Concentration and segregation of particles and bubbles by turbulence

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1 The Problem

Understanding the spatial distribution of finite-size massive particles, such as heavy impurities, dust, droplets, neutrally buoyant particles or bubbles suspended in incompressible, turbulent flows is a relevant issue in industrial engineering and atmospheric physics. In a turbulent flow vortices act as centrifuges ejecting particles heavier than the fluid and entrapping lighter ones [1, 2]. This phenomenon produces on one side clusterization (also dubbed *preferential concentration*) on the other segregation (*de-mixing*) of particle species differing in size and densities.

Stated in a rather simplified form, i.e., assuming spherical, not-deformable particles smaller than the smallest scale of turbulence and gravity negligible, the equation of motion for a particle is [3]:

$$\ddot{\mathbf{x}} = \beta\left(\partial_t \mathbf{u} + (\mathbf{u}\cdot\partial)\mathbf{u}\right) - \left(\dot{\mathbf{x}} - \mathbf{u}\right)/\tau. \tag{1}$$

Here $\mathbf{u} = \mathbf{u}(\mathbf{x}(t), t)$ is the fluid velocity field described by the incompressible Navier-Stokes (NS) equation, while the parameters β and τ account for the physical properties of the particle. Specifically, β is a dimensionless number connected to the ratio between the particle density (ρ_p) and the fluid one (ρ_f), defined as $\beta \equiv 3\rho_f/(\rho_f + 2\rho_p)$. τ instead is the typical particle response-time, that is $\tau \equiv a^2/(3\beta\nu)$, with, a, the particle radius and, ν, the fluid kinematic viscosity. Equation (1) coupled to NS can be considered an accurate physical model as long as the particle suspension is dilute, namely it is almost collisionless and it does not exert feedback on the fluid, that is to say, it is passively advected by the flow.

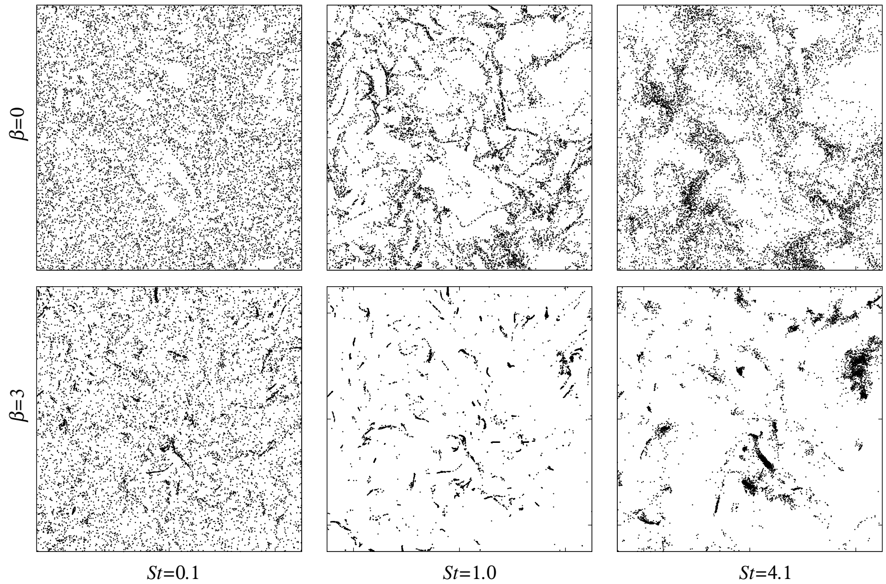

Fig. 1. Slices $320 \times 320 \times 8$ in size from a 512^3 DNS. Both very heavy particles, $\beta = 0$ (top), and bubbles, $\beta = 3$ (bottom) at different Stokes numbers, $St = 0.1, 1, 4.1$ (left to right) are reported. The underlying fluid flow field is the same in all cases. All particles were injected homogeneously into the fluid domain roughly one large-eddy-turnover-time before the snapshots.

2 A numerical study

We address the problem numerically. Here we present results from a series of direct numerical simulations (DNS) where passive suspensions of particles of variable density and size are tracked in a homogenous isotropic turbulent flow. We track up to $\sim$500 sets of particles, corresponding to couples of values in the parameter-space β-St, where $St \equiv \tau/\tau_\eta$ stands for the Stokes number and τ_η is the dissipative time-scale. The total number of particles per type ranges between 10^5-10^6. Numerics are performed at different resolutions, 128^3 and 512^3, corresponding respectively to $Re_\lambda =$ 65-185, and extended in time for few large-eddy-turnover times. As shown in fig.1, non-homogeneities in the particle/bubble distributions, their dependence on the Stokes number and demixing between different species are already evident from plain visualizations.

Correlation dimension and concentration conditioned to flow topology

To gain better insight into the small-scale features of clustering, we study the probability, $P_2(r)$, that the distance between two particles is less than r. In the small-distance limit such probability has a power law behavior, $P_2(r) \sim r^{D_2}$.

The exponent D_2, called correlation dimension of the spatial distribution, can be used as an estimator for the dimension of the set on which particles accumulates. Whether particle distribute *locally* uniformly D_2 equals the spatial dimension 3. If instead $D_2 < 3$ we say that particles accumulate onto a fractal set.

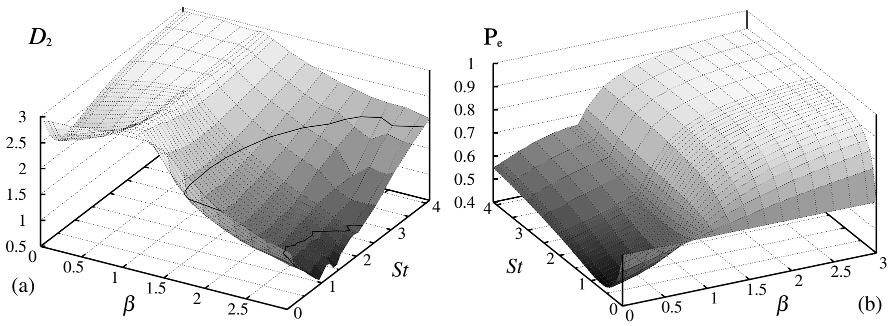

Fig. 2. (a) The correlation dimension D_2 as a function of the density parameter β and the Stokes number. Isolines are drawn at $D_2 = 1, 2$. (b) The probability P_e to find a particle in elliptic regions of the flow versus β and St. Note that for fluid tracers ($St = 0$ particles) it is here $P_e \simeq 0.6$, i.e., as expected elliptic regions in a turbulent flow extend over larger volumes than strain regions.

We observe that both heavy and light particles at small St numbers concentrate on fractal sets, see fig.2(a), the minimum of D_2 being around $St \simeq 1$. Heavy particles always have D_2 above 2, light particles instead reach even D_2 values below 1. Indeed, the extremely strong agglomeration occurring for light particles produces here decimation of statistics and noisy D_2 results for ($\beta \gtrsim 2$). Nevertheless, we may conclude that at small-scales filament-like clusters are expected for light particles while heavy particles agglomerate on surface-shaped regions.

Segregation is addressed by looking at particle concentrations conditioned to the local topology of the flow field. Vortical (also called *elliptic*) regions of the flow are defined as the positions where the eigenvalues of the local strain matrix $(\partial_i u_j)$ have imaginary parts [4]. The measure of the probability (P_e) to find a particle, of given β-St value, in an elliptic region of the flow is reported in fig.2(b). Heavy particles are lacking in vortical regions, while extremely light particles concentrate almost completely in elliptic regions.

References

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