Some considerations on Determinism, Chaos and Reductionism

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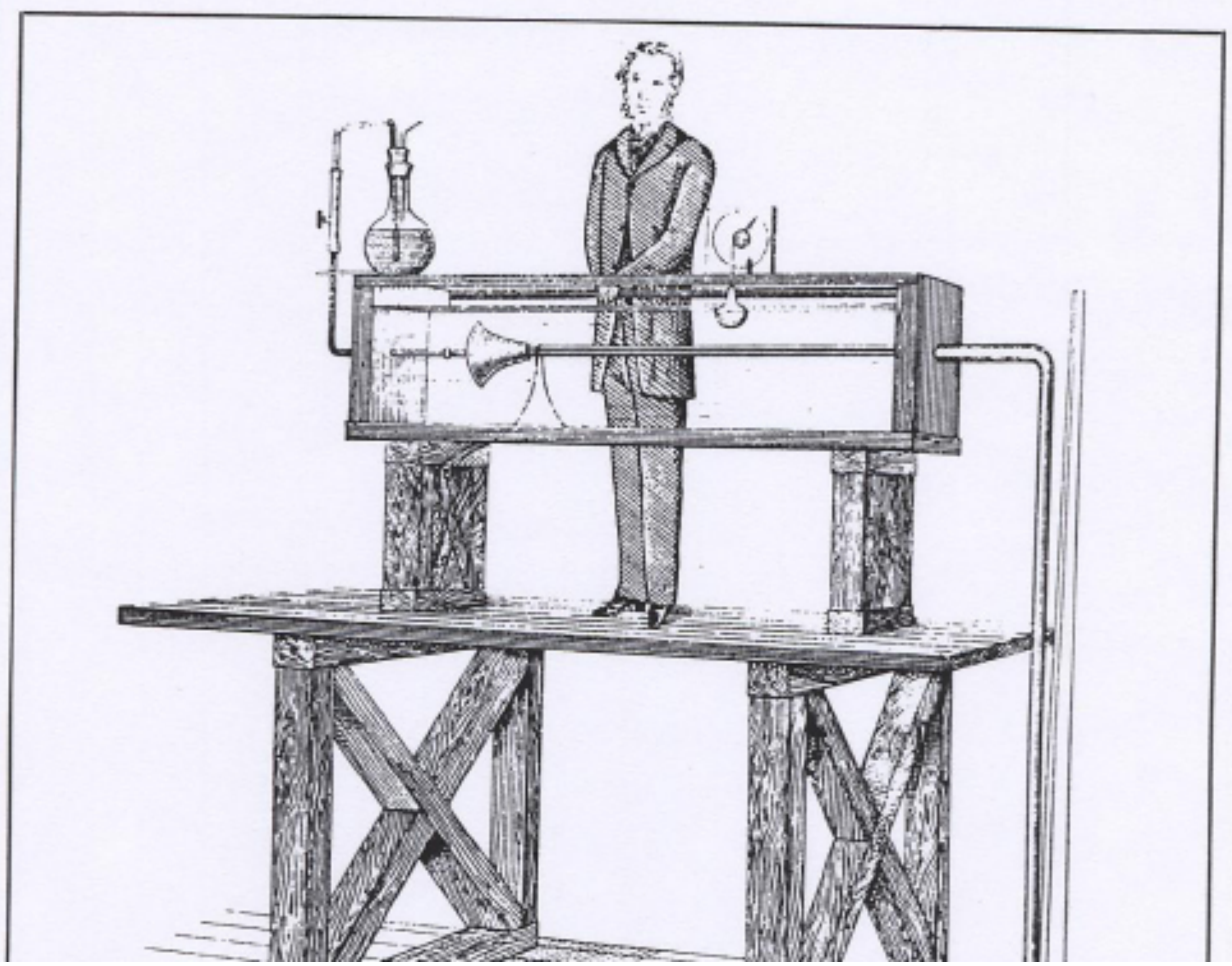
The gentle side of Angelo Vulpiani

