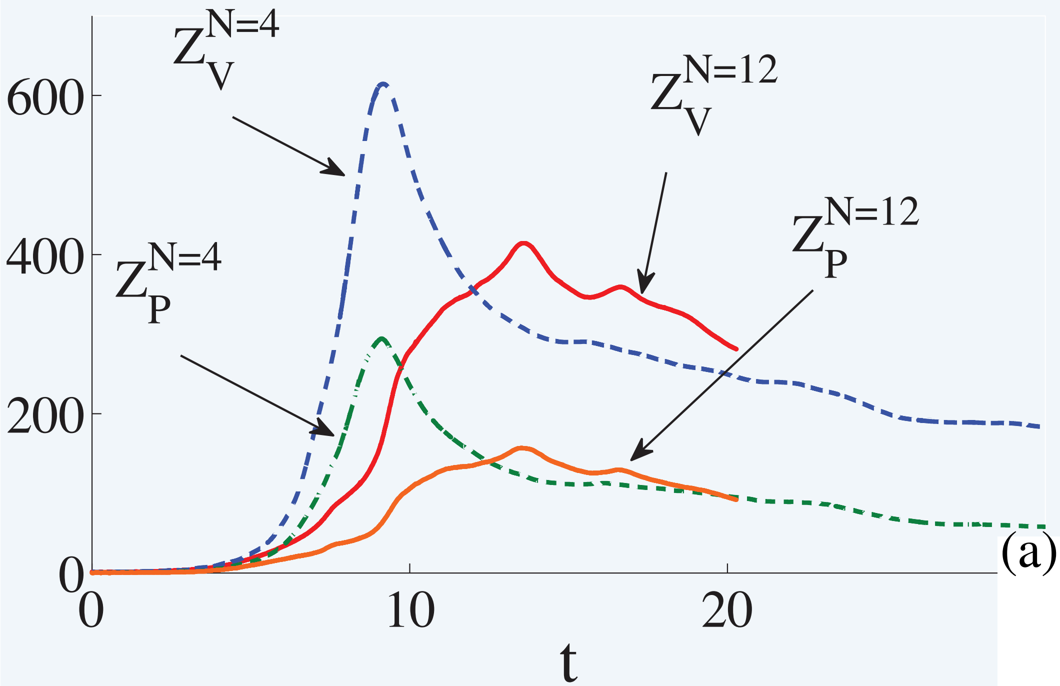
$\nu = \varkappa$ — *Unit Prandtl nb.*

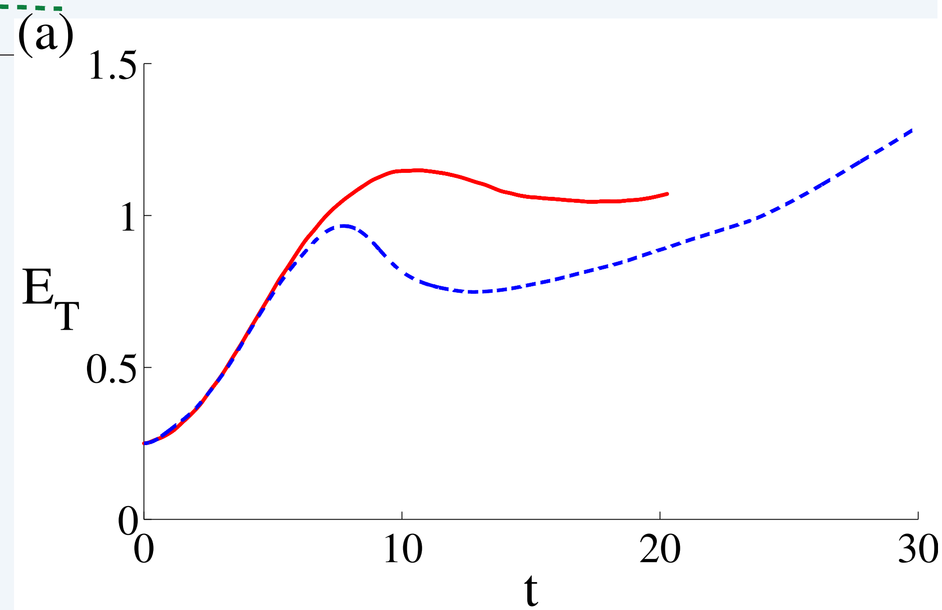
Energy and enstrophy

DNS 2048^3, Re=24000

N=12, Fr=0.03, R_B=22

N=4, Fr=0.09, R_B=200

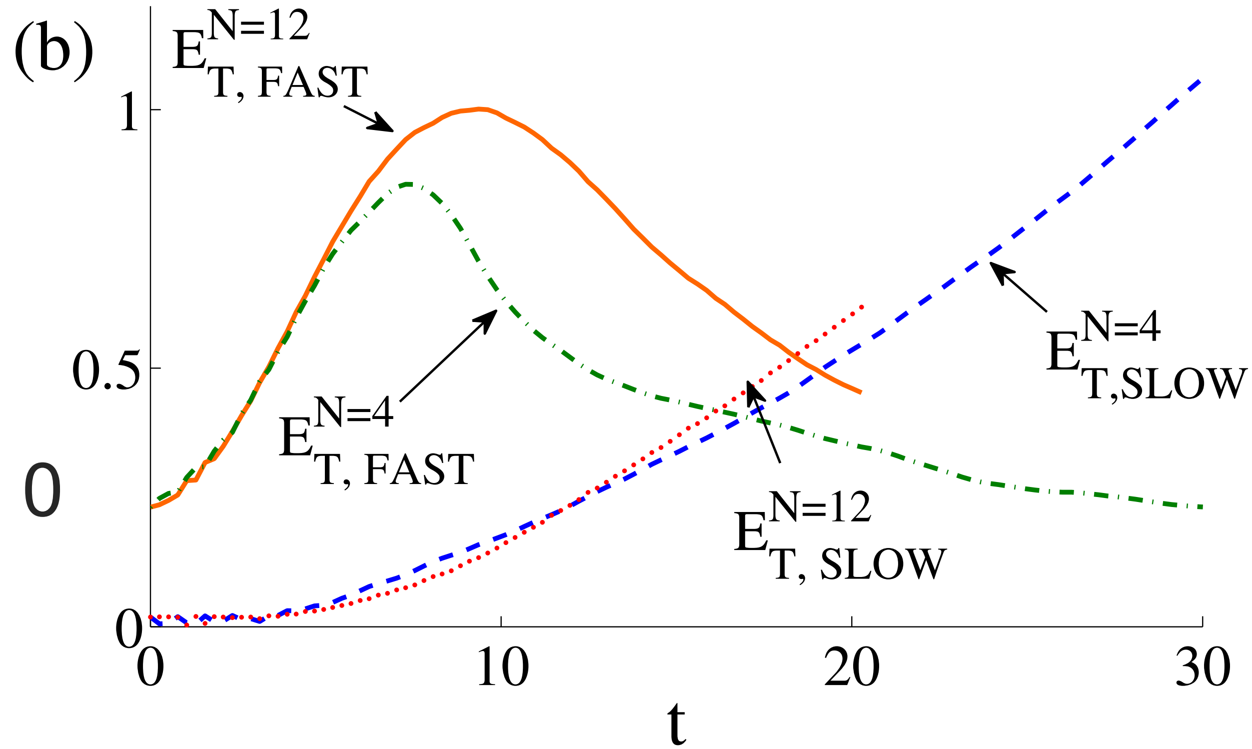


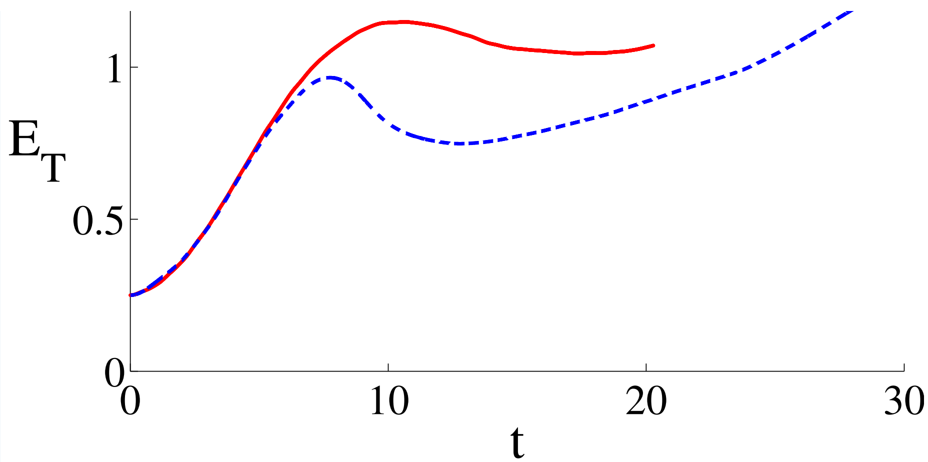


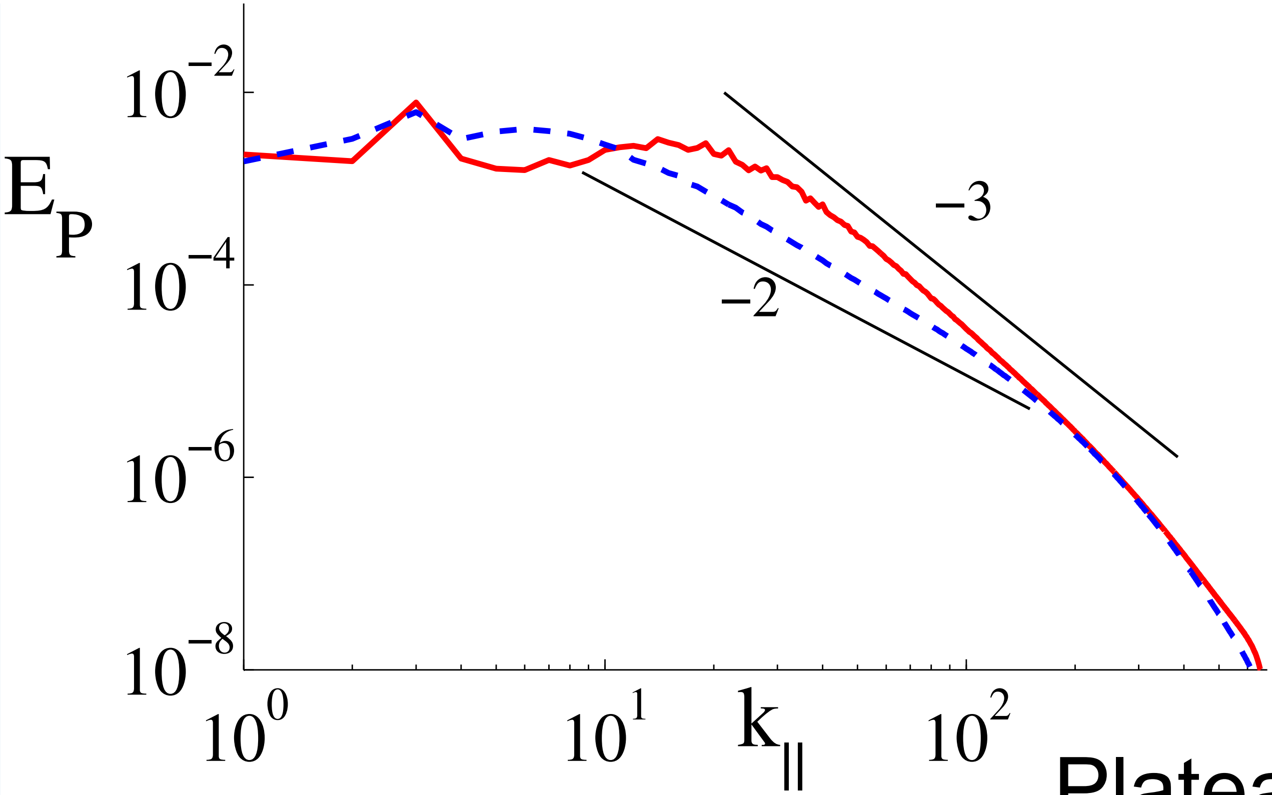
Kinetic and potential energy

$\omega_k = N\, k_{perp} / k$

Slow mode: $k_{perp} = 0$





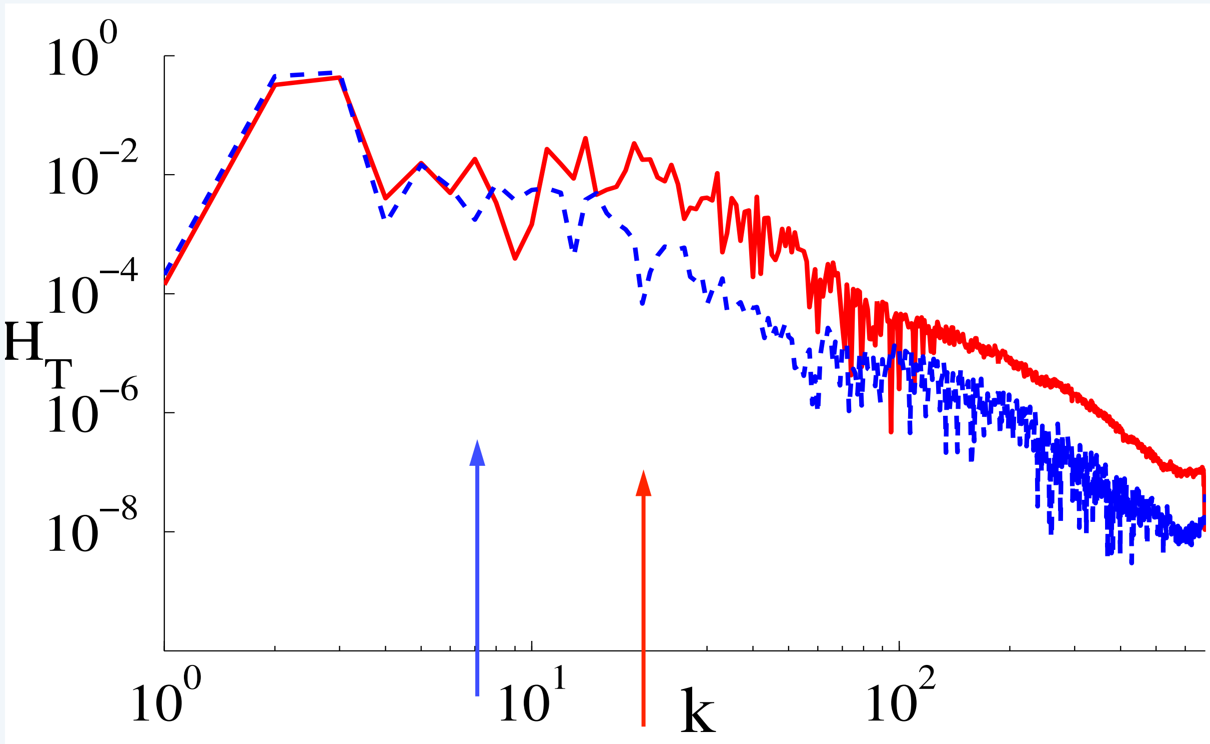


Plateau until

k_B~N/U, *the buoyancy wvnb*

Flat spectra with a break at L_B are also observed in oceanic and atmospheric data

(D'Asaro & Lien 2000)



