Finite-Size Lyapunov Exponents: applications to transport in the oceans

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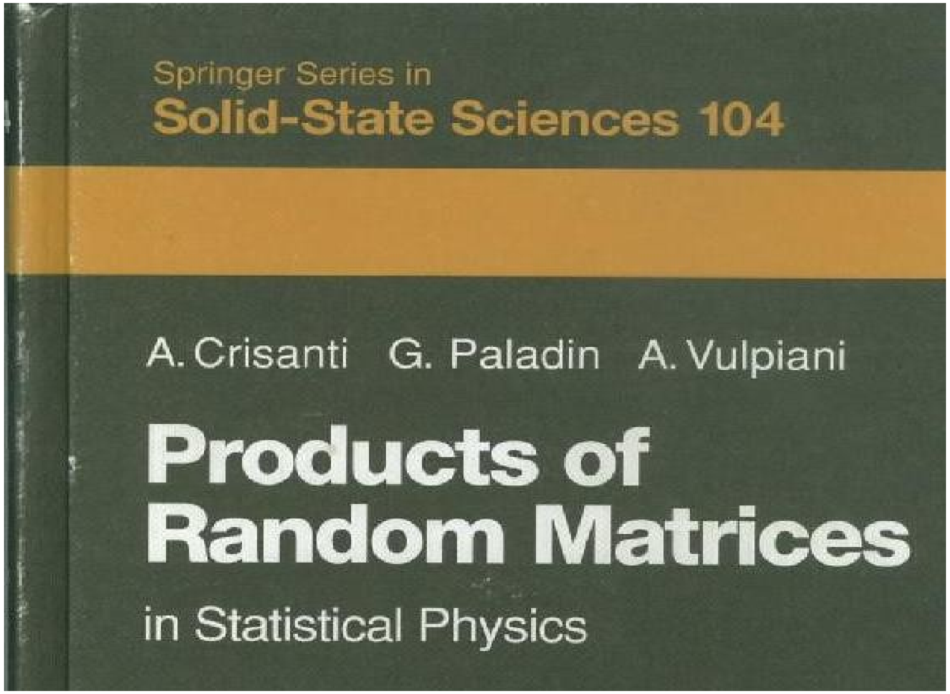
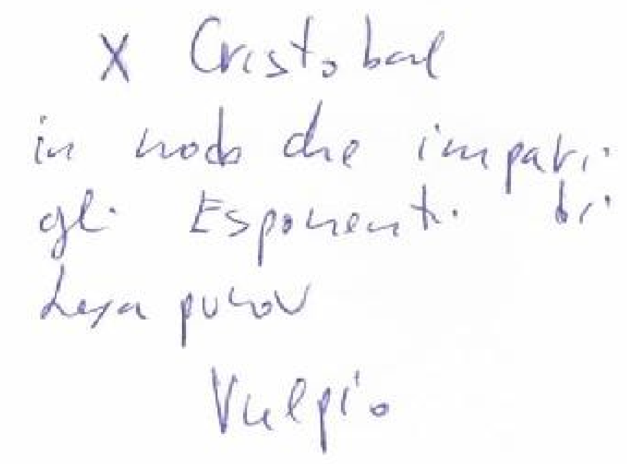
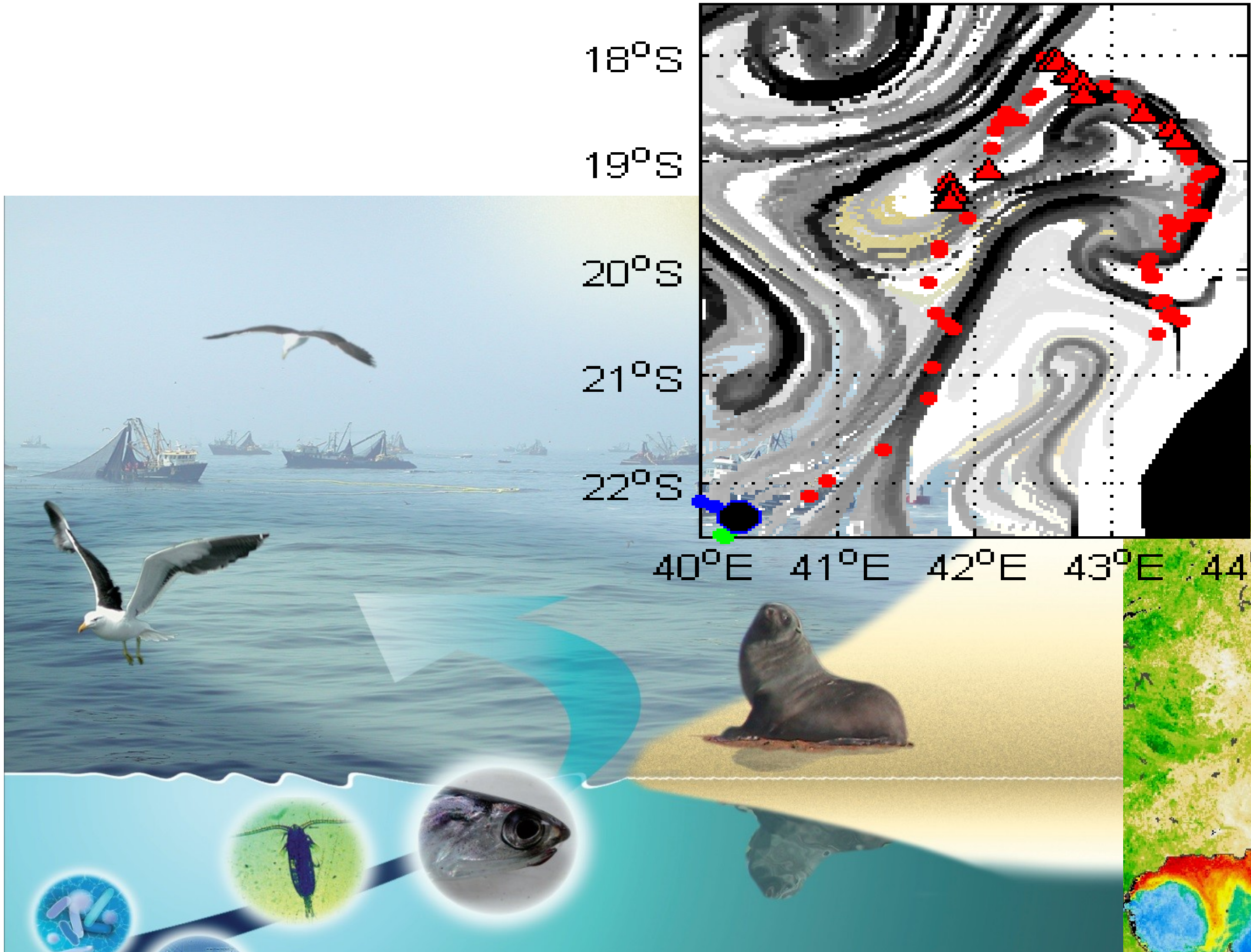




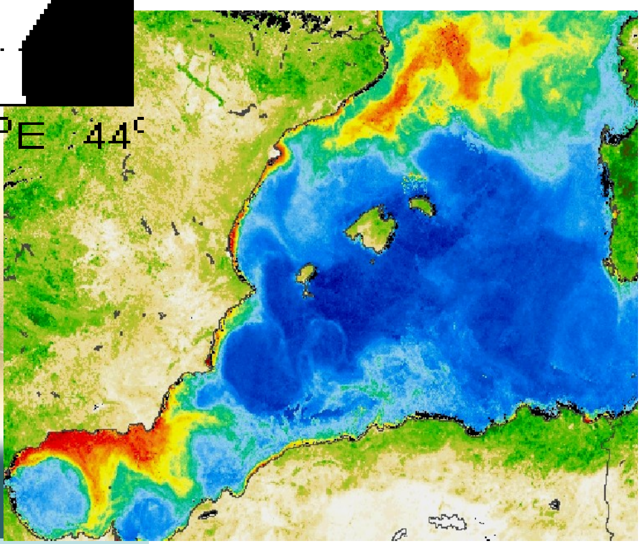
For Cristobal, to learn about Lyapunov Exponents

- Aurell E, Boffetta G, Crisanti A, Paladin G and Vulpiani A, 1996, Phys. Rev. Lett. 77 1262.
- Aurell E, Boffetta G, Crisanti A, Paladin G and Vulpiani A, 1997, J. Phys. A: Math. Gen. 30 1

- Boffetta, Lacorata, Redaelli, Vulpiani, Physica D (2001).



Frigatebirds over the Mozambique channel, September 2003
Tew-Kai et al.
PNAS 106, 8245 (2009)



Chlorophyll from SeaWiFs, monthly composite April 2000

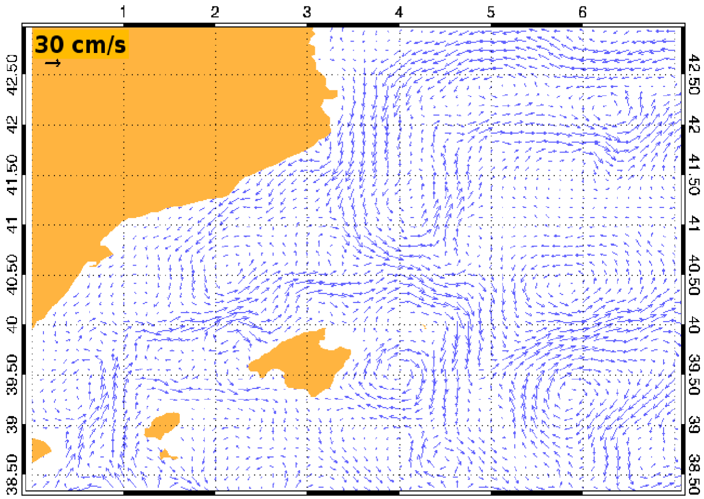
Flow/biology interaction from the first trophic levels --phytoplankton-- to the top predators

OUTLINE

- The dynamical systems approach to fluid transport: manifolds, Lyapunov Exponents, Lagrangian Coherent Structures (LCS) …
- Finite-size Lyapunov exponents. Impact of flow structures (fluid stretching) on:
  - Phytoplankton.
  - Some studies of 3d LCS.
  - Top marine predators: Frigatebirds.
  - Oxygen Minimum Zones.

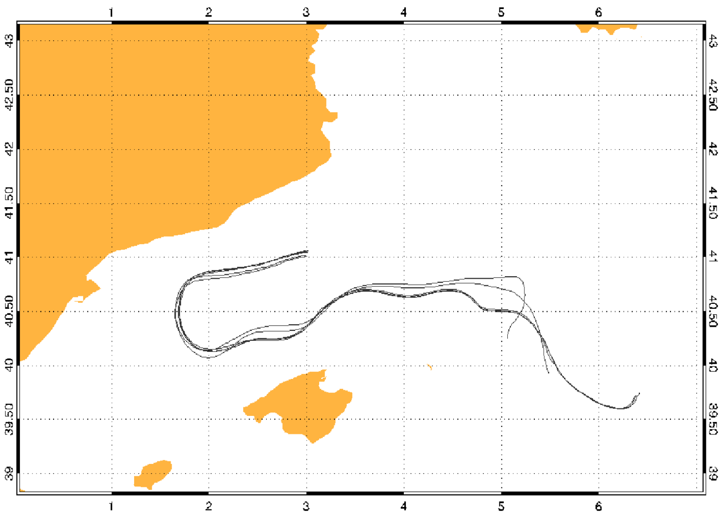
The dynamical systems approach to fluid transport

Eulerian description



One deals at any time with velocity field at any spatial point in the fluid.

Lagrangian description



One deals with trajectories of fluid particles.

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

Connection between the Eulerian and Lagrangian description

Lagrangian dynamical system:

deduction of the phase portrait from the velocity field

Phase portrait of the velocity field ⟷ ***Physical meaning in fluid dynamics***

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

ODEs

Phase portrait

KAM tori, stable/unstable manifolds

Chaotic sets

.....

=>

Velocity field

Transport organization.

Transport barriers, eddies, fronts and avenues

Mixing regions

.....

Aref, H. (1984). Stirring by chaotic advection. Journal of fluid mechanics, 143, 1-21.

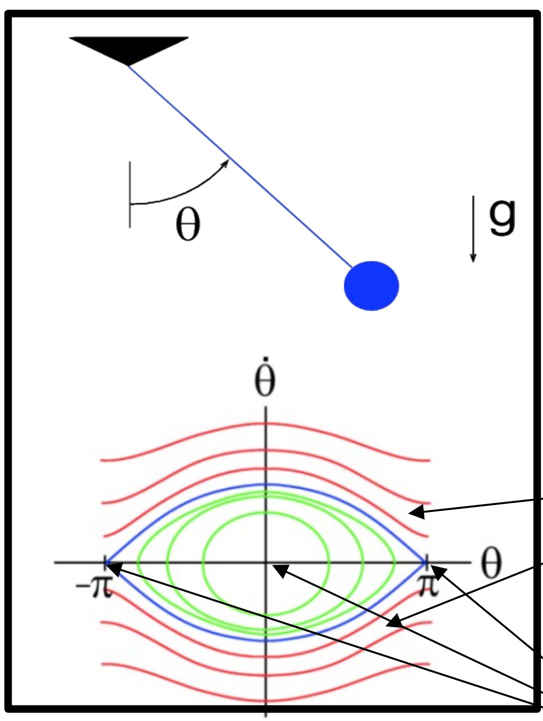
Crisanti, A., Falcioni, M., Vulpiani, A., & Paladin, G. (1991). Lagrangian chaos: transport, mixing and diffusion in fluids. La Rivista del Nuovo Cimento, 14(12), 1-80.