Gianni Battimelli e Angelo Vulpiani

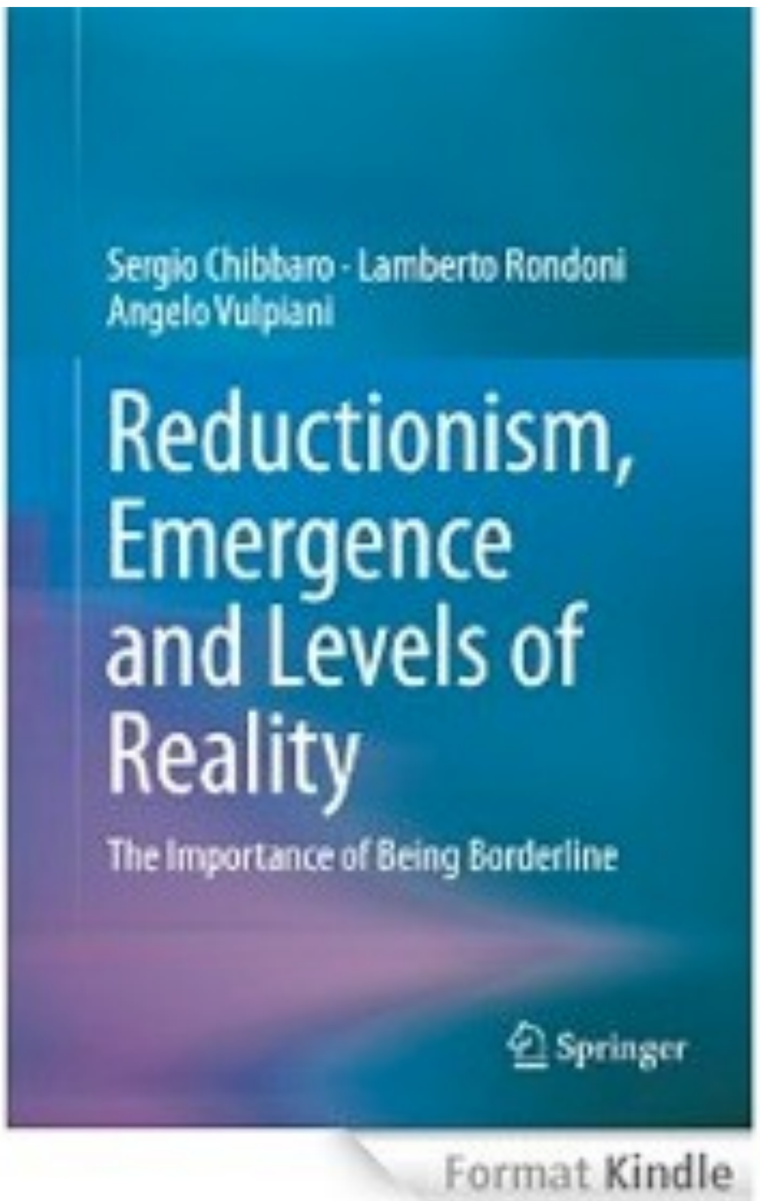
La turbolenza, buco nero della fisica classica

«Abbiamo una conoscenza peggiore di ciò che accade in un millimetro cubo di aria che non di ciò che accade dentro un nucleo atomico»



SAPERE - AGOSTO 1985 -

Shameless self-advertising...



Special acknowledgements

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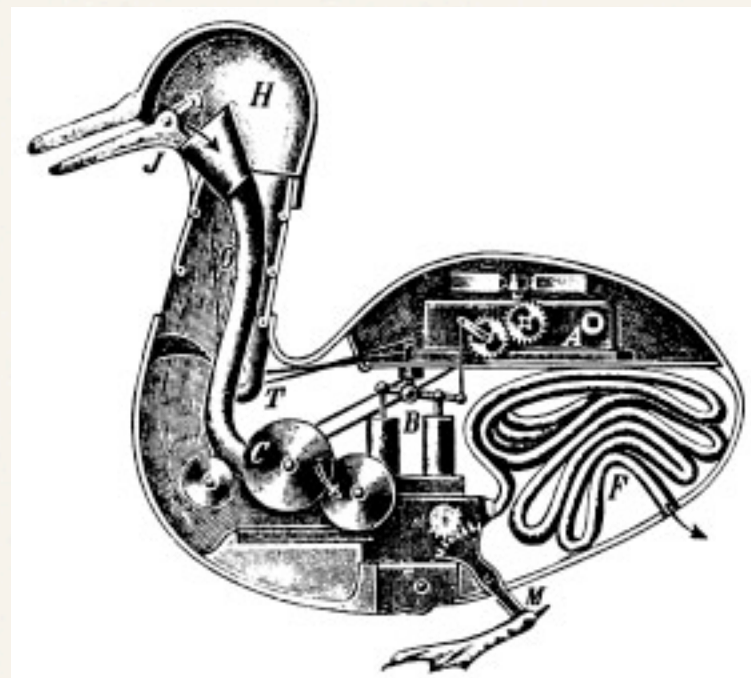
MV Berry

OG Jepps

Overview

General debate: Reductionism



[Jacques de Vaucanson 1738]

The whole is nothing but the sum of the parts

Philosophical examples

Biosphere
Species
Organisms
Organs
Cells
Process of replication
Genetic Transcription
Biochemical cycles
Biomolecules
Molecules

Human culture
Phase sequences
Complex assemblies
...
...
Assemblies of assemplies of assemplies
Assemplies of assemplies
Assemplies of neurons
Nerve impulses
Nerve membranes
Membrane proteins
Molecules

Question: do levels of reality exist?

Unifying principles?

