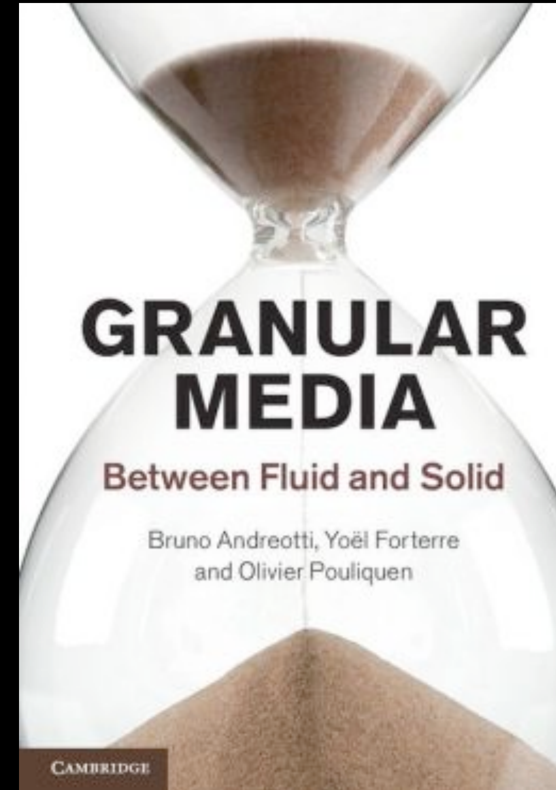
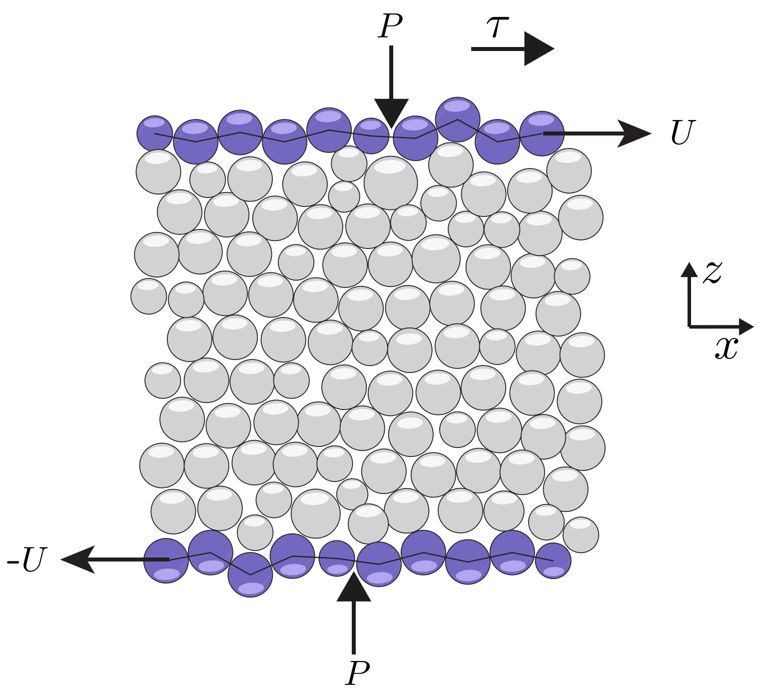
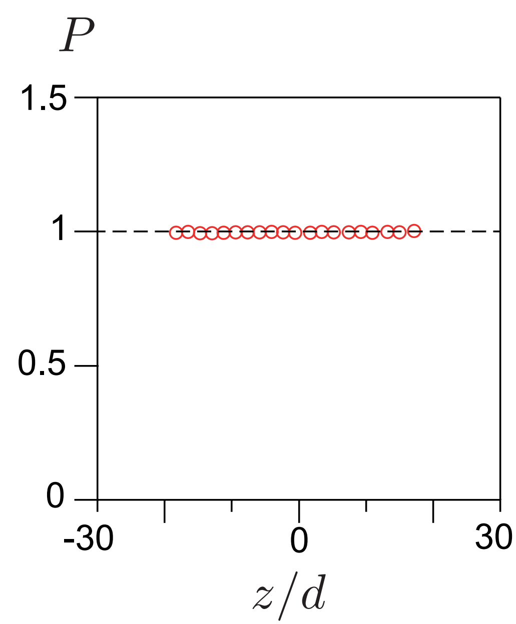
B. Andreotti
M. Bouzid
P. Claudin
E. Clément
M. Trulsson

Nonlocal rheology of granular flows

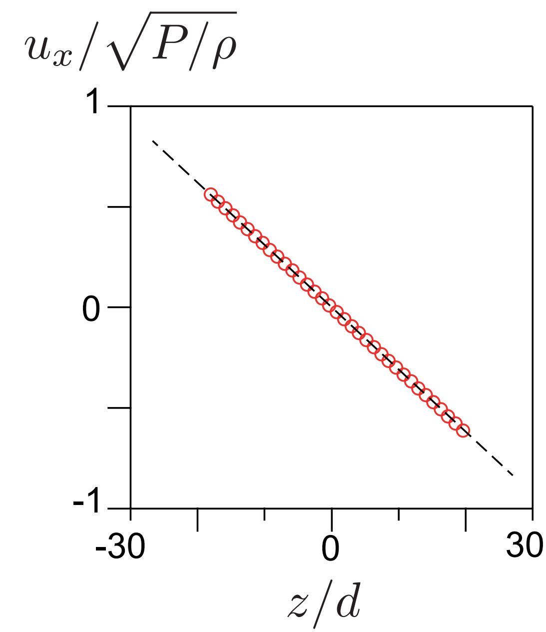


Homogeneous shear flow of rigid particles





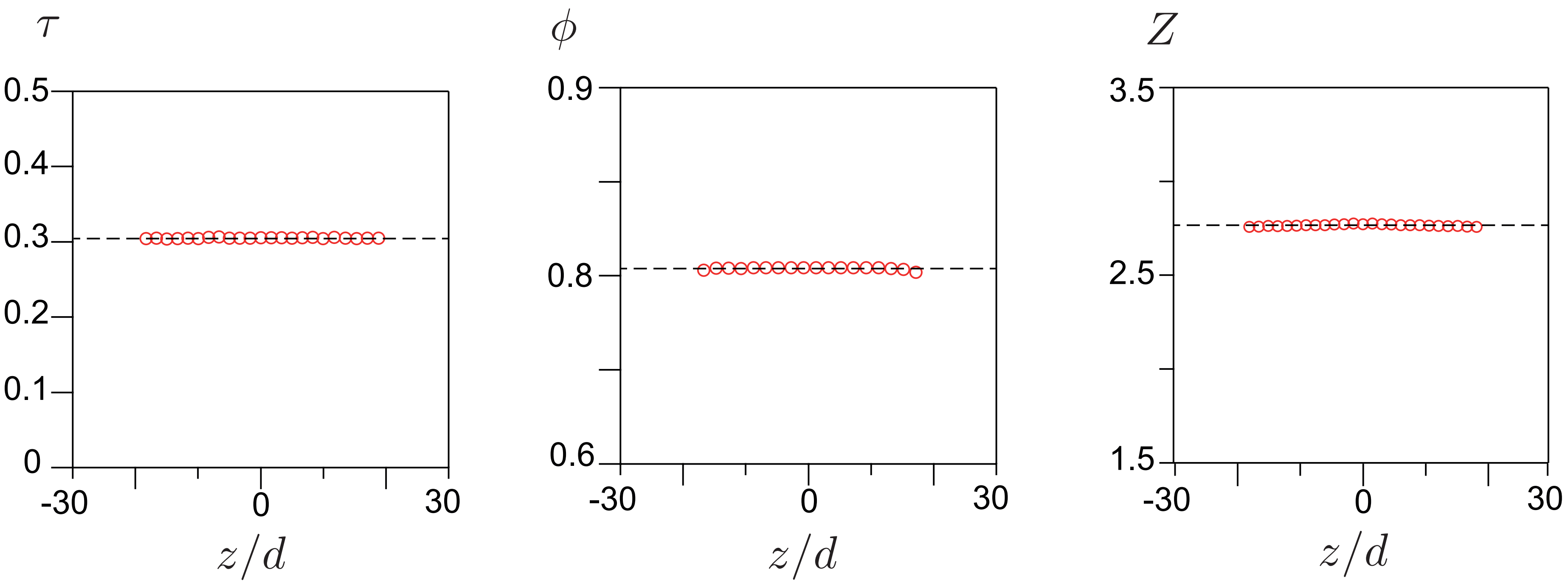
Normal stress profile



Velocity profile

Order parameter: $$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_p}}$$ Inertial number