WHICH ARE THE RELEVANT LINES?

Trajectories of two-dimensional time-independent flows are organized by the fixed points of the dynamical system $\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t))$

Global flow geometry understood by studying **invariant manifolds** of the **fixed points**.

Pendulum



Stable manifold: trajectories asymptote the fixed point $t \to \infty$

Unstable manifold: trajectories asymptote the fixed point $t \to -\infty$

If hyperbolic: Stable and unstable manifolds $\to$ separate regions of distinct motion

Manifolds

Fixed points

How to find separatrices in time-dependent flows?
Separatrices divide regions of qualitatively different dynamics

↓

We measure stretching with Lyapunov exponents

Separatrices ⟶ Lagrangian Coherent Structures (LCS)

PARTICLE TRAJECTORIES CANNOT CROSS THEM, SO THEY ARE A TEMPLATE FOR TRANSPORT.

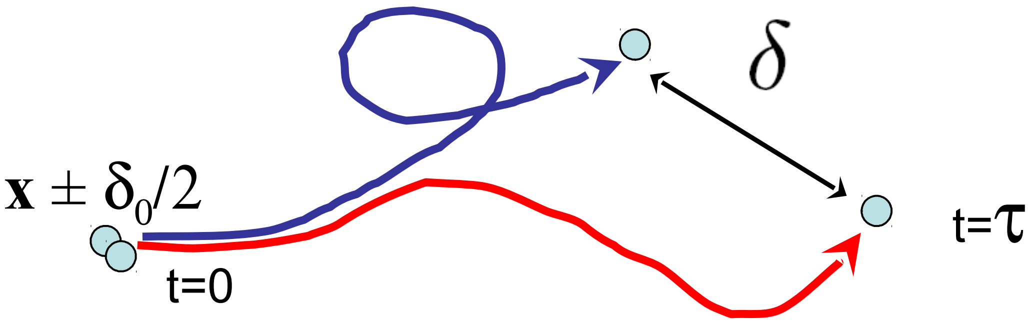
Finite-size Lyapunov exponent (FSLE)

$$\lambda(t) = \lim_{\|\delta(0)\| \to 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$

Finite-**time** Lyapunov exponent

$$\lambda = \lim_{t \to \infty} \lambda(t)$$

Lyapunov exponent



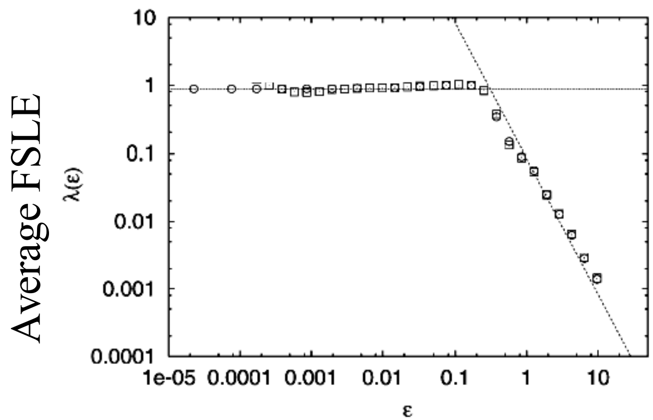
$$\lambda(\delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}$$

All the quantities are also functions of the initial position and time:

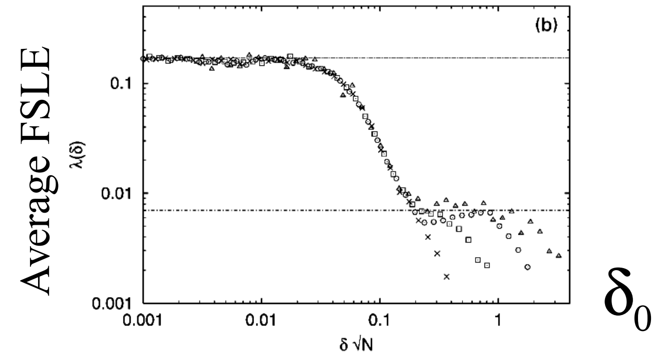
$$\lambda(\mathbf{x}, t, \delta_0, \delta_f)$$

The FSLE was originally introduced to quantify dispersion from non-infinitesimal initial separations (Aurell et al. 1997)

Chaotic map

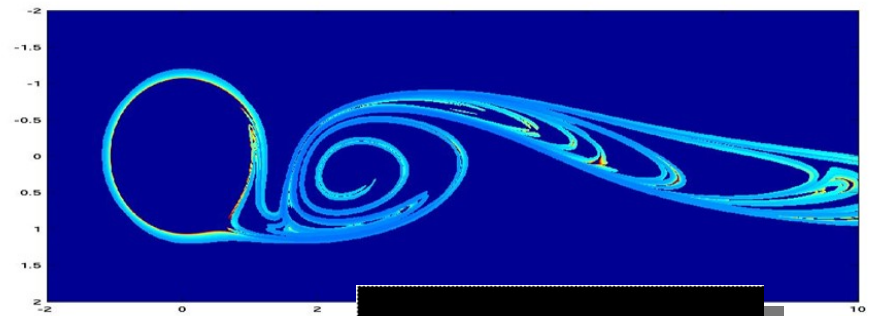


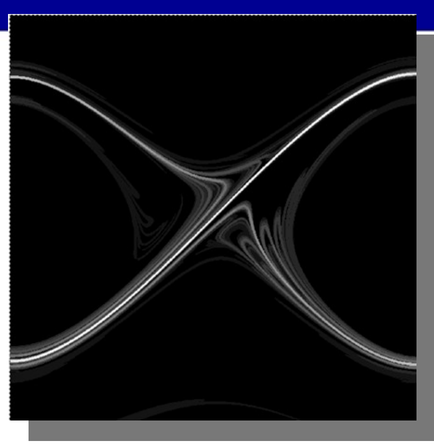
System with several time scales



δ_0

(coupled maps)



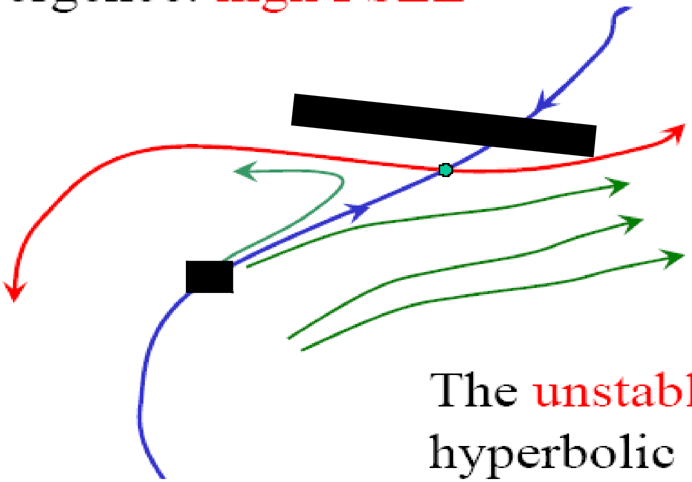


But as for FTLEs (Haller, 2001) there heuristic arguments (Joseph and Legras, 2002) relating **lines with high values of backward and forward FSLE** to **attracting** and to **repelling material lines**

≈ Lagrangian Coherent Structures

Catching Lagrangian Coherent structures with ridges (maxima) of FSLEs

The idea is that initial conditions close to the stable manifold of a hyperbolic trajectory or set will show strong divergence: high FSLE



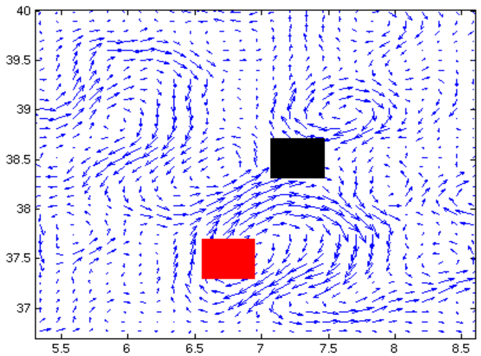
Stable and unstable manifods are also called Lagrangian Coherent Structures

The unstable manifold of hyperbolic sets would be marked by high FSLE in the time backwards direction

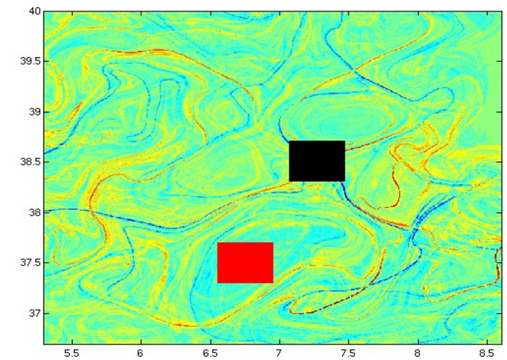
The spatial dependence of the FSLE allows the detection of stable and unstable manifolds of hyperbolic objects

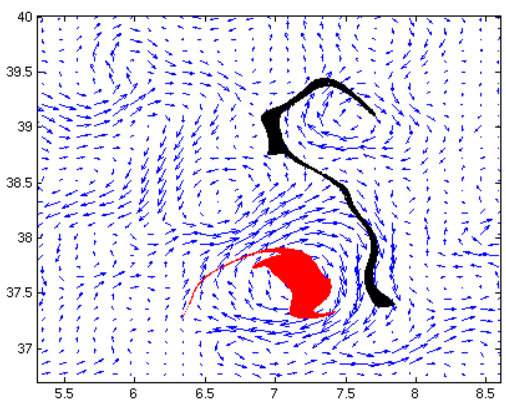
Velocity field

FSLEs

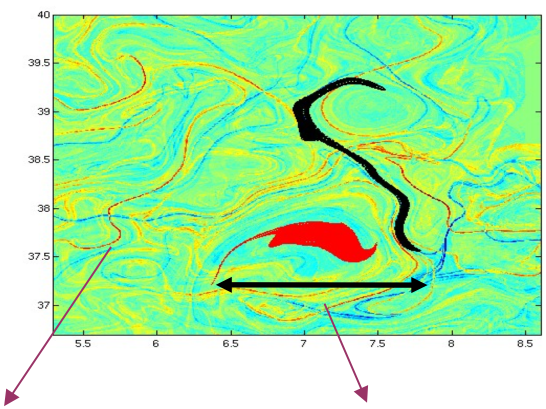


Initial conditions





After several days



Submesoscale filament

Mesoscale eddy

The strongest lines are seen to organize tracer flow

Lagrangian Coherent Structures

The Economist

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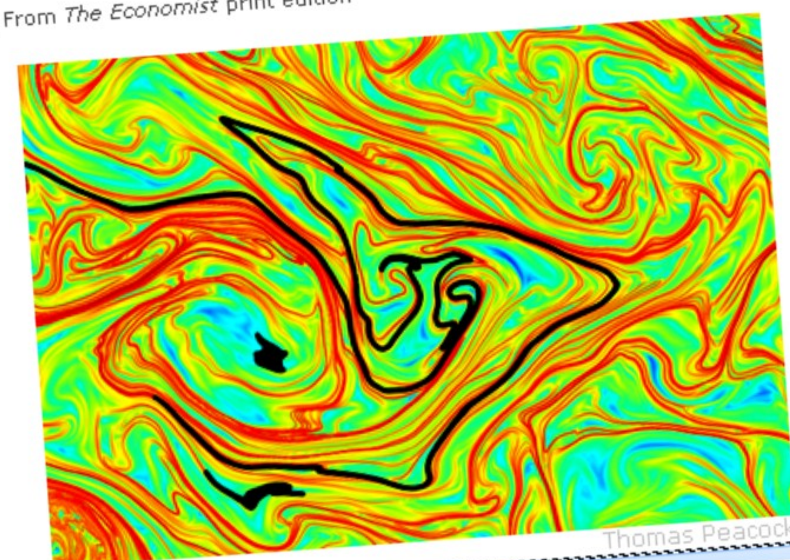
Lagrangian coherent structures

The skeleton of water

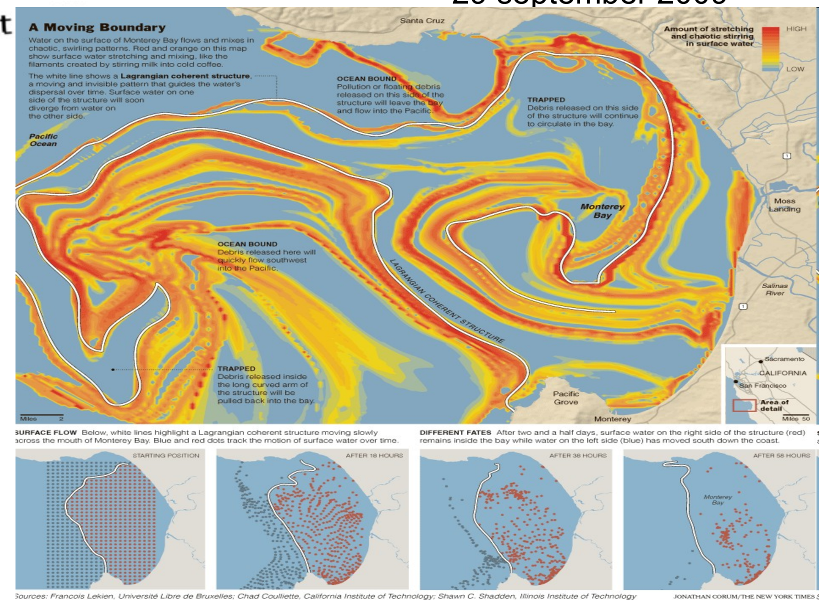
Research is revealing a hidden structure within liquids and gases that the movement of everything from pollution to aeroplanes

Nov 12th 2009 | From *The Economist* print edition



The New York Times

29 september 2009



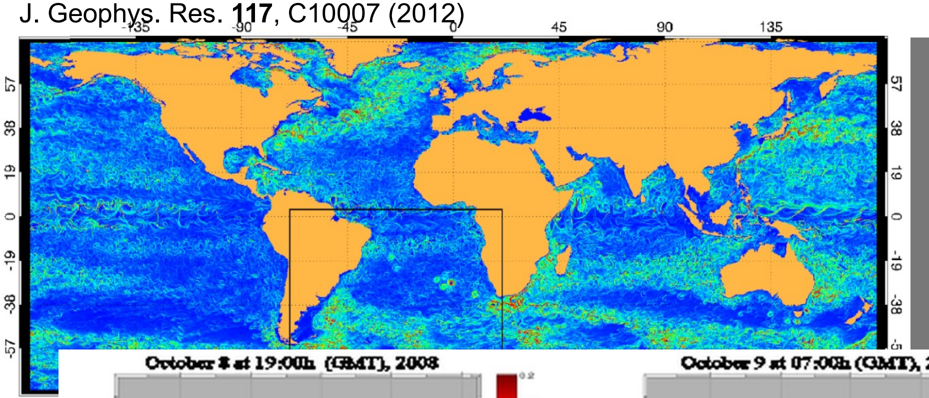
G. Haller, Ann. Rev. Fluid Mechanics (2015).