History

- ✦Thalès de Milet (624 b.C.)
- ✦Anaximène (VI° b.C)
- ✦Héraclite (~500 b.C.)
- ✦Empédocle d'Agrigente (~450 b.C)
- ✦Pythagoras
- ✦Democritus

} Greek Monism

Few unifying principles

Palomar

The lawn is a collection of grasses – this is how the problem must be formulated – that includes a subcollection of cultivated grasses and a subcollection of spontaneous grasses... The two subcollections, in their turn, include various species, each of which is a subcollection, or rather it is a collection that includes the subcollection of its own members, which are members also of the lawn and the sub-collection of those alien to the lawn... is "the lawn" what we see or do we see one grass plus one grass plus one grass...? What we call "seeing the lawn" is only an effect of our coarse and slapdash senses; a collection exists only because it is formed of discrete elements. There is no point in counting them, the number does not matter; what matters is grasping in one glance the individual little plants, one by one, in their individualities and differences (Calvino 1983).

Debate

Atkins, Midgley and Edelman:
Theism (and the implicit rejection of reductionism) is a system of knowledge based on ignorance, and that twin of ignorance, fear.

(Cornwell 1995).

"reductionism" is not the enfant terrible of present-day scientific, physicalistic culture: the desire to connect something to its origin – especially the desire to connect the world to its divine origin – is reductionism. Theology is the fundamental form of reductionism. Indeed, theology reduces the essence of the world to God, in the same fashion that in science, one day, reductionists will reduce all human reality to movements of elementary particles.

Severino (1997),

P Anderson «More is different»

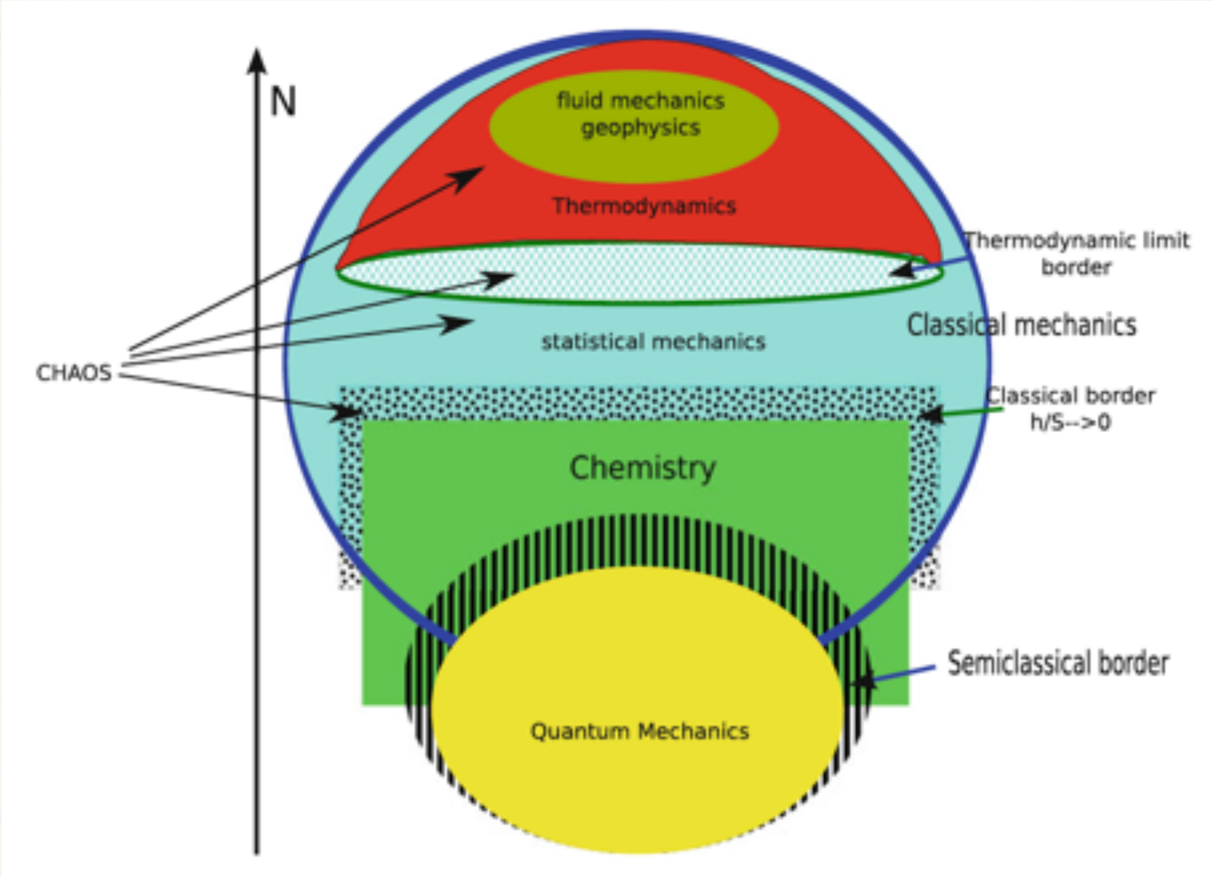
Purposes

- Is there any evidence of actual reduction of theories? Are the examples that are found in books on the philosophy of science too simplistic or not completely correct?

- Relation between theories at different levels

- Is anti-reductionism mystical?

