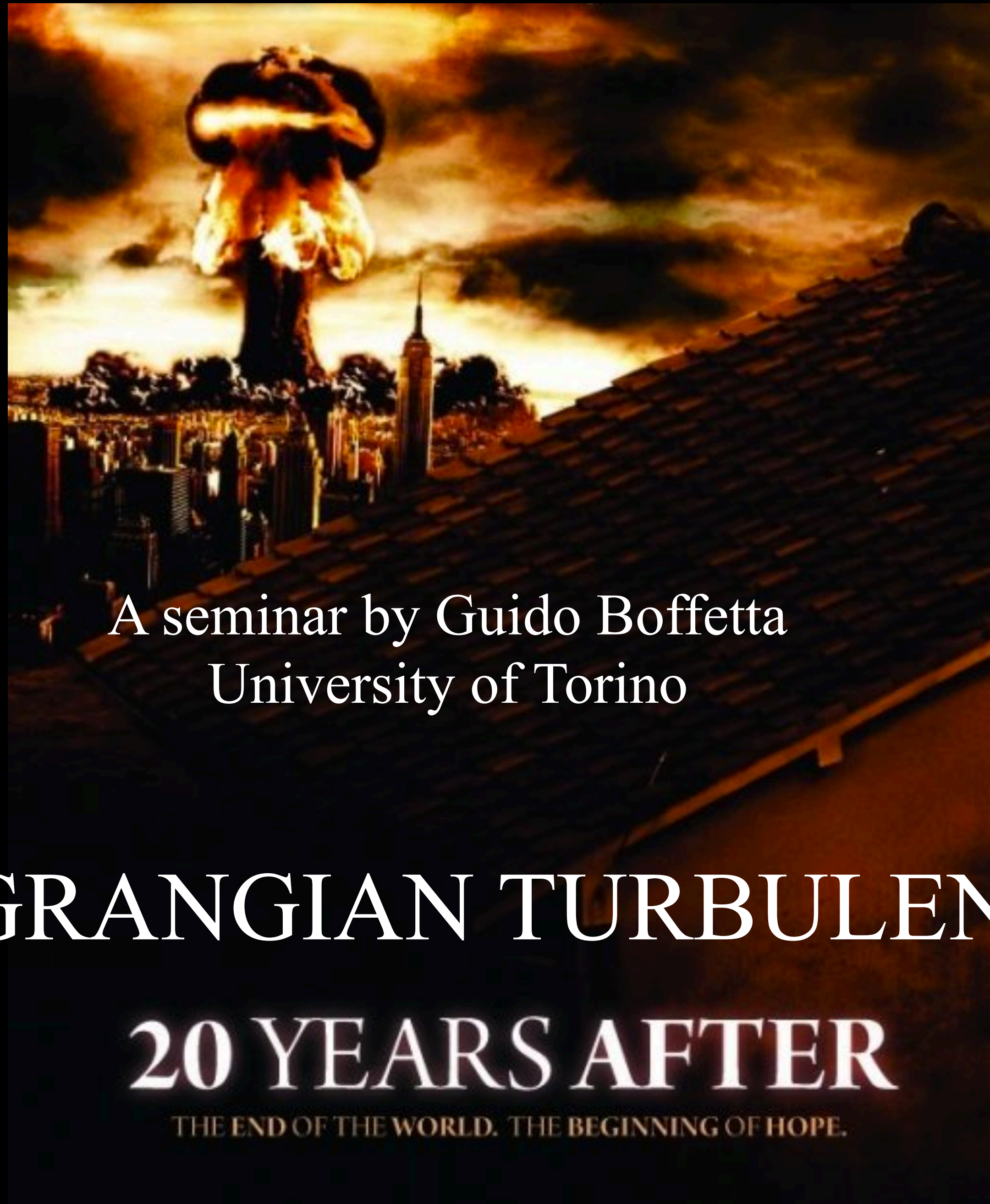
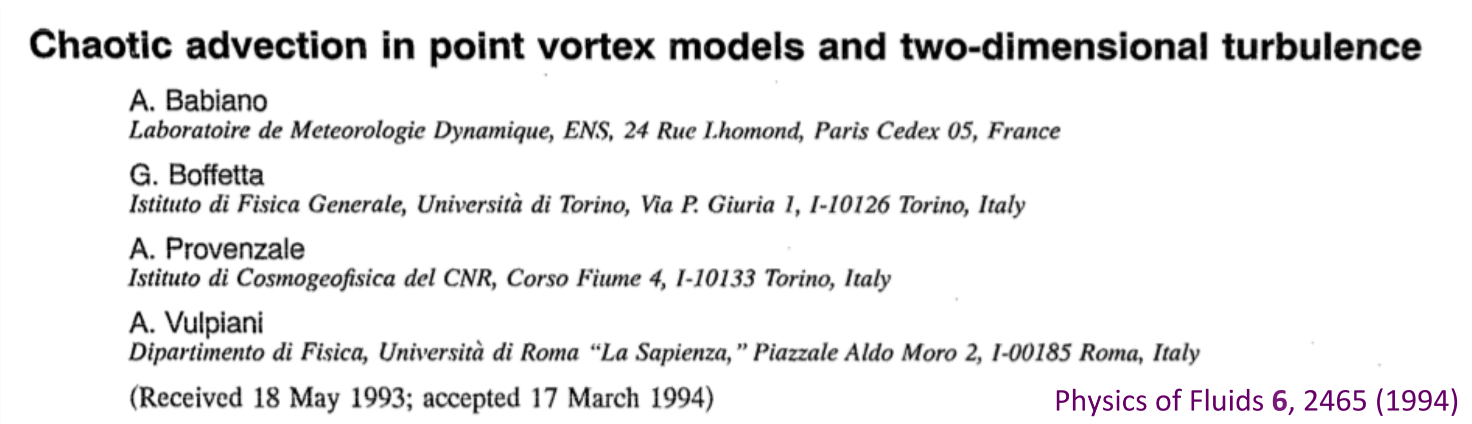


A seminar by Guido Boffetta
University of Torino

LAGRANGIAN TURBULENCE:

http://www.to.infn.it/~boffetta


Chaotic advection in point vortex models and two-dimensional turbulence

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(Received 18 May 1993; accepted 17 March 1994)

Physics of Fluids **6**, 2465 (1994)

P. Sommelier, 3333 m

20 years of Lagrangian statistics

Outline

Historical part:

- Lagrangian and Eulerian statistics
- Exit time statistics
- Applications in turbulence

Modern part:

- Active Lagrangian tracers
- Swimming in turbulence

Prehistory

Chaotic advection in point vortex models and two-dimensional turbulence

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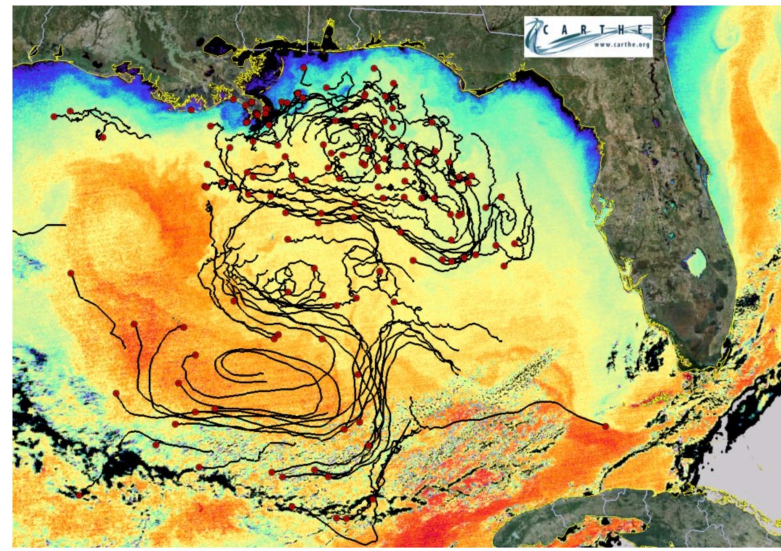
Physics of Fluids **6**, 2465 (1994)

General question: **how to connect Eulerian and Lagrangian statistics in flows**

Relevant for applications: e.g. reconstruct circulation from drifter trajectories



In 2D: a nonautonomous Hamiltonian system

$$\frac{dx}{dt} = +\frac{\partial\psi}{\partial y}$$
$$\frac{dy}{dt} = -\frac{\partial\psi}{\partial x}$$

$\psi(x,y,t)$ is the stream function

chaotic trajectories are possible for regular (time periodic) 2d flows (1+1/2 dof)

Regular (non chaotic) trajectories are possible even for chaotic flows

Chaotic advection in point vortex models and two-dimensional turbulence

Point vortex model: discrete N-particle model for 2D ideal fluid [Kirchhoff, Onsager...]

$$\omega(\mathbf{x},t) = \sum_{\alpha=1}^{N} \Gamma_\alpha \delta(\mathbf{x} - \mathbf{x}_\alpha(t))$$

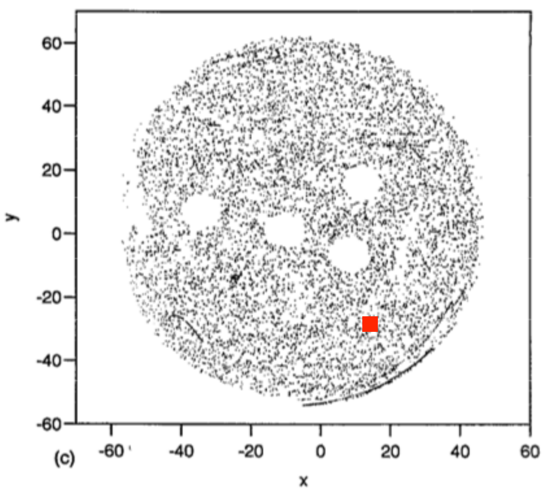
the motion of point vortices $x_\alpha(t)$ is ruled by the Hamiltonian

$$H = -\frac{1}{4\pi} \sum_{\alpha \neq \beta} \Gamma_\alpha \Gamma_\beta \ln r_{\alpha\beta}$$

Lars Onsager (1903-1976)

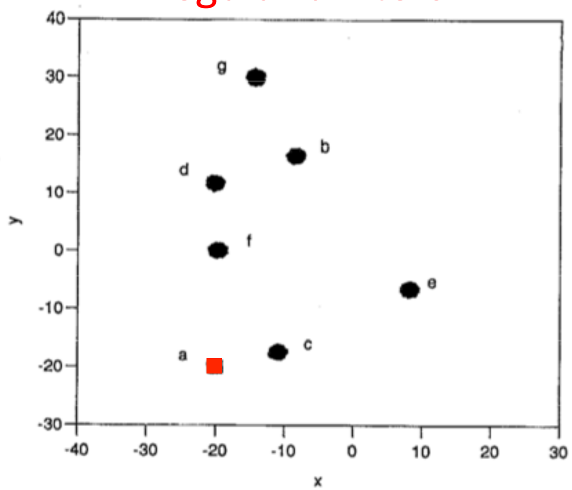
For N≥4 the motion of vortices (the flow) is chaotic: how is the (Lagrangian) motion of a drifter ?