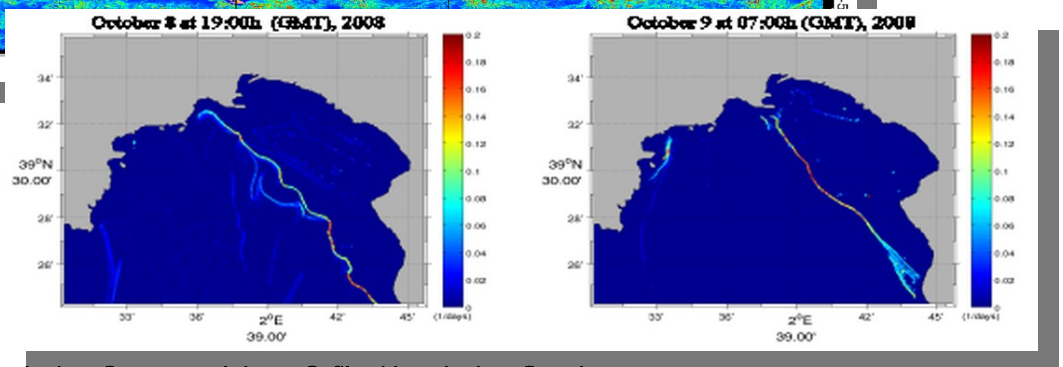
Lagrangian approaches to transport and mixing

- Geometric, local, … : FTLE, FSLE, geodesics, variational theory, M function, …
- Set-oriented, probabilistic ,….: Transfer operator, coherent sets, eigenvectors and singular vectors, networks, …
- Detailed view of single events
- Statistical (climatological) descriptions

Our guiding goal here:
Review the FSLE as a proper tool to study the impact of flow transport and mixing on biological processes

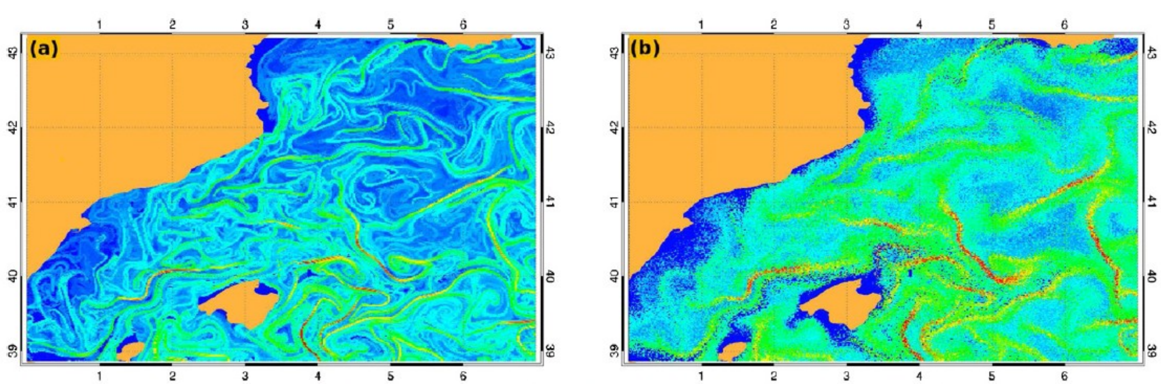
Hernández-Carrasco, López, Hernández-García, Turiel,
J. Geophys. Res. **117**, C10007 (2012)





Hernández-Carrasco, López,Orfila, Hernández-García,
Nonlinear Processes in Geophysics **20**, 921-933 (2013)

Bahía de Palma



Any advantage in using FSLE in Lagrangian studies?

- Easy switching between local and statistical approaches
- In oceanographic contexts it is usually straightforward to identify the relevant spatial scales: Rossby radius, coastal features
- Trajectories can be nonsmooth (noise …)

Disadvantages:

- No distinction between hyperbolic, shear, … structures
- Lack of analytical approaches (but see Tzella and Haynes, PRE 2010, Karrasch and Haller, Chaos 2013)
- As for FTLE, not all high FSLE structures have a clear impact on flows. Need to check with actual particle trajectories

Hernández-Carrasco et al.
Ocean Mod. 36, 208 (2011)

Mesoscales (1-100 km and 1day- 1year) are crucial for oceanic processes

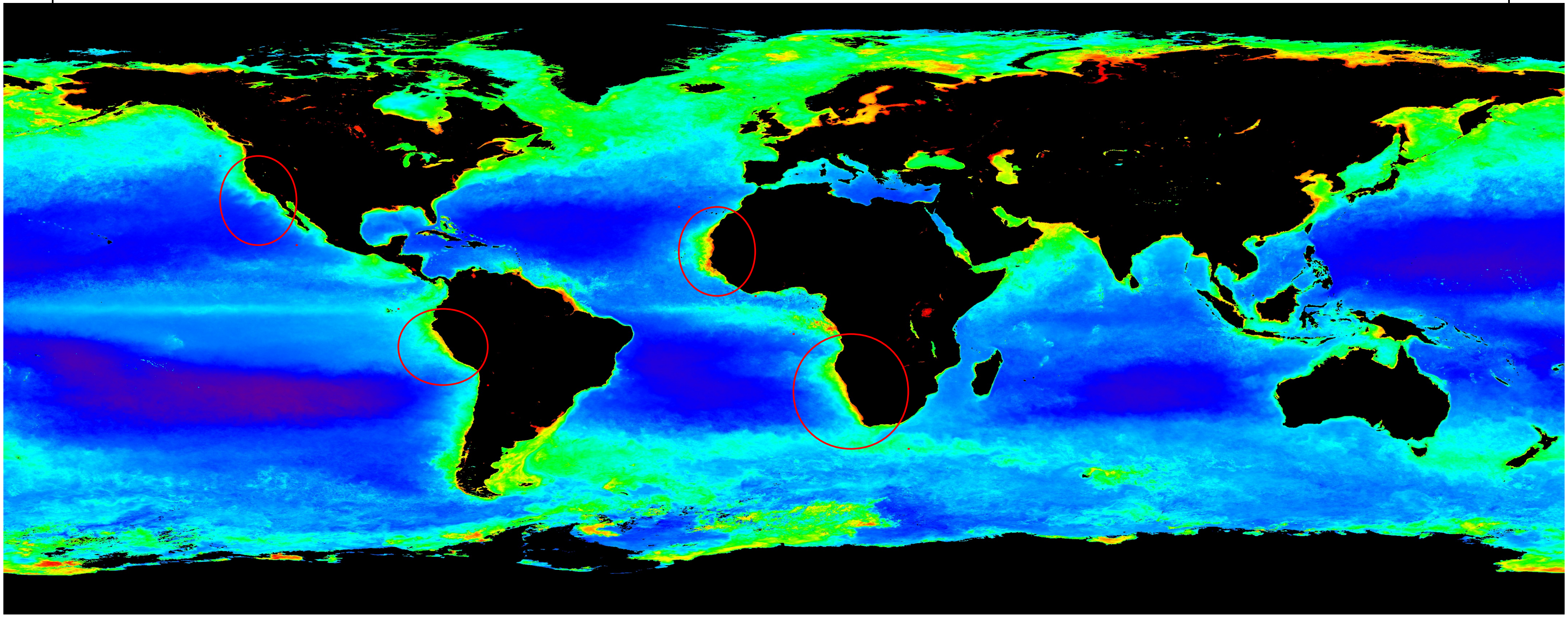
VELOCITY DATA FROM ALTIMETRY DATA (SATELLITE)

See movie.gif

Note the MULTIFRACTAL character, and the access to SUB-MESOSCALE

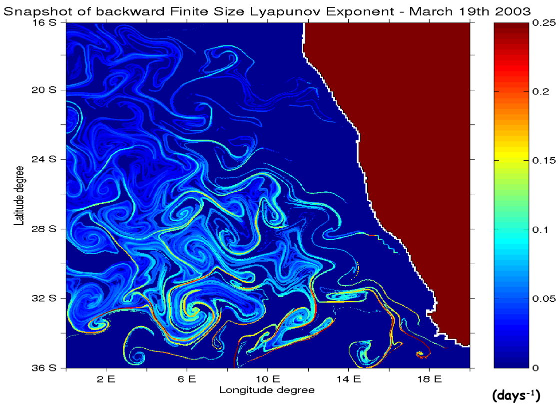
FSLE AND PHYTOPLANKTON

Chlorophyll-a (≈ phytoplankton) from space

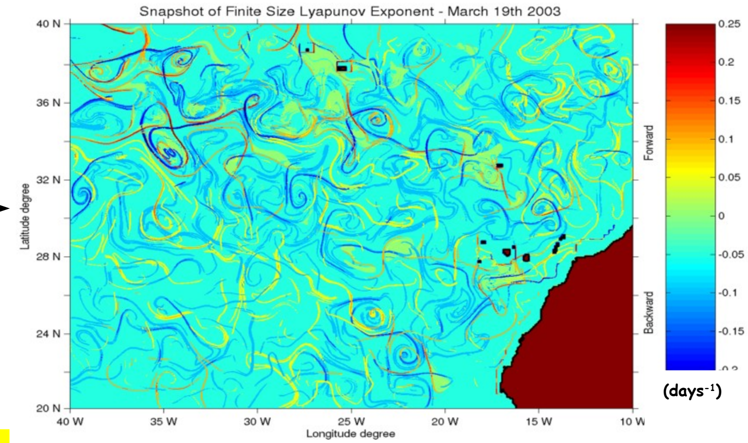


- Importance of upwelling areas due to:
  - large contribution in the world ocean productivity and biomasses.
  - several and intense human activities (about half of the world fisheries).
- High variability of the physico-chemical properties of the ocean.
- Vulnerability, especially in a global climate change context.