- What is the point of embedding science Y within science X, if predictions concerning science Y cannot be made starting from science X?
- Investigate some examples from physics



Emergent properties at *borders*

↕

Singular limits

Berry, Primas, Batterman

Main References

Boffetta, Cencini, Falcioni, Vulpiani 2002

Cencini, Cecconi, Vulpiani 2009

General remarks on Determinism

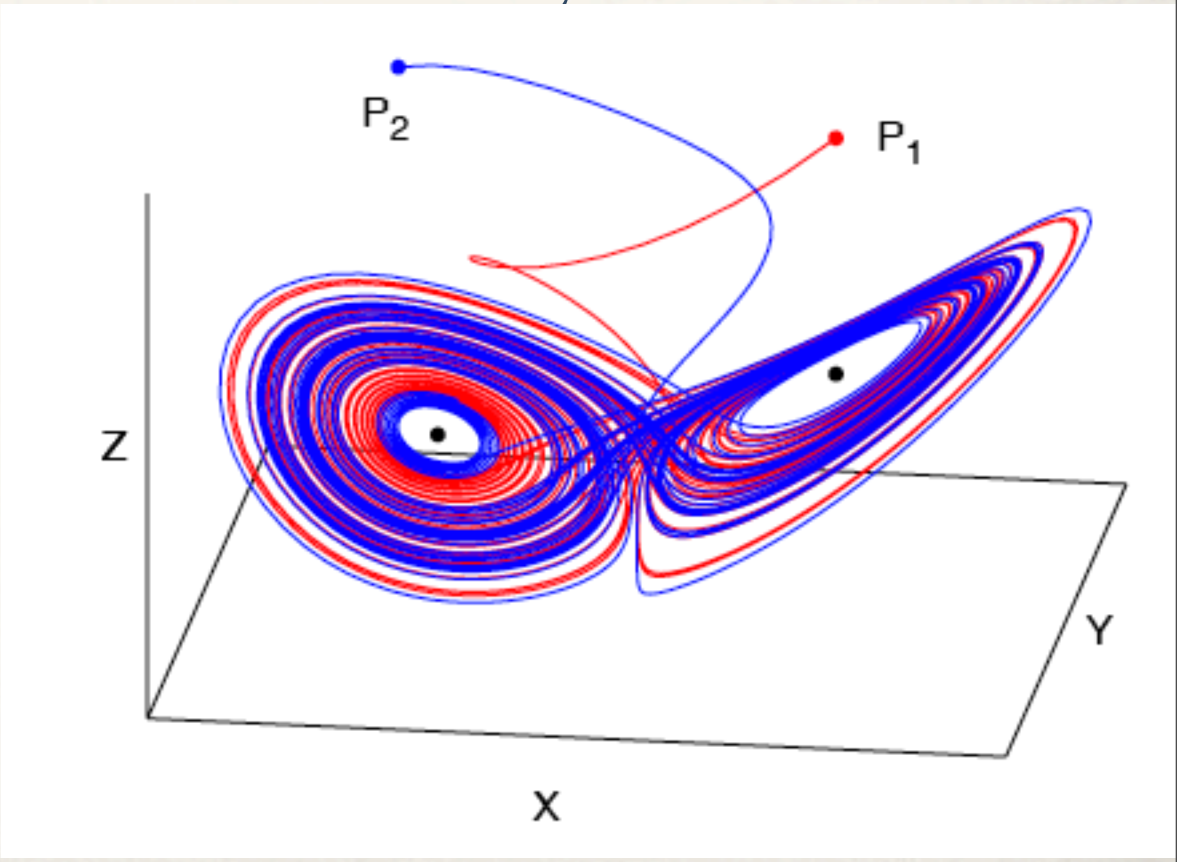
Determinism as dystopia (Popper) → Reductionism

Mechanistic determinism against chaos, desorder

Main point: discovery of simple models with complex behaviour (Lorenz 63)

- Deterministic chaos changed the picture of the "Elementary blocks"
- Similarity Chaos and stochasticity
- Coarse-graining: singular limits
- Chaos: probability in deterministic world

(Kojeve 1990)



Determinism and Predictability

Laplace heritage

A Philosophical Essay on Probabilities" (Laplace 1829)

> *We ought then to regard the present state of the universe as the effect of its anterior state and the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it – an intelligence sufficiently vast to submit this data to analysis – it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present in its eyes.*

Mathematical intelligence: importance of predictions

> *[t]hey tell me you have written this large book on the system of the universe, and have never even mentioned its Creator. Laplace answered: Sire, I have no need of this assumption. To that Napoleon replied: Ah! That is a beautiful assumption, it explains many things, and Laplace: This hypothesis, Sire, explains everything, but does not permit the prediction of anything. As a scholar, I must provide you with works permitting predictions.*

Determinism and Predictability

• *Determinism:* the metaphysical assumption of a deterministic causal structure of nature

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{f}(\boldsymbol{x}(t), t)$$ (Newton, Cauchy)

• *Exact predictability:* the practical possibility of making predictions through mathematical laws.

$$\boldsymbol{x}(n+1) = \boldsymbol{f}(\boldsymbol{x}(n))$$ $\mathbf{x}(0) = \mathbf{X}_0$

• *Mechanistic reductionism:* the possibility of explaining (at least in principle) any phenomenon from the motion of its elementary constituents

In principle all phenomena
Essence of Laplace's mechanistic (causal) determinism

Successes: deduction Kepler'law, Newton gravitation, discovery Neptune
Delaunay computation of moon trajectory

Determinism and Predictability

Does the World seem really deterministic? Irregular behaviors are so «regular»...