Re~2.5x10^4 , 2048^3 grids

N=4 *(Fr~0.1)*

N=12 *(Fr~0.03)*

Flat spectra of helicity
in the atmosphere
when very stable,
at night in the planetary
boundary layer

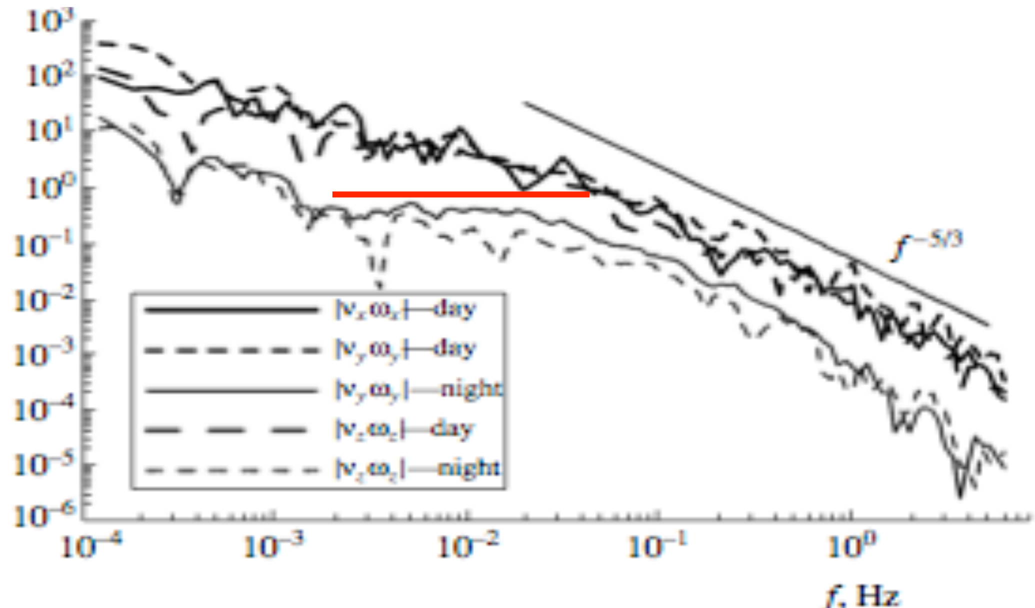


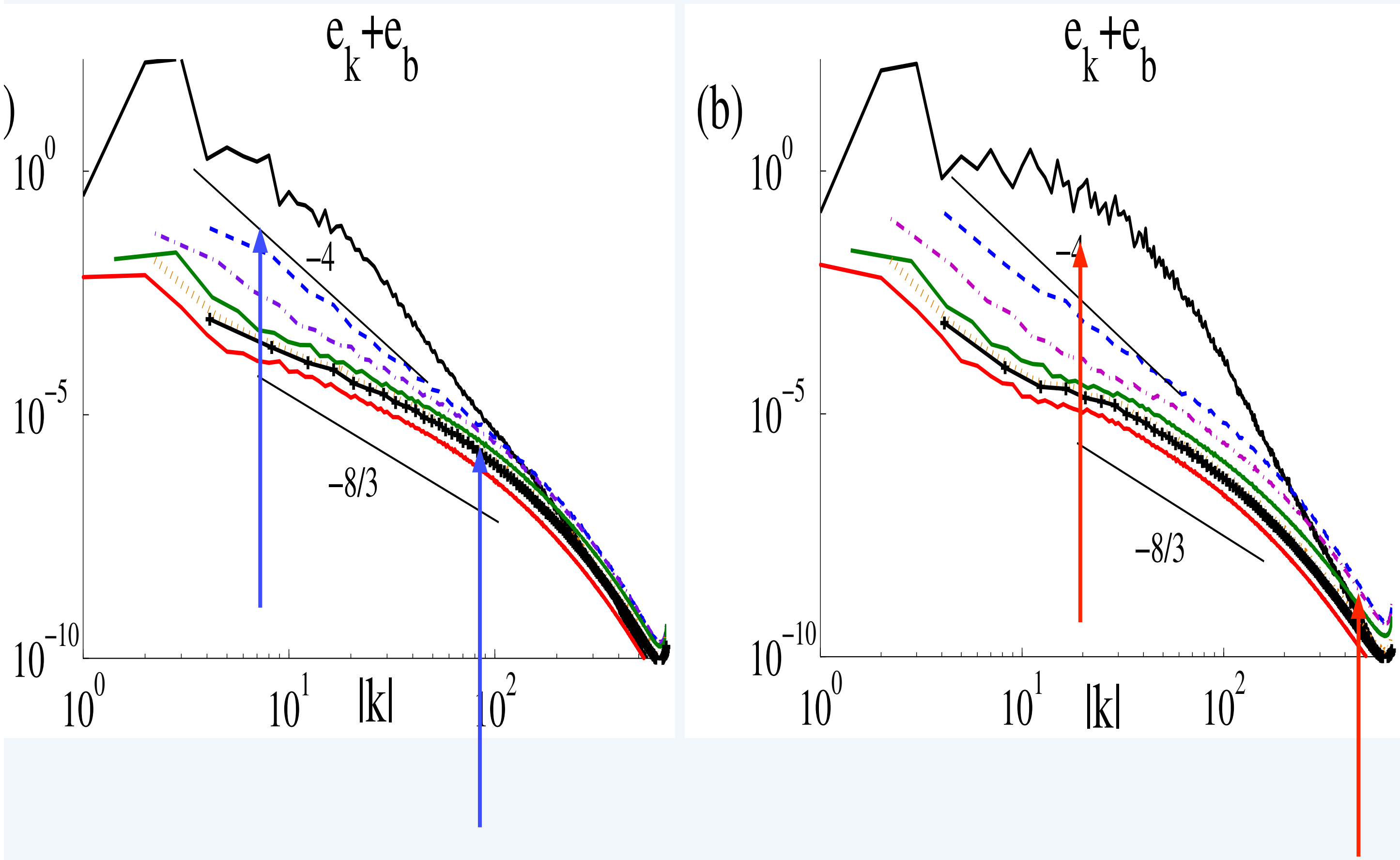
Fig. 4. Spectra of helicity components.

Koprov 2005

Re~2.5x10^4 2048^3 grids

N= 4 & N= 12

Fr~0.1 & *Fr~0.03*



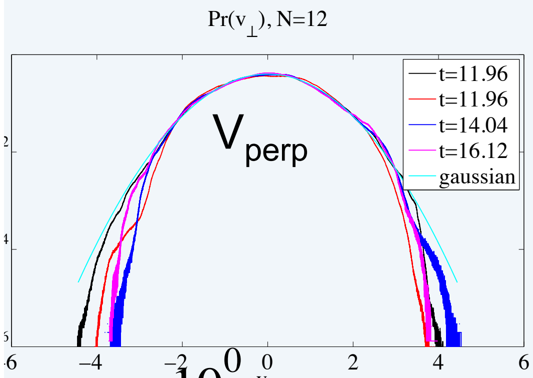
2d total energy spectra for co-latitudes ϕ

ϕ =0 ($k_{//}$)

$\phi=\pi/2$ (k_{perp})

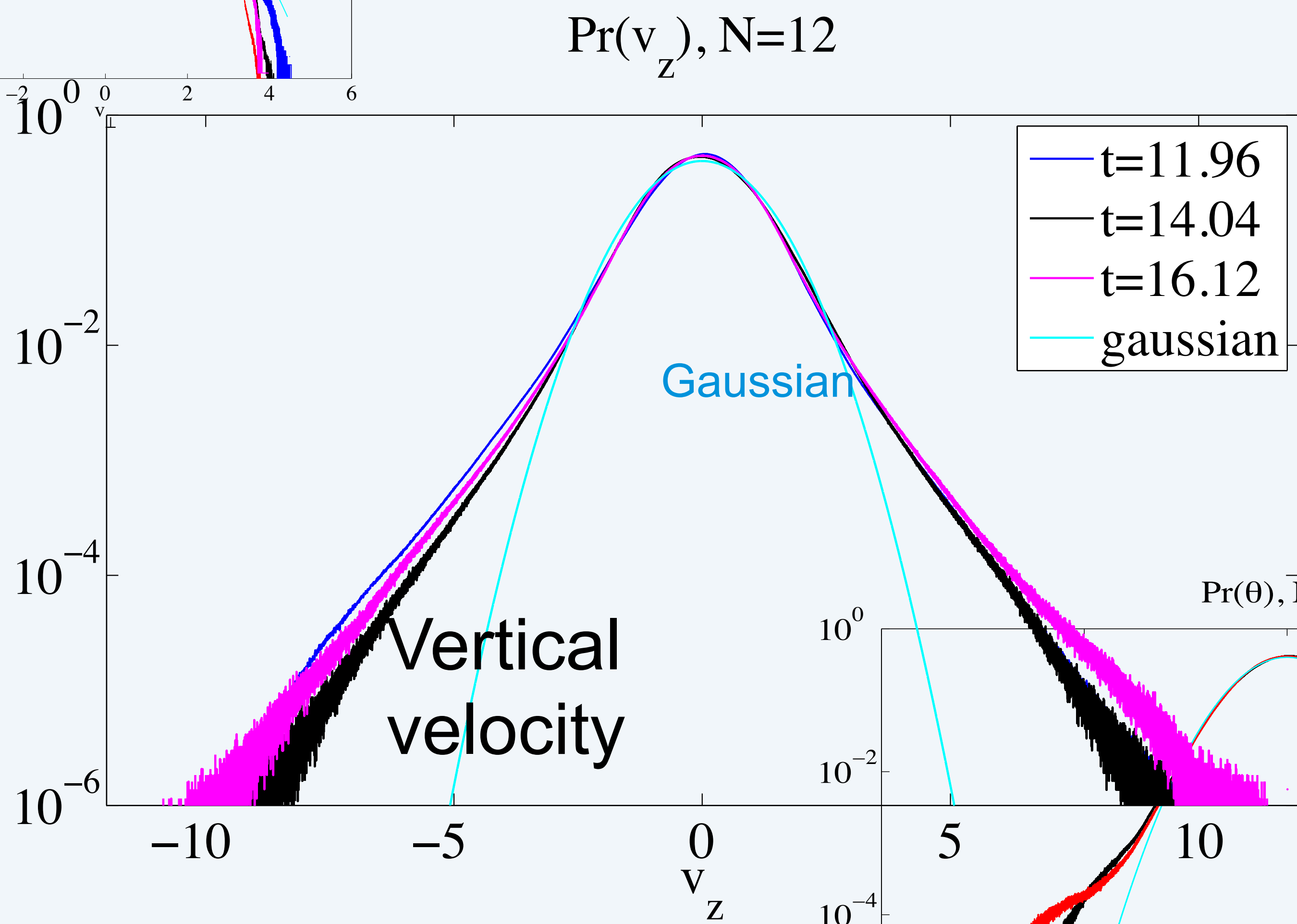
Isotropy at k_{Oz}~$[N^3/\varepsilon]^{1/2}$: K41 beyond the Ozmidov scale