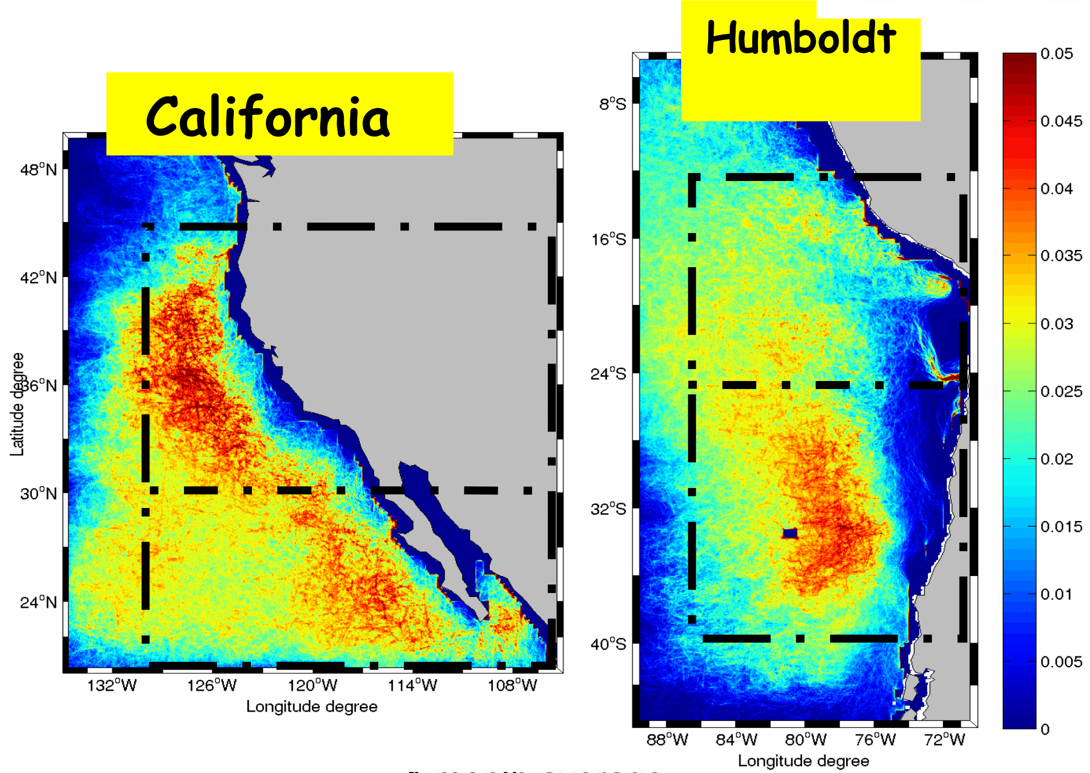
BENGUELA



CANARY

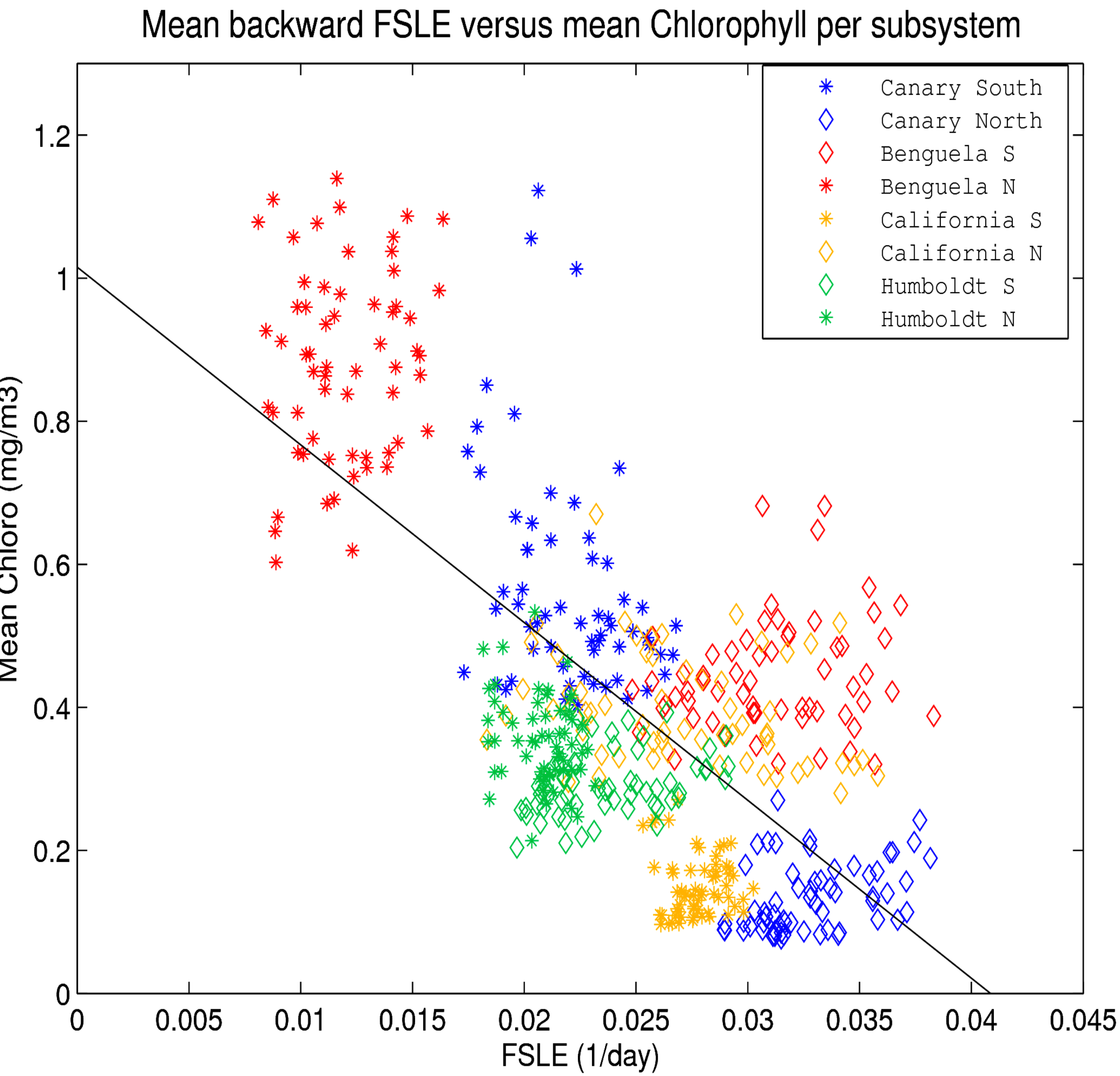


Snapshots



1 month average

FSLE and phytoplankton



- Negative correlation
- Clustering
- Less turbulent systems are characterized by: LOW FSLE / HIGH CHLOROPHYLL.
- Most turbulent systems: HIGH FSLE / LOW CHLOROPHYLL.

Opposite to behavior seen in less enriched systems

Also confirmed by Gruber et al (Nature Geoscience, 2011)

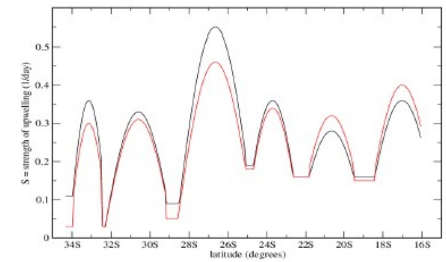
- Dominance of (small) upwelling vertical velocities in the less turbulent subsystem.
- Thus, probably the influence of horizontal stirring on plankton is only indirect: need to understand ***the 3d flow structure***.

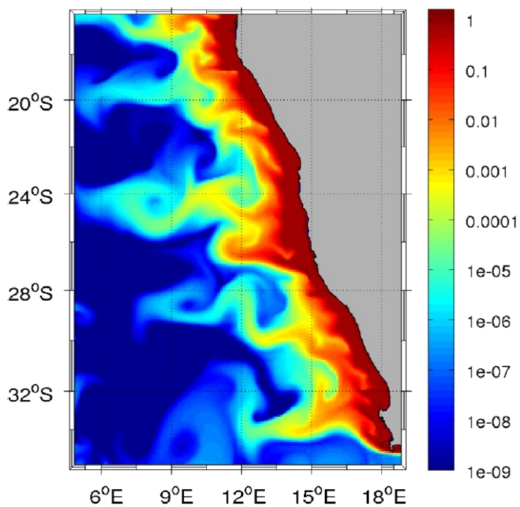
Hernández-Carrasco, Rossi, Hernández-García, Garçon and López
The reduction of plankton biomass induced by mesoscale stirring: A modeling study in the Benguela upwelling.
Deep-Sea Research I, 83, 65-80 (2013)

Advection-Reaction-Diffusion Equations

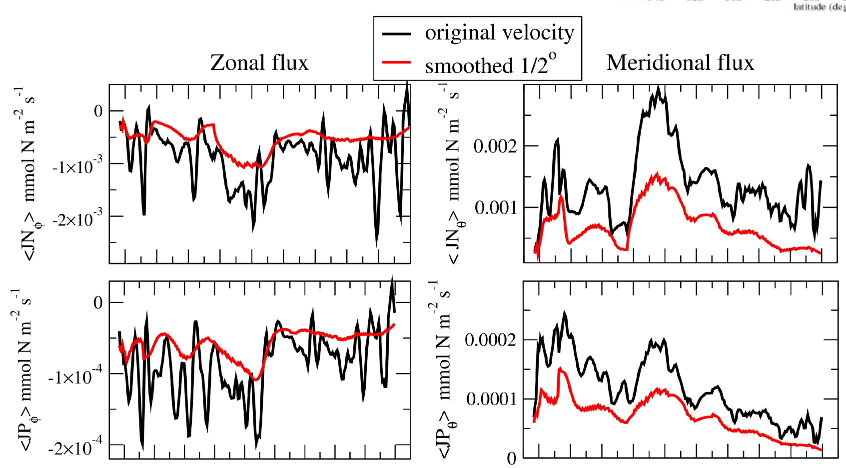
$$\frac{\partial N}{\partial t} + \mathbf{v}\nabla N = F_N + D\nabla^2 N$$
$$\frac{\partial P}{\partial t} + \mathbf{v}\nabla P = F_P + D\nabla^2 P$$
$$\frac{\partial Z}{\partial t} + \mathbf{v}\nabla Z = F_Z + D\nabla^2 Z$$

$$F_N = \Phi_N - \beta\frac{N}{\kappa_N + N}P + +\mu_N\left((1-\gamma)\frac{\alpha\eta P^2}{\alpha+\eta P^2}Z + \mu_P P + \mu_z Z^2\right)$$
$$F_P = \beta\frac{N}{\kappa_N + N}P - \frac{\alpha\eta P^2}{\alpha+\eta P^2}Z - \mu_P P$$
$$F_Z = -\gamma\frac{\alpha\eta P^2}{\alpha+\eta P^2}Z - \mu_Z Z^2$$





P (mg/m³)

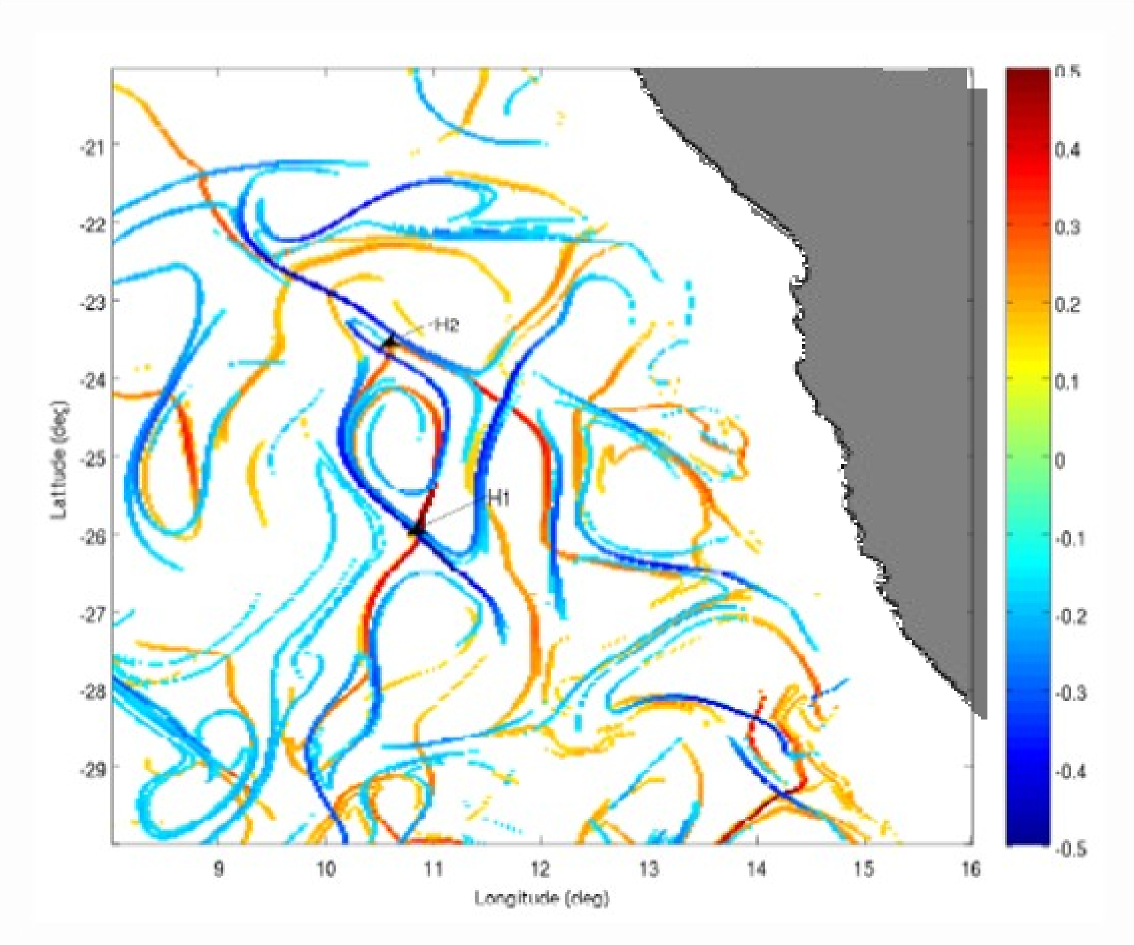


Conclusion: Mesoscale turbulence greatly enhances nutrient flux out of upwelling cells. More turbulence -> less nutrients available