Possible solution: many complicated equations (Landau 44)

Determinism is different from pedictability

Maxwell 1873

It is a metaphysical doctrine that from the same antecedents follow the same consequences. No one can gainsay this. But it is not of much use in a world like this, in which the same antecedents never again concur, and nothing ever happens twice... The physical axiom which has a somewhat similar aspect is "that from like antecedents follow like consequences". But here we have passed .... from absolute accuracy to a more or less rough approximation

1. the impossibility of proving (or refuting) the deterministic character of the laws of nature;
2. the practical impossibility of making long-term predictions for a class of phenomena, referred to here as chaotic, despite their deterministic nature.

The issue of continous dependence on initial and boundary conditions Duhem (1901) on Hadamard

Poincaré

Determinism and Predictability: summary

Ontological determinism cannot be proven

the ontological determinism à la Laplace can neither be proved nor disproved on the basis of observations.

van Kampen 1991

Popper on «scientific determinism»:
Laplace's «intelligence» has to be outside our world

Determinism: causal vision in mathematical terms, laws are differential equations

- Macroscopic phenomena are chaotic, «random»
- Microscopic phenomena appear probabilistic: ontologically non-deterministic

Are deterministic phenomena always predictable?
What does prediction mean?

An excursus on Chaos

Predictability a tough affair

Three bodies: Poincaré

the effect of a very distant single electron on massive bodies Berry 1978

Chaotic system:

(i) the evolution is given by a deterministic rule, for example, by a set of differential equations;

(ii) solutions sensitively depend on the initial conditions: i.e. two initially almost identical states $\mathbf{X}(0)$ and $\mathbf{X}'(0)$ characterised by a very small initial displacement $|\mathbf{X}(0) - \mathbf{X}'(0)| = \delta_0$, separate at an exponential rate:

$$|\mathbf{X}(t) - \mathbf{X}(t)'| \sim \delta_0 e^{\lambda t}$$

where λ is positive and is called the Lyapunov exponent;
(iii) the evolution of the state $\mathbf{X}(t)$ is not periodic and appears quite irregular, similar in many respects to that of random systems.

About reductionism:
Turbulence (Landau 44)

"Simple elementary bricks"
not true

"Elementary bricks" are
sometimes complex
Ruelle & Takens 1971

An excursus on Chaos

Sensitive dependence on i.c.: $$T_p \sim \frac{1}{\lambda} \ln \frac{\Delta}{\delta_0}$$

Not arbitrary predictions
Chaos = Unpredictable

→ Probability

the reductionistic idea that
complex systems can be analysed as an
agglomerate of simple elements
incorrect.

Generalisation for finite time-resolution

FTLE Ishigara 83, Benzi et al 85
FSLE Aurell et al. 96,97; Torcini et al. 95; Kantz & Lee 2000

Information and Complexity

Information theory

ergodic, stationary source s(t)

$$C_N = (s(1), s(2), \ldots, s(N)) \quad P(C_N)$$

$$H_N = -\sum_{\{C_N\}} P(C_N) \ln P(C_N)$$

$$h_N = H_{N+1} - H_N$$

$$h_{\mathrm{Sh}} = \lim_{N\to\infty} h_N = \lim_{N\to\infty} \frac{H_N}{N}$$

Block entropy

Shannon entropy

$$h_{Sh} > 0$$

Source complex, 'surprise'

Property of the source

Shannon-Mcmillan thm

If N is large enough, the set of all possible N-words, $\Omega(N) \equiv \{\mathcal{W}_N\}$ can be partitioned into two classes $\Omega_1(N)$ and $\Omega_0(N)$ such that if $\mathcal{W}_N \in \Omega_1(N)$ then $P(\mathcal{W}_N) \sim \exp(-N h_{Sh})$ and

$$\sum_{\mathcal{W}_N \in \Omega_1(N)} P(\mathcal{W}_N) \underset{N\to\infty}{\longrightarrow} 1$$

while

$$\sum_{\mathcal{W}_N \in \Omega_0(N)} P(\mathcal{W}_N) \underset{N\to\infty}{\longrightarrow} 0 \,.$$

$$N_{eff}(N) \sim \exp(N h_{Sh})$$

Shannon-Fano code
complex=incompressible

$$\lim_{N\to\infty} \frac{\langle \mathfrak{L}_N \rangle}{N} = \frac{h_{Sh}}{\ln 2}$$

Information and Complexity

Complexity of a binary sequence

11111111111111...	ordered
10101010101010...	ordered
00101000110100...	complex

In terms of simple rules

```
WRITE 1 N TIMES
```

```
WRITE 10 N/2 TIMES
```

```
WRITE 0  WRITE 0  WRITE 1  WRITE 0  WRITE 1  ...
```

Given the sequence $a_1, a_2, a_3, \ldots, a_n$

among all possible programs which generate this sequence, one considers that with the smallest number of instructions.