Flow characteristics



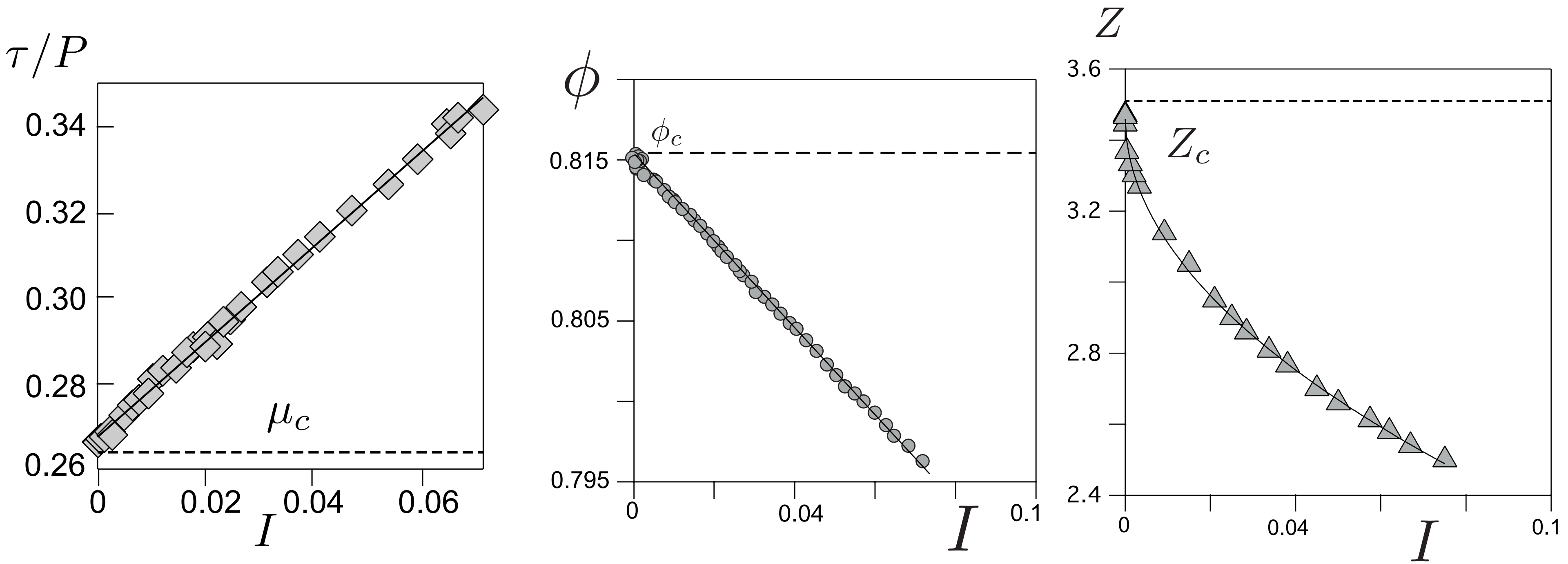
Shear stress profile | Volume fraction profile | Coordination number

What is the parameter controlling the transition from solid to liquid states ?

Solid $\tau/P < \mu_c$

Liquid $\tau/P > \mu_c$

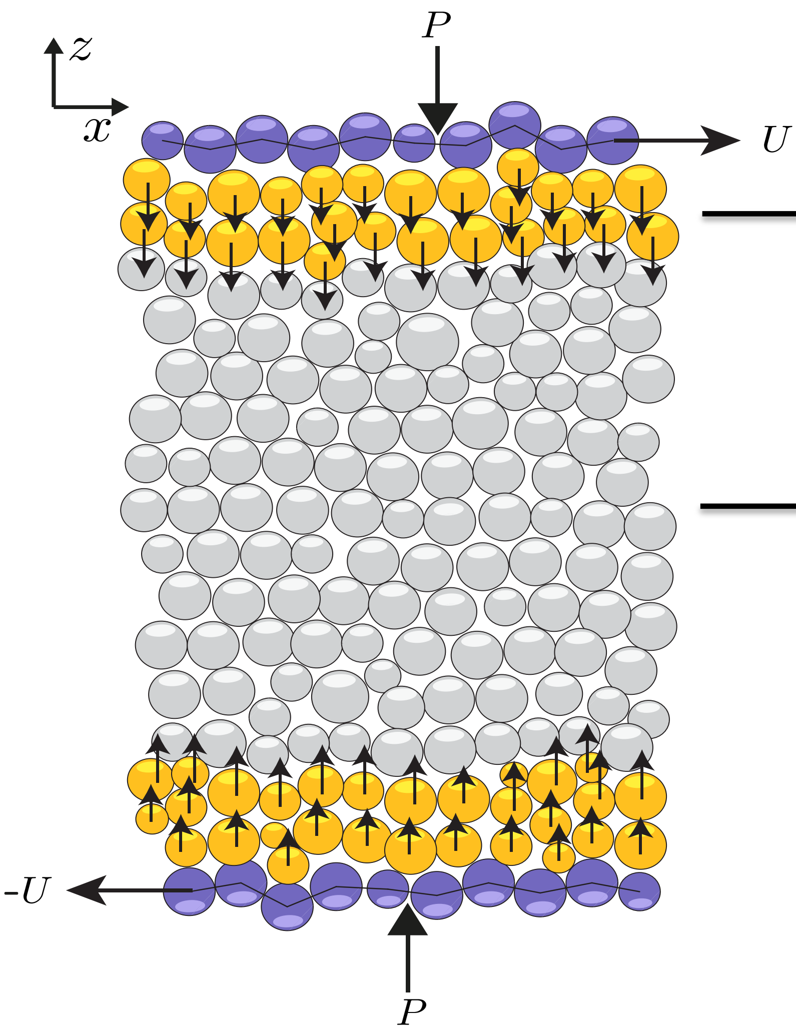
Local rheology



$$\frac{\tau}{P} - \mu_c \sim I \qquad \phi_c - \phi \sim I \qquad Z_c - Z \sim I^{\alpha}$$

Solid	$\tau/P < \mu_c$	$\phi \geq \phi_c$	$Z > Z_c$
Liquid	$\tau/P > \mu_c$	$\phi < \phi_c$	$Z < Z_c$