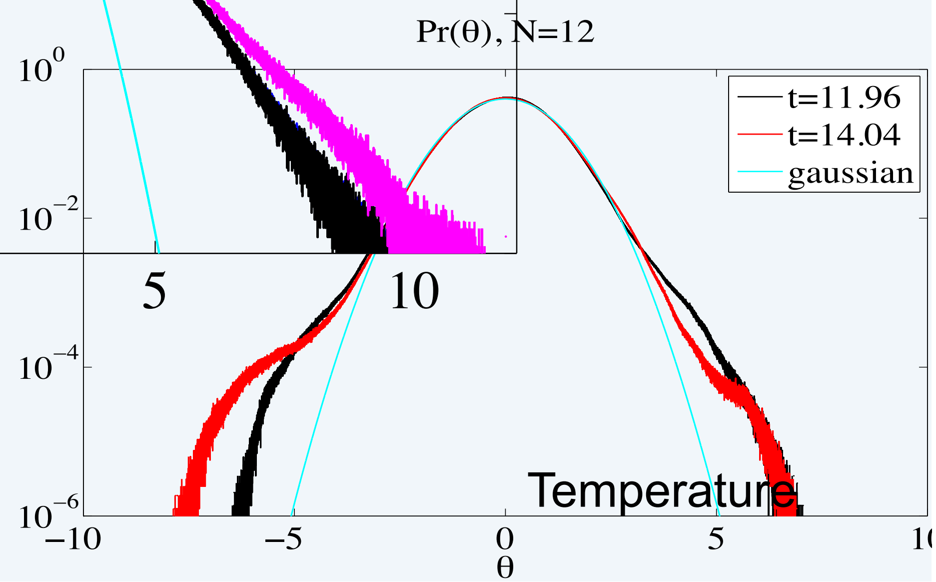
Velocity PDF, different time outputs



DNS 2048^3,
Re=24000

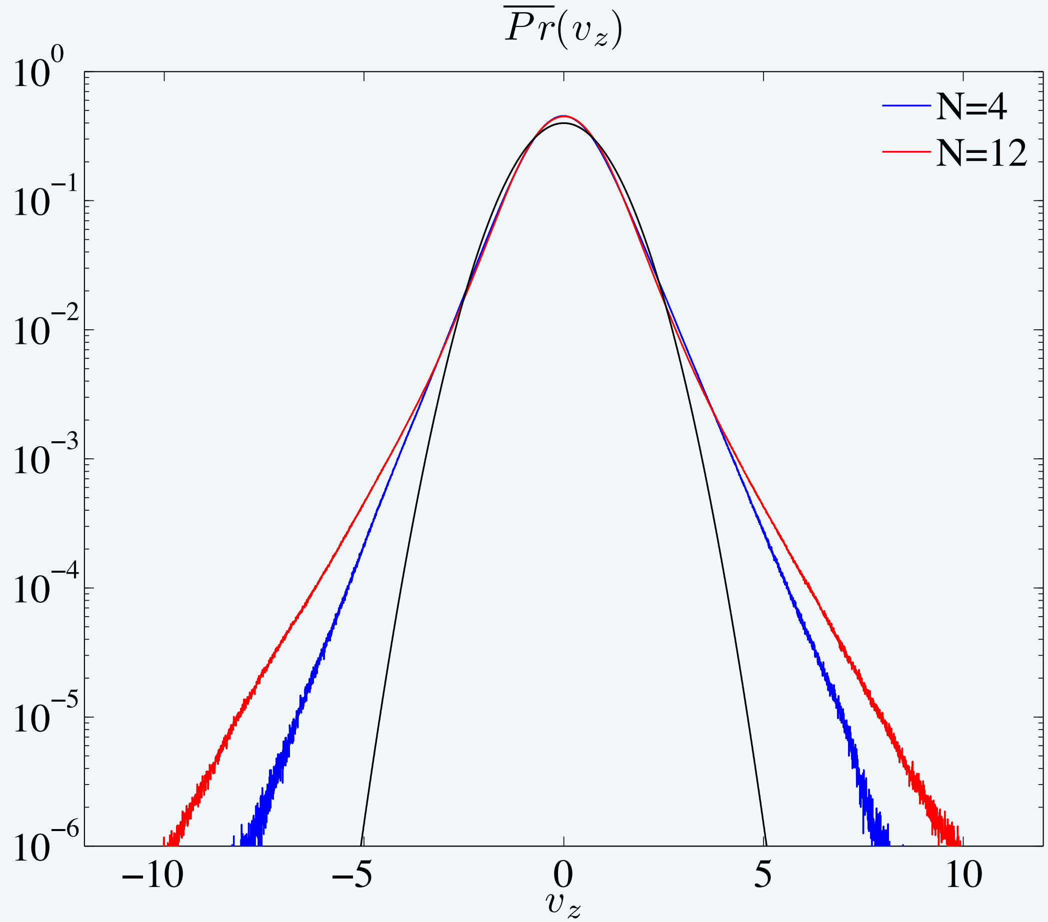
N=12,
Fr=0.03,
R_B=22





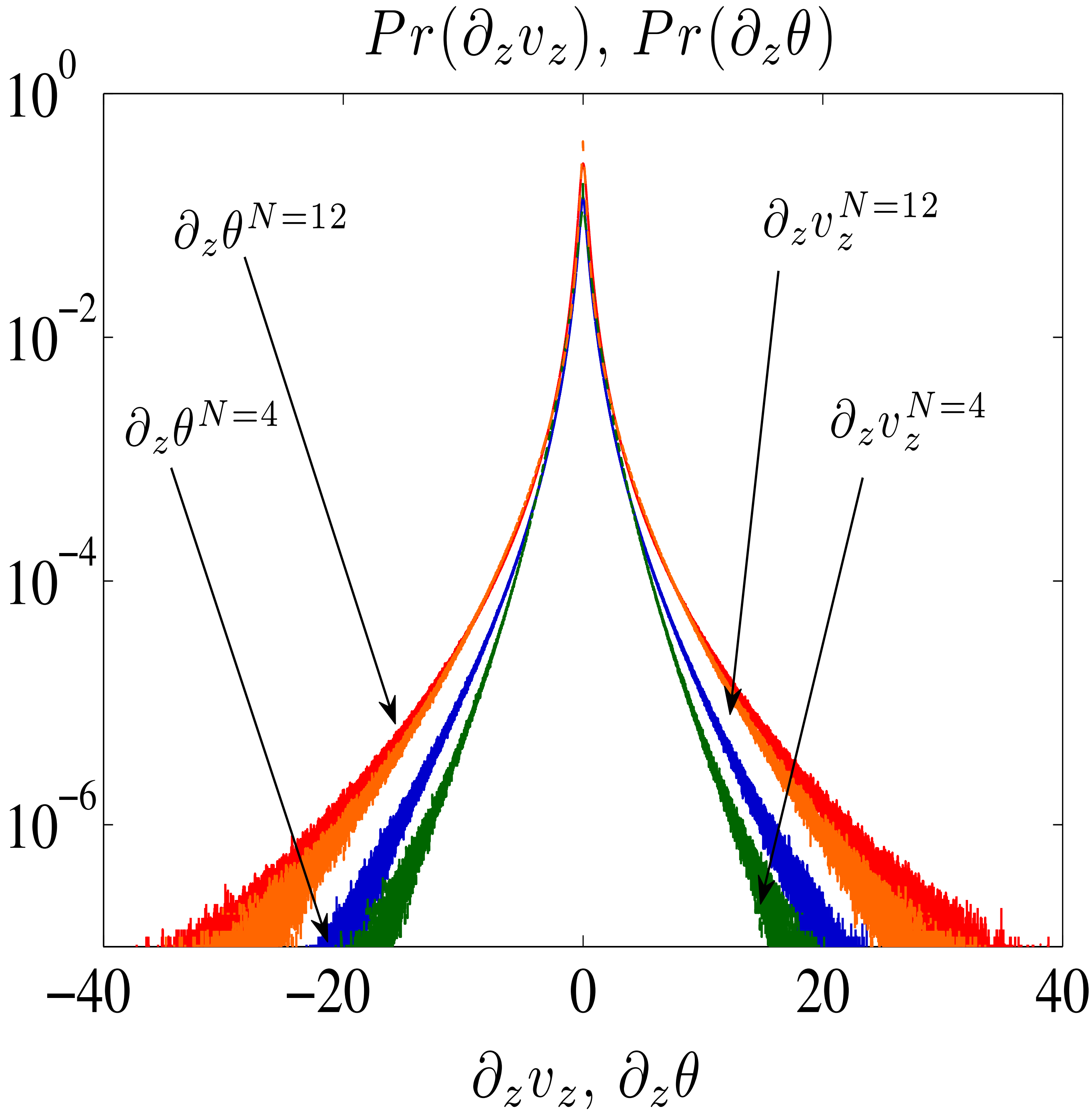
Rorai et al. 2015

Vertical velocity
Time average



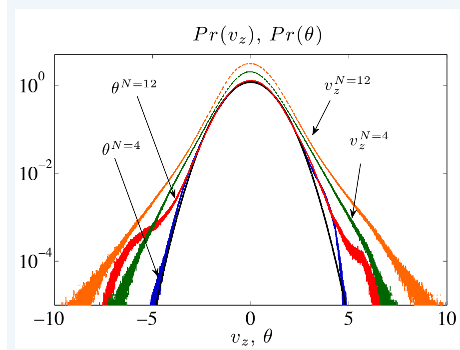
DNS 2048^3,
Re=24000

N=12,
Fr=0.03,
R_B=22

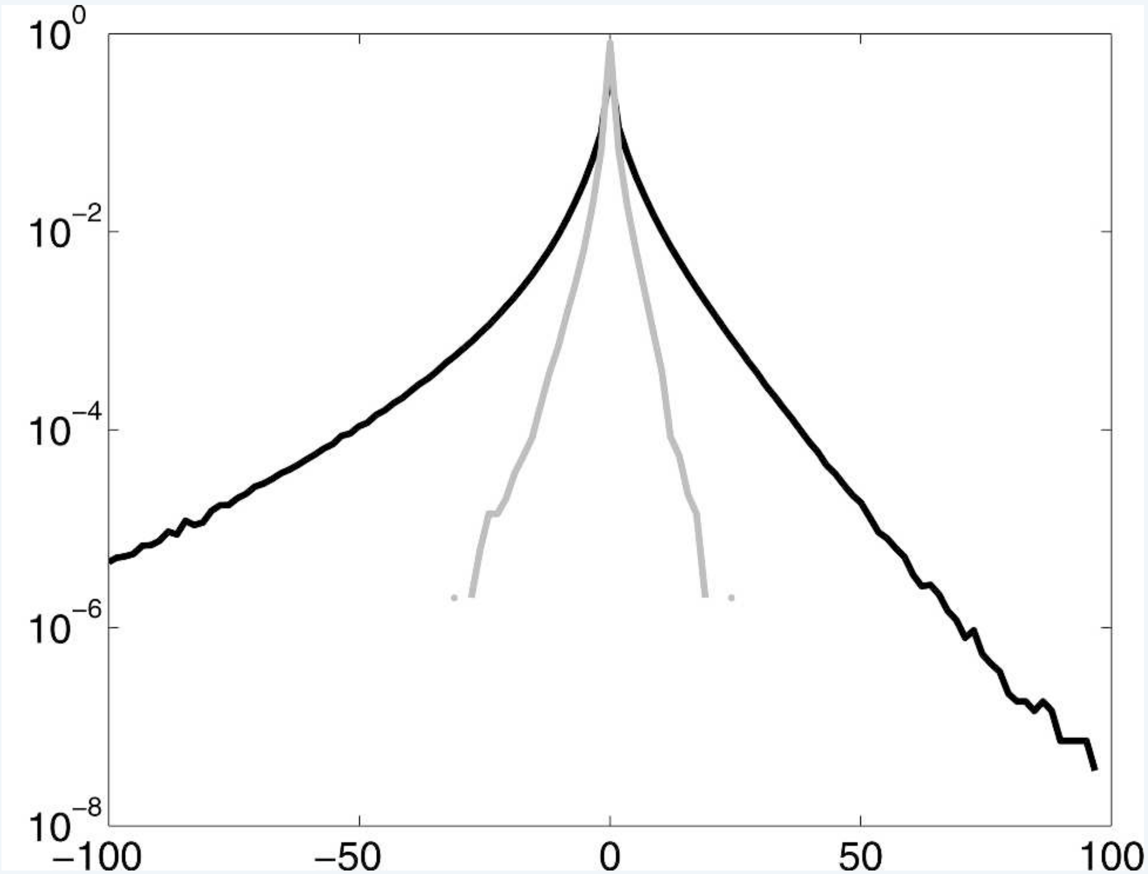


DNS 2048^3

Re=24000
Fr=0.1 & 0.03
R_B=240 & 22



PdF of vertical velocity in an oceanic model



Two depths
ROMS with atmospheric forcing
~ 1000^2 X 40 res.
California current

Wings associated with frontogenesis

Capet et al., 2008a

A MODEL

Vitesse **u** = (u_x, u_y, w)