$K^{(N)}$ the number of these instructions, the algorithmic complexity of the sequence is defined by

$$K = \lim_{N \to \infty} \frac{K^{(N)}}{N}$$

Kolmogorov, Chaitin and Solomonoff

Information and algorithmic complexity are conceptually different

Grassberger, '86, '89

Chaos and Complexity

$x(t+1) = 2x(t) \; mod \, 1$

Bernoulli map

$\lambda = \ln(2)$

$T_p \sim \frac{1}{\lambda} \ln \frac{\Delta}{\delta_0}$

real number in [0,1]

$x(0) = \frac{a_1}{2} + \frac{a_2}{4} + \cdots + \frac{a_n}{2^n} + \cdots$

a_n takes either the value 0 or the value 1

$x(0) = 0.110010100101100100101001011 10\ldots$

Time evolution

$x(1) = 0.100101001011001001010010111 0\ldots$

$x(2) = 0.00101001011001001010010111 0\ldots$

$x(3) = 0.0101001011001001010010111 0\ldots$

Initial conditions crucial

$\{a_1, a_2, \ldots, a_n, \ldots\}$

Chaos and Complexity

$x(t+1) = 2x(t) \ mod \ 1$ Transmission with accuracy Δ $0 < t < T$

$x(0)$ and the rule → $\delta_0 \sim 2^{-T}\Delta$

→ complexity $\{a_1, a_2, \ldots, a_n, \ldots\}$

- algorithmic complexity impossible to determine (Gödel incompleteness Theorem)
- "almost all" binary sequences are complex (Martin-Löf 1966)

the details of the time evolution are well hidden in the initial condition in general complex.

$$N_{bits} \approx \frac{\lambda T}{\ln 2}$$

Chaos -> Singular limit (Berry 1994)

Incompressible = Complex

stochastic macroscopic properties emerge from chaos

Chaos, Complexity and Probability

$\mathscr{A} = \{A_1, \ldots, A_N\}$ Partition of phase space

$\{\mathbf{x}(0),\ \mathbf{x}(1),\ \mathbf{x}(2),\ \ldots, \mathbf{x}(n), \ldots\}$ Trajectory

$\{\sigma(\mathbf{x}(0)),\ \sigma(\mathbf{x}(1)),\ \sigma(\mathbf{x}(2)), \ldots, \sigma(\mathbf{x}(n)), \ldots\} = \{i_0, i_1, i_2, \ldots, i_n, \ldots\}$ Symbolic sequence

$i_n \in \{1, 2, \ldots, N\}$ $\sigma(\mathbf{x}(k)) = i_k \text{ if } \mathbf{x}(k) \in A_{i_k}$

<u>Coarse-grained analysis</u>

$C_m = (i_1 i_2 \ldots i_m)$ 'word' with probability $P(C_m)$

$$H_m(\mathscr{A}) = -\sum_{C_m} P(C_m) \ln P(C_m)$$

block entropy of the m-sequence
a measure of '*surprise*'
or information

Chaos, Complexity and Probability

partition entropy

$$h_S(\mathscr{A}) = \lim_{m\to\infty} (H_{m+1} - H_m)$$

$$h_{KS} = \sup_{\mathscr{A}} h_S(\mathscr{A})$$

Kolmogorov-Sinai entropy

$$h_{KS} = \lim_{\varepsilon\to 0} h(\mathscr{A}_\varepsilon)$$

ϵ-partition

$\epsilon^d \quad \mathcal{A}_\epsilon$

singular limit

$h_{KS} > 0$ Complex, 'random'

Chaos=random=complex

$$h_{\mathrm{KS}} \leqslant \sum_{\lambda_i > 0} \lambda_i$$

Pesin's relation

complex=unpredictable

Chaos, Complexity and Probability

Algorithmic complexity of a dynamical system trajectory **X**(t)

$$\kappa(\mathbf{X})$$

Algorithmic complexity *vs KS entropy*

$$\langle \kappa(\mathbf{X}) \rangle = \frac{h_{KS}}{\ln 2}$$

Brudno 83
White 92

KS-entropy quantifies not only the richness, or surprise, of a dynamical system but also the difficulty of describing (almost) everyone of its typical sequences.

complex = incompressible = unpredictable

Chaos and Probability Revisited

Debate on determinism, randomness, complexity (Amsterdamski et al. 1990)

ex. Thom vs Prigogine: necessity of deterministic rules

What Chaos teaches us: the issue is predictability

Role of probability:

- Maxwell-Boltzmann -> statistical description of thermodynamics
  The large number makes things different *qualitatively*
- Chaos -> statistical approach also for low-dimensional systems