3d LAGRANGIAN COHERENT STRUCTURES IN THE OCEAN

Three-dimensional characterization flow and eddies in Benguela

J.H. Bettencourt, C. Lopez, E. Hernandez-Garcia, Ocean Modelling 51 (2012) 73–83

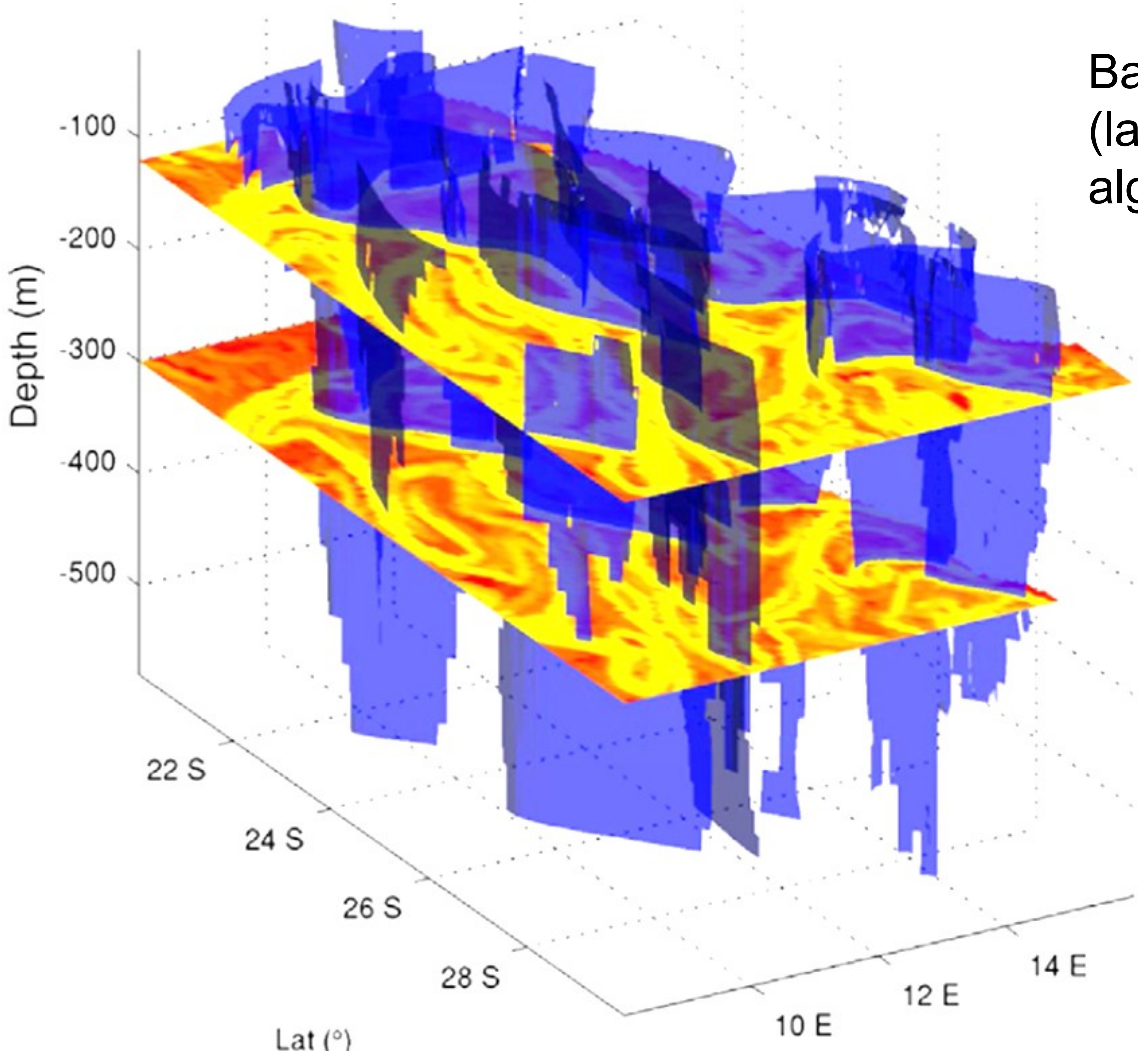


ROMS model:
(from Gutknecht et al.(2013))
2 years of simulation, climatologically forced.

Horizontal resolution
1/12 degrees (8 km)
32 vertical terrain-following levels

Forward and backward FSLE fields
δ_0=2 km ; δ_f=100 km

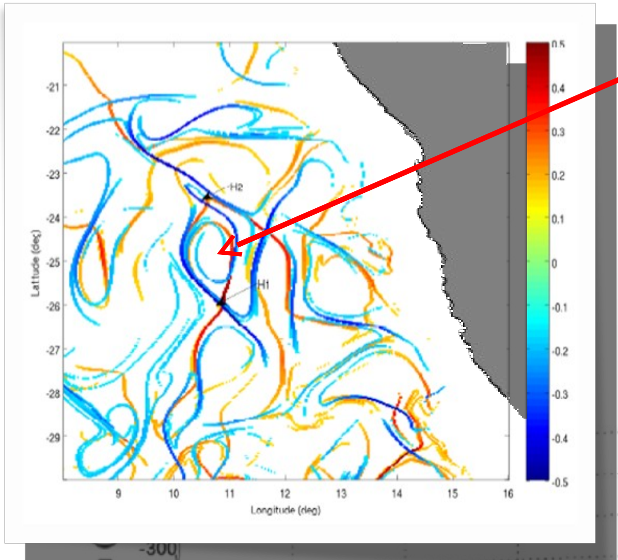
Particles released in horizontal planes every 20 m and integrated in 3D



Backward FSLE from a (largest) ridge extracting algorithm

Curtain-like structure as arising when vertical shear of horizontal velocities much smaller than horizontal velocities

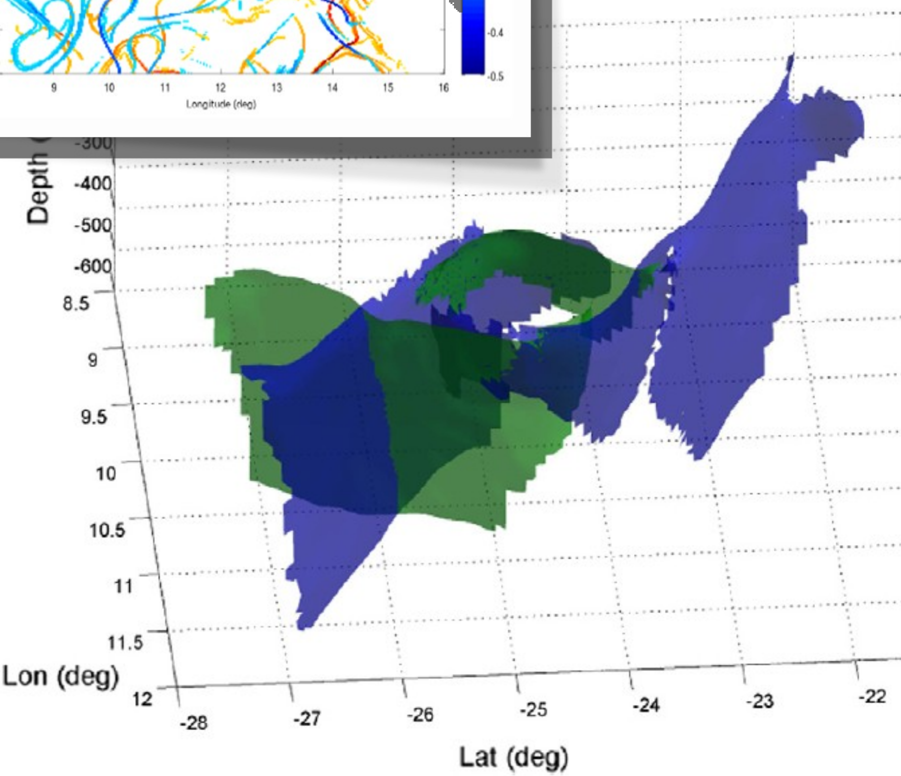
(Branicki, Mancho, Wiggins, Physica D 240 (2011) 282–304)

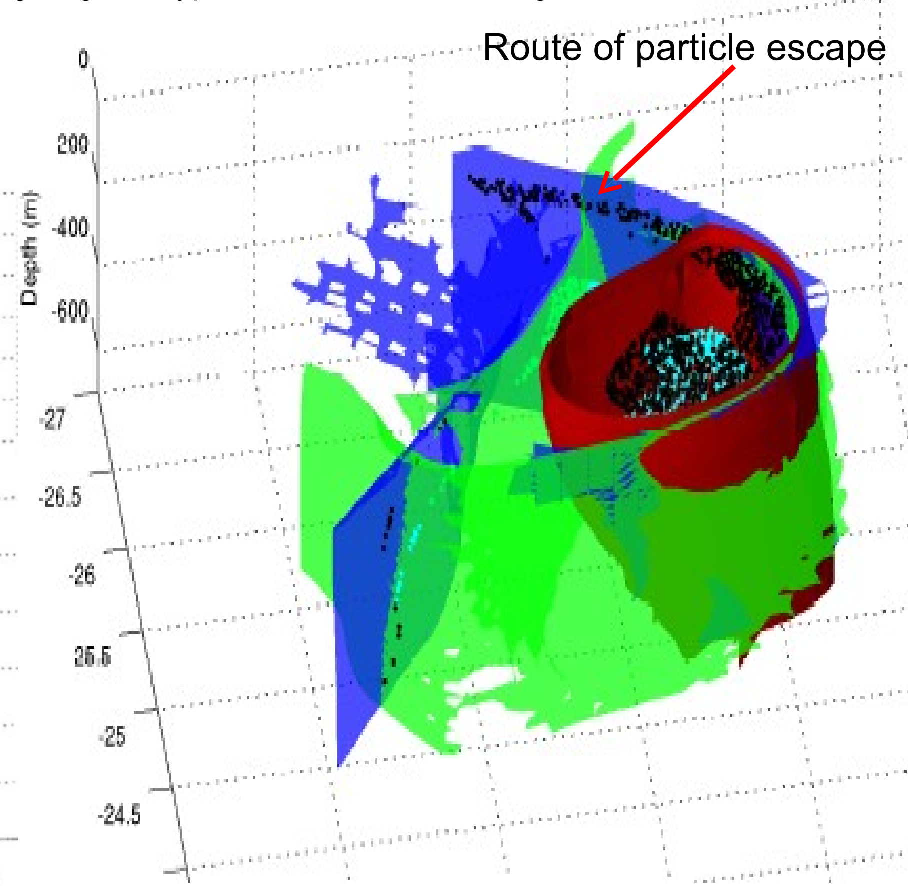


Particular eddy enclosed by hyperbolic manifolds

BACKWARDS FSLE
FORWARD FSLE
Q-criterion isosurface

FSLE methodology is giving the hyperbolic filamentation region, not the coherent core