Longitudinal differences of fluctuations of velocity over a distance l :

$$\delta u_x(\ell) = \langle u_x(x+\ell) - u_x(x) \rangle$$

$$D_t \delta u(l) = - \delta u^2 / l$$

Stratified turbulence model (N is the Brunt-Vaissala frequency):
vertical differences of fluctuations of vertical velocity w
and temperature θ over a vertical distance $l = l_{//}$

$$\frac{d\delta w}{dt} = -\frac{\delta w^2}{\ell} - N\delta\theta,$$

$$\frac{d\delta\theta}{dt} = -\frac{\delta w\delta\theta}{\ell} + N\delta w.$$

→ 3 regimes

* N large: harmonic oscillator of frequency N

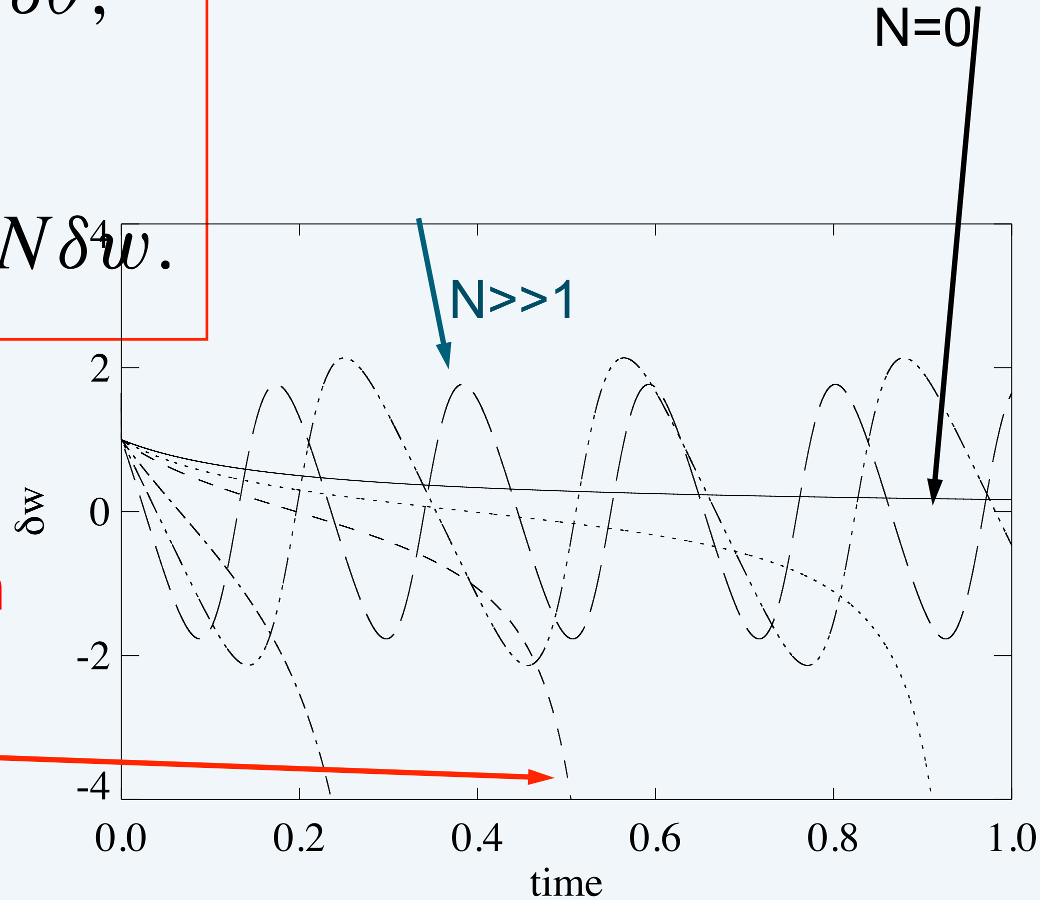
* N small: strong turbulence case

- $N\theta l_{//} \sim w^2$, $N l_{//} \sim \theta$: balance compatible with *saturated* spectrum $E_w(k_{//}) \sim E_\theta(k_{//}) \sim N^2 k_{//}^{-3}$

Stratified turbulence model (N is the Brunt-Vaissala frequency):
vertical differences of fluctuations of vertical velocity w
and temperature θ over a vertical distance $l_{//}$

$$\frac{d\delta w}{dt} = -\frac{\delta w^2}{\ell} - N\delta\theta,$$

$$\frac{d\delta\theta}{dt} = -\frac{\delta w\delta\theta}{\ell} + N\delta w.$$



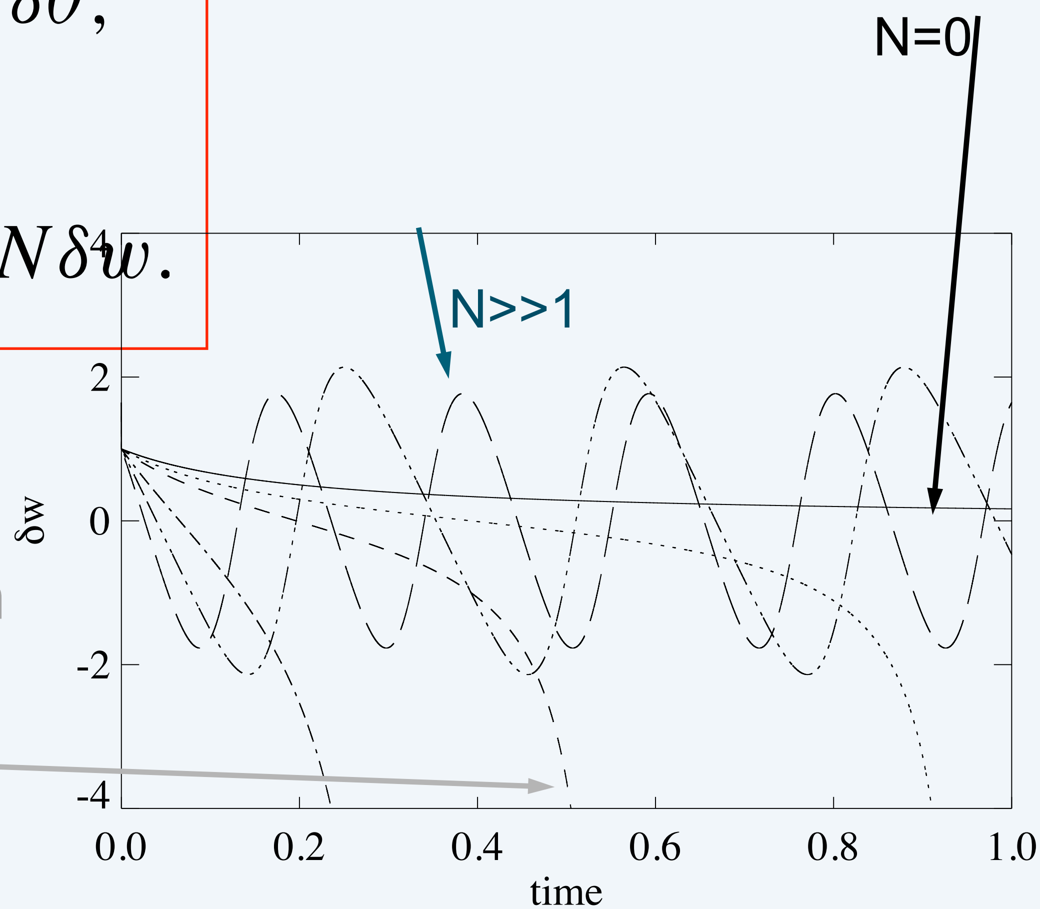
Intermediate N: faster growth of negative gradients (*saturated regime*)

Rorai et al. 2015

Stratified turbulence model (N is the Brunt-Vaissala frequency): vertical differences of fluctuations of vertical velocity w and temperature θ over a vertical distance $l_{//}$

$$\frac{d\delta w}{dt} = -\frac{\delta w^2}{\ell} - N\delta\theta,$$

$$\frac{d\delta\theta}{dt} = -\frac{\delta w\delta\theta}{\ell} + N\delta w.$$



Intermediate N: faster growth of negative gradients (*saturated regime*)

- Add transverse velocities
- Add rotation *(Li 2010)*, passive scalar, …

Rorai et al. 2015

Conclusion

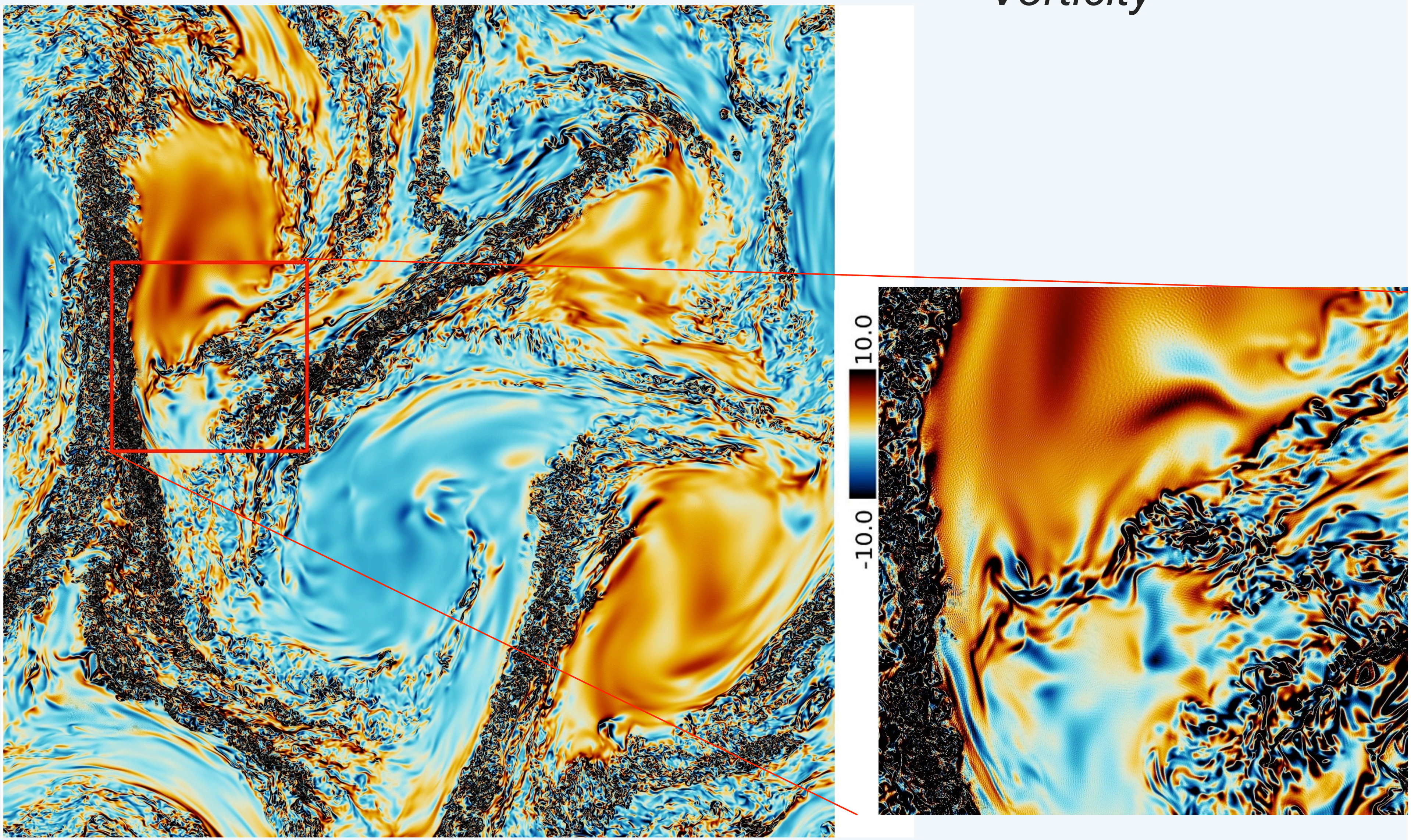
Turbulent flows can produce strong velocities, as observed e.g. in the nocturnal (very stable) planetary boundary layer