chaotic drifters



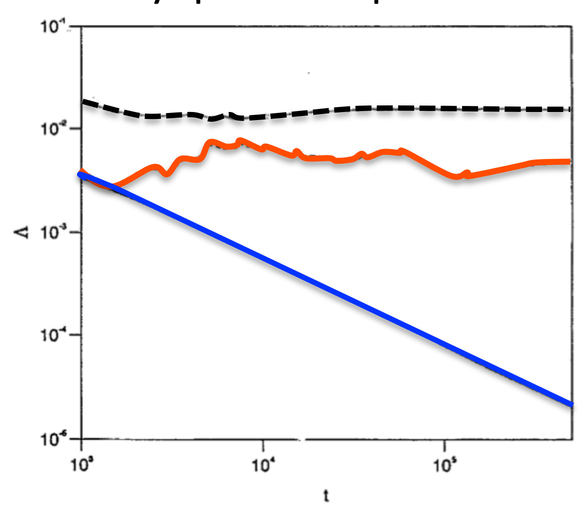
trajectories starting far from vortices visit all the available phase space

regular drifters



trajectories starting close to a vortex remain trapped

Lyapunov exponent

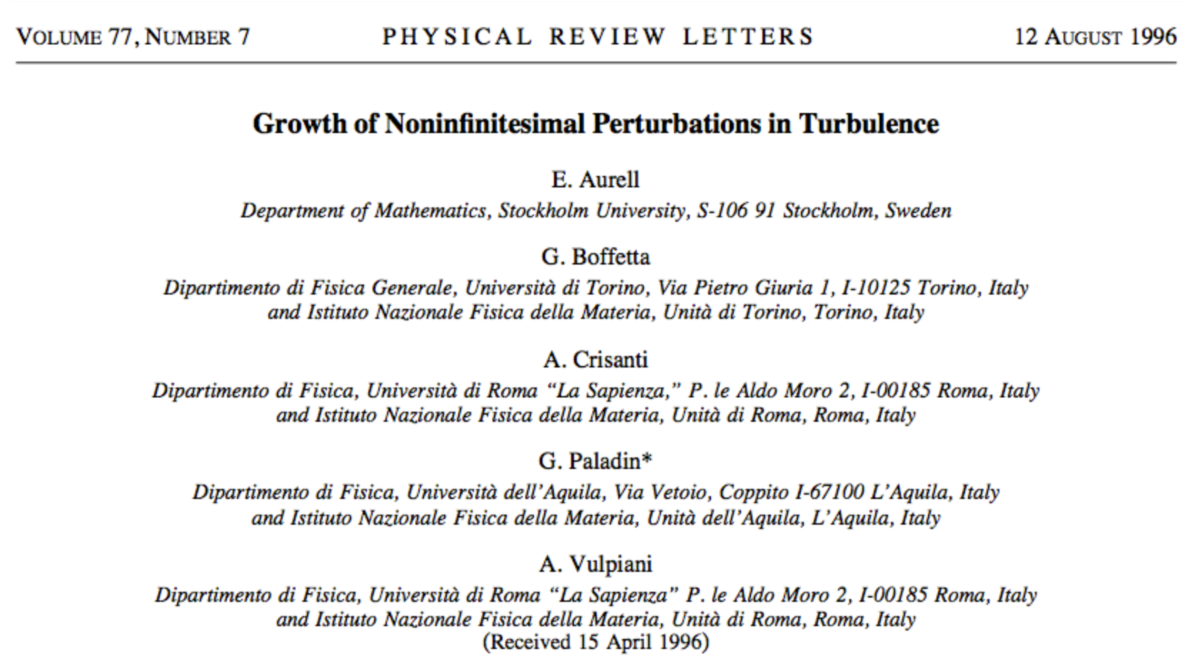


chaotic λ>0 Eulerian: λ>0

regular λ=0

Exit time statistics

The Lyapunov exponent measures the rate of separation of infinitesimally close trajectories
In many applications the growth of finite separations is more relevant


VOLUME 77, NUMBER 7 PHYSICAL REVIEW LETTERS 12 AUGUST 1996

Growth of Noninfinitesimal Perturbations in Turbulence

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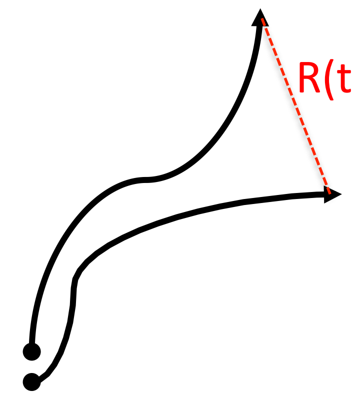
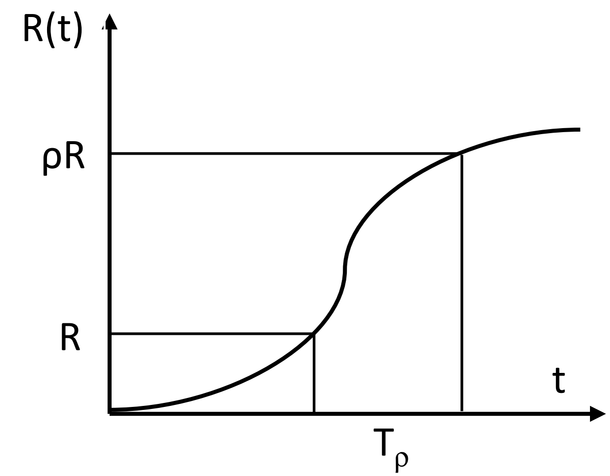
(Received 15 April 1996)



Exit time $T_\rho(R)$: time for separation to grow from R to ρR

$$R(t+T_\rho)=\rho R(t)$$



Finite size Lyapunov exponent

$$\lambda(R) \equiv \frac{1}{\langle T_\rho(R)\rangle}\ln\rho$$

for infinitesimal separations

$$\lim_{R\to 0}\lambda(R)=\lambda$$

Dispersion of passive tracers in closed basins: Beyond the diffusion coefficient

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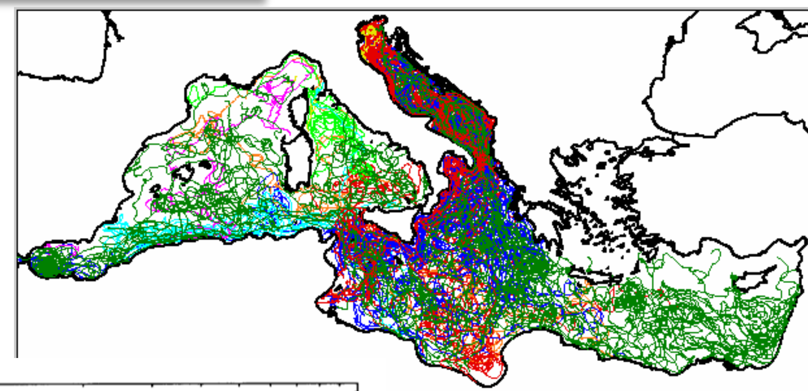
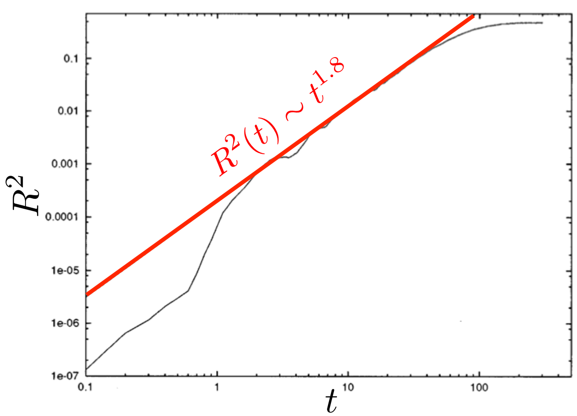
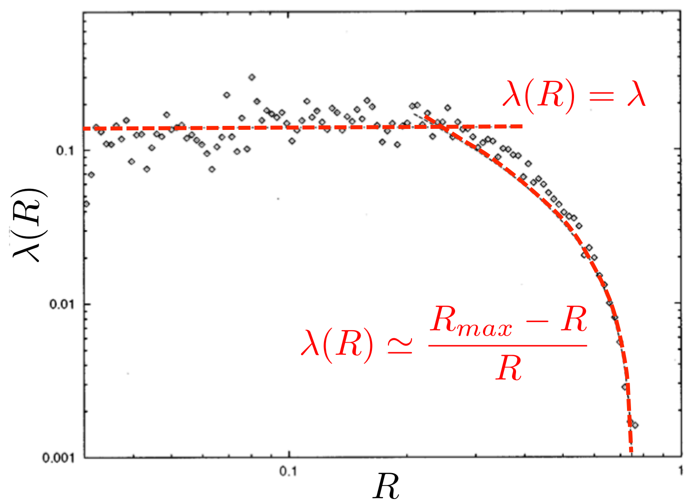
Physics of Fluids **9**, 3162(1997)



Dispersion of Lagrangian drifter in finite basins:
no scale separation between flow correlation scale L_U and domain scale L_B : no diffusive behavior



Apparent anomalous behavior
$\langle R^2(t)\rangle \sim t^{2\nu}$ at intermediate times

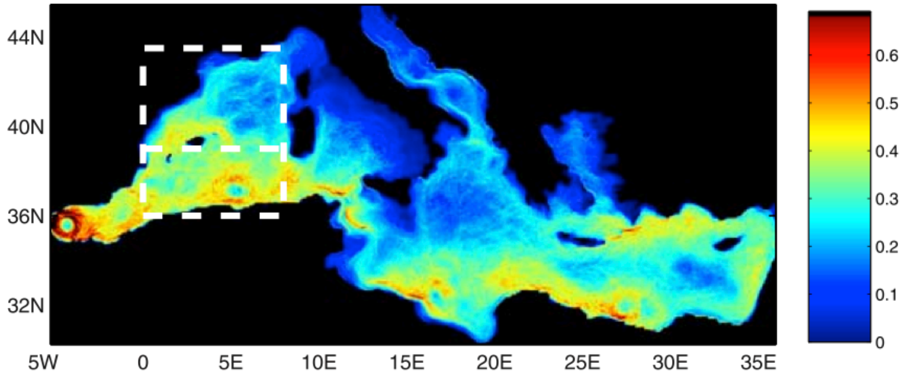


No anomalous behavior with FSLE:
chaos + saturation
to maximal separation

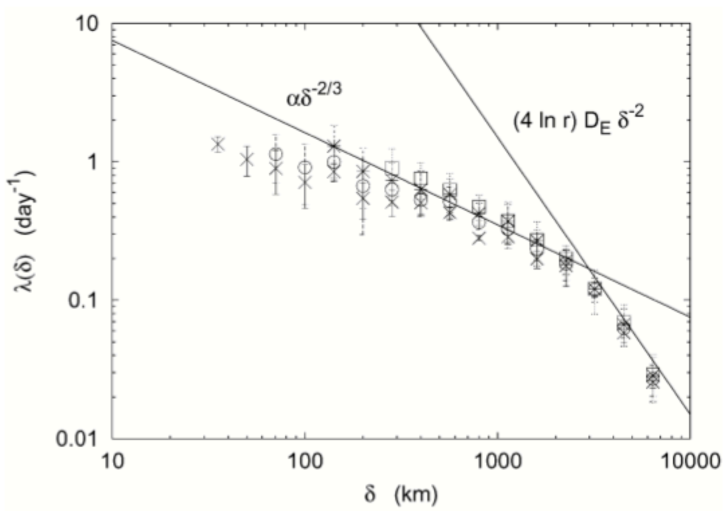
Use of the Finite Size Lyapunov Exponent

Mixing properties of the Mediterranean Sea

F. d'Ovidio *et al*, "Mixing structures in the Mediterranean Sea from finite-size Lyapunov exponents" Geophys. Res. Lett. **31**, L17203 (2004)



Relative dispersion and inverse cascade in the stratosphere



EOLE project: 483 balloons in the southern hemisphere

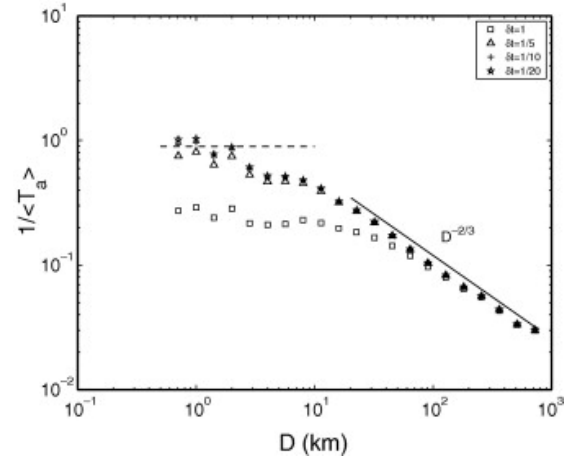
G. Lacorata, E. Aurell, B. Legras, A. Vulpiani, "Evidence for a $k^{-5/3}$ Spectrum from the EOLE Lagrangian Balloons in the Low Stratosphere", J. Atmos. Sci. 61, 2936 (2004)