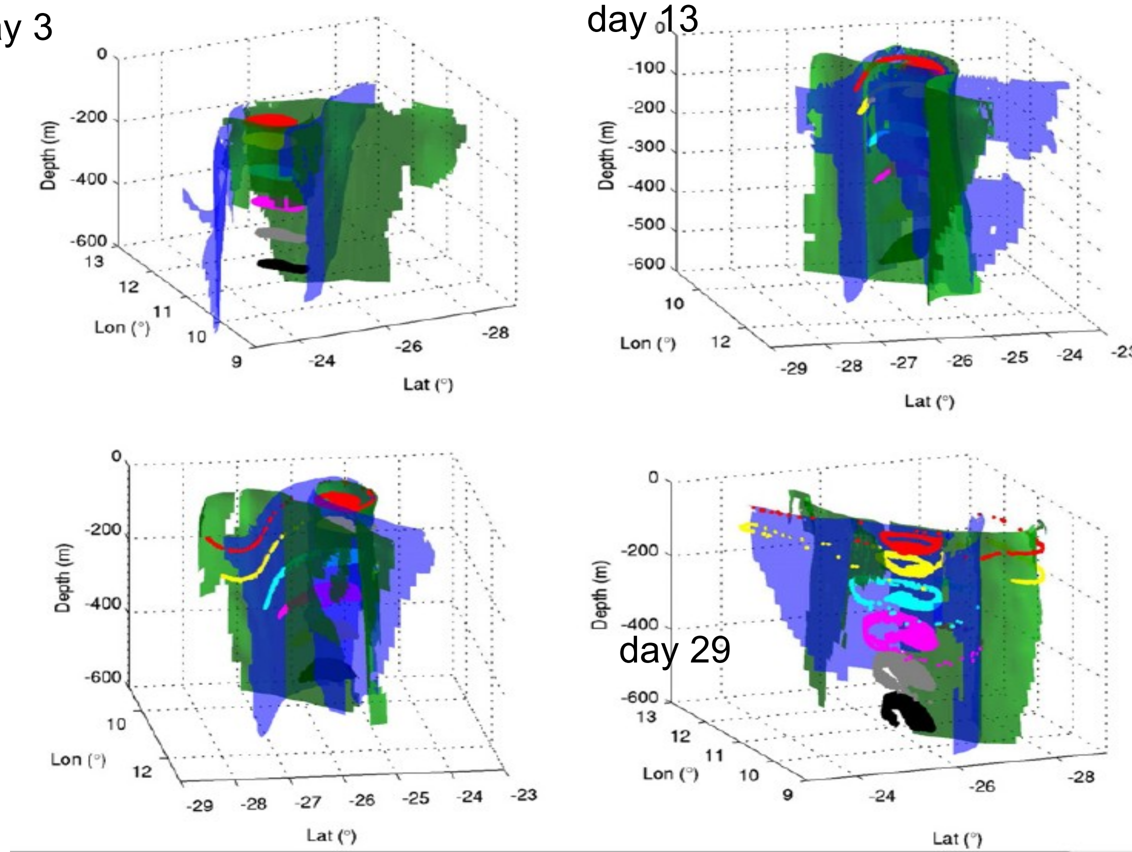




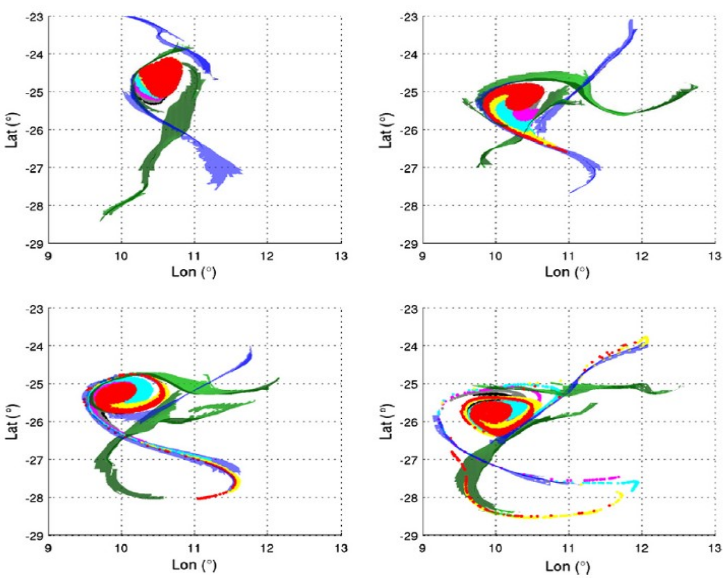
BACKWARDS FSLE
FORWARD FSLE

Red: 40 m
yellow: 100 m
cyan: 200 m
magenta: 300 m
grey: 400 m
black: 500 m

day 3



day 19



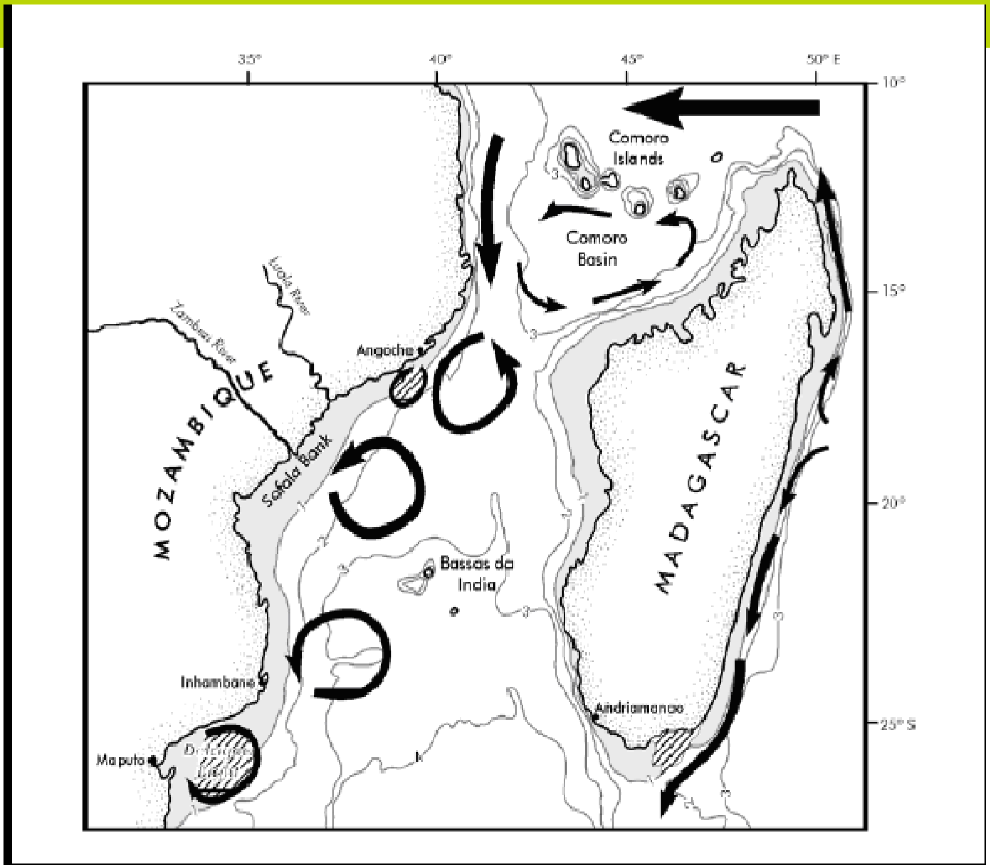
- Lagrangian Coherent Structures give the skeleton of transport.
- This certainly influences abiotic quantities: temperature, nutrients.
- This certainly influences plankton distribution .
- From there, impact is expected in plankton consumers, their predators, … cascades up along the food chain …

Do top marine predators track Lagrangian Coherent structures?

Great Frigatebirds : Fregata minor

DATA

Dynamics in the Mozambique channel (MC)



Satellite transmitter and altimeter : total weight : 1 to 3% mass of adults (maxi 45 g)

-Strong mesoscale activity in the MC.

- Surface velocity data in the channel.

- 8 birds from Europa island fitted with satellite transmitters (august-september 2003).

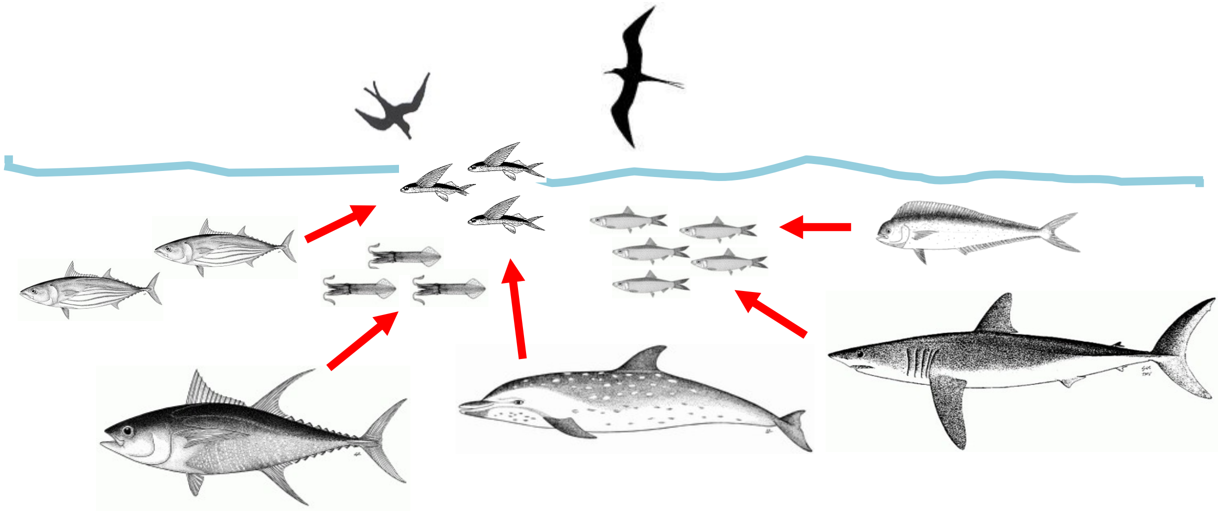
- Foraging trips:

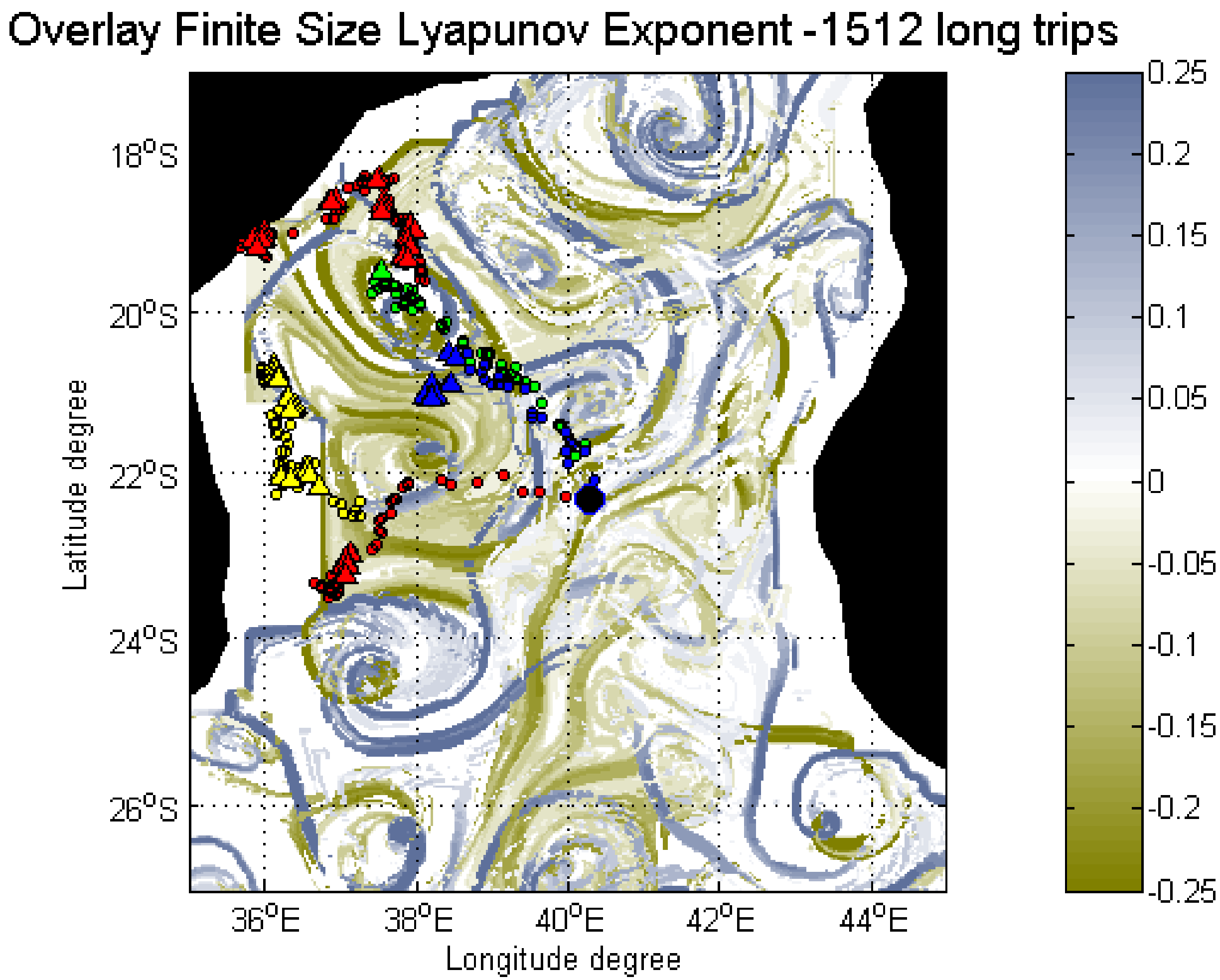
50 trips : 17 long trips (> 614 km), 33 short trips

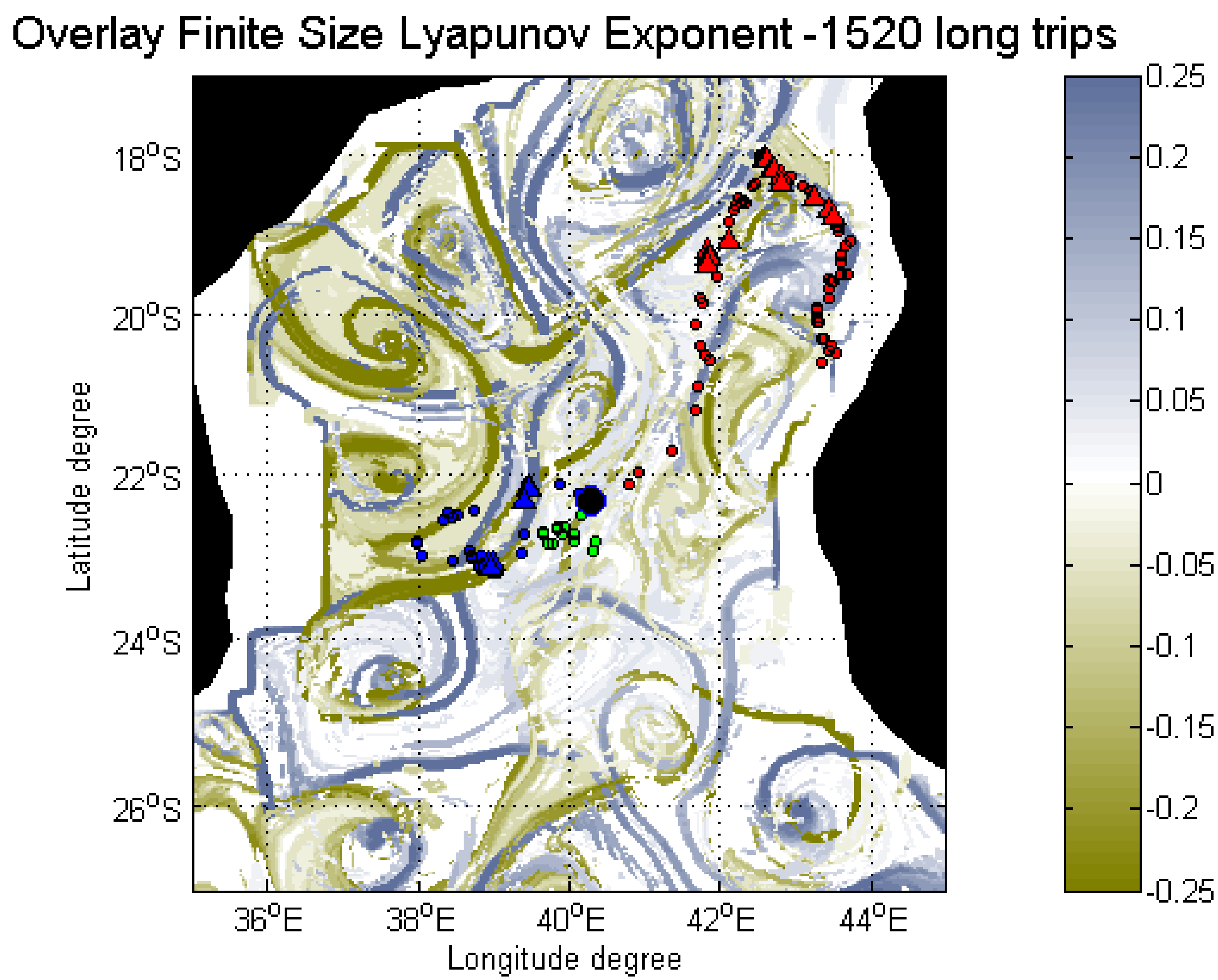
1600 Argos positions

Great frigatebird (*fregata minor*):

- Large seabirds (light weight < 5 kg and large wings > 2m). Use thermals to soar before gliding over long distances and time (days/nights over weeks).
- Traveling at high altitudes to locate patches of prey and come close to surface to feed (reduced flight speed indicates foraging).
- Feeding occurs only during daytime (peaks in the morning and evening).
- Unable to dive or rest on the water surface (permeable plumage) → in association with subsurface predators (tuna, …): **fisheries indicators**