Part III

The emergence of Bolgiano-Obukhov scaling in rotating stratified turbulence

Rosenberg et al., ArXiv:1409.4254

Vorticity

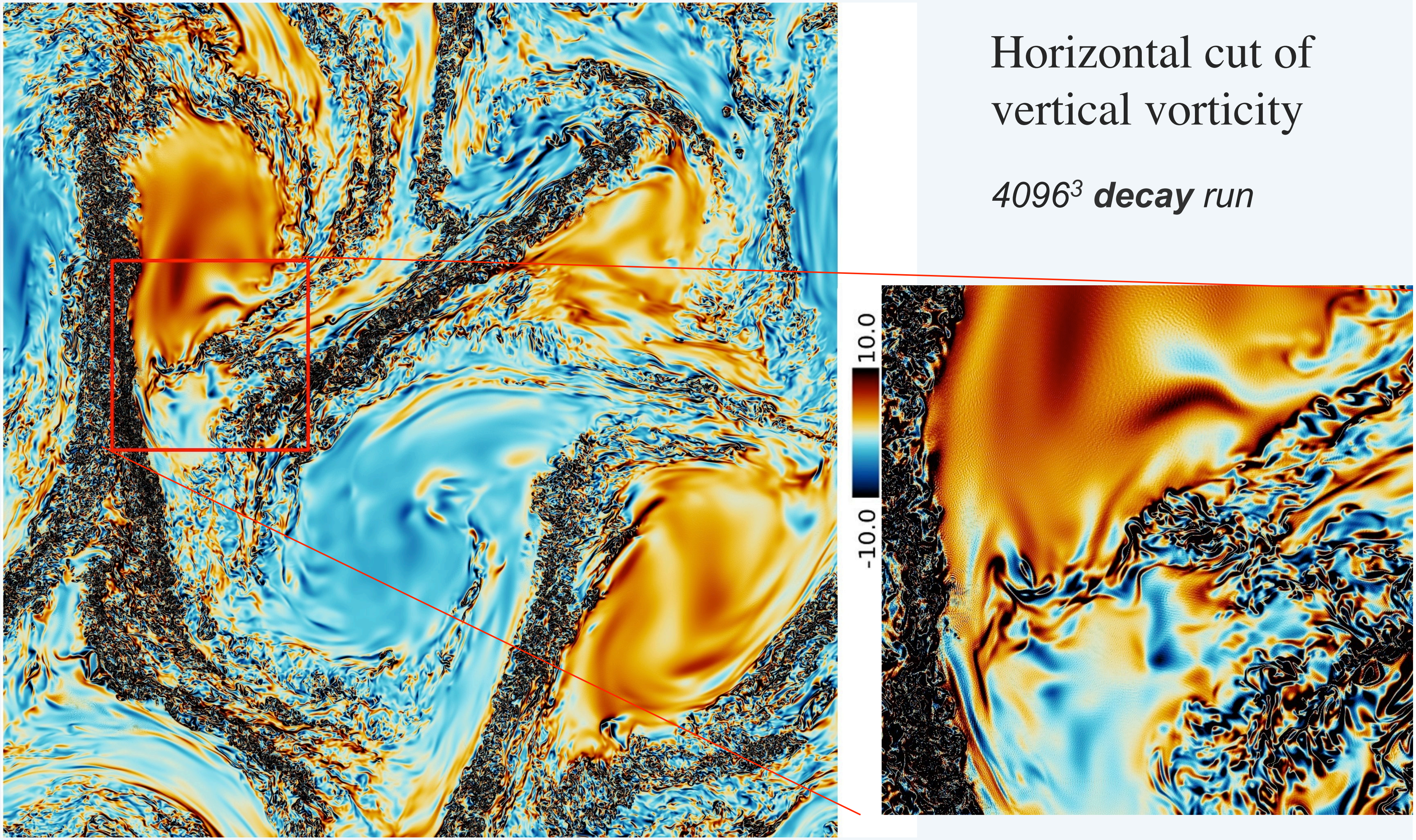


Rosenberg, OakRidge, 2015

Rotating stratified flow

Horizontal cut of vertical vorticity

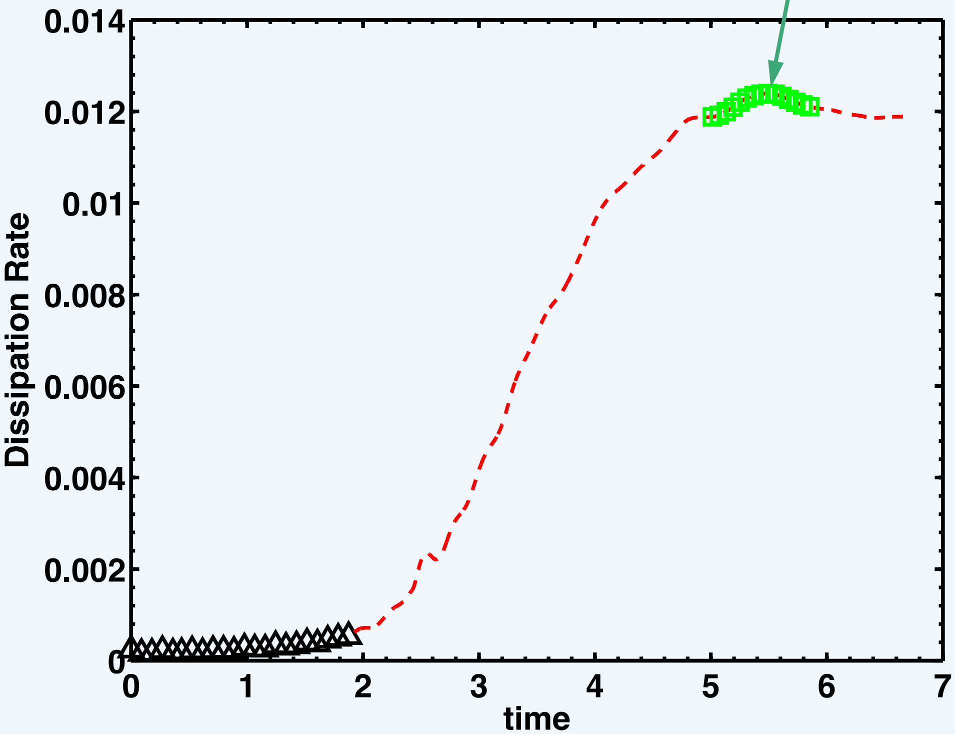
*4096^3 **decay** run*



Rosenberg, OakRidge, 2015

The short 4096^3 run

Peak of dissipation



N/f=4.95

Fr=0.024, Ro=0.12

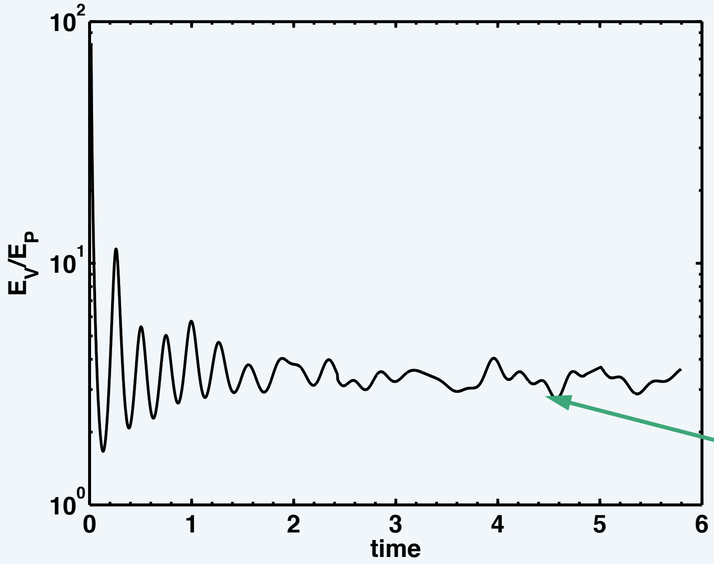
Re=55000, R_B=32

Decay, k_0=2.5

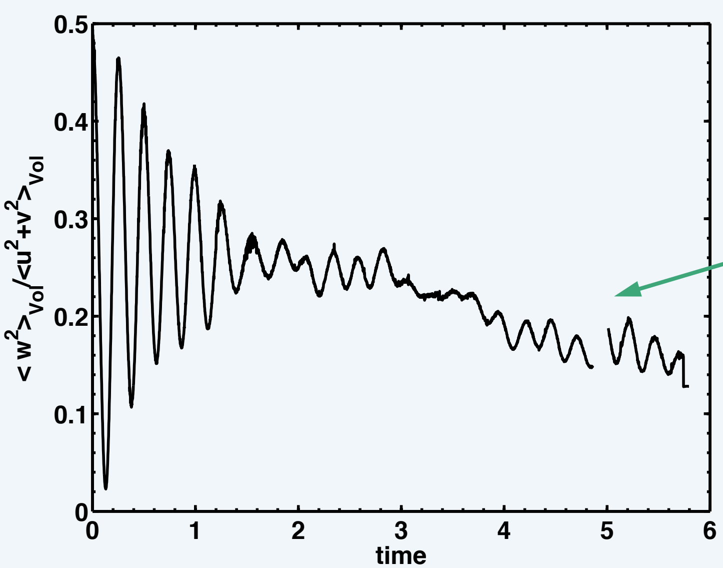
Triangles: 1536^3 grid

- - - - : 3072^3 grid

. . . Green: 4096^3 grid



Energy ratios



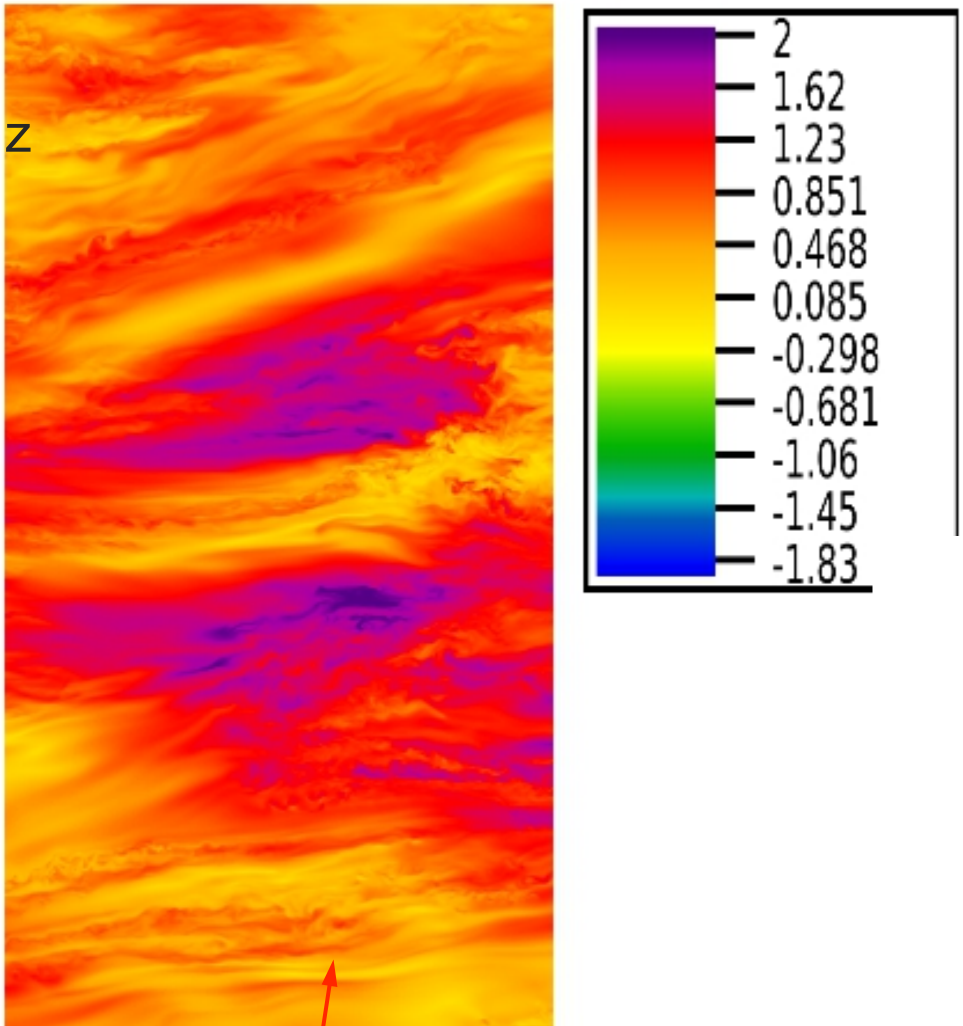
Kinetic to potential: ~ 3

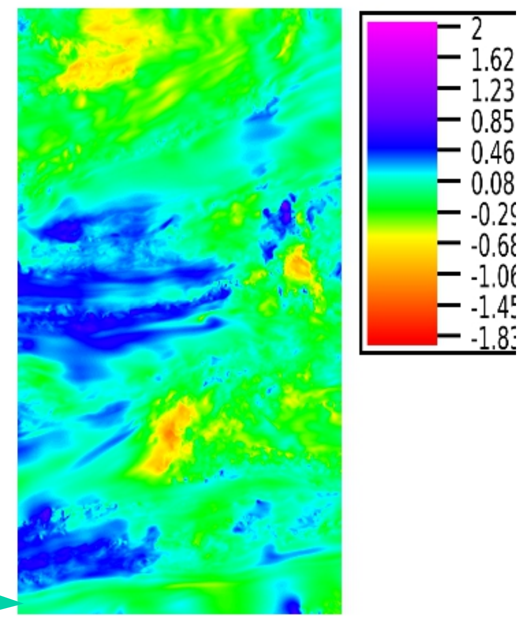
And

Vertical to horizontal: ~ 0.12

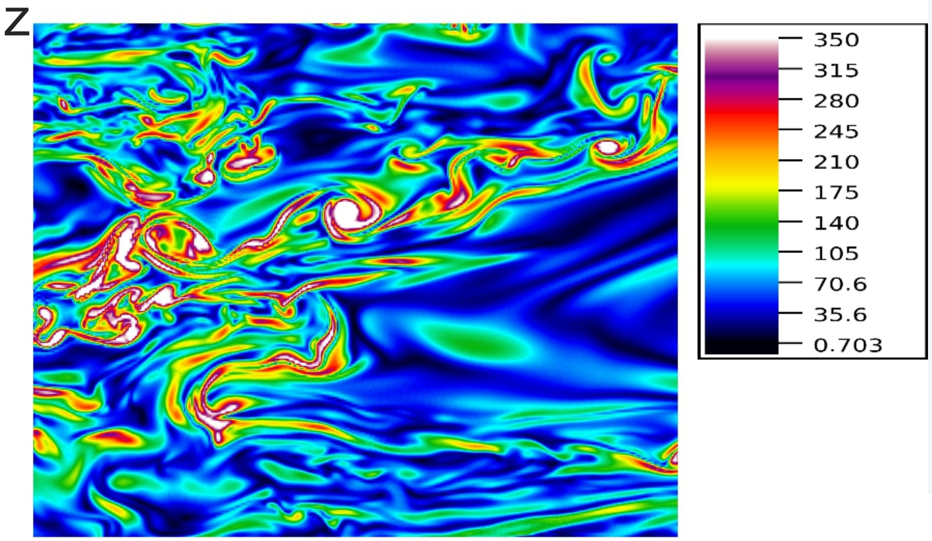
N/f=4.95, Fr=0.024
Ro=0.12
Re=55000, R_B=32
K_0=2.5, decay