Use of the Finite Size Lyapunov Exponent / 2

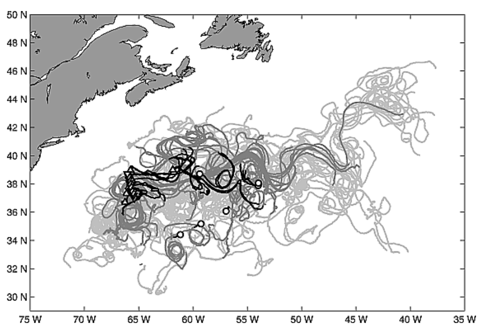
Relative dispersion in the Gulf of Mexico

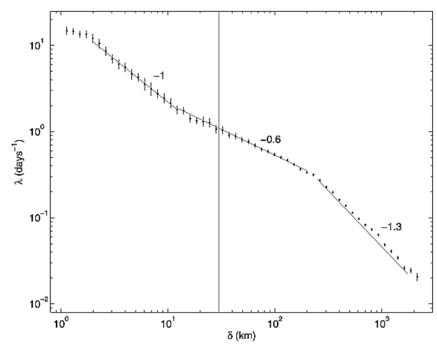
700 surface drifters deployed at pairs

J.H. LaCasce, “Statistics from Lagrangian observations” Prog. Oceanography **77**, 1 (2008)



Dispersion of surface drifters in the Gulf stream

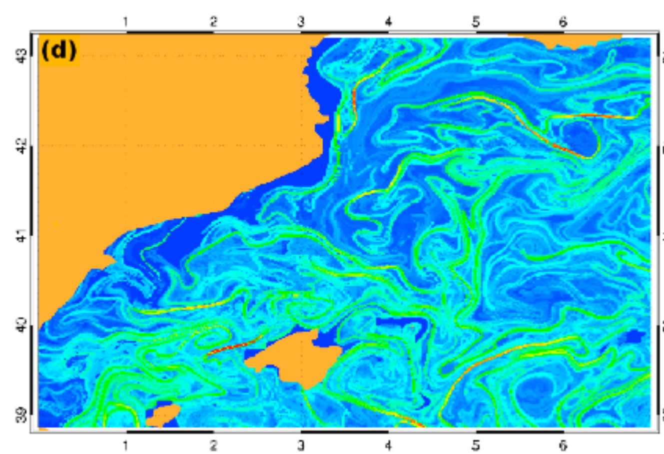




R. Lumpkin, S. Elipot, “Surface drifter pair spreading in the North Atlantic”, J. Geophys. Res. **115**, C12017 (2010)

Numerical study of dispersion in the Balearic sea

I. Hernández-Carrasco, C. López, E. Hernández-García, A. Turiel, “How reliable are finite-size Lyapunov exponents for the assessment of ocean dynamics?” Ocean Mod. 36, 208 (2011)



Relative dispersion in turbulence

First empirical evidence of Kolmogorov scaling in turbulence (in 1926)

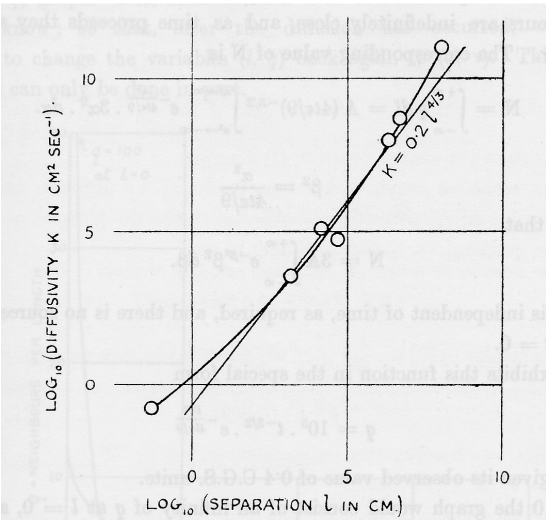


Fig. 8.

Diffusivity in the atmosphere

$$K(R) \simeq R^{4/3}$$

From Kolmogorov scaling:

$$K(R) = \frac{dR^2}{dt} \simeq R\delta v(R) \simeq \varepsilon^{1/3} R^{4/3}$$

L.F. Richardson (1881-1953)

Diffusion equation for *distance neighbor function* p(**R**,t)

$$\frac{\partial p(\mathbf{R},t)}{\partial t} = \frac{\partial}{\partial R_i}\left[K(R)\frac{\partial p(\mathbf{R},t)}{\partial R_i}\right]$$

Solution with δ initial condition: $p(\mathbf{R},t) = \frac{C}{(\alpha t)^{9/2}} \exp\left(-\frac{9R^{2/3}}{4\alpha t}\right)$

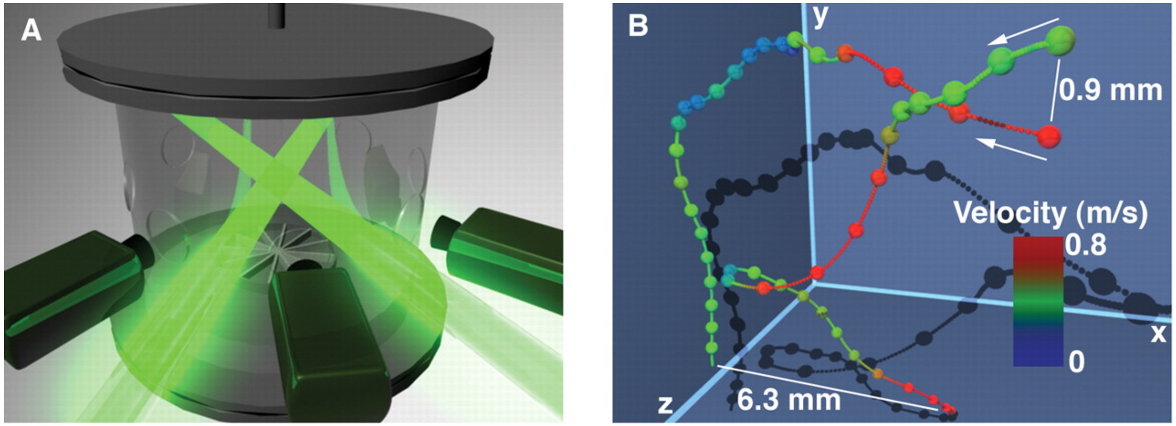
Explosive separation of trajectories:

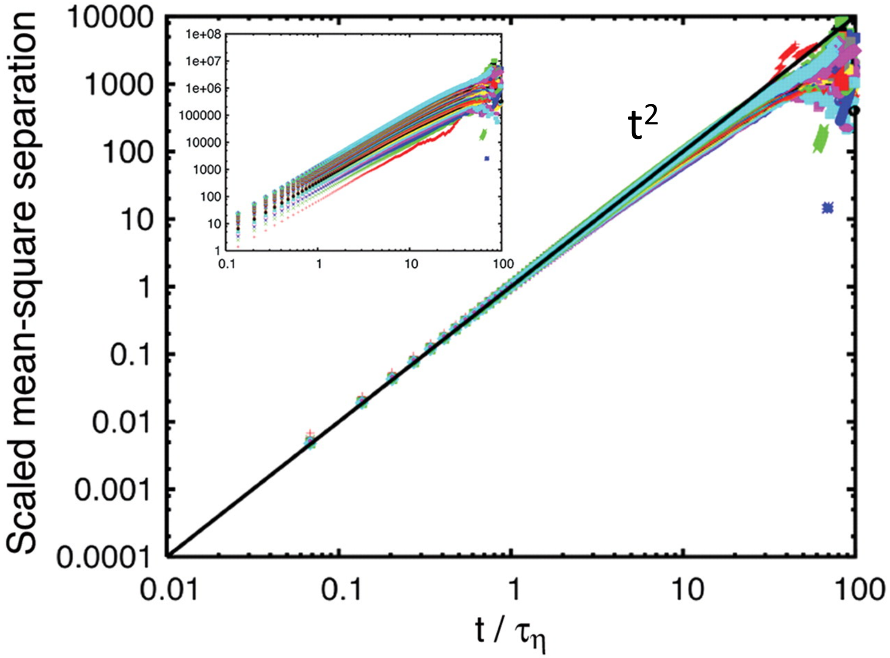
$$\langle R^2(t)\rangle = g\varepsilon t^3$$

(Richardson's law)

Exit times in turbulence

Difficulties in observing Richardson's law because of strong fluctuations at small scales





Recent laboratory experiments at high Reynolds numbers failed to observe Richardson's t^3 law (they observe t^2, ballistic)

M.Bourgoin *et al*, Science **311**, 5762 (2006)

Exit times in turbulence

Difficulties in observing Richardson's law because of strong fluctuations at small scales

Pair dispersion in synthetic fully developed turbulence

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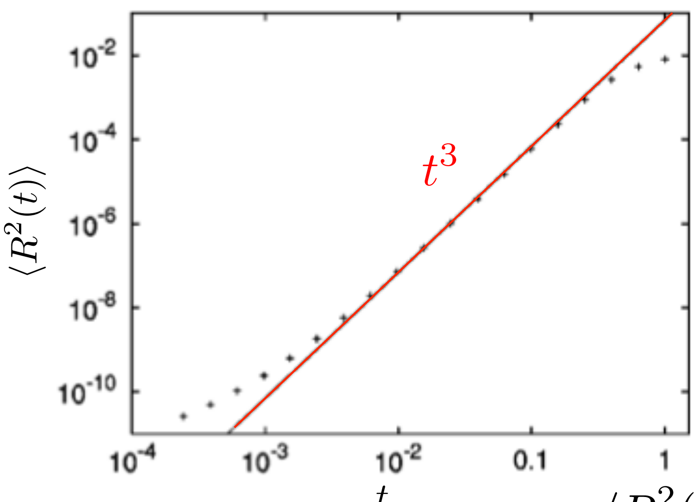
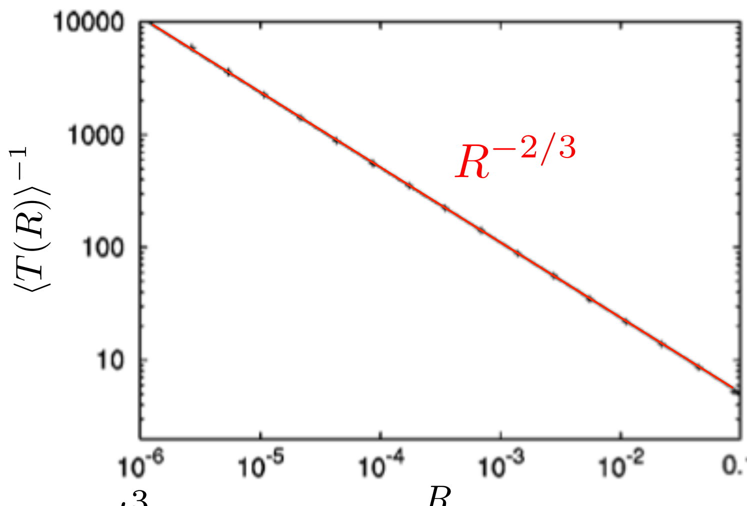
(Received 1 June 1999)

Phys. Rev. E **60**, 6734 (1999)