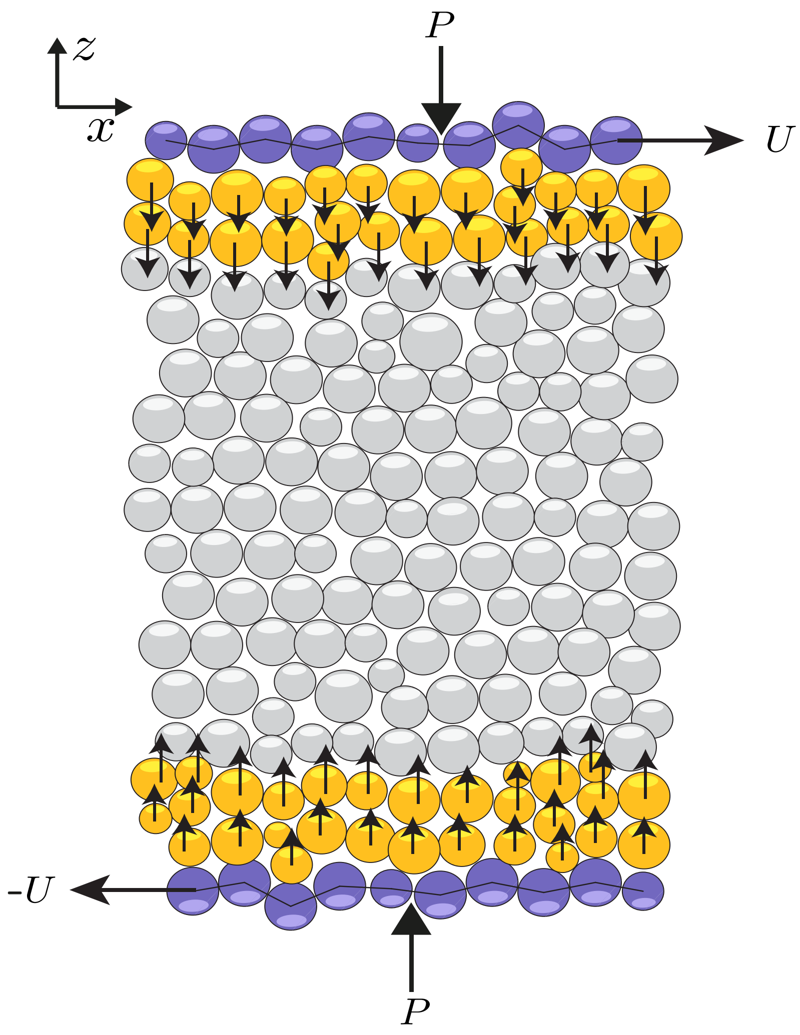
Can one create a solid/liquid interface?

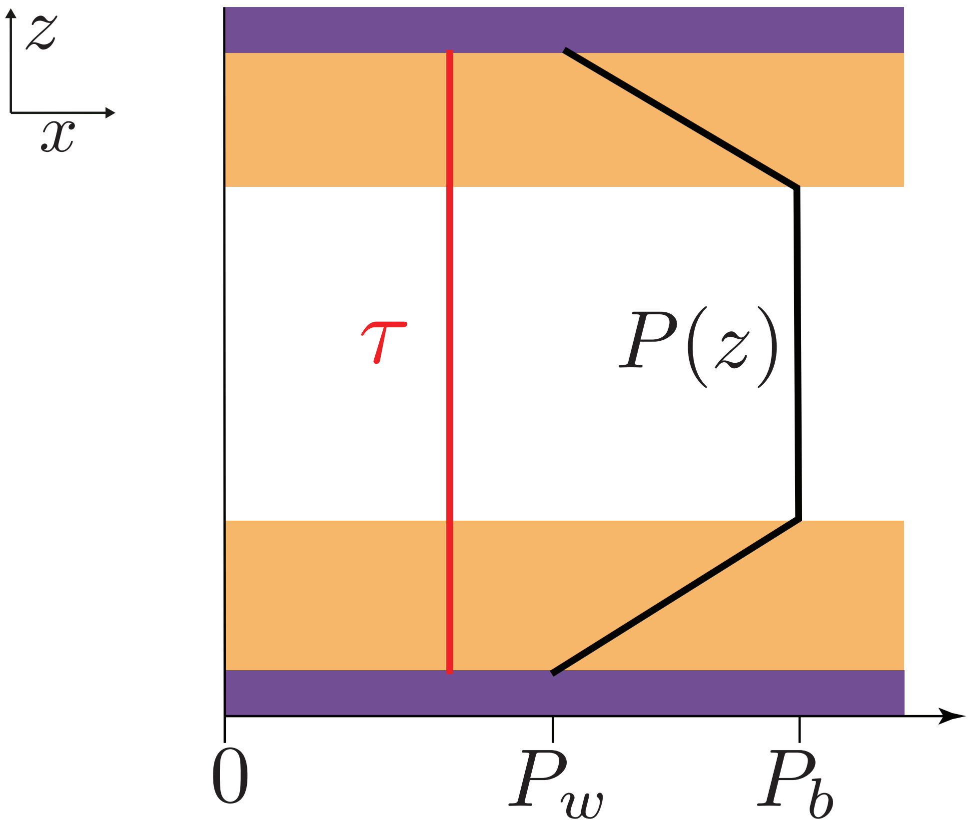


$\tau/P > \mu_c$ (liquid)

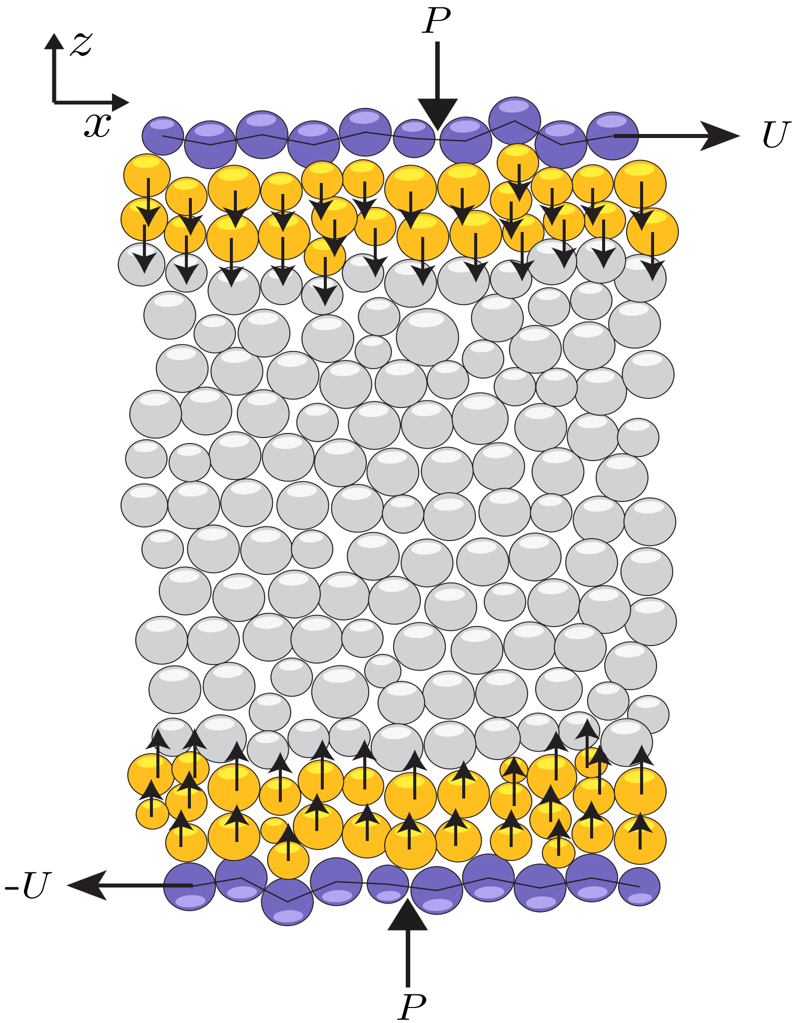
$\tau/P < \mu_c$ (solid)