Vertical cuts
Sub-volumes





Perpendicular
& Vertical velocity
Sub-volume: 0.7 X 0.4 X 0.04 L_{box}^3

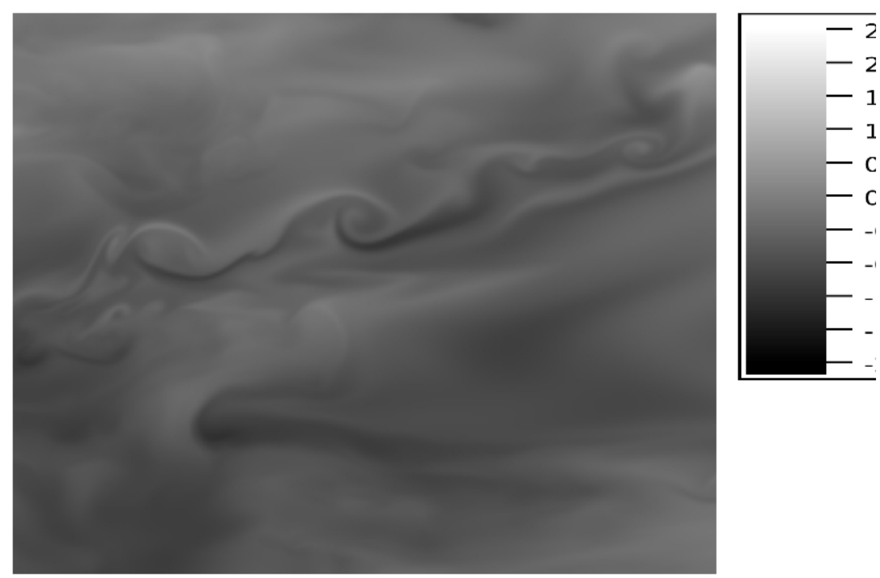


$N/f=4.95$

$Fr=0.024$, $Ro=0.12$

$Re=55000$, $R_B=32$

$K_0=2.5$, decay



Vertical vorticity

Sub-volume: 0.12 X 0.1 X 0.01

Temperature fluctuations

Stably stratified turbulence: Bolgiano-Obukhov 1959 scaling

Main hypotheses: * Energy source for cascade is a constant buoyancy flux

* Isotropy

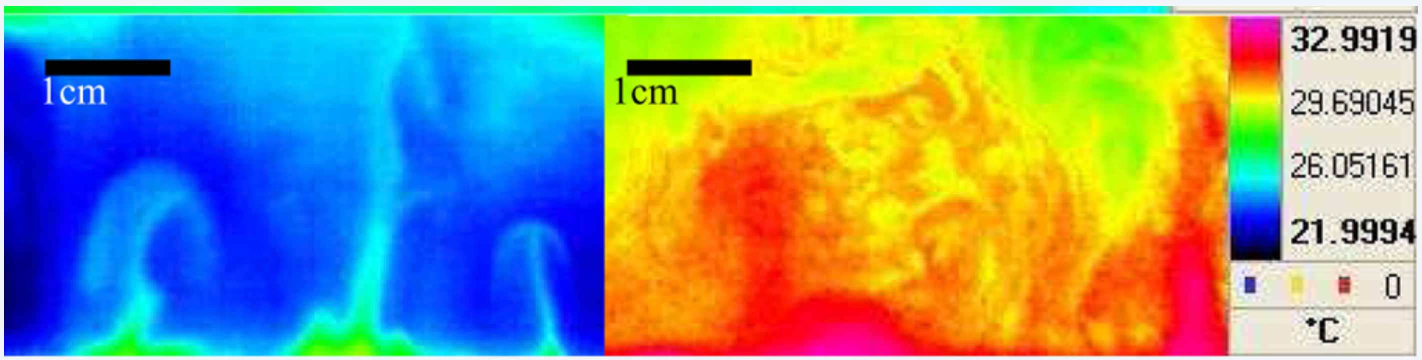
Kinetic & potential energy: $E_{V,P}(k) = f(k, \varepsilon_P)$
with $\varepsilon_P = DE_P/DT$ of dimension L^2T^{-5}

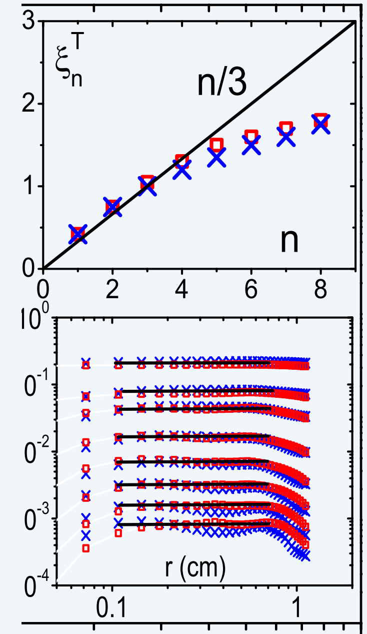
→ $E_V(k) = \varepsilon_P^{2/5}\ k^{-11/5}$
→ $E_P(k) = \varepsilon_P^{4/5}\ k^{-7/5}$

→ Recovery of a Kolmogorov spectrum for $K_{BO} \sim \varepsilon_P^{3/4} \varepsilon_V^{-5/4}$

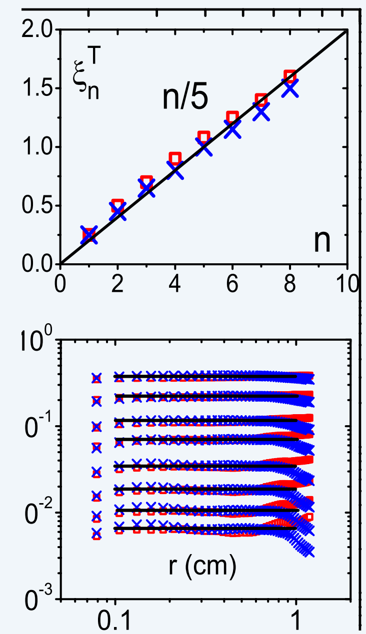
Seychelles et al. (2010) 2D soap bubble experiment

Temperature structure functions





$\Delta T =$ 21C or 50C



Boffetta et al. (2012): DNS of Rayleigh-Taylor turbulence, quasi 2D: 4096x128x8192

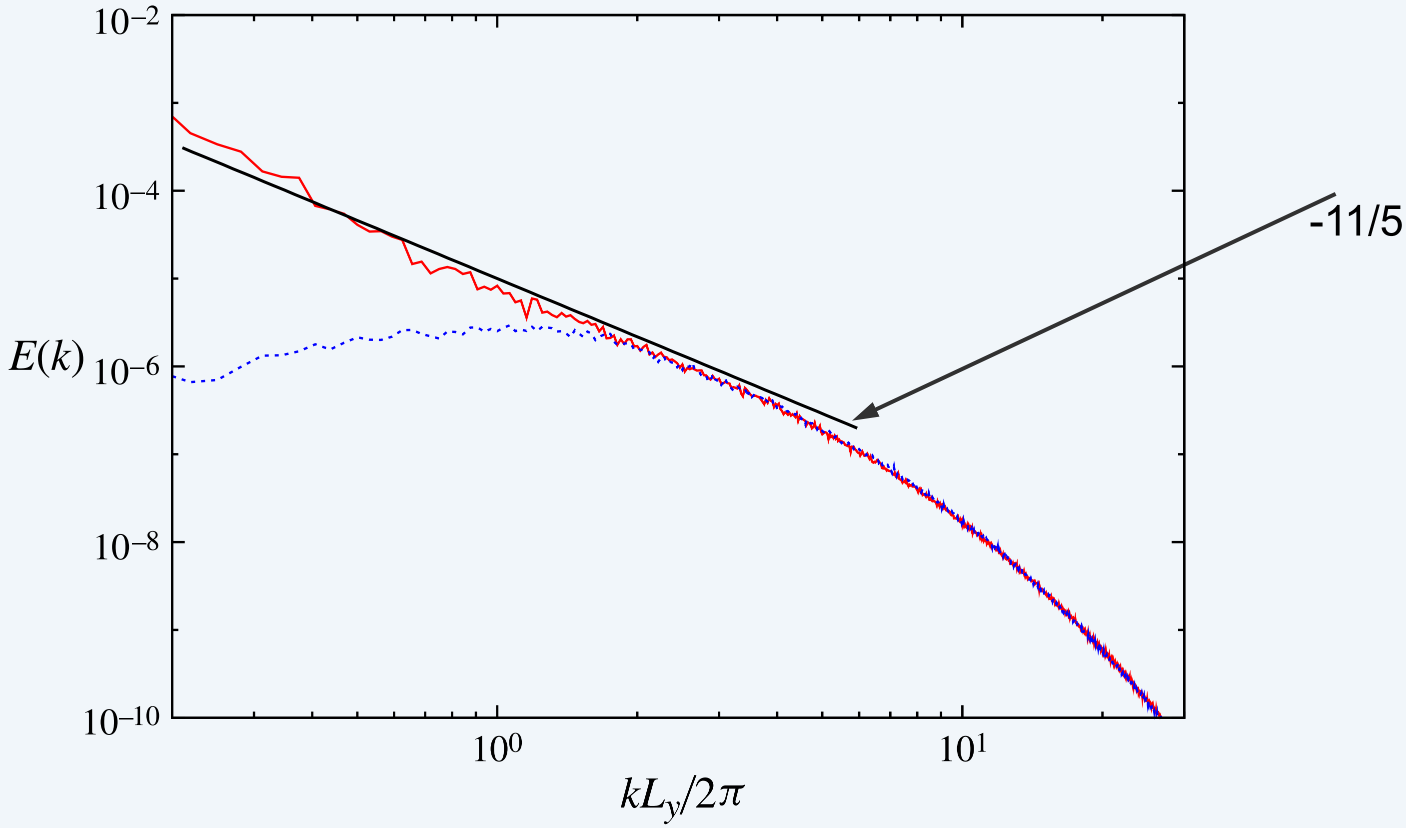
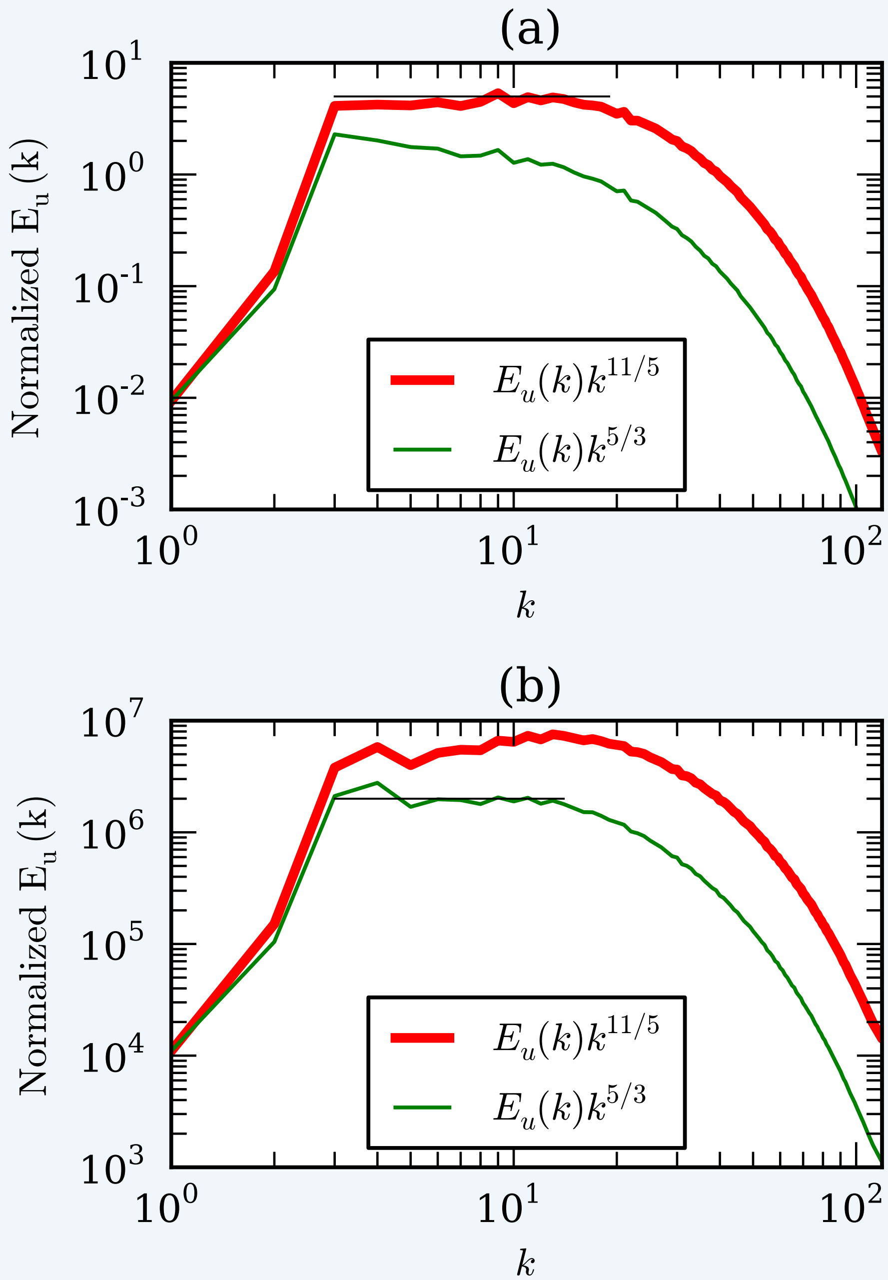