Use the exit time statistics to avoid contamination between different scales





$$\langle R^2(t)\rangle = g\varepsilon t^3$$

Synthetic turbulence

much better scaling

Exit time statistics in direct numerical simulations of Navier-Stokes equations

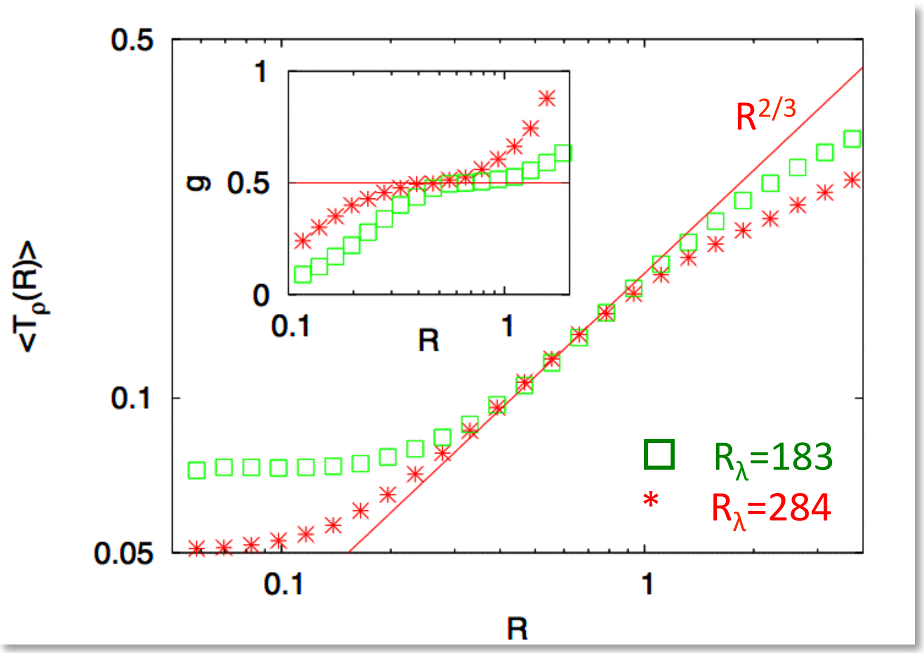
Assuming the Richardson model for relative dispersion the mean exit time is given by a first passage problem as

$$\langle T_\rho(R)\rangle = \frac{3}{2d}\frac{\rho^{2/3}-1}{\alpha\varepsilon^{1/3}\rho^{2/3}}R^{2/3}$$

Richardson constant

$$g = \frac{143}{81}\frac{(\rho^{2/3}-1)^3}{\rho^2}\frac{R^2}{\varepsilon\langle T_\rho(R)\rangle^3} \simeq 0.5$$

$$\langle R^2(t)\rangle = g\varepsilon t^3$$



G.Boffetta and I.M.Sokolov, PRL **88**, 094501 (2002)
L.Biferale *et al*, Phys. Fluids **17** 115101 (2005)

20 years of Lagrangian statistics

Outline

Modern part:
- Active Lagrangian tracers
- Swimming in turbulence

In collaboration with
M. Cencini (ISC, CNR Roma), F. De Lillo (Universita' di Torino)
R. Stocker, M. Barry (MIT, Cambridge, USA)
W.M. Durham (University of Oxford, UK), E. Climent (CNRS Toulouse, FR)

W.M. Durham et al, *Nature Comm* **4**, 2148 (2013)
F. De Lillo et al, *Phys Rev Lett* **112**, 044502 (2014).

Phytoplankton: Active Lagrangian Tracers

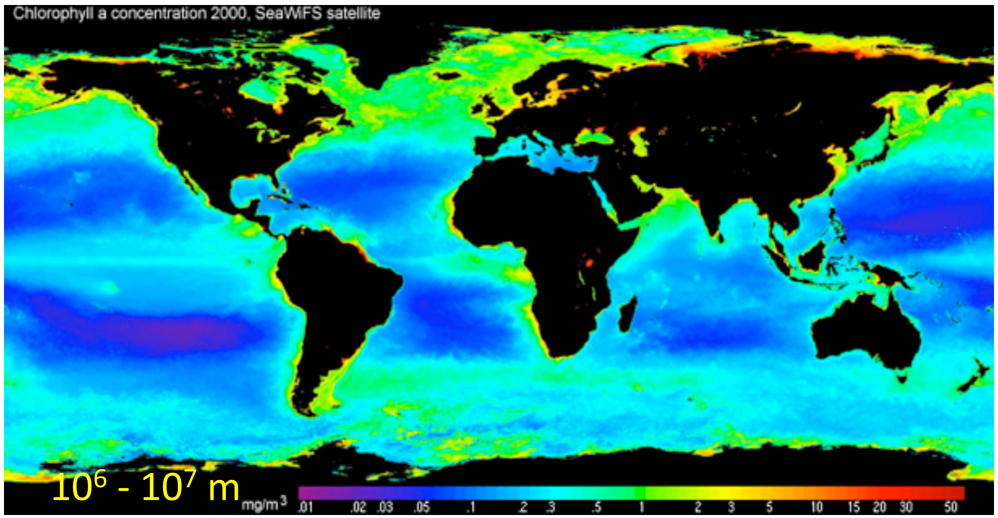
- unicellular organisms of many forms and sizes between 1 and 100 μm
- about 5000 species
- about 50% of photosynthetic activity on Earth
- at the basis of the marine food web
- harmful algal blooms from toxic species
- many species are able to swim
- patchiness at different scales

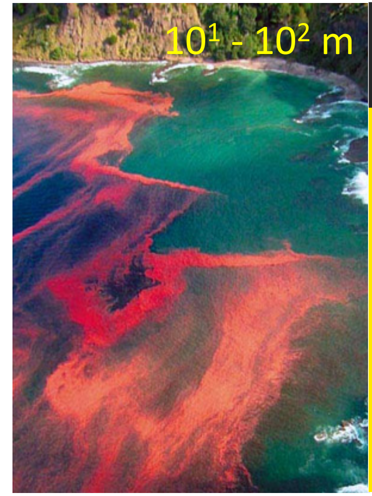
Turbulence mediates many processes crucial to the ecology of phytoplankton, including motility, nutrient uptake, and cell-cell encounters.

Motivations

Collective phenomena in micro-swimmers

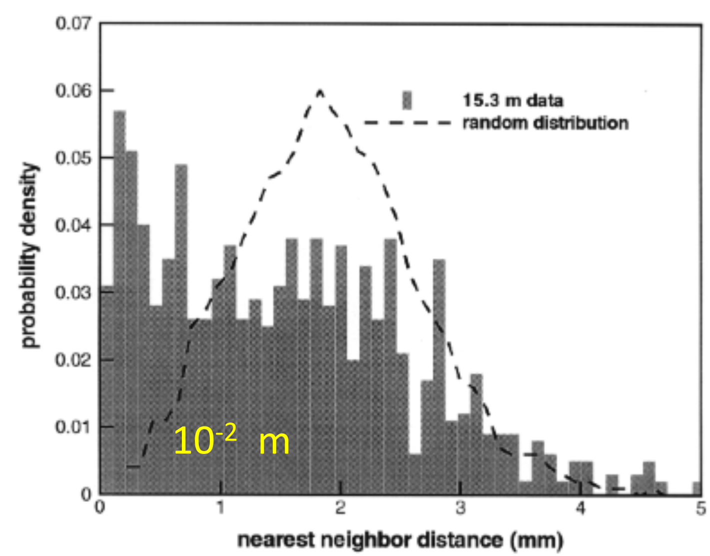
- plankton patchiness over many scales
- (toxic) algal blooms and “thin layers”





Red tide of *Karenia brevis*

Motile phytoplankton is found to be **more patchy** than non-motile one

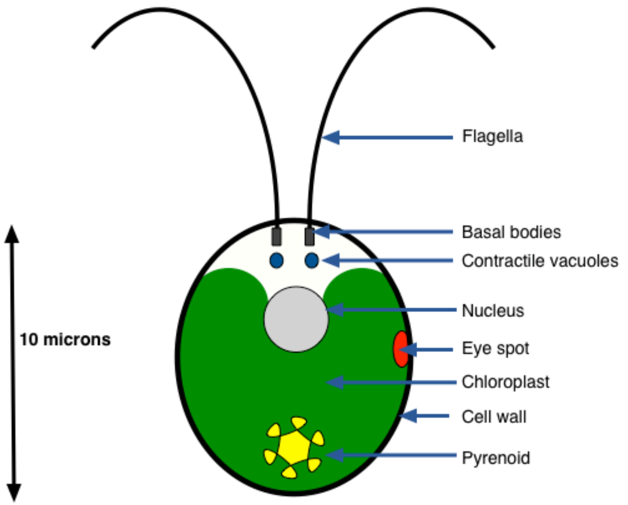


E.Malkiel, O.Alquaddoomi, J.Katz
Meas. Sci. Technol. **10** (1999)

Figure 10. Measured nearest-neighbour distances and expected values from a random distribution.

Swimming algae

The genus Chlamydomonas, unicellular flagellate, a model organism for molecular biology.

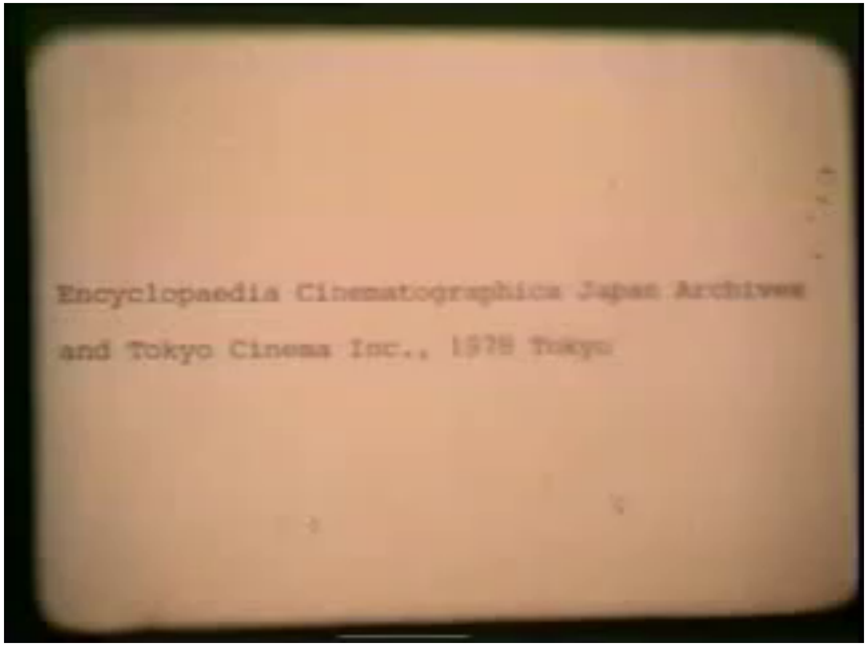


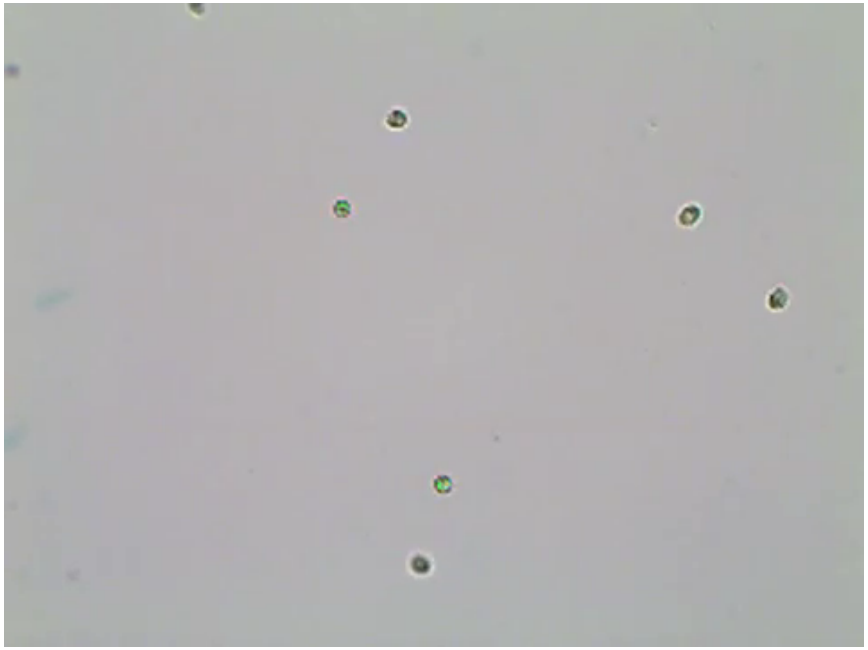
Both sexual and asexual reproduction

Eyespot: light receptor for phototactic (and photophobic) behaviors (cell rotates at about 2 Hz to detect direction)

