Rome 2015-03-24

Clustering of chiral particles in flows with broken parity invariance

K. Gustavsson[1)], L. Biferale[1)]

1) Department of Physics, University of Tor Vergata, Italy

Motion of an 'isotropic helicoid'

Equations for velocity $\boldsymbol{v}$ and angular velocity $\boldsymbol{\omega}$ for small isotropic helicoid:

Happel & Brenner, Low Reynolds number hydrodynamics (1963)

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_\mathrm{p}}\left[\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v} + \frac{2a}{9}C_0(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega})\right]$$

$$\dot{\boldsymbol{\omega}} = \frac{1}{\tau_\mathrm{p}}\left[\frac{10}{3}(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega}) + \frac{5}{9a}C_0(\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v})\right]$$

Stokes' law translation – rotation coupling (scalar)

- $\boldsymbol{u}$ Fluid velocity
- $\boldsymbol{\Omega}$ Half fluid vorticity
- τ_p Particle relaxation time
- $a = \sqrt{5I_0/(2m)}$ Particle 'size' (defined by mass m and moment of inertia I_0)
- C_0 Helicoidality

Equations break spatial reflection symmetry ($\boldsymbol{\Omega}$ and $\boldsymbol{\omega}$ pseudovectors)

Dimensionless parameters

Stokes number $\mathrm{St} \equiv \frac{\tau_\mathrm{p}}{\tau_\eta}$ Size $\overline{a} \equiv \frac{a}{\eta}$ Helicoidality C_0

with τ_η and η smallest time- and length scales of flow.

Constraint on C_0 :

Using $\boldsymbol{v}_\pm = \boldsymbol{v} + B\boldsymbol{\omega}$, $\boldsymbol{u}_\pm = \boldsymbol{u} + B\boldsymbol{\Omega}$ with $B \equiv \frac{a(21 \pm \sqrt{441 + 40C_0^2})}{10C_0}$

The equations of motion becomes

$$\dot{\boldsymbol{v}}_+ = \frac{1}{\tau_\mathrm{p}} \frac{39 + \sqrt{441 + 40C_0^2}}{18} (\boldsymbol{u}_+ - \boldsymbol{v}_+)$$

$$\dot{\boldsymbol{v}}_- = \frac{1}{\tau_\mathrm{p}} \frac{20(27 - C_0^2)}{9(39 + \sqrt{441 + 40C_0^2})} (\boldsymbol{u}_- - \boldsymbol{v}_-)$$

Solution blows up unless $-\sqrt{27} < C_0 < \sqrt{27}$

St and $\overline{a}$ constrained by particle density higher than that of the fluid and geometrical size must be smaller than η .

Example of an isotropic helicoid

Recipe from Lord Kelvin:

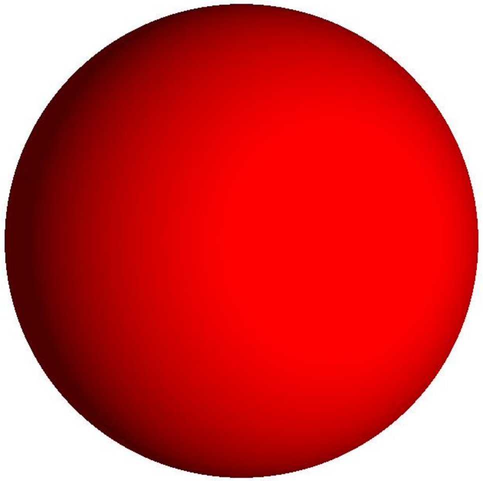
"An isotropic helicoid can be made by attaching projecting vanes to the surface of a globe in proper positions; for instance cutting at 45º each, at the middles of the twelve quadrants of any three great circles dividing the globe into eight quadrantal triangles."

Kelvin, Phil. Mag. **42** (1871)

Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

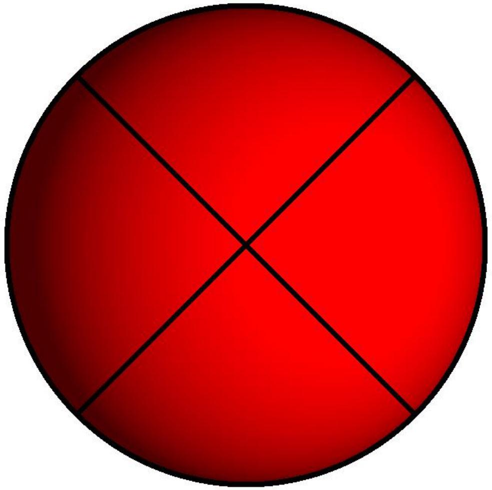
Start with a sphere



Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

✓ Start with a sphere
Draw 3 great circles

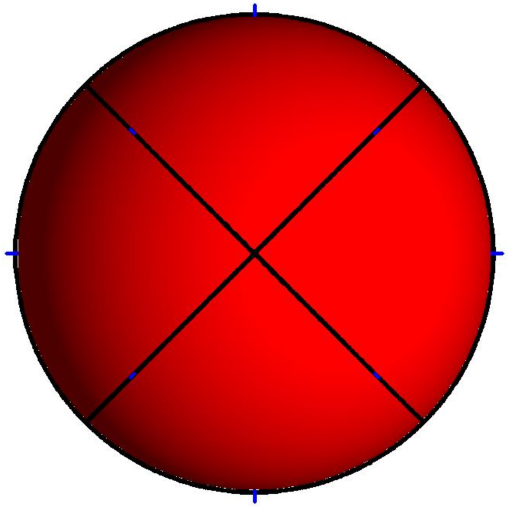


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

- ✓ Start with a sphere
- ✓ Draw 3 great circles

Identify 12 vane positions at midpoints of quarter-arcs

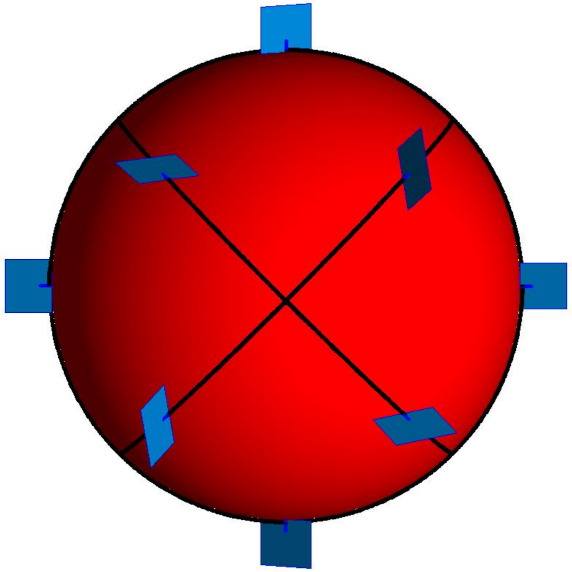


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

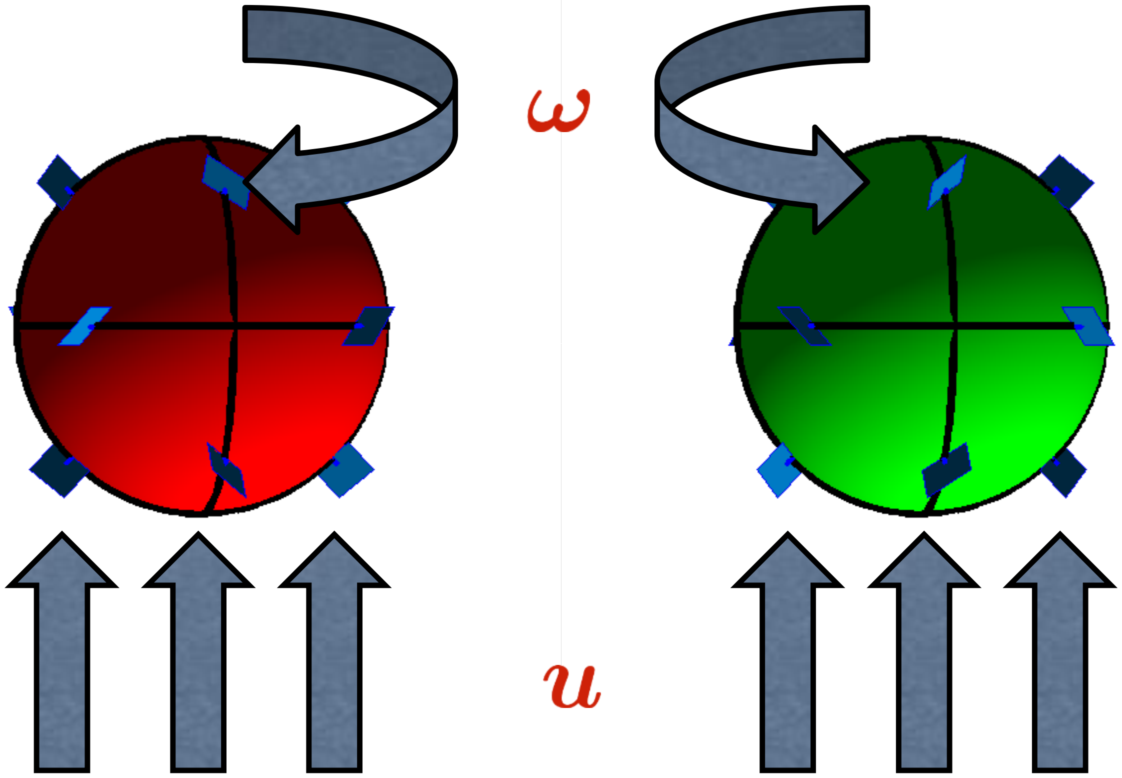
- ✓ Start with a sphere
- ✓ Draw 3 great circles
- ✓ Identify 12 vane positions at midpoints of quarter-arcs