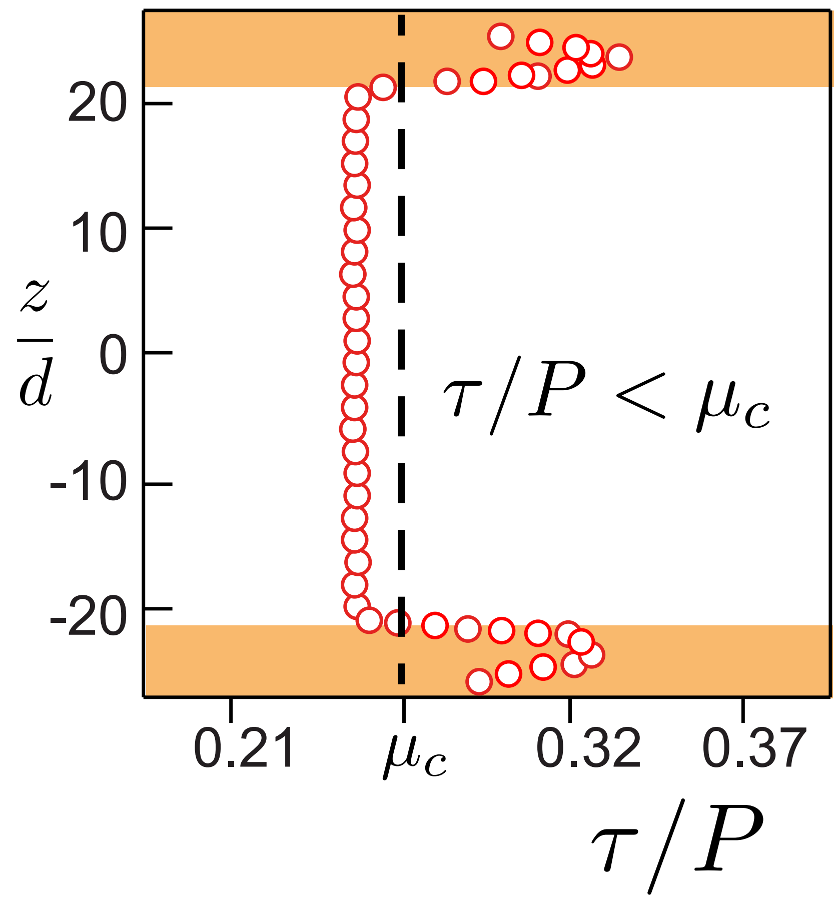
Can one create a solid/liquid interface?



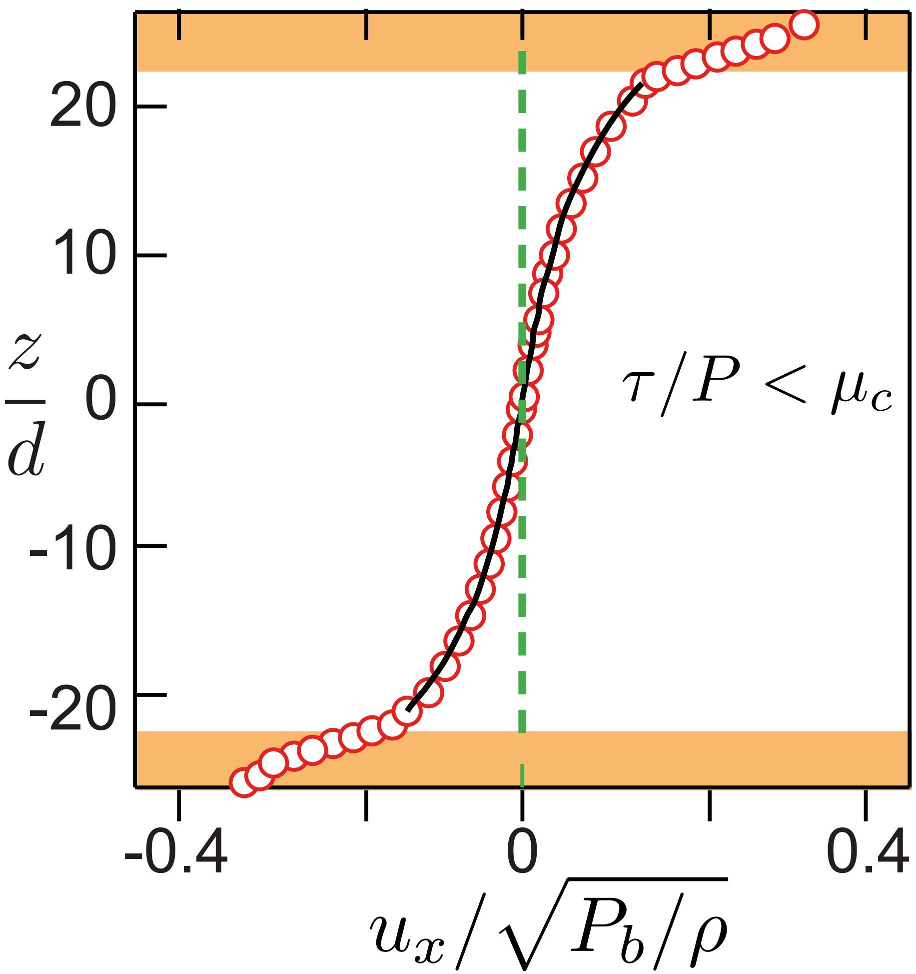


Can one create a solid/liquid interface?





E pur si muove!