Slightly heavy, unbalanced weight (bottom heavy due to chloroplast distribution): they naturally swim against gravity (gyrotaxis)

Good swimmers: about 10 body-length per second

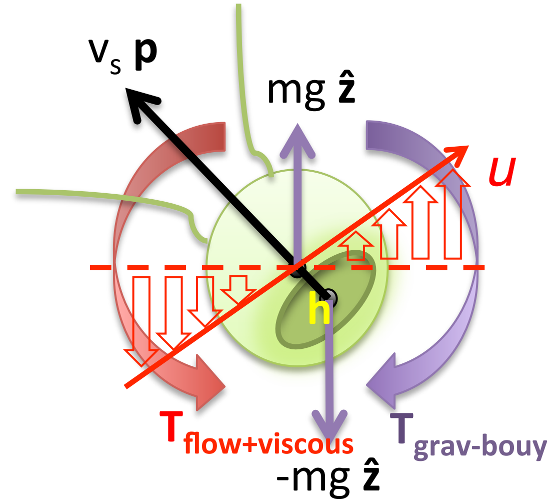




Gyrotactic model for bottom-heavy cells

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x},t) + v_s \mathbf{p}$$

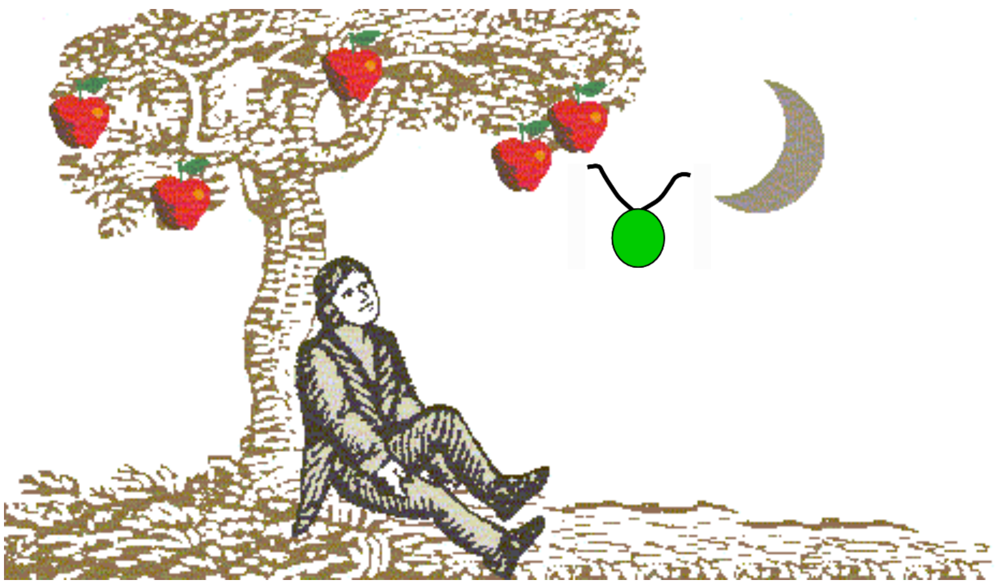
$$\frac{d\mathbf{p}}{dt} = -\frac{1}{2v_o}\left[\mathbf{A} - (\mathbf{A}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega \times \mathbf{p}$$



$$\mathbf{A} = \mathbf{g} - \mathbf{a}(\mathbf{x},t)$$

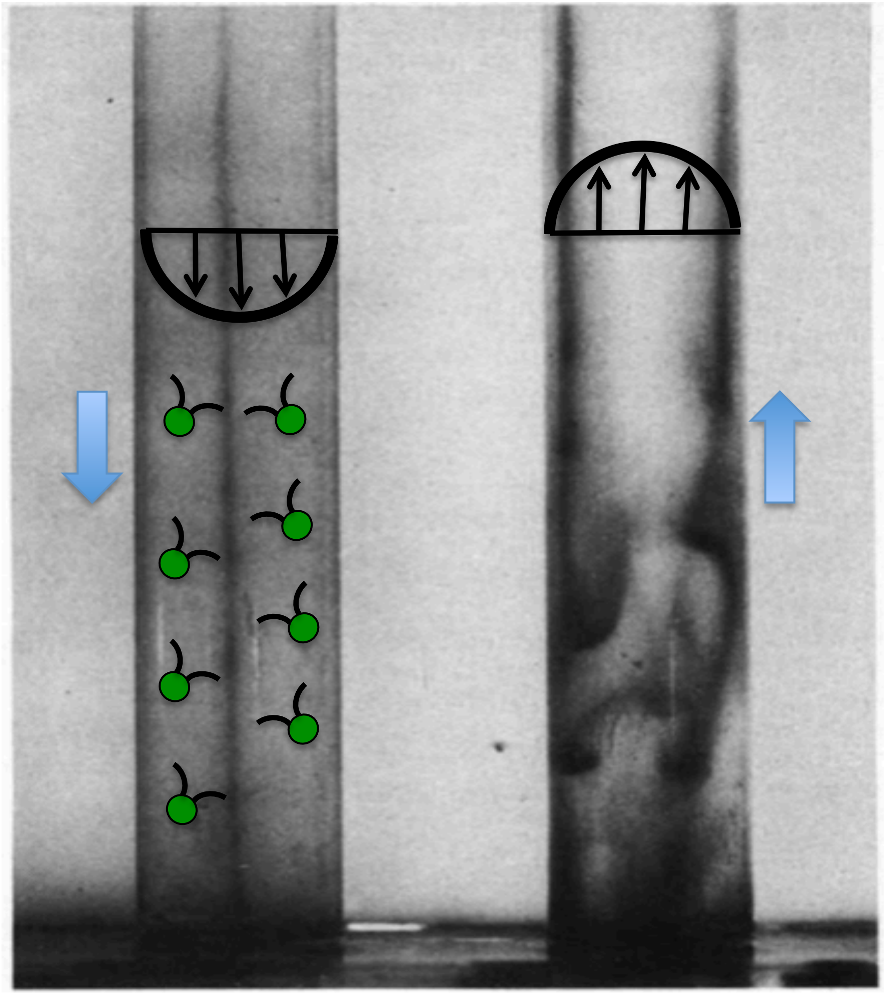
$$v_s \simeq 100\,\mu m\, s^{-1}$$

$$v_0 = \frac{3\nu}{h} = O(1) m\, s^{-1}$$



JO Kessler, Nature **313**, 218 (1985)
T.J. Pedley, J.O. Kessler, Proc. Roy Soc. B **231** (1987)

Gyrotactic focusing in (laminar) pipe flows



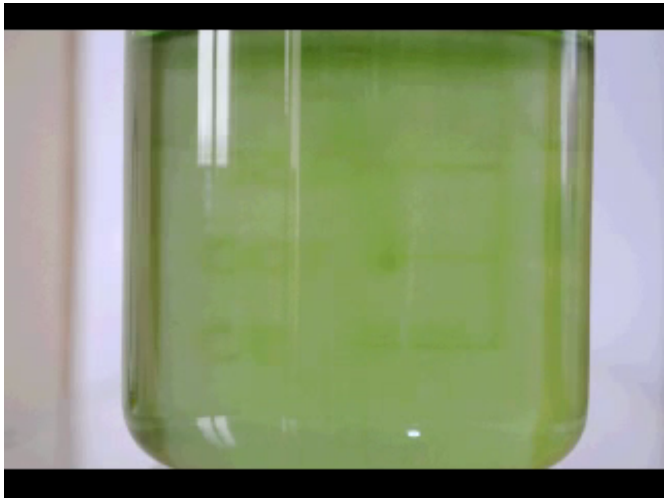
JO Kessler, Nature **313**, 218 (1985)

$$a = 0$$

$$\frac{d\mathbf{p}}{dt} = -\frac{1}{2v_o}\left[\mathbf{g} - (\mathbf{g}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega \times \mathbf{p}$$

Swimming cells concentrate
in the center of downwelling flows

Concentration in bioconvection



Acceleration in turbulence

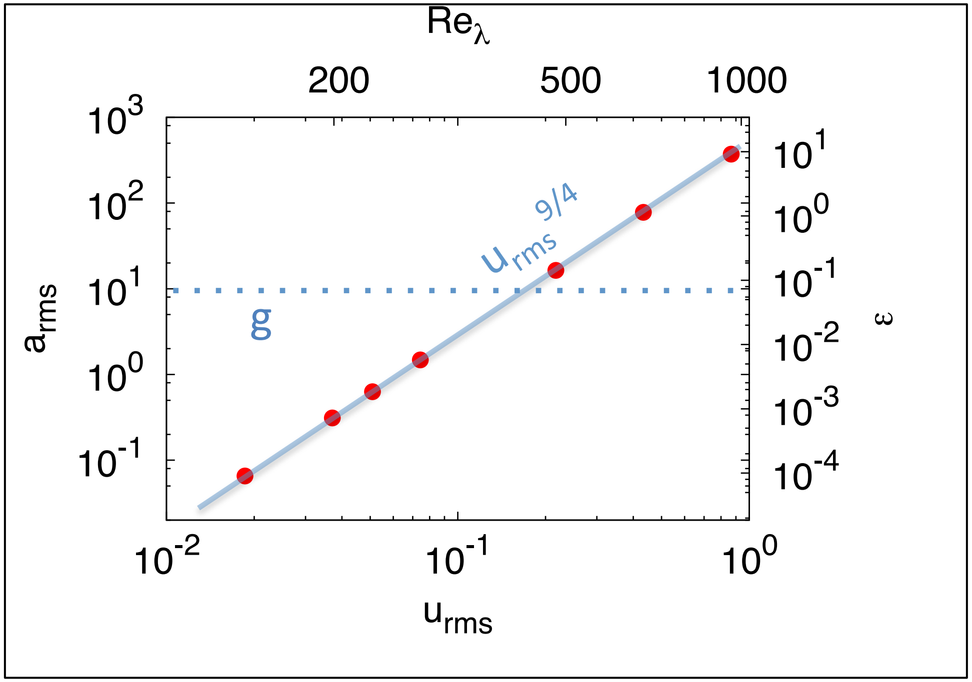
Laboratory experiments

$$\langle a^2 \rangle = a_0 \epsilon^{3/2} \nu^{-1/2}$$

with $a_0 \simeq 5-6$ and $\nu_{H_2O} \simeq 10^{-6} m^2 s^{-1}$

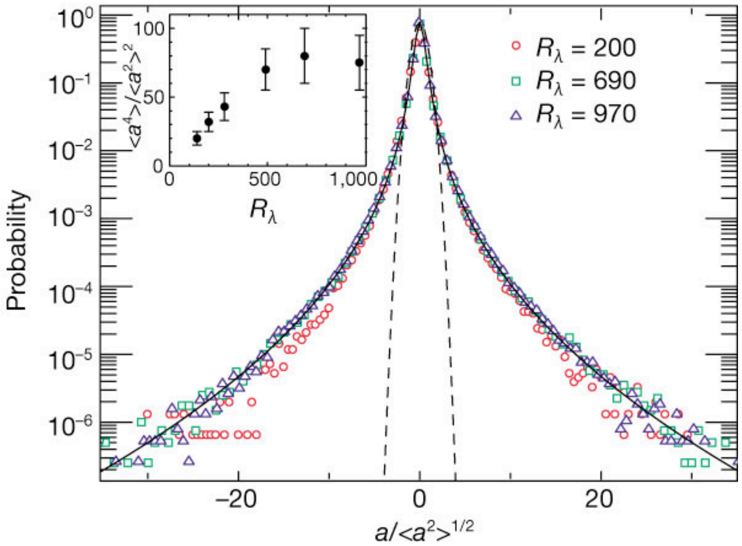
Kinetic energy dissipation in the oceanic mixing layer $\epsilon \leq 10^{-4} m^2 s^{-3}$ which means

$$a_{rms} \simeq 0.1\, m s^{-2} \ll g$$



from Voth *et al*, JFM **469**, 121 (2002)