Put a vane on each vane position (45° to arc line)



Chirality

In a constant flow u, the isotropic helicoid starts spinning around the flow direction with angular velocity ω.
The spinning direction depends on the chirality of the vanes.



Left handed $C_0 < 0$ *Right handed* $C_0 > 0$

Clustering at small St

Expand compressibility of particle-velocity field $\boldsymbol{\nabla}\cdot\boldsymbol{v}$ to first order in St

$$\boldsymbol{\nabla}\cdot\boldsymbol{v} = -\frac{27}{27-C_0^2}\tau_{\mathrm{p}}\left[\mathrm{Tr}\big(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\big) - \frac{1}{15}\mathrm{a}\mathrm{C}_0\mathrm{Tr}\big(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{\nabla}\boldsymbol{\Omega}^{\mathrm{T}}\big)\right]$$

Centrifuge effect with modified amplitude
Maxey, J. Fluid Mech. **174** (1987)

Term due to parity breaking of system

Reflection-invariant systems have $\langle\mathrm{Tr}\big(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{\nabla}\boldsymbol{\Omega}^{\mathrm{T}}\big)\rangle = 0$

Isotropic helicoids violate that relation $\langle\mathrm{Tr}\big(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{\nabla}\boldsymbol{\Omega}^{\mathrm{T}}\big)\rangle \propto \tau_{\mathrm{p}}\mathrm{C}_0$

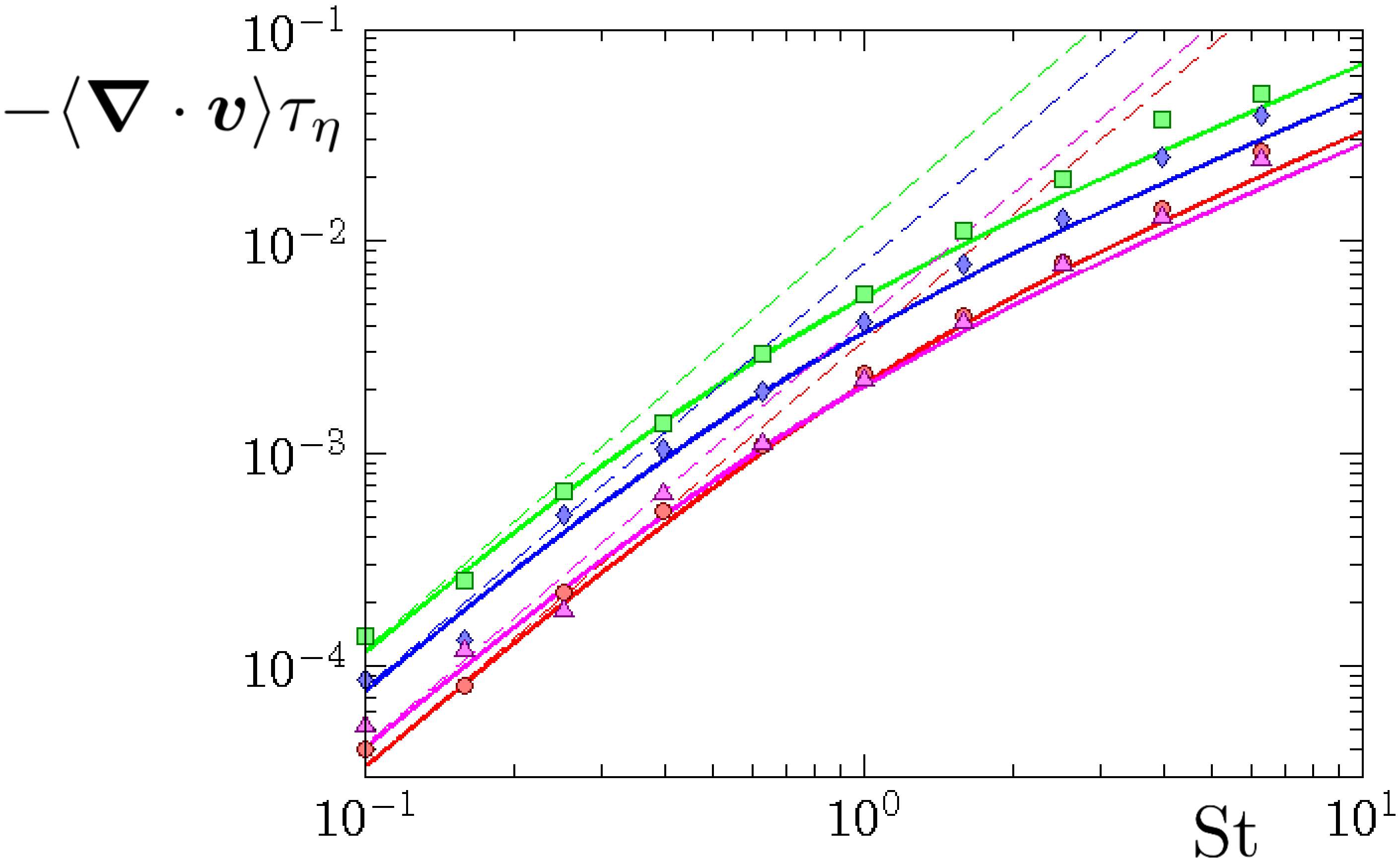
Same for parity-breaking flows $\langle\mathrm{Tr}\big(\boldsymbol{\nabla}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{\nabla}\boldsymbol{\Omega}^{\mathrm{T}}\big)\rangle \propto \tau_{\mathrm{p}}\mathrm{K}$