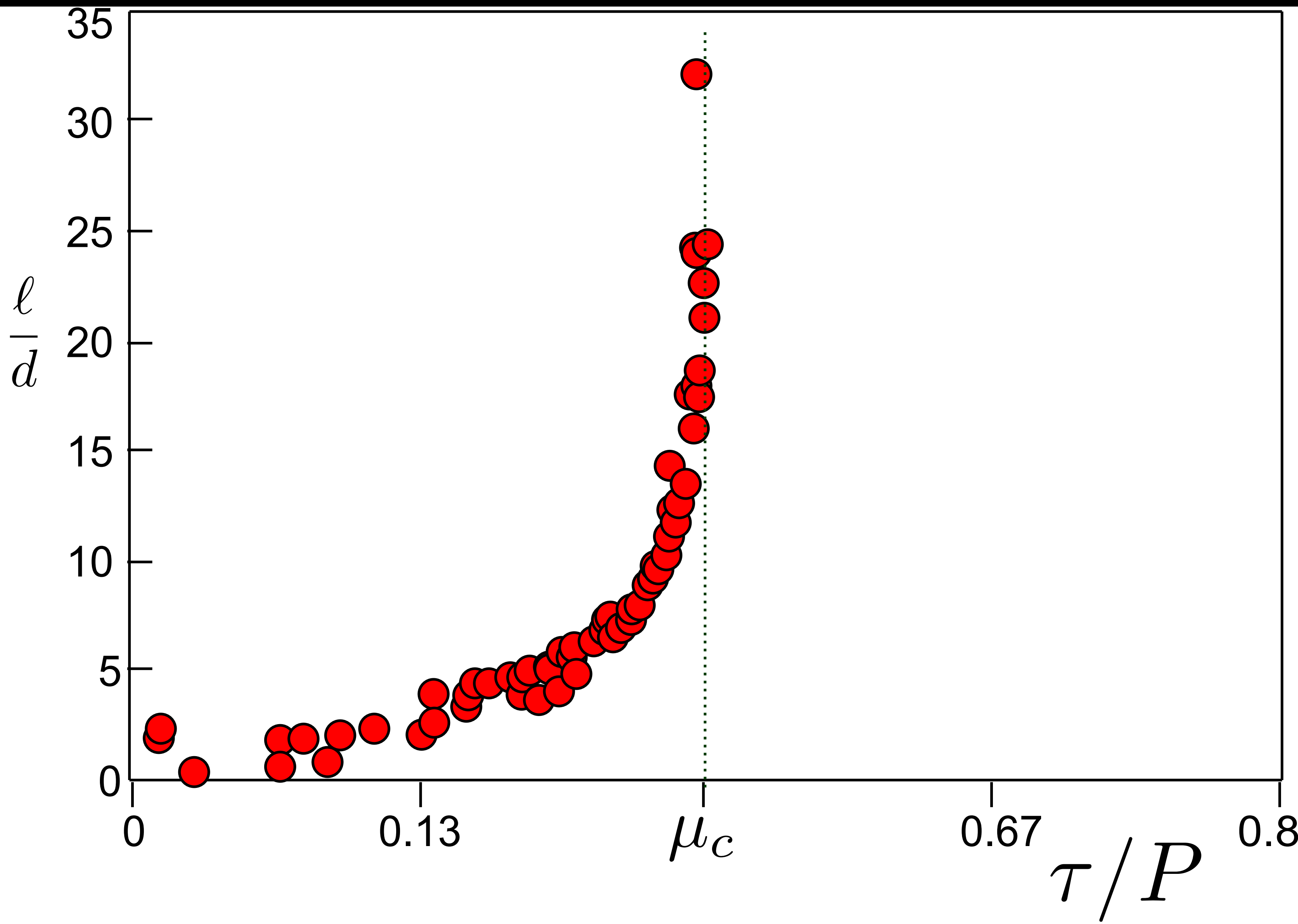


τ/P does not control the solid-liquid transition.

Linear relaxation:

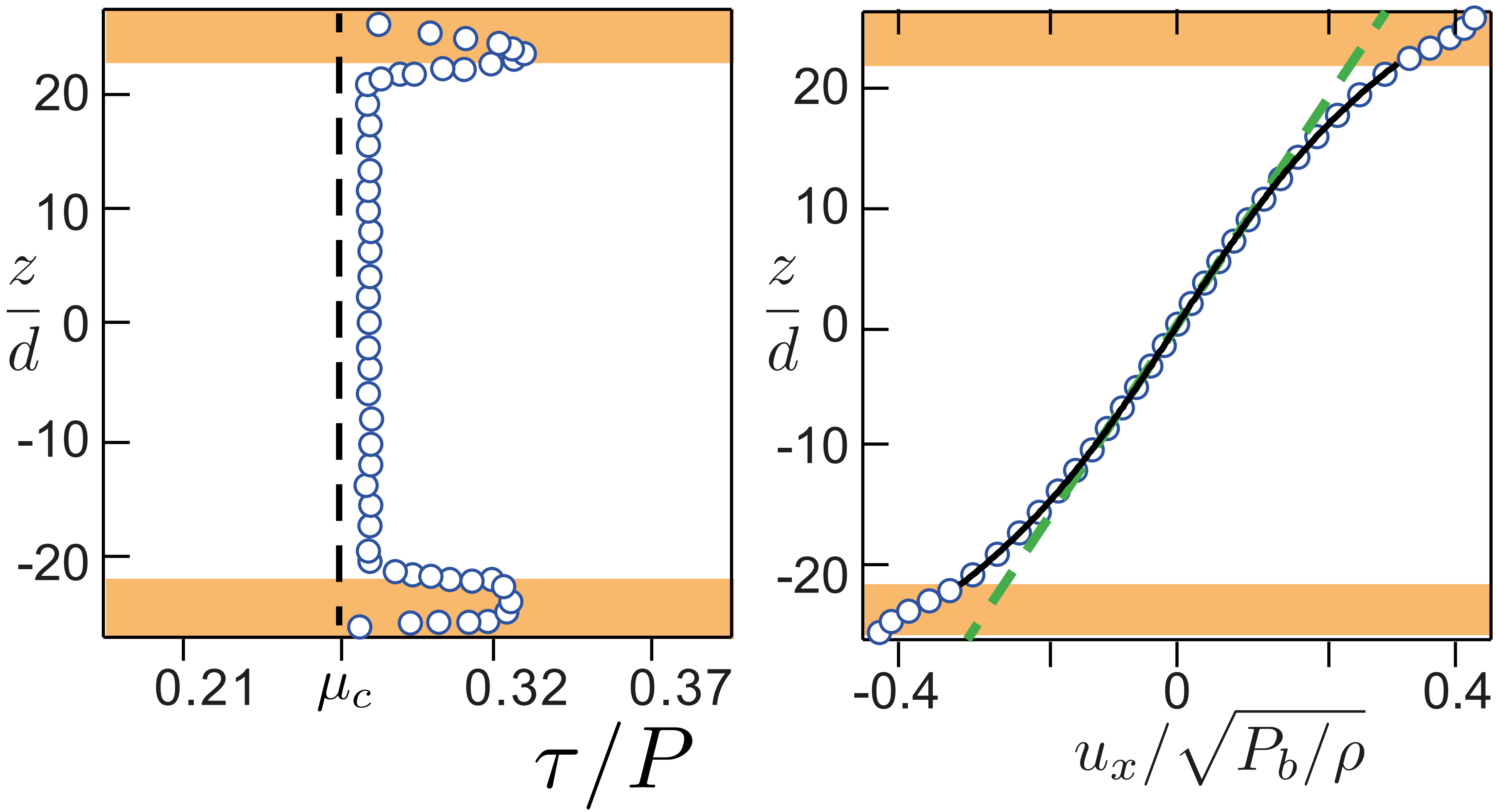
$$e^{\pm z/\ell}$$

Relaxation length of the shear rate

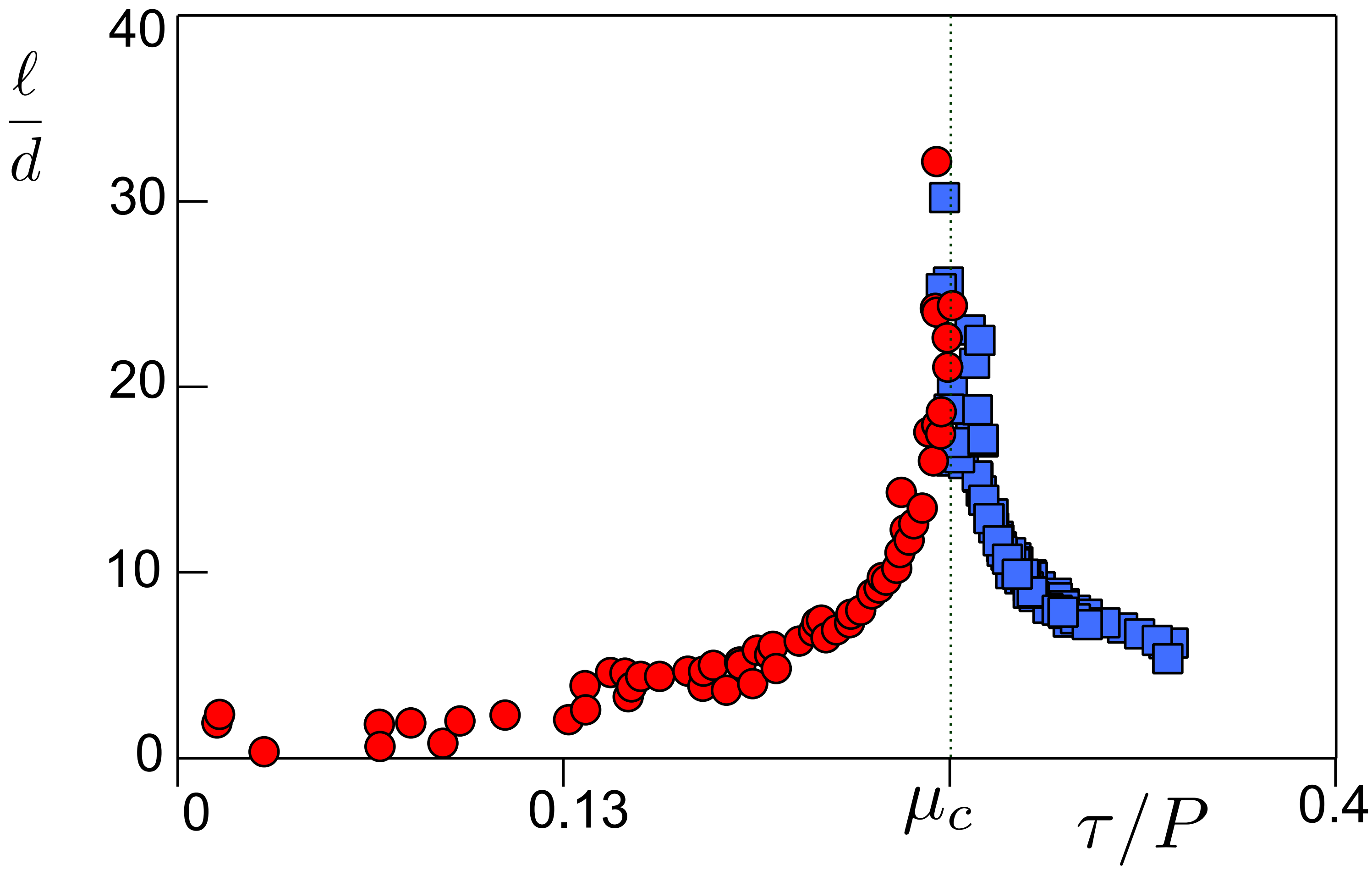


Above yielding conditions

$\tau / P > \mu_c$



Relaxation length of the shear rate



A single liquid phase: the divergence is NOT the signature of change of state.

Fluidity

- Order parameter: fluidity

$$f = 0 \qquad \text{Solid}$$

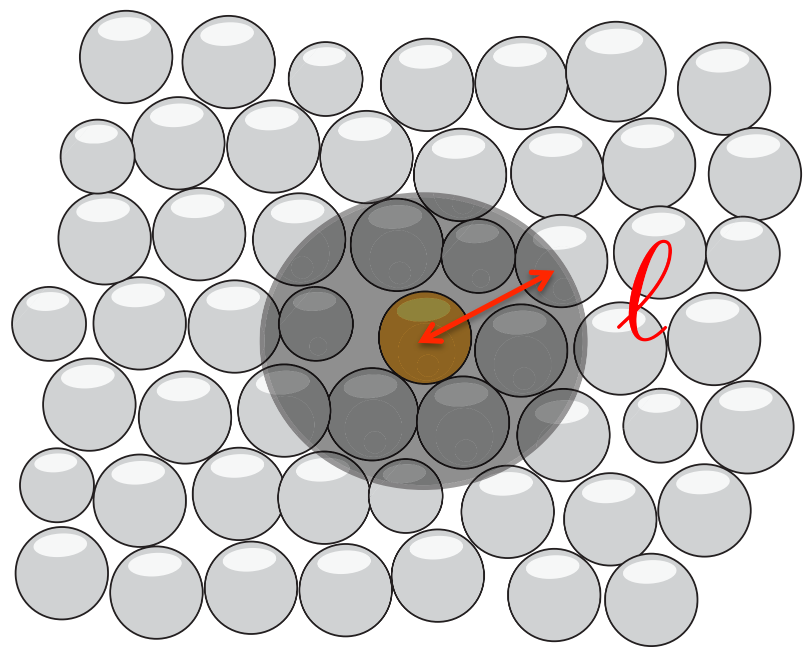
$$f \neq 0 \qquad \text{Liquid}$$

- Fluidity must depend on state variables only

$\phi \qquad I \qquad Z \qquad$ ~~τ~~

- Relative environment fluidity

$$\kappa \equiv \frac{\nabla^2 f}{f}$$

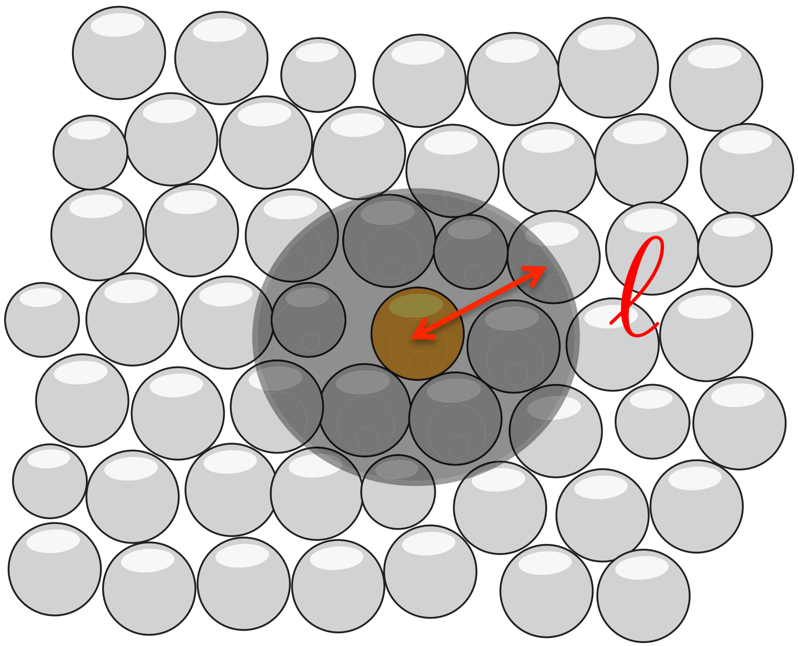


Non-local rheology

$$\frac{\tau}{P} = \mu(I)[1 - \chi(\kappa)]$$

$$\kappa \equiv \frac{\nabla^2 f}{f}$$

$$\chi(\kappa) = \nu\kappa + \mathcal{O}(\kappa^2)$$

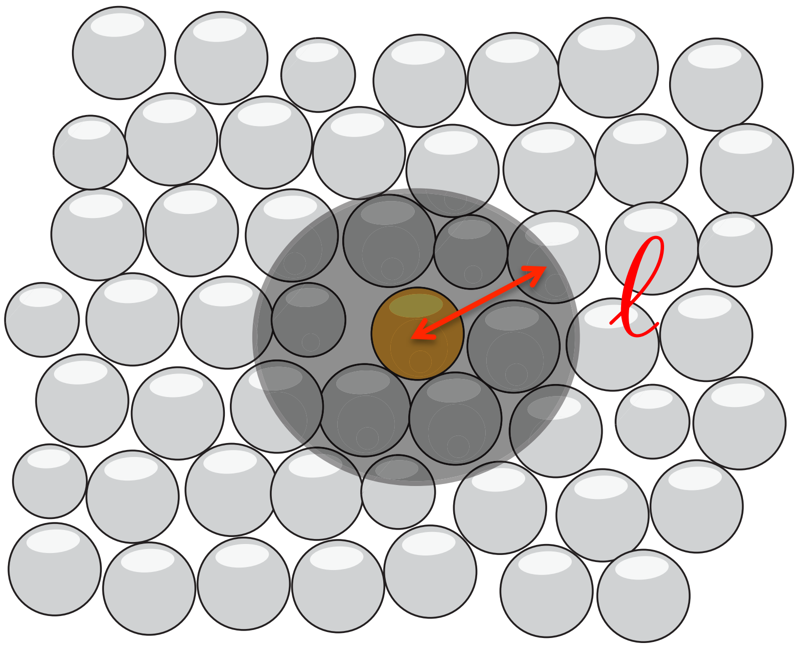


Non-local rheology

$$\frac{\tau}{P} = \mu(I)[1 - \chi(\kappa)]$$

$$f = I = \frac{|\dot{\gamma}|d}{\sqrt{P/\rho_p}}$$

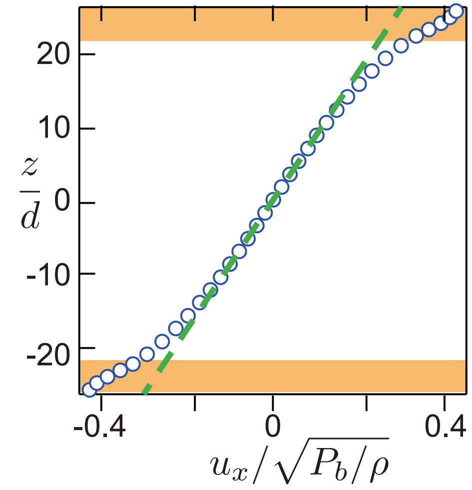
$$\chi(\kappa) = \nu\kappa + \mathcal{O}(\kappa^2)$$