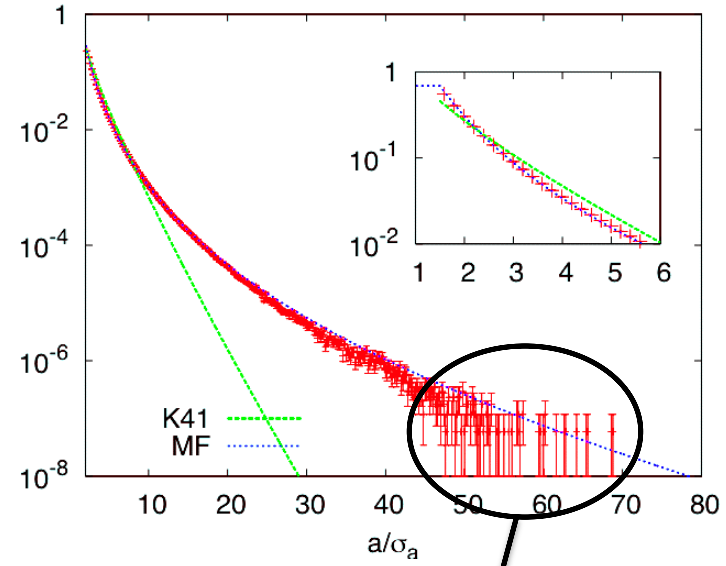
Experimental and numerical data show that local acceleration in turbulence is extremely intermittent with PDF with very wide tails



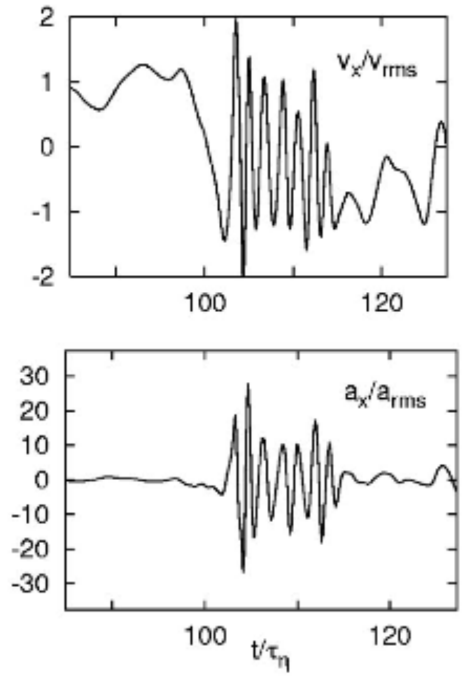
Even if $a_{rms} < g$, local fluid acceleration can exceed gravity

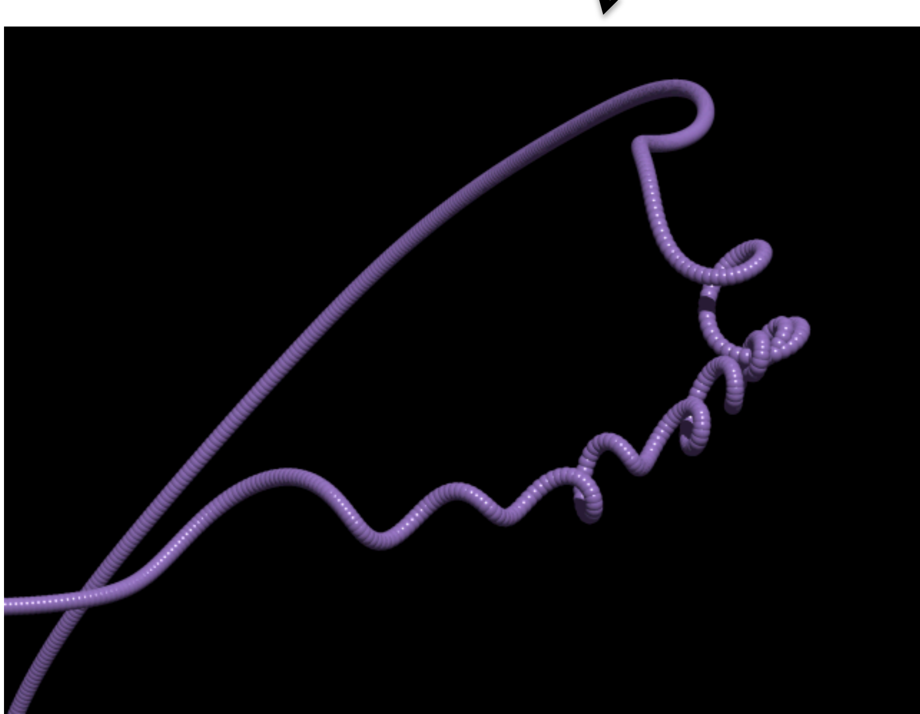
La Porta *et al*, Nature **409**, 1017 (2001)

Origin of extreme accelerations: turbulent vortices



Trapping of particles in small scale vortices





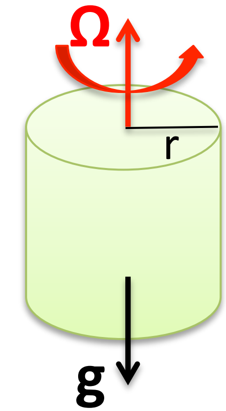
Maximal acceleration inside vortices

Biferale, Boffetta, Celani, Devenish, Lanotte, Toschi
Phys. Rev. Lett. **93**, 064502 (2004)

The effects of fluid accelerations: a toy model for a vortex

Cylinder in solid body rotation

$$\mathbf{u}(\mathbf{x}) = (-\Omega y, \Omega x, 0) \qquad \omega = (0, 0, 2\Omega) \qquad \mathbf{a}(\mathbf{x}) = (-\Omega^2 x, -\Omega^2 y, 0)$$

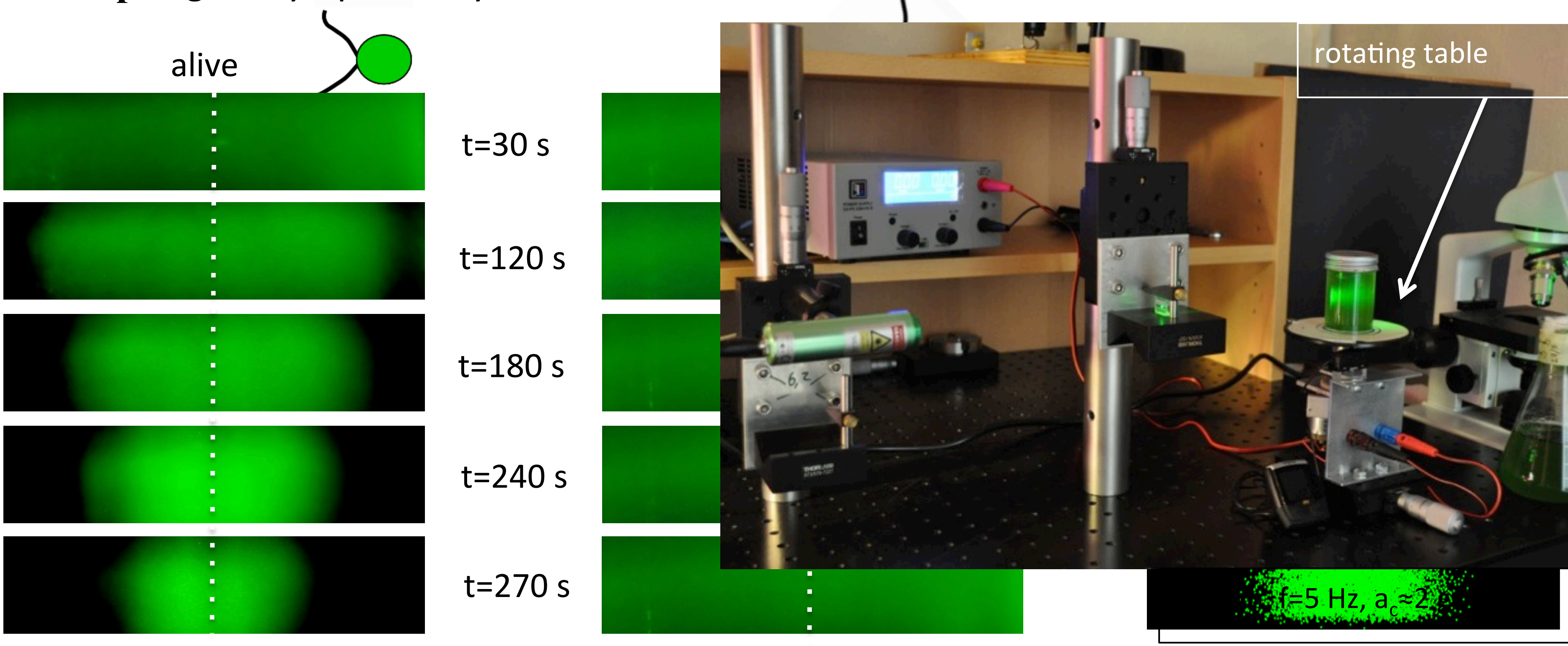


equilibrium swimming direction (stationary) $\frac{d\mathbf{p}}{dt} = 0$

cylindrical coordinates $\begin{cases} \mathbf{p}_r = -\gamma \mathbf{r} \\ p_z = \sqrt{1 - \gamma^2 r^2} \end{cases}$ $\qquad \gamma = \Omega^2 / g$

gives the trajectory $r(t) = r(0) e^{-\gamma v_s t}$ cells concentrate around the axis

and $\mathbf{p}$ aligns asymptotically in the direction z



Simulations of turbulence

Simulation of the complete set of equations

Three dimensionless numbers

$$Re_\lambda = \frac{u_{rms}\lambda}{\nu}$$ controls the weight of turbulent acceleration $$\alpha \equiv \frac{a_{rms}}{g}$$

$$\Phi = \frac{v_s}{v_k}$$ swimming number

$$\Psi = \frac{\omega_{rms} v_o}{A_{rms}}$$ stability number

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

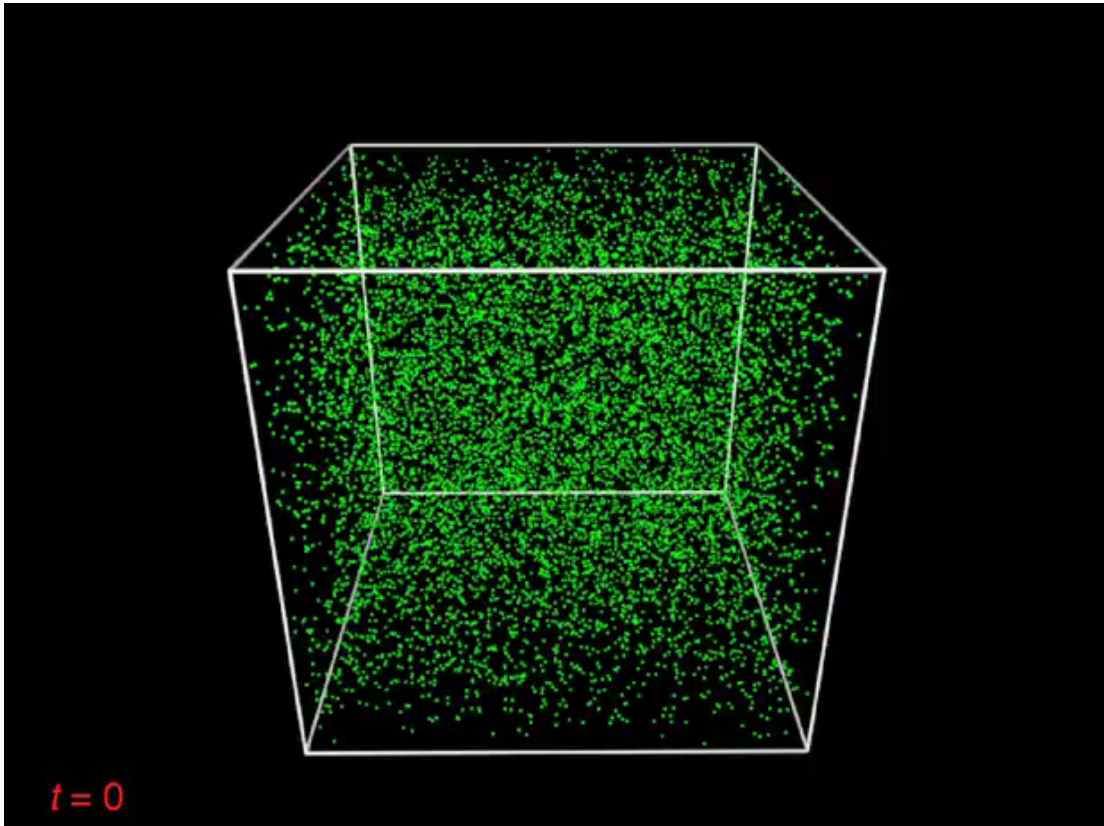
$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x},t) + v_s\mathbf{p}$$

$$\frac{d\mathbf{p}}{dt} = \frac{1}{2v_o}\left[\mathbf{A} - (\mathbf{A}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega\times\mathbf{p}$$