Helicity parameter K

$K>0$ Right-handed structures ($\boldsymbol{u}\cdot\boldsymbol{\Omega}>0$) more common

$K<0$ Left-handed structures ($\boldsymbol{u}\cdot\boldsymbol{\Omega}<0$) more common

Clustering at small St in random flow



$$\mathrm{Ku} \equiv \frac{u_0 \tau_\eta}{\eta} = 0.1$$

$$\overline{a} = 1$$

Small-Ku theory

Gustavsson & Mehlig EPL **96** (2011)

Small- St limit

- Spherical particle ($C_0 = 0$) in neutral flow
- Right-handed particle ($C_0 = 3$) in left-handed flow
- Right-handed particle ($C_0 = 3$) in neutral flow
- Right-handed particle ($C_0 = 3$) in left-handed flow

Clustering at small St in random flow

$$\langle \nabla \cdot \boldsymbol{v} \rangle \tau_\eta = -\frac{27\,\mathrm{Ku}^4\,\mathrm{St}^2}{13(27-C_0^2)(27(10+13\,\mathrm{St}+3\,\mathrm{St}^2)-10C_0^2)^3}\Big[\frac{6656}{5\pi}\overline{a}^2 C_0^2 K^2 (27-C_0^2)(27(10+39\,\mathrm{St}+15\,\mathrm{St}^2)-10C_0^2)$$

$$-192\overline{a}C_0 K\sqrt{2/\pi}(1300C_0^4-27C_0^2(2600+5070\,\mathrm{St}+2457\,\mathrm{St}^2+324\,\mathrm{St}^3)+729(1300+5070\,\mathrm{St}+4654\,\mathrm{St}^2+1845\,\mathrm{St}^3+351\,\mathrm{St}^4+27\,\mathrm{St}^5))$$

$$+4550\overline{a}^2C_0^6+852930(10+3\,\mathrm{St})^3(1+3\,\mathrm{St}+\mathrm{St}^2)-45C_0^4(21\overline{a}^2(260+507\,\mathrm{St}+195\,\mathrm{St}^2+18\,\mathrm{St}^3)-20(1300+27\,\mathrm{St}^3))$$

$$+243C_0^2(10+3\,\mathrm{St})(21\overline{a}^2(65+234\,\mathrm{St}+247\,\mathrm{St}^2+87\,\mathrm{St}^3+9\,\mathrm{St}^4)-10(2600+4290\,\mathrm{St}+1677\,\mathrm{St}^2+261\,\mathrm{St}^3+27\,\mathrm{St}^4))\Big]$$