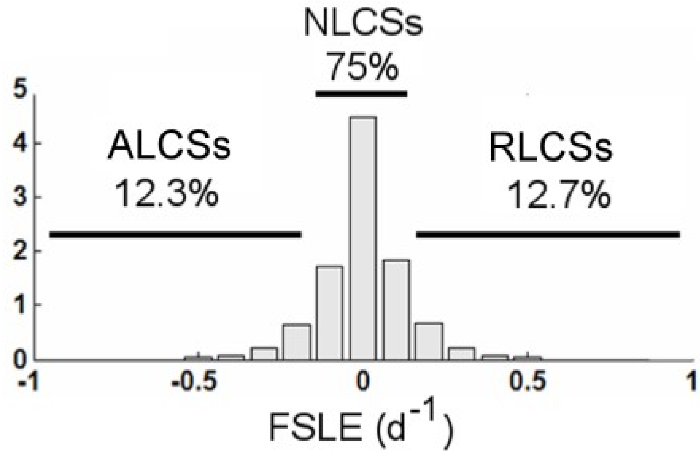



Overlay Finite Size Lyapunov Exponent -1512 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

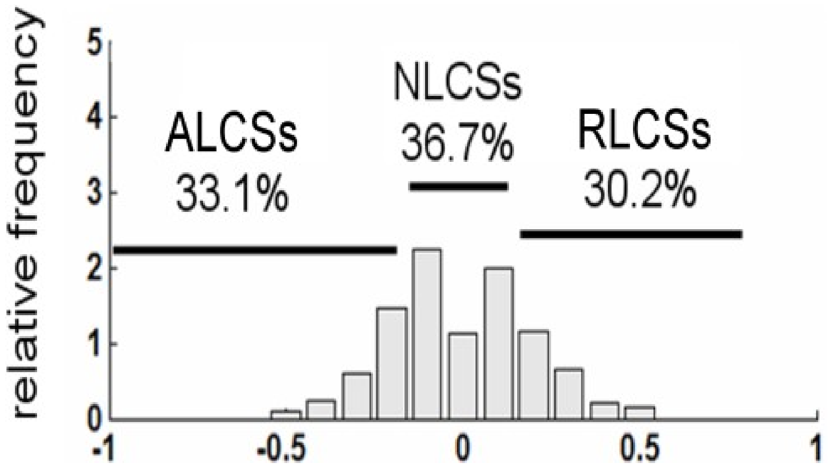

Overlay Finite Size Lyapunov Exponent -1520 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Histograms of FSLE values

On the whole area



On the birds positions



ALCS: attracting LCS, i.e. FSLE (backwards) < - 0.1 day^{-1}

RLCS: repelling LCS, i.e. FSLE (forwards) > 0.1 day^{-1}

NLCS: not LCS (small FSLE)

Despite LCS occupy only 25% of space, 63% of bird's positions are on them

Results of statistical tests:

- Frigate birds fly on top of LCSs both for travelling as for foraging
- No significant difference between day and night positions
- No significant difference between come and return trip

Frigatebirds ‘follow’ LCSs not only to find there prey, but as biological corridors which bring them to foraging places

HOW AND WHY?

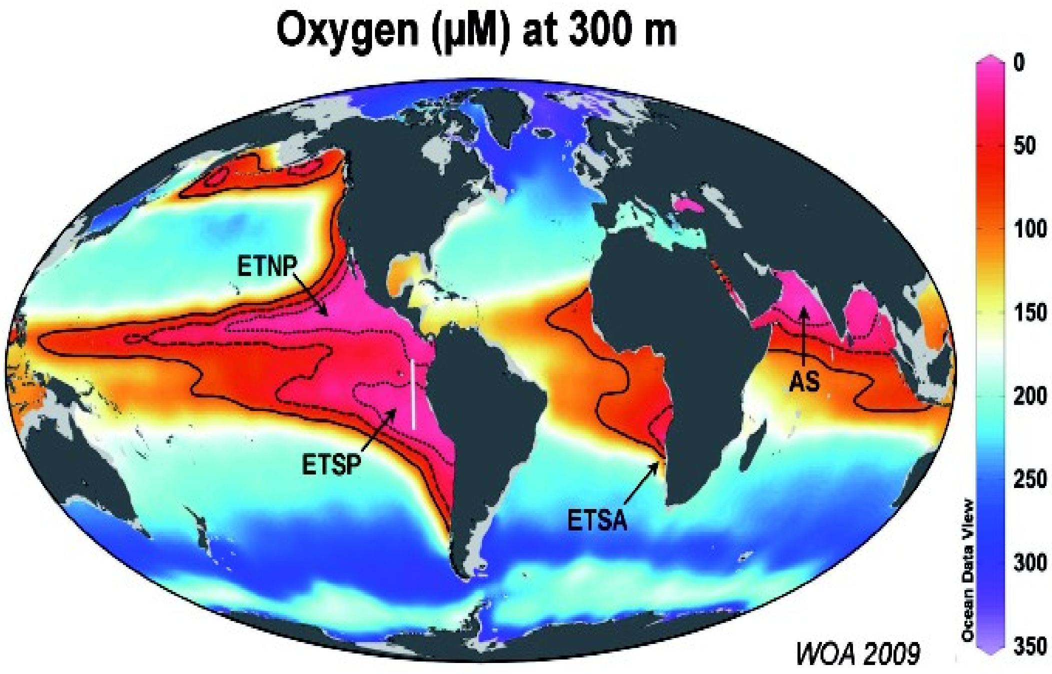
Aggregation of prey on LCSs? or aggregation of subsurface predators?

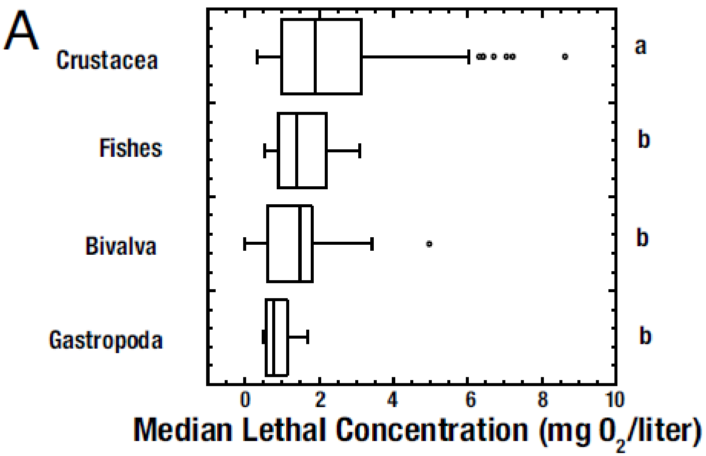
Olfatory clues (DMS produced by zooplankton) ? thermal air currents?

Puzzling issue: no significant difference between attracting and repelling LCSs

- Tangencies between manifolds?
- Interleaving between them?
- 3d dynamics associated both to ALCS and RLCS?
- Do they simply avoid low FSLE regions?

FSLE and Oxygen Minimum zones





Hypoxic levels - O2 < 88 μM

Vaquer-Sunyer & Duarte (2008)

Respiration and nitrification of some bacteria consume oxigen.

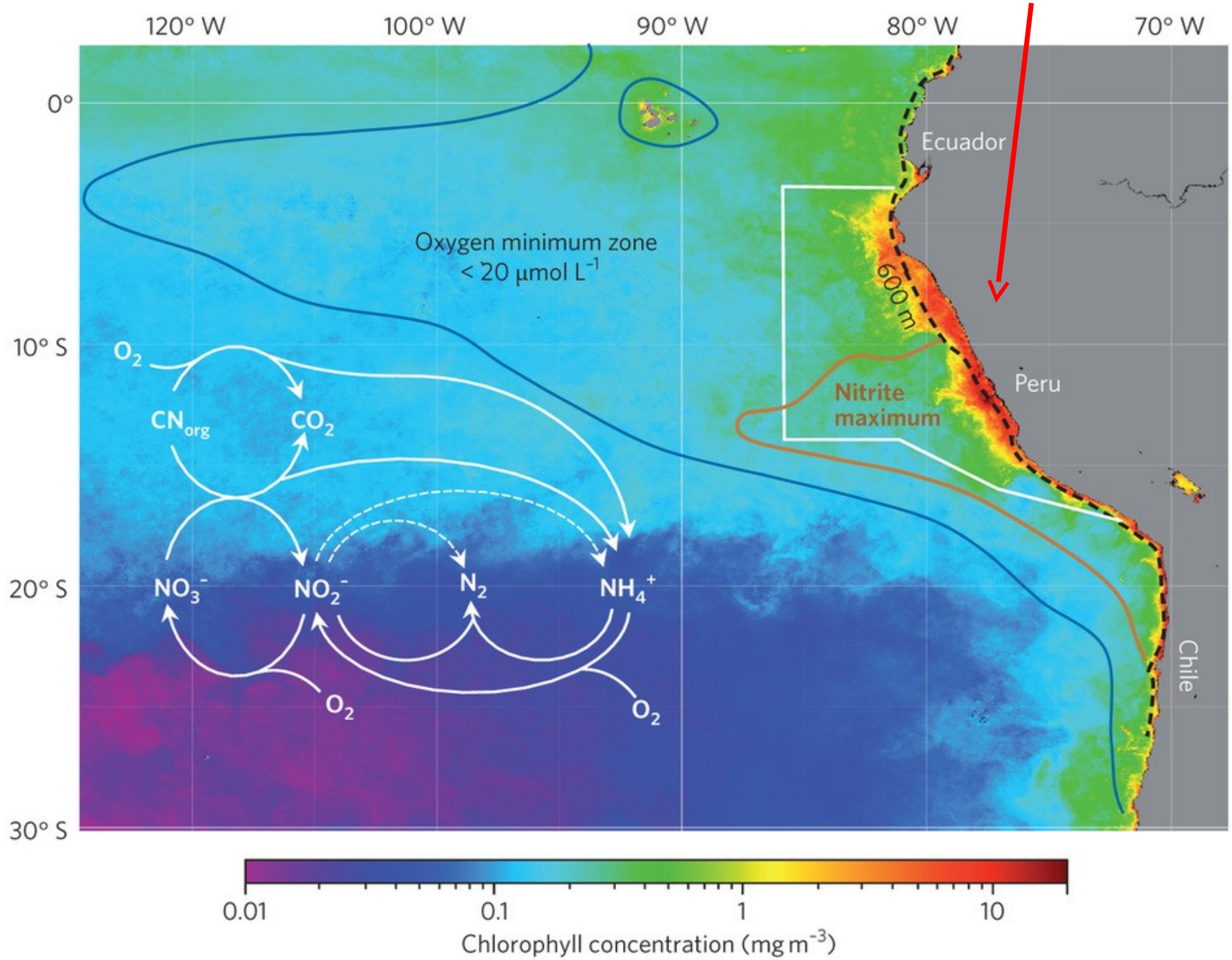
Increased stratification associated to global warming will make things worse

Role of flow:

Large scale patterns induce low ventilation areas.

What about horizontal stirring and mixing?

Coastal upwelling



Thamdrup (2013)

ROMS hydrodynamic model:

3D primitive equations

Hydrostatic

Terrain following

Forced by climatology

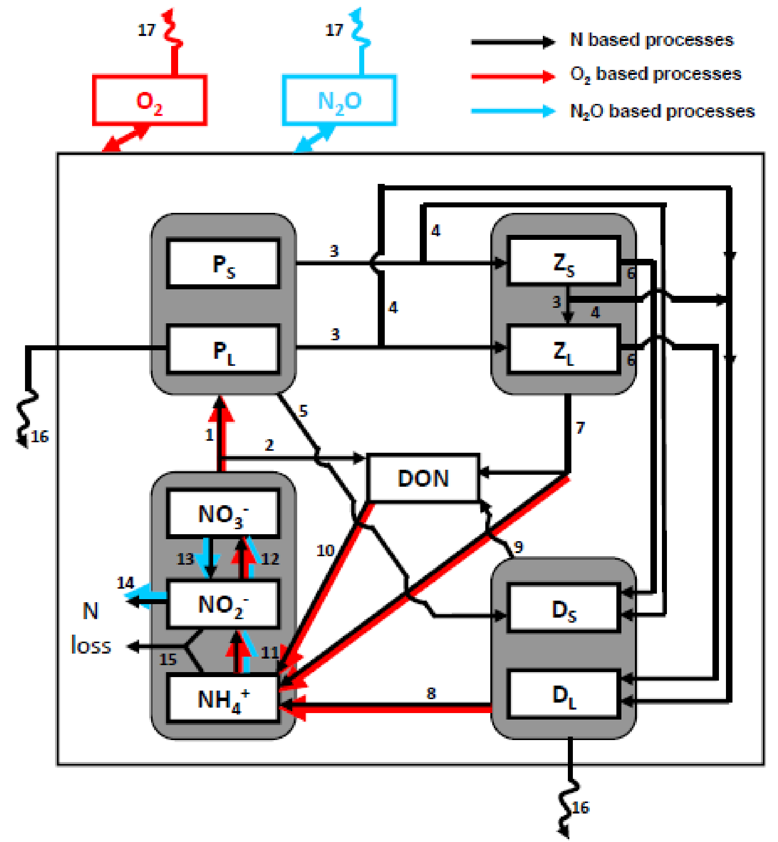
horizontal resolution of 1/9 degrees (~ 12 km)