FIGURE 9. (Colour online) Kinetic energy spectra computed at $t = 35\tau$. The thin continuous line represents the components $E_u(k) + E_w(k)$, the dotted line is $E_v(k)$. Spectra are computed by 2D Fourier transforming the velocity field on (x, z)-planes and by averaging over the y-direction. The straight line represents Bolgiano scaling $k^{-11/5}$.

1024^3 run, no rotation, Re=650, forced at large scale



Kinetic energy spectra compensated by K41 (5/3) & BO (11/5)

← $Ri=1/Fr^2 = 4x10^{-7}$
$R_B >> 1$

← $Ri=1/Fr^2 = 0.5$
$R_B=1300$

Kumar et al. (2014)

Hurst exponents from dropsondes in the troposphere for horizontal wind varying in the vertical

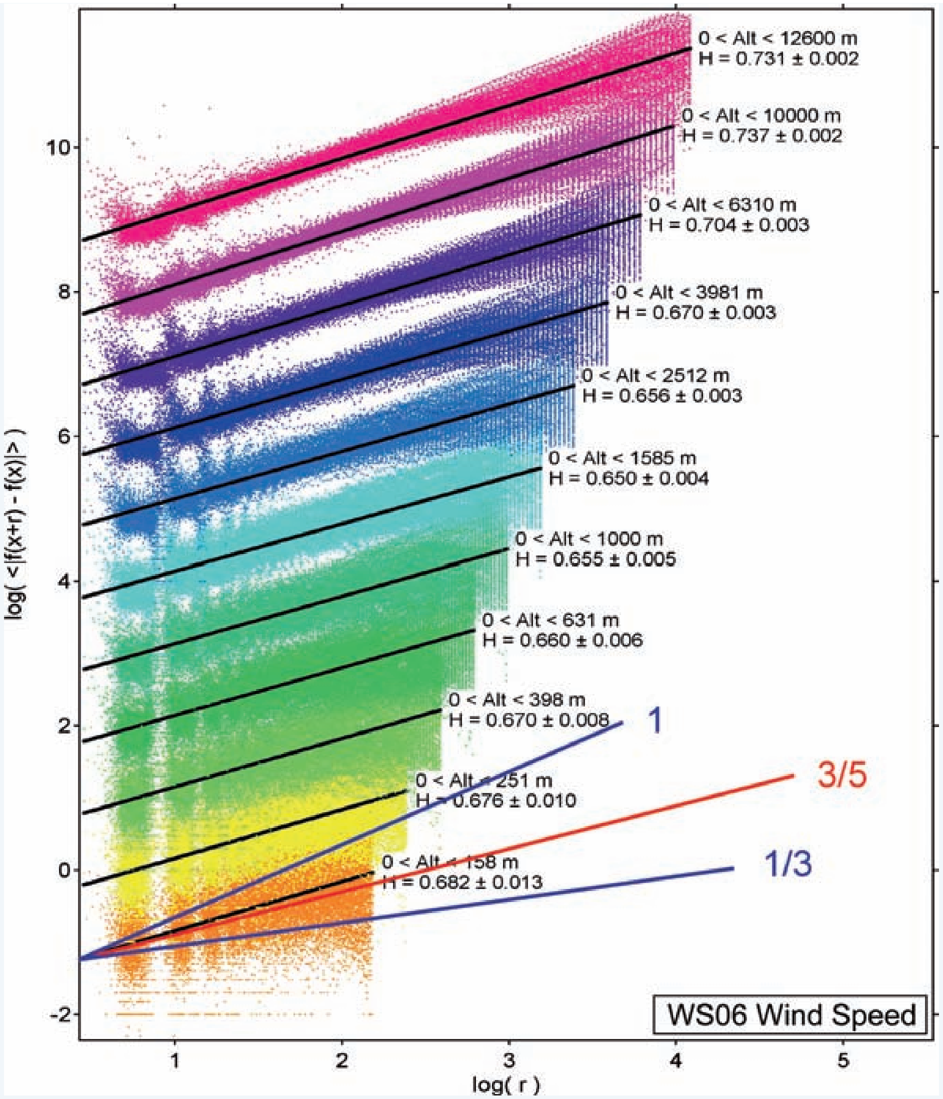
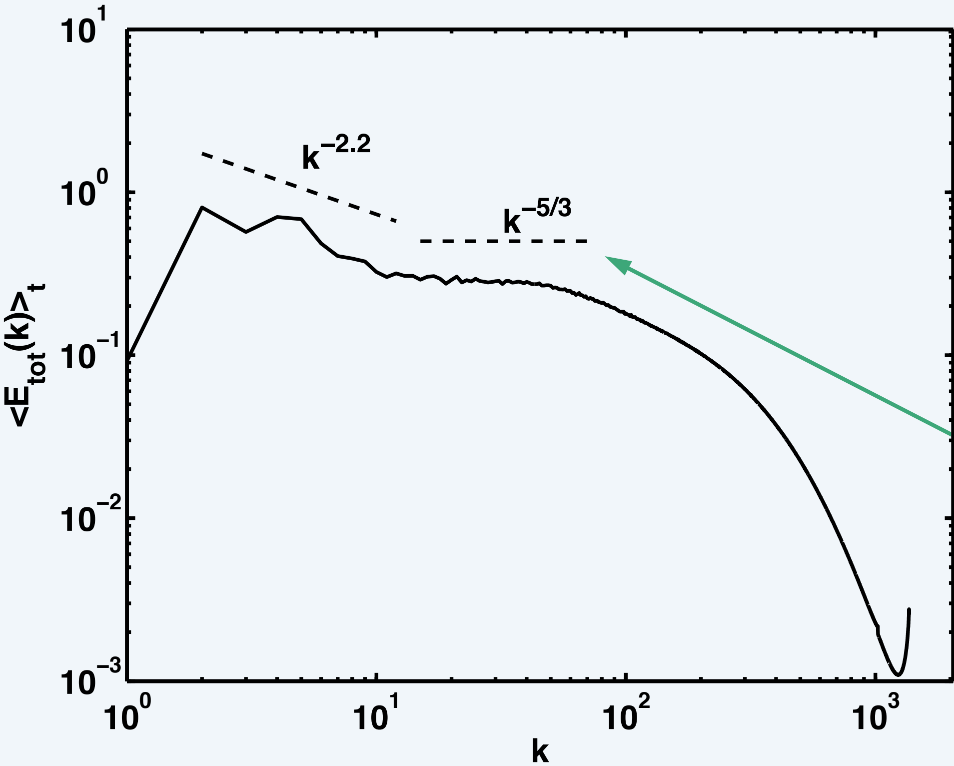
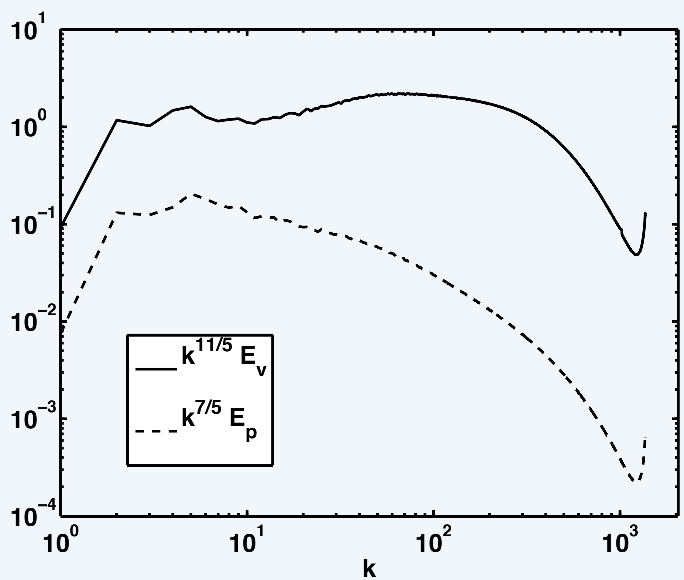


Figure 12. Total of 315 dropsondes dropped during Winter Storms 2006, in the area 21°N–60° N, 128° W–172° W. The vertical scaling of the horizontal wind is shown. The fits are rms to the vertical shears across layers of thickness increasing logarithmically upwards. The reference lines have slopes corresponding to $H_v = 1$ (gravity waves), $H_v = 3/5$ (BO) and $1/3$ (Kolmogorov). The H_v corresponding to each rms fit is given. The data for each level are offset by 1 order of magnitude to aid legibility. While BO is a good fit in the lower troposphere, in the upper troposphere the presence of jet streams leads to a systematic increase when the upper troposphere is included. In any event, isotropic turbulence is always precluded. See Lovejoy *et al.* (2007) and Tuck (2008). Note: rms, root mean square; BO, Bolgiano–Obukhov.