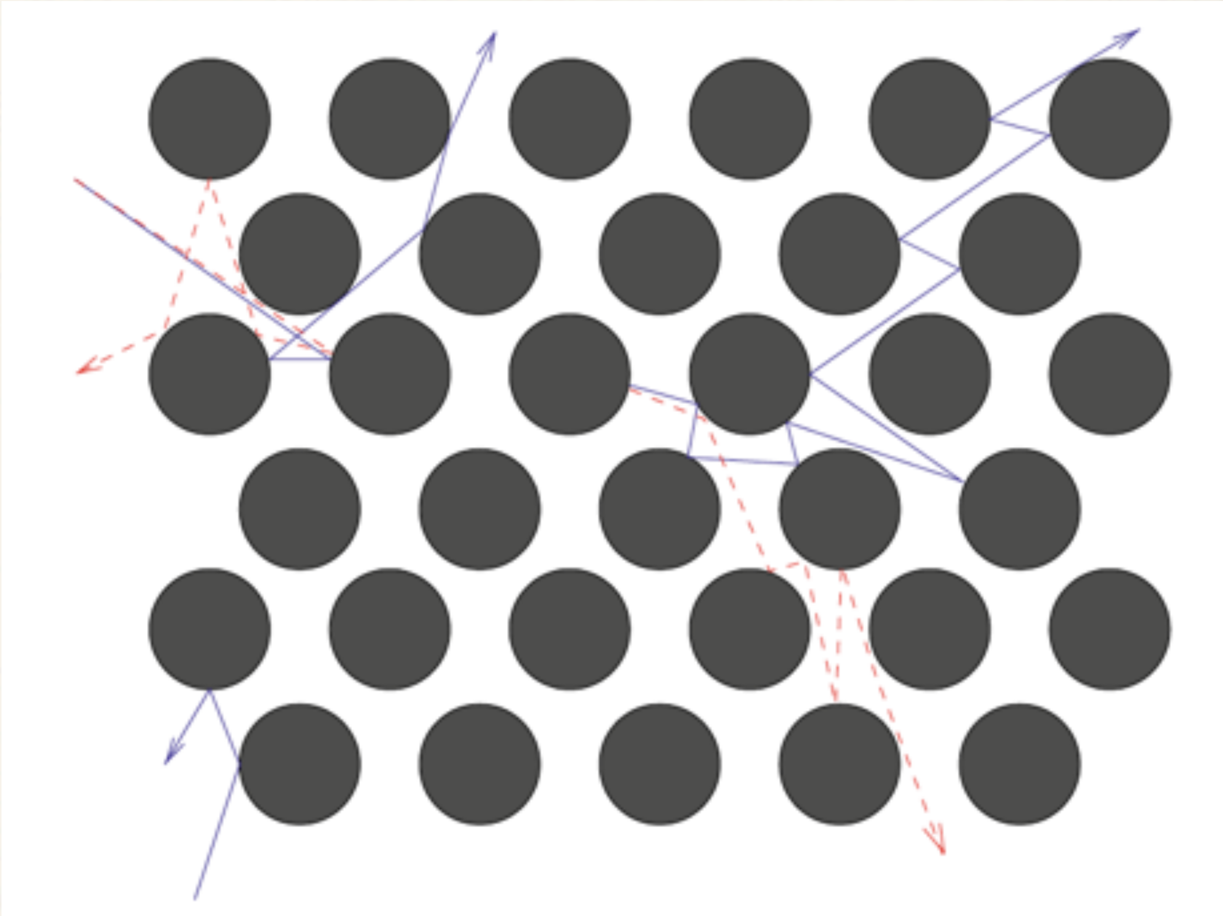
New qualitative laws

Probability as an emergent property

Chaos and Probability Revisited

Sinai billiard = Brownian motion

Dorfman `99



Impossible to decide at
finite resolution and high-dimensionality

(i) Laws governing the universe are inherently random, and the determinism that is believed to be observed is in fact a result of the probabilistic nature implied by the large number of degrees of freedom;
(ii) the fundamental laws are deterministic, and seemingly random phenomena appear so due to deterministic chaos.

Conclusions

(a) Complex(unpredictable) behaviours are not necessarily produced by complicated structures, such as structures made of many components, but are common in simple and low dimensional dynamics;

(b) The methodological approach [“micro-reductionism” in the words of Smith (1998)], which seeks to understand and control dynamics by determining the equations ruling the interactions of its parts, can fail. We may say that: knowing the Navier-Stokes equation does not solve the problem of understanding turbulence.

Statistical approach

The computer will enable us to divide the atmosphere at any moment into stable regions and unstable regions. Stable regions we can predict. Unstable regions we can control (von Neumann)

(i)Accurate numerical simulations which approximate the solution of equations representing, or thought to represent, a given phenomenon.
(ii)Numerical implementations of models which, retaining the basic features of a real system, are crude simplifications, or phenomenological caricatures of “realistic” models.