Relaxation length (i)

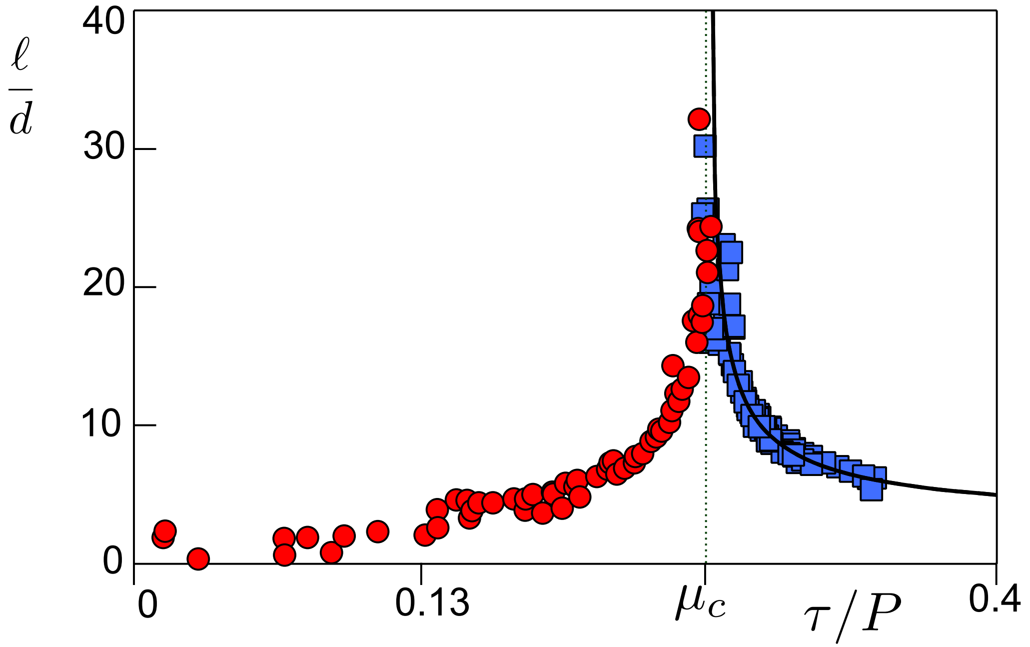
Linearisation around the base state:

$$I = I_b + \delta I$$



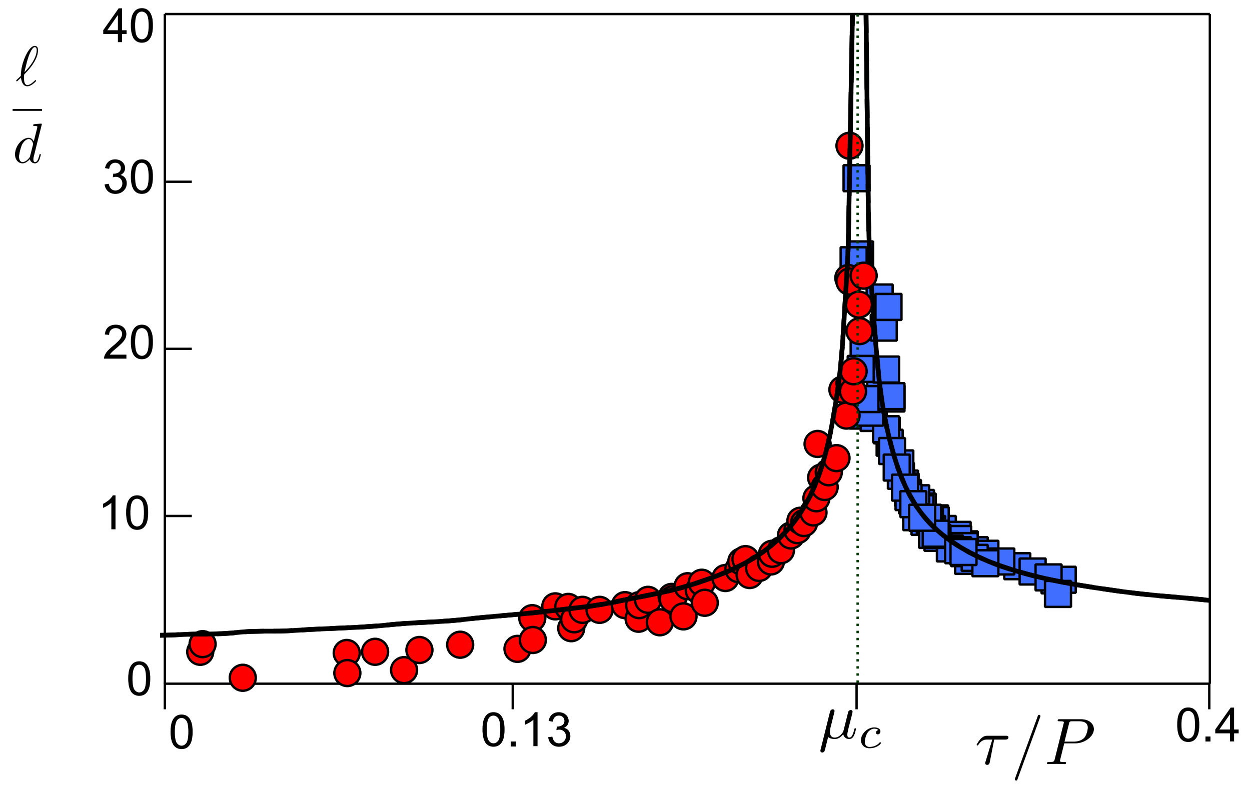
$$\ell^2 \frac{d^2 \delta I}{d^2 z} - \delta I = 0$$

$$\ell = \sqrt{\frac{\nu \tau / P}{\tau / P - \mu_c}}$$



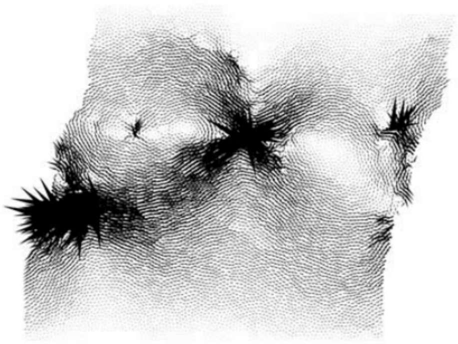
Relaxation length (ii)

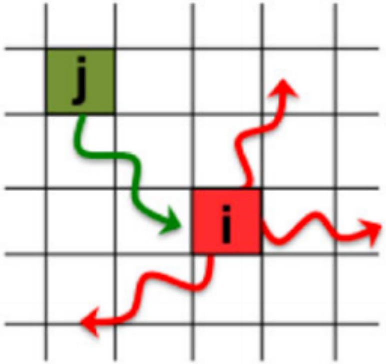
Linéarisation around the fluid marginal state $I = 0^+$



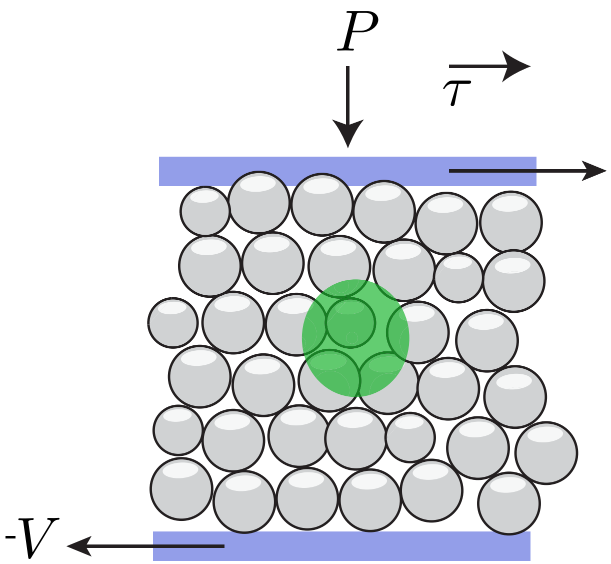
Agreement without any adjustable parameter

What is the origin of non-locality?