32 terrain-following vertical levels

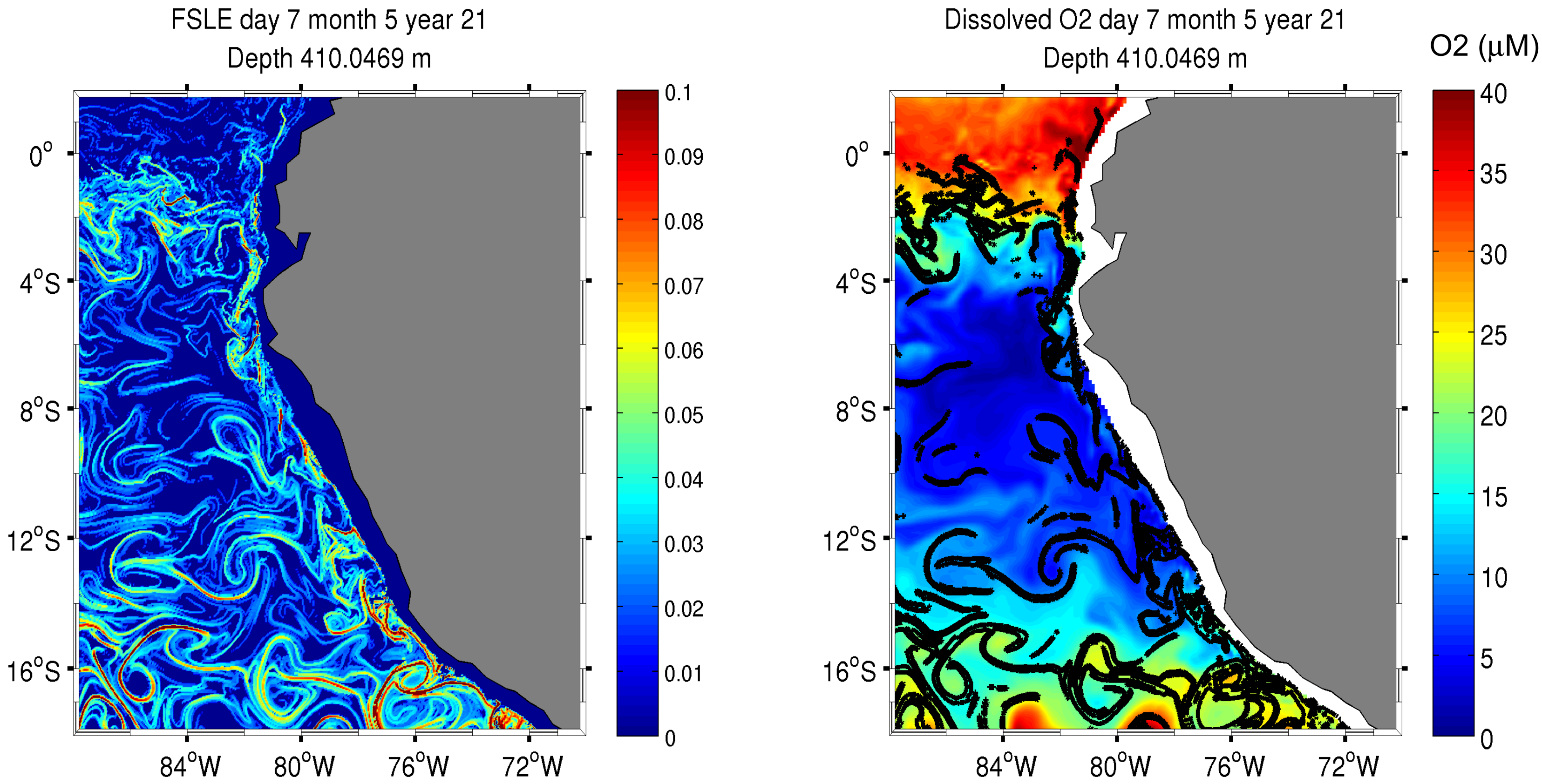
BioEBUS biogeoch. model:

$$\frac{\partial C_i}{\partial t} = -\nabla \cdot (\mathbf{u} C_i) + K_h \nabla^2 C_i + \frac{\partial}{\partial z}\left(K_z \frac{\partial C_i}{\partial z}\right) + SMS(C_i)$$

(Gutknecht et al, 2013)



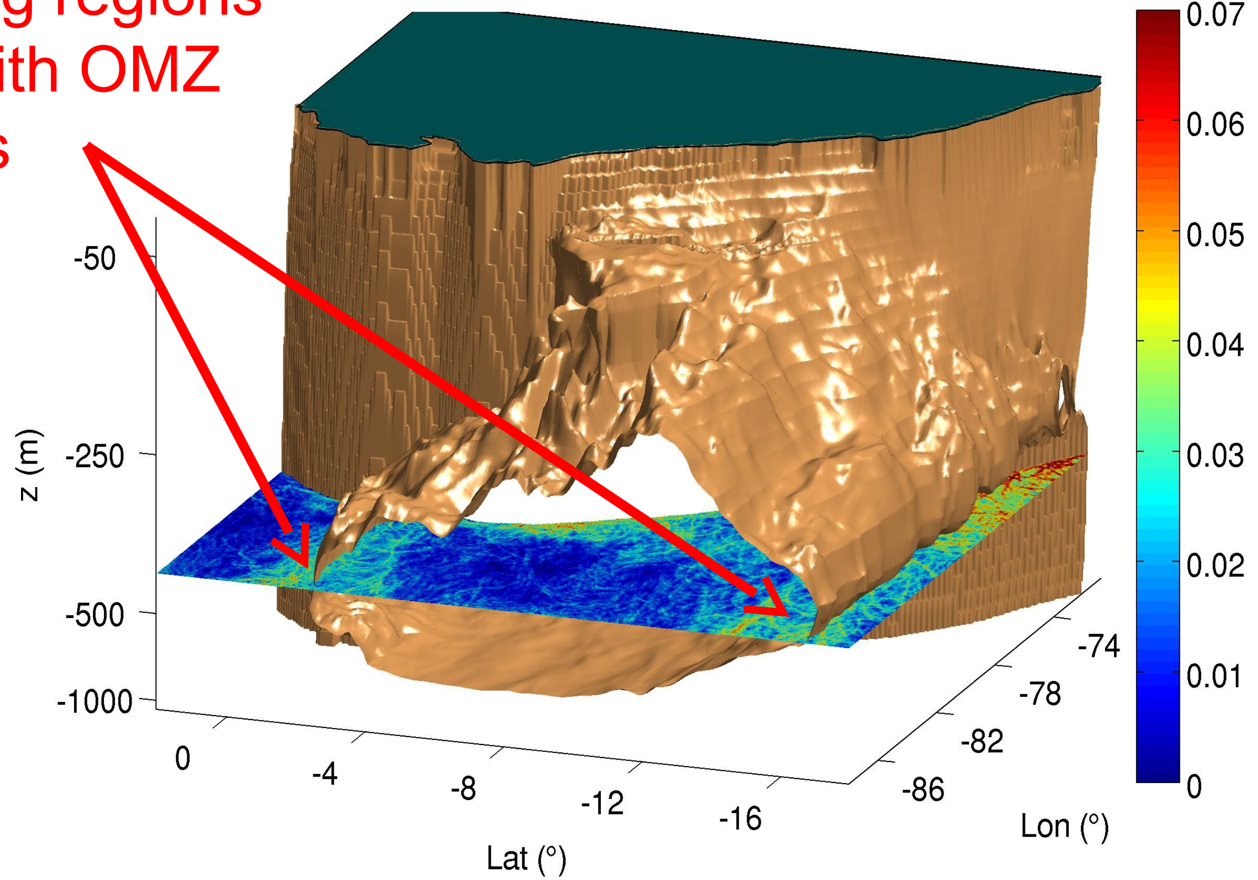
1. Assimilation of nutrients
2. Exudation
3. Grazing
4. Fecal pellets
5. Mortality of phytoplankton
6. Mortality of zooplankton
7. Excretion
8. Decomposition of detritus
9. Hydrolysis
10. Decomposition of DON
11,12. Nitrification
13,14. Denitrification
15. Anammox
16. Vertical sinking
17. Sea-air flux



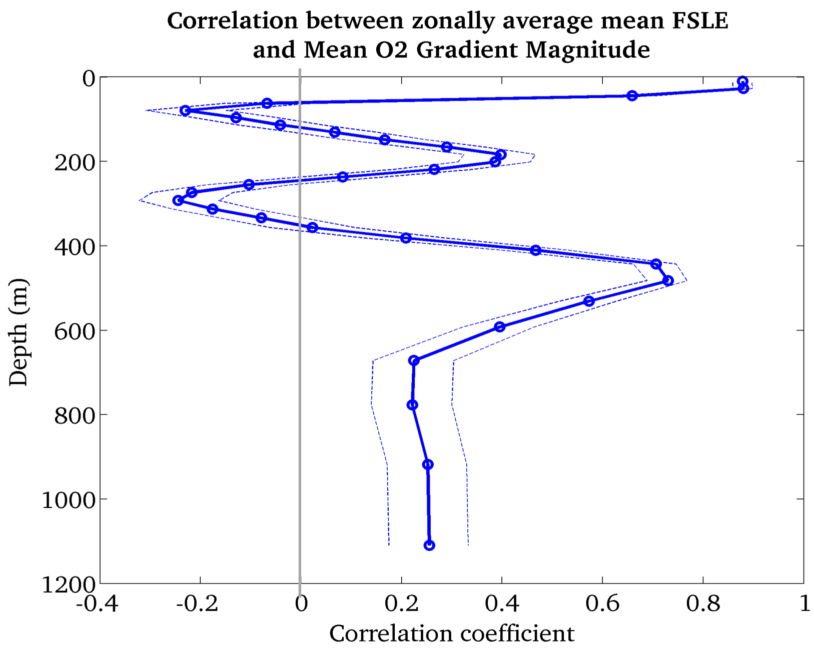
Backward FSLE (day^{-1})
Particles released in horizontal planes and integrated in 3D
δ_0=4 km ; δ_f=100 km

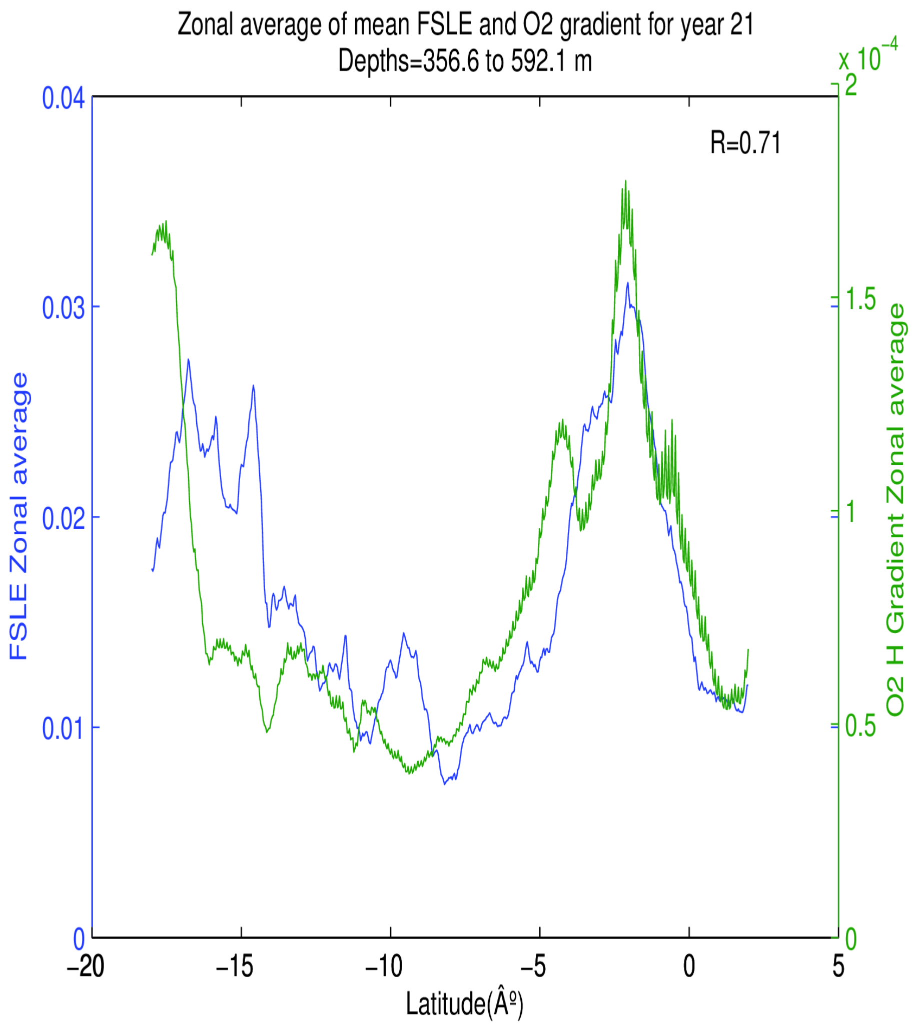
High stirring regions coincide with OMZ boundaries

Mean Oxigen Minimum Zone boundary at 20 μM



O2 gradients and FSLE





SOME CONCLUSIONS

FINITE-SIZE LYAPUNOV EXPONENT FIELDS

- Able to reveal globally the dynamical structures in the flow: main hyperbolic trajectories, their manifolds, …
- Simple enough to be applied in a practical way to real and complex ocean velocity fields.
- Reveals impact of fluid flow on biological dynamics at all scales: from plankton to top predators.
- 3d studies start understanding the vertical dynamics of the oceans.

Thanks for listening.

Tanti augri Angelo

Thanks to Angelo for:

- Showing me some of his Science and giving me the opportunity to collaborate with him.

- Give me the opportunity to know many wonderful people (now good friends).