H=1: gravity waves
H=3/5: Bolgiano-Obukhov
H=1/3: Kolmogorov 1941

Hovde et al. 2011

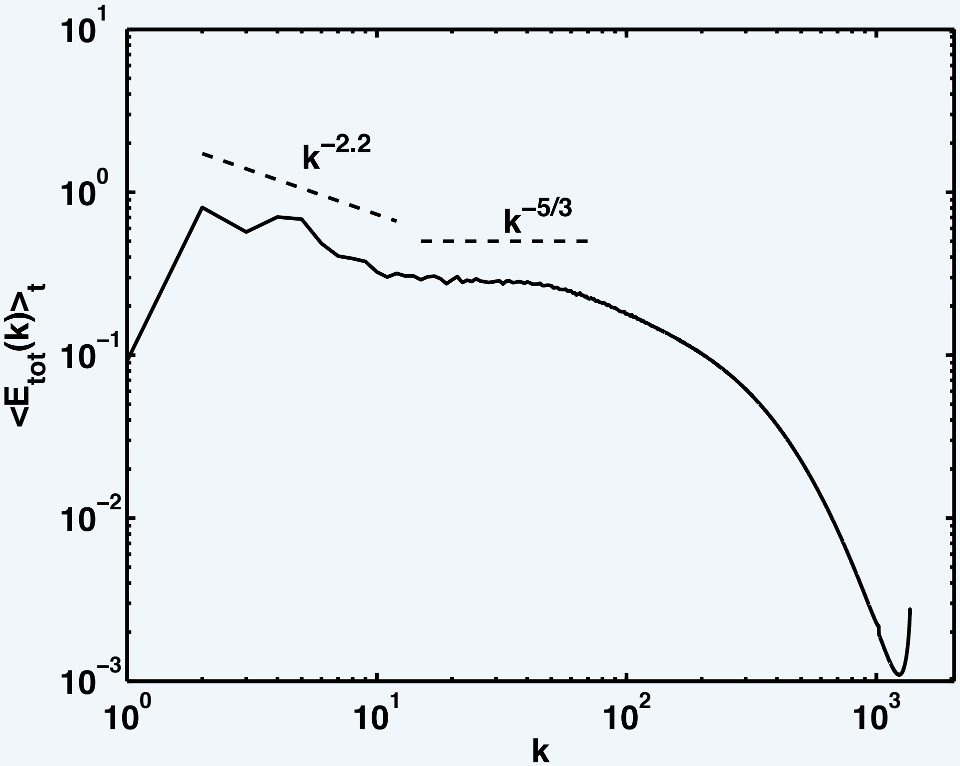


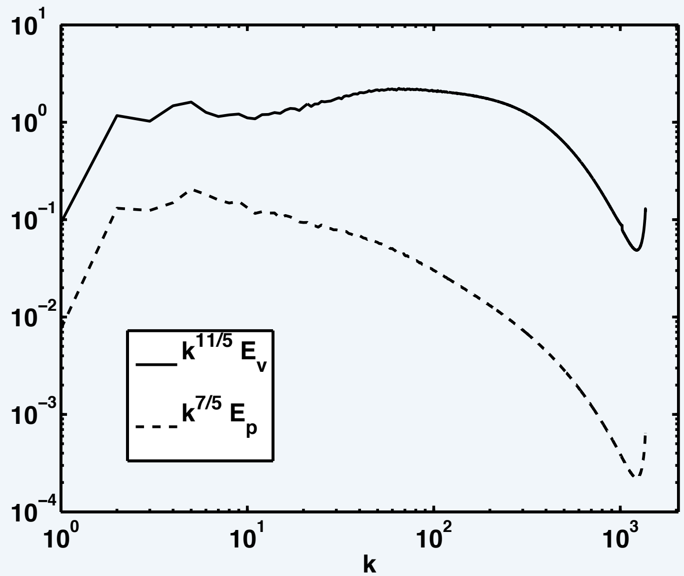


N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, decay

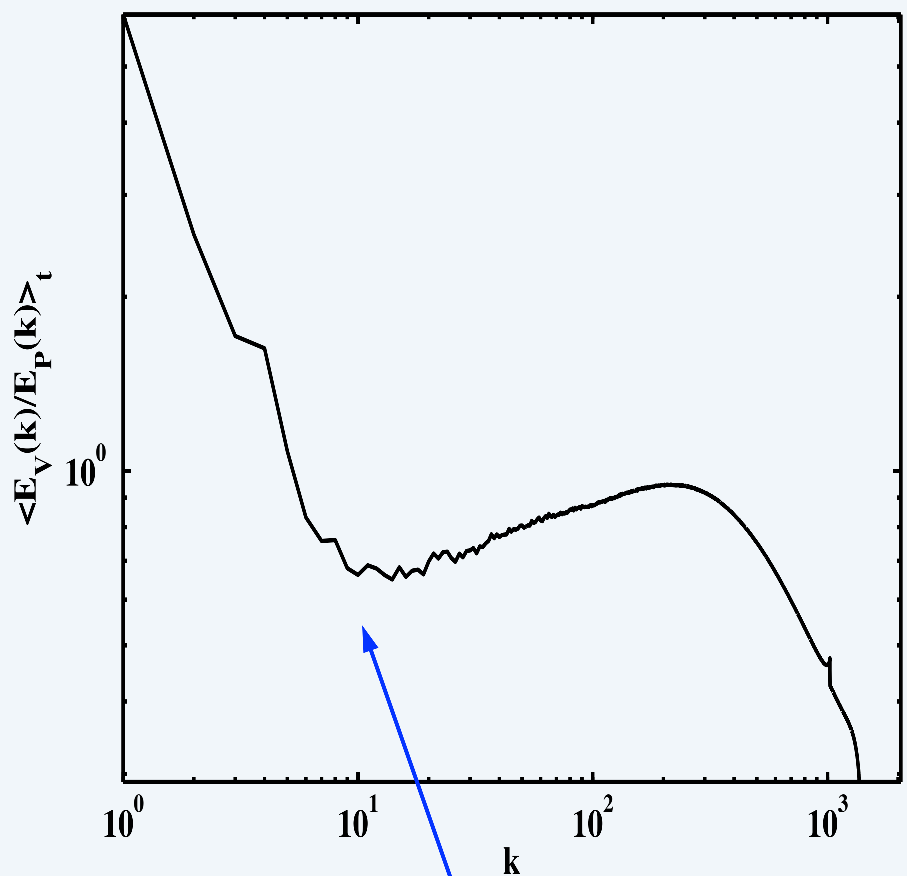
5/3-compensated
total energy
isotropic spectrum

Energy spectra:
kinetic (___) or
potential (- - -)
compensated
by 11/5 or 7/5:
Bolgiano-Obukhov scaling





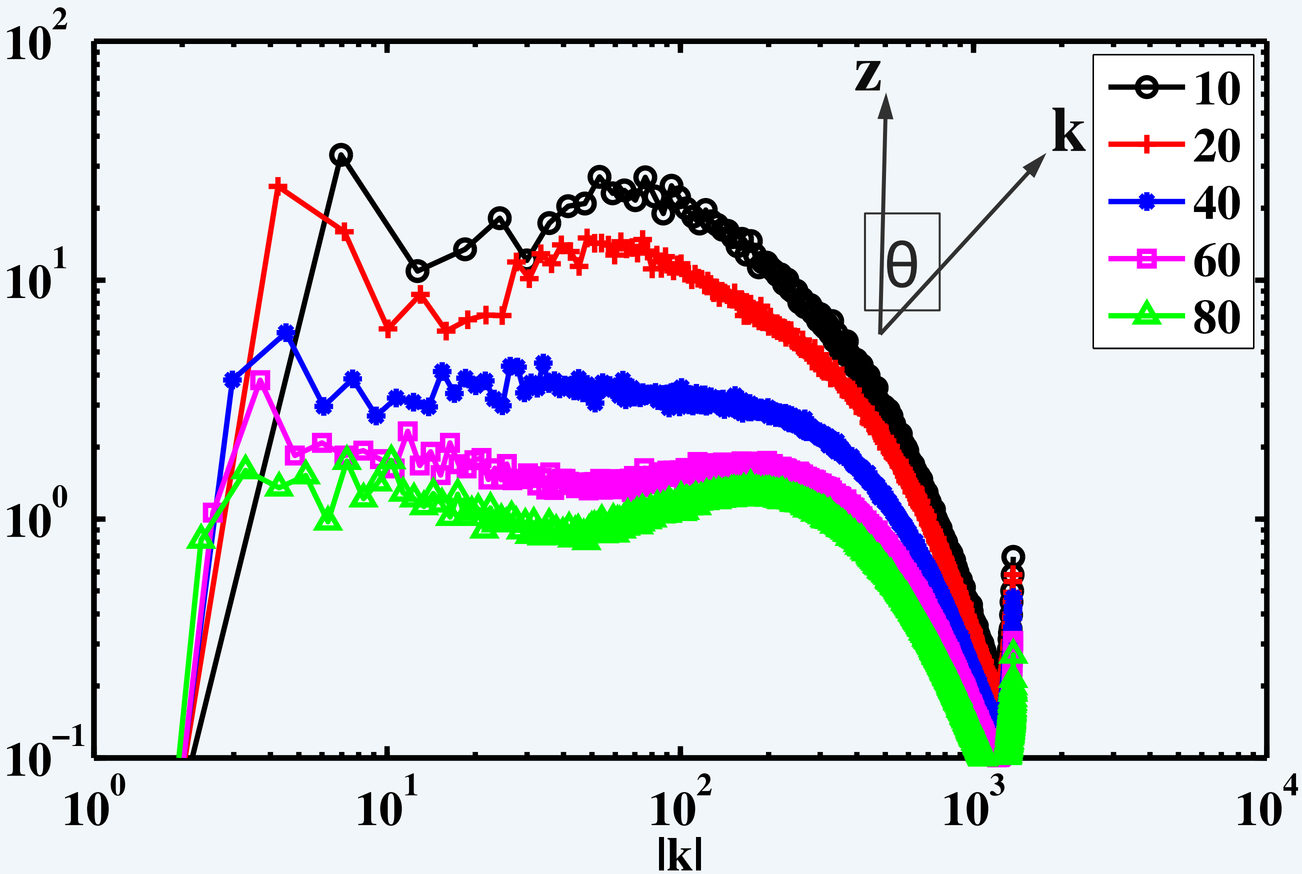
Ratio of spectra of potential & kinetic energy ~ k $^{-4/5}$



$$K_{BO} \sim \varepsilon_P^{3/4}\varepsilon_V^{-5/4} \quad \sim 11$$

Compensated total energy angular spectra

N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, decay



e(|k|,θ) with E(k)=Σ e(|k|,θ)dθ

Rosenberg et al. 2015

Vertical buoyancy flux:

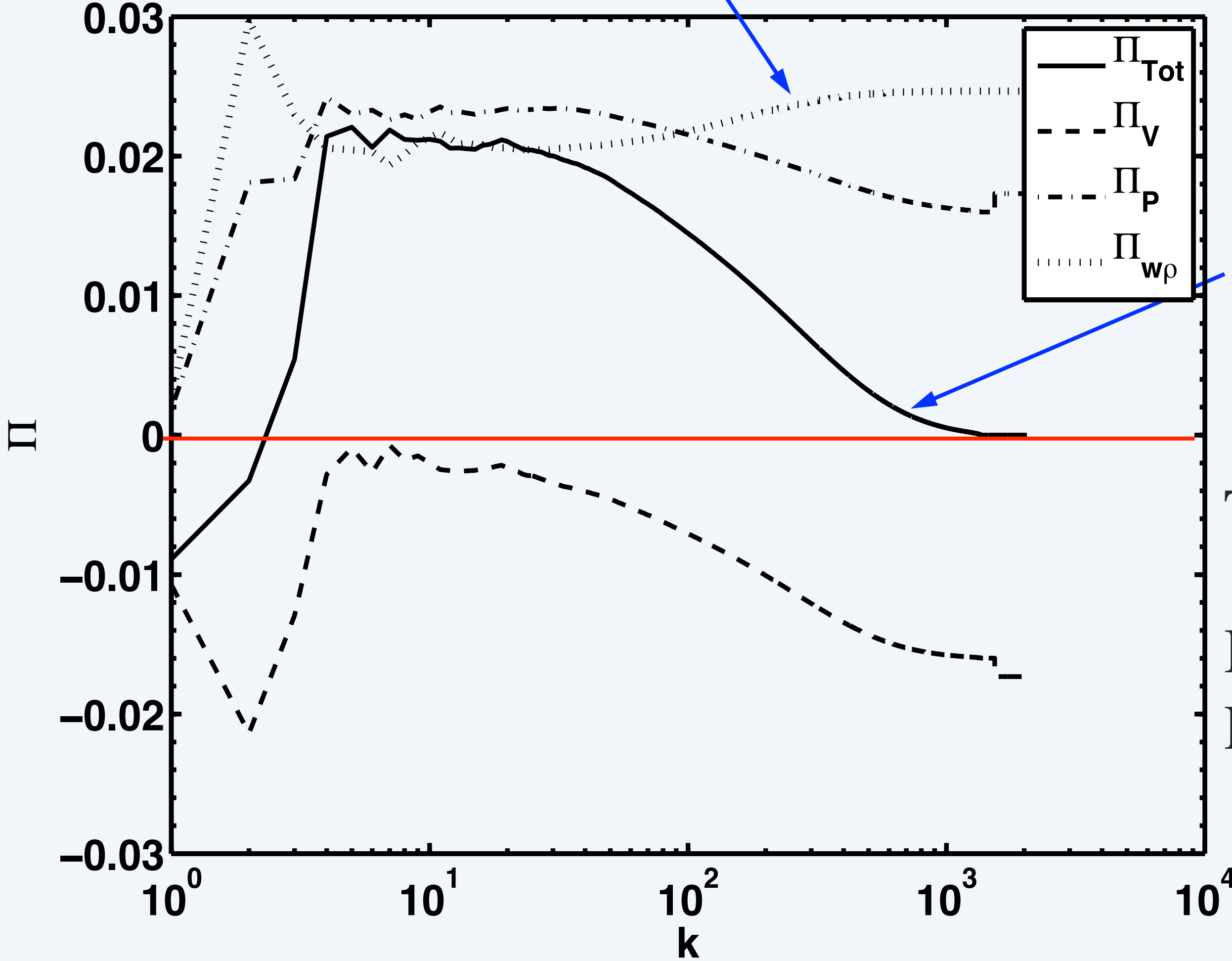
$$\Pi_{w\rho}(k) = \sum_{k'=0}^{k'=k} \sum_{k'<|k''|<k'+1} Re(\hat{w}(\mathbf{k}'')\hat{\rho}(\mathbf{k}'')^*) \ ,$$

4096^3 data, N/f=4.95

Fr=0.024, Ro=0.12

Re=55000, R_B=32

K_0=2.5, decay



Isotropic energy flux

and its components

Total (____): $\Pi_T \sim \Pi_V + \Pi_P$

Kinetic (---): $\Pi_V \sim \mathbf{u}.(\mathbf{u}.\text{grad } \mathbf{u})$

Potential (-. -.): $\Pi_P \sim \varrho \mathbf{u}.\text{grad } \varrho$

Buoyancy flux $\Pi_{w\varrho}$: ….

Bolgiano, Marseille meeting, 1962

``Important progress appears likely in the next few years."

Conclusion

Bolgiano-Obukhov scaling taking into account the potential energy input can be observed in some cases including with geostrophically-balanced initial conditions

Forced case?