Gyrotactic swimmers as a dissipative system

$$\sum_{i=1}^{d}\left(\frac{\partial \dot{X}_i}{\partial X_i} + \frac{\partial \dot{\mathbf{p}}_i}{\partial \mathbf{p}_i}\right) = -\frac{d-1}{2v_o}\left(g\mathbf{p}_z + \mathbf{a}\cdot\mathbf{p}\right)$$

as **p** orients in the direction of local acceleration, swimming cells concentrate on a (fractal) subset of the phase space



Turbulence at small Reynolds numbers ($a_{rms} << g$)

Typical conditions in the ocean mixing layer $\varepsilon = 10^{-7}\, m^2 s^{-3}$

$$\eta = (\nu^3/\varepsilon)^{1/4} \simeq 2\, mm$$

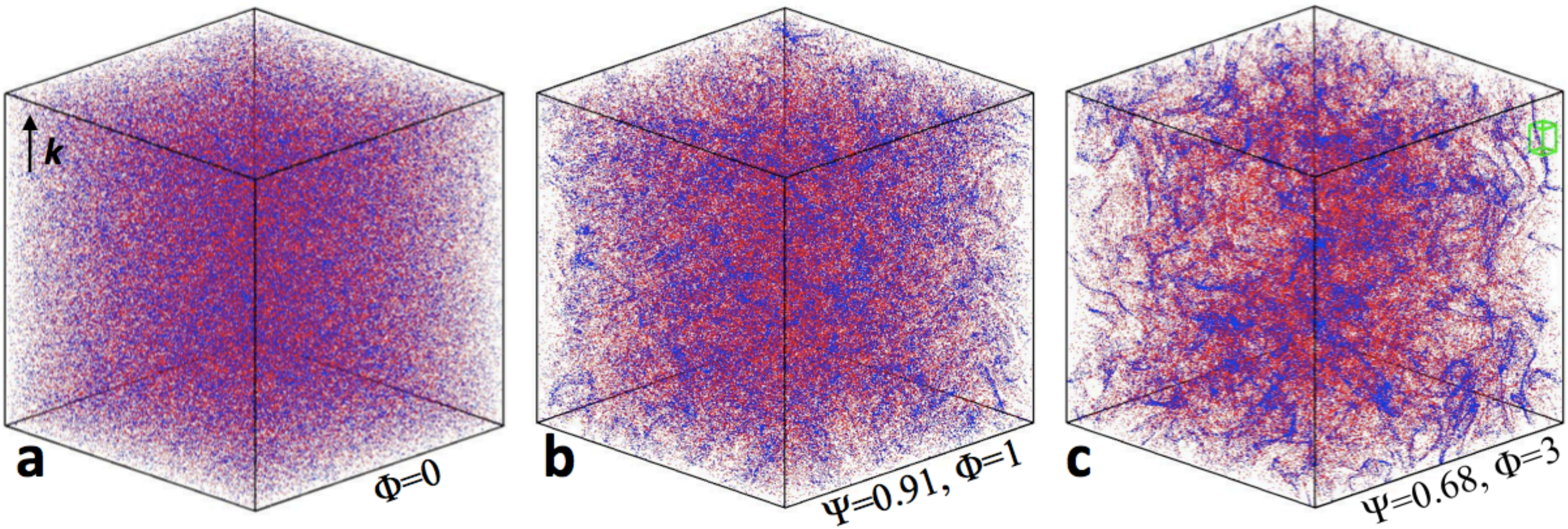
$$\tau_k = (\nu/\varepsilon)^{1/2} \simeq 3\, s$$

$$v_k = (\nu\varepsilon)^{1/4} \simeq 0.5\, mm\, s^{-1}$$

$$\Phi = \frac{v_s}{v_k} \simeq 0.4$$

$$\Psi = \frac{\omega_{rms} v_o}{g} \simeq 0.3$$

$Re_\lambda = 65$



$n > 2\langle n \rangle$

How clustering depends on parameters ?

Where do cells cluster ?

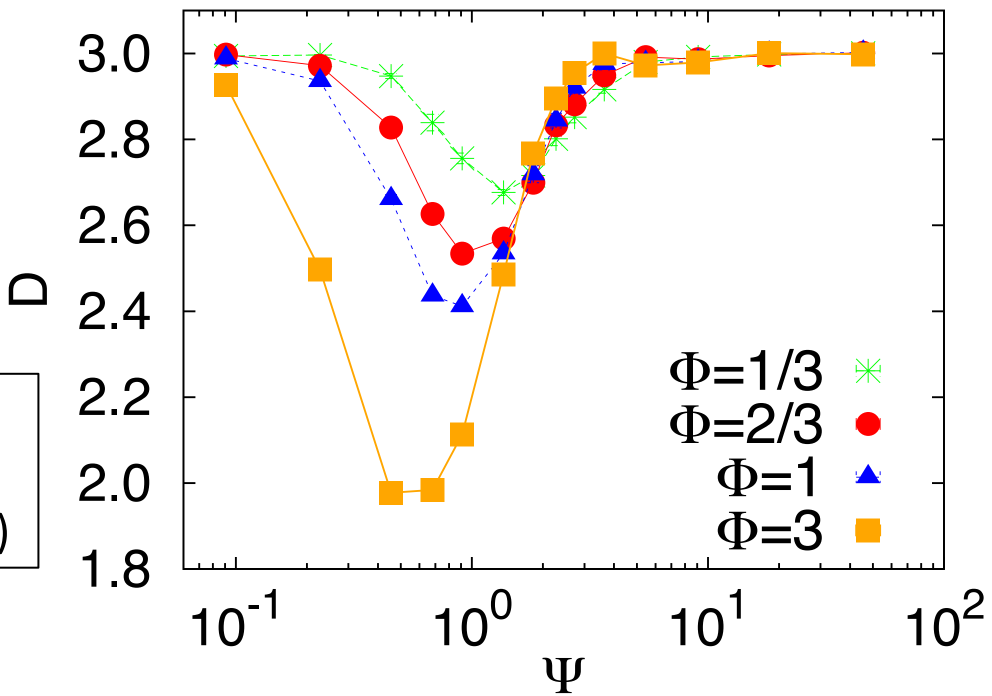
10^6 cells

Fractal clustering

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x},t) + \Phi\mathbf{p}$$
$$\frac{d\mathbf{p}}{dt} = \frac{1}{2\Psi}\left[\mathbf{k} - (\mathbf{k}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega \times \mathbf{p}$$

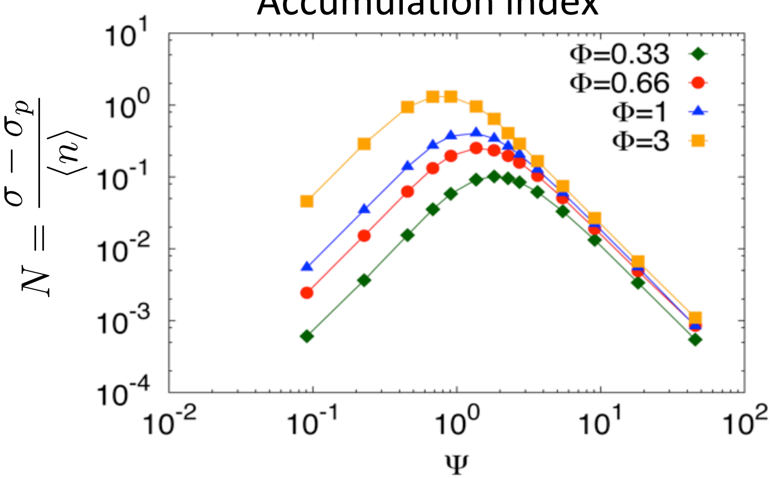
Homogeneous distribution in both limits $\Psi \to 0$ (uniform vertical swimming) and $\Psi \to \infty$ (swimming in random directions)

Correlation dimension



Clustering is maximum (D minimum) for $\Psi \simeq 1$ and increases with Φ

Accumulation index



$$N = \frac{\sigma - \sigma_p}{\langle n \rangle}$$

$$\sigma = \left(\langle n^2 \rangle - \langle n \rangle^2\right)^{1/2}$$

swimming number	$\Phi = \dfrac{v_s}{v_k}$
stability number	$\Psi = \dfrac{\omega_{rms} v_o}{g}$

How clustering depends on Ψ

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x},t) + \Phi\mathbf{p}$$

$$\frac{d\mathbf{p}}{dt} = \frac{1}{2\Psi}\left[\mathbf{k} - (\mathbf{k}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega\times\mathbf{p}$$

For small Ψ, at first order we have

$$\mathbf{p} = (\Psi\omega_y, -\Psi\omega_x, 1)$$

passive tracers in an effective velocity field

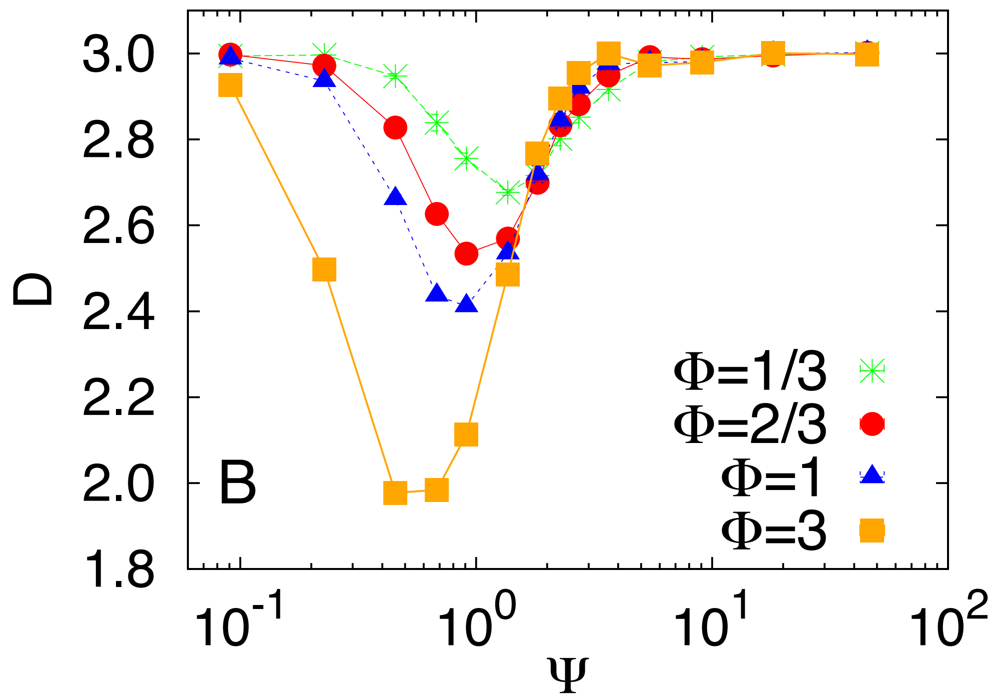
$$\mathbf{v} = \mathbf{u} + \Phi\mathbf{p}$$

with divergence

$$\nabla\cdot\mathbf{v} = \Phi\nabla\cdot\mathbf{p} = -\Psi\Phi\nabla^2 u_z$$

Fractal codimension $3 - D \propto (\nabla\cdot\mathbf{v})^2$

and therefore $D = 3 - a(\Phi\Psi)^2$



A weakly compressible velocity field

$$\boldsymbol{v} = \boldsymbol{u} + \delta\boldsymbol{w}$$

with $\nabla\cdot\boldsymbol{u} = 0 \qquad \nabla\cdot\boldsymbol{w} \neq 0$

tracers cluster on a fractal set with codimension given by $d - D_2 \simeq \delta^2$

G Falkovich, A Fouxon, MG Stepanov, *Nature* **419**, 151-154 (2002).
I Fouxon, *Phys. Rev. Lett.* **108**, 134502 (2012).