Philosophical Conclusions

Ontic vs Epistemic *Determinism is ontic, determinability is epistemic* (Atmanspacher 2002)

There are certain classes of phenomena... in which a small error in the data only introduces a small error in the result...There are other classes of phenomena which are more complicated, and in which cases of instability may occur. (Maxwell)

Scientific determinism is the doctrine that the state of any closed physical system at any future instant can be predicted. (Popper)

Philosophers ...

- Impossible to distinguish deterministic systems from stochastic with finite resolution (ε-entropy, FSLE)

True nature?

Philosophical Conclusions

- Ergodicity justifies frequentist probability

> *probabilistic concepts are extraneous to a deterministic description of the world* (Popper)

I do not believe that you are right in your thesis that it is impossible to derive statistical conclusions from a deterministic theory. Only think of classical statistical mechanics (gas theory, or the theory of Brownian movement) (Einstein)

Statistical properties illusory?

- h_{KS} λ Objective, intrinsic properties.

Information epistemic, chaos ontic

Angelo's lesson

L'importante è esagerare

(the importance is to exagerate !)

Summary

1.Deterministic systems, even with just a few degrees of freedom, may be sensitive to initial conditions, hence unpredictable except in the short term.

2.Chaotic systems are complex. Complexity can be rigorously defined in terms of algorithmic complexity, which is a notion of incompressibility hence of unpredictability. Moreover, almost all initial conditions of a generic deterministically chaotic system are complex, hence almost all trajectories are complex.

3.The elementary bricks of complex systems may have far from elementary behaviour, and be complex themselves.

4.A probabilistic description is needed both for chaotic systems and for systems with many degrees of freedom. In both cases, new statistical laws emerge from the underlying deterministic framework.

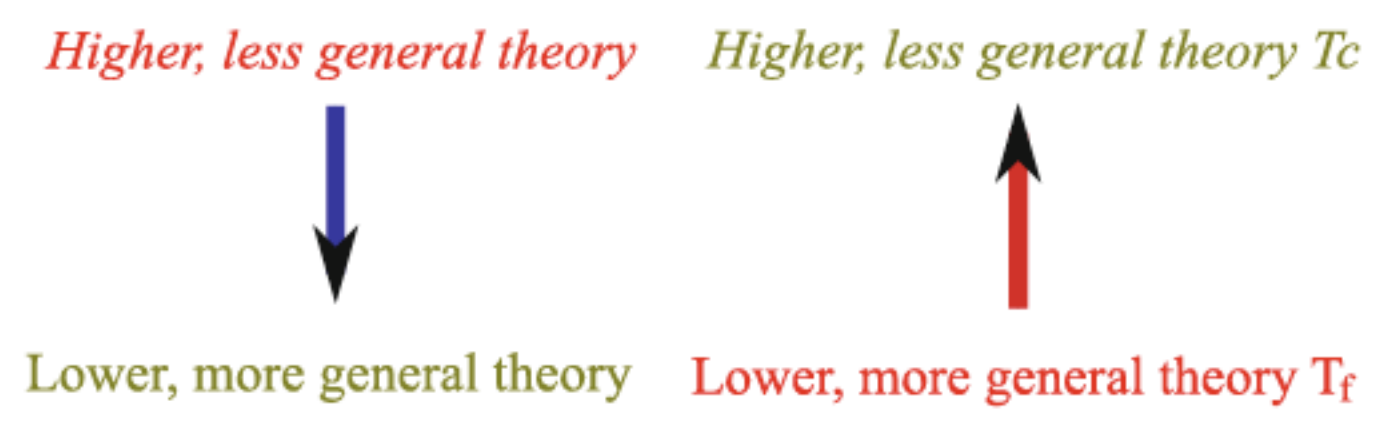
5.If a given phenomenon appears irregular or disordered, it is practically impossible to check whether this is due to chaos, to the presence of many interacting degrees of freedoms, or to some intrinsic randomness.

6.Analogously to the case of the singular limit of statistical mechanics, the singular nature of the chaotic limit allows neither practically nor conceptually the reduction of chaotic macroscopic phenomena to deterministic mechanistic laws. From a philosophical perspective, this is another case of strong emergence.

Philosophical Conclusions

- Lower-level laws (in the basic, reducing science)
- Bridge principles
- Boundary conditions
- Higher-level laws (in the secondary, reduced science).

Nagel, Schaffner



$$\boxed{\lim_{\delta\to 0} T_f(\delta) = T_c}$$

Singular limit

Kim (2000) Bishop and Atmanspacher (2006)

At a certain level, the description of properties (including its laws) offers both necessary and sufficient conditions to rigourously derive the description of properties at a higher level. **reduction**.
2. At a certain level, the description of properties (including its laws) offers necessary but not sufficient conditions to derive the description of properties at a higher level. **emergence**.
3. At a certain level, the description of properties (including its laws) offers sufficient but not necessary conditions to derive the description of properties at a higherlevel. **supervenience**.
4. At a certain level, the description of properties (including its laws) offers neither necessary nor sufficient conditions to derive the description of properties at a higher-level. **radical emergence**

Singular limits <=> emergent properties