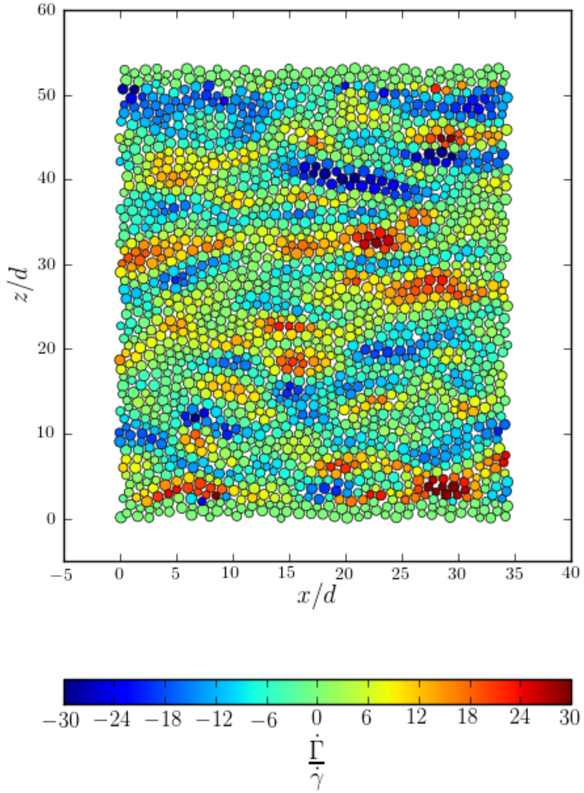
Are there localised plastic events ?



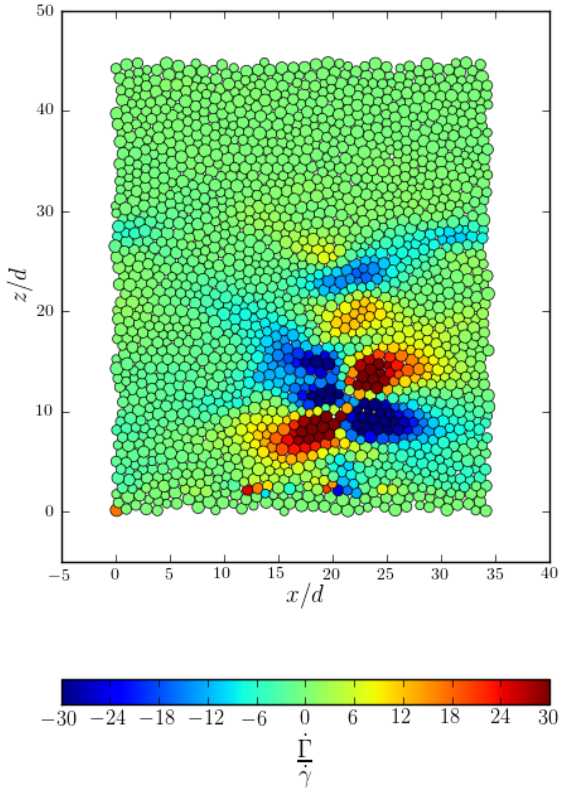
$\dot{\Gamma}(\vec{r}_i, t)$ Local contribution of a region to the mean shear rate $\dot{\gamma}$

Rigid vs soft particles

Rigid particles



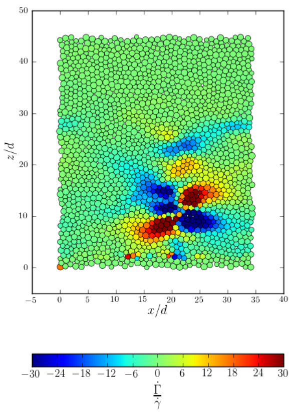
Soft particles

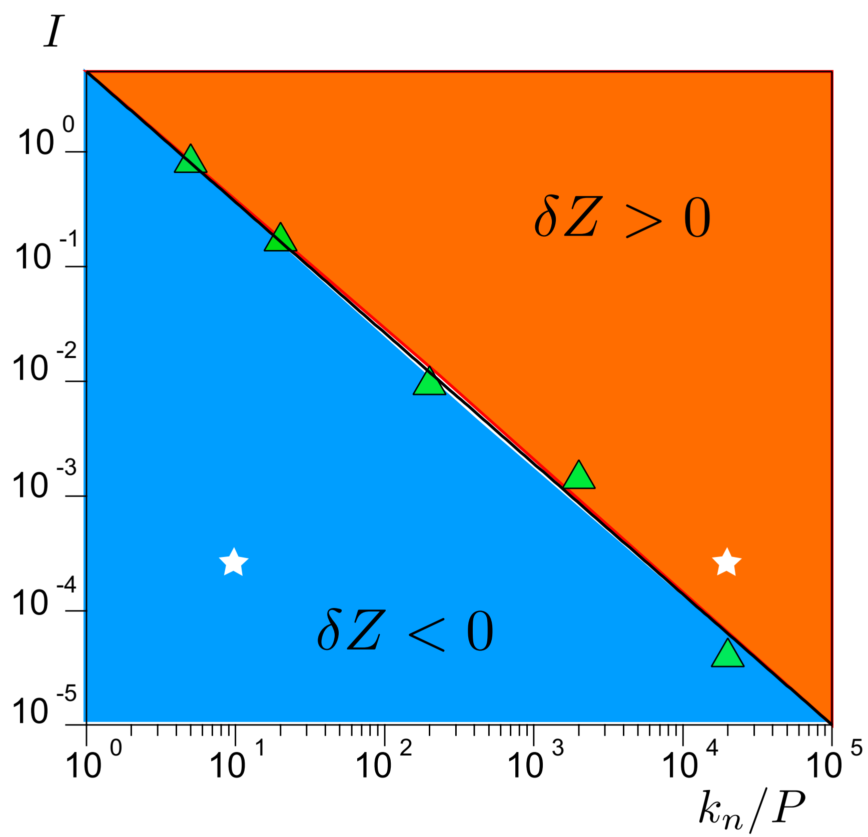


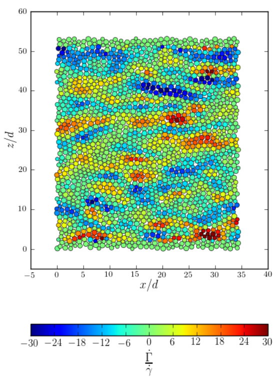
- No localized plastic events
- Permanent spatial heterogeneities

- Localized plastic events
- Very intermittent dynamics

Generalised isostaticity







The isostatic limit gives the cross-over between:

- non-locality controlled by localized plastic events
- non-locality controlled by non-affine motions