How clustering depends on Ψ

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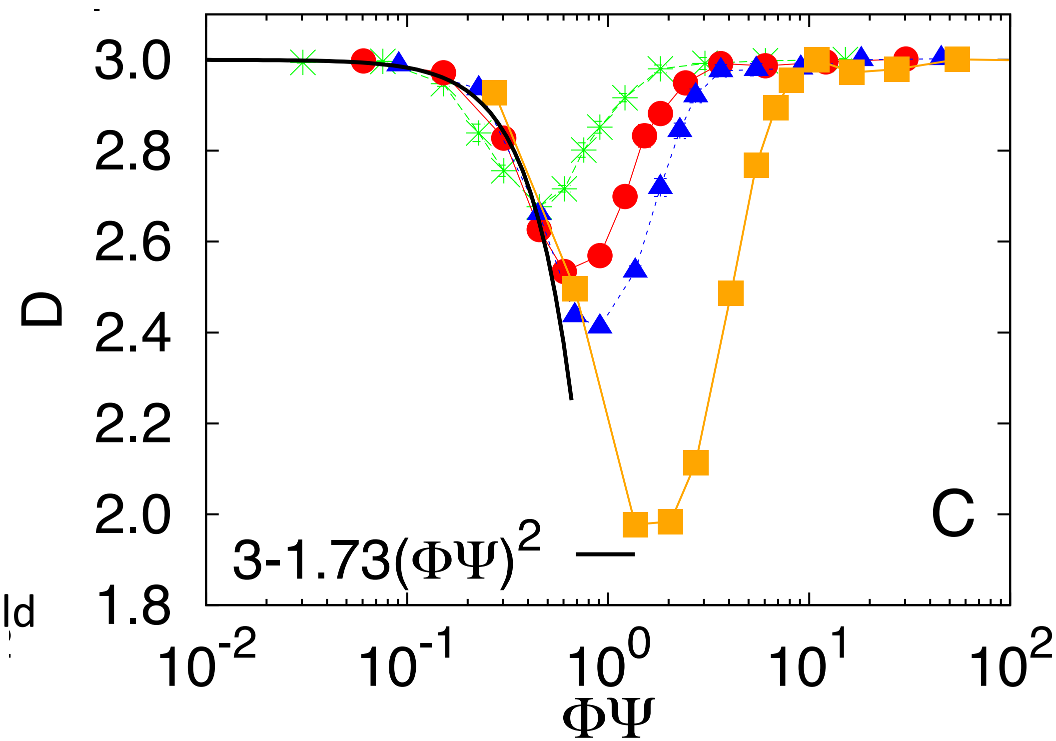

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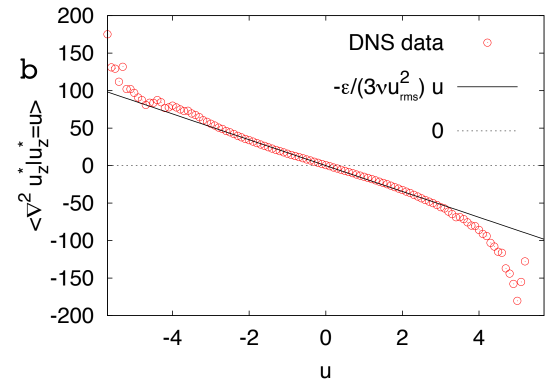
Where do cells cluster ?

Swimmers as tracers transported by a weakly compressible flow

$$\nabla \cdot \mathbf{v} = \Phi \nabla \cdot \mathbf{p} = -\Psi \Phi \nabla^2 u_z$$

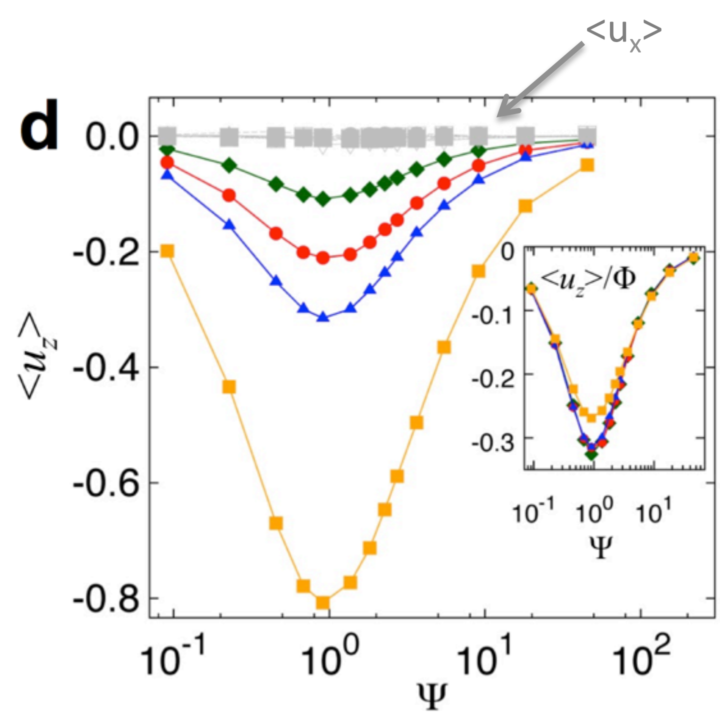
concentrate on regions where $\nabla^2 u_z > 0$

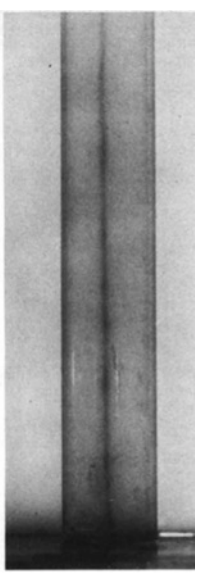


In homogeneous, isotropic turbulence

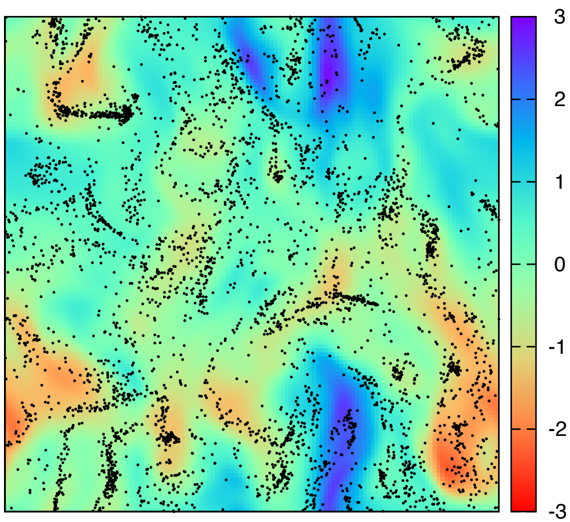
$$\epsilon = \nu \langle (\nabla \mathbf{u})^2 \rangle = -3\nu \langle u_z \nabla^2 u_z \rangle$$

and therefore $\nabla^2 u_z > 0$ means $u_z < 0$





Swimming cells accumulate in downwelling regions, where $u_z < 0$



Clustering at increasing Reynolds numbers

$\Phi = 1/3$

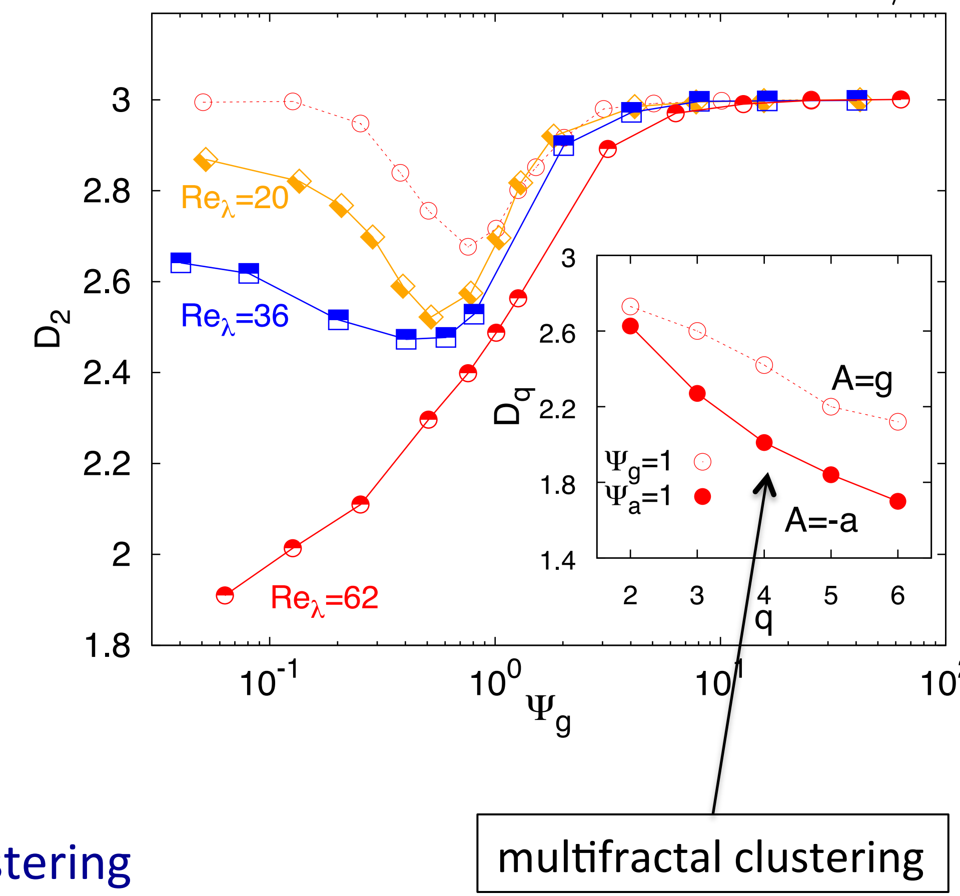
- Clustering increases with Reynolds
- Minimum of D disappears for large Re (when α≈1)

$$\alpha \equiv \frac{a_{rms}}{g}$$

$\alpha = 0.34$ $Re_\lambda = 20$

$\alpha = 0.50$ $Re_\lambda = 36$

$\alpha = 0.84$ $Re_\lambda = 62$



Turbulent accelerations enhance cell clustering

At small Ψ the swimming direction aligns towards strong acceleration regions which are not uniform

What is the role of fluid acceleration ?

Clustering in the limit g=0

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x},t) + \Phi\mathbf{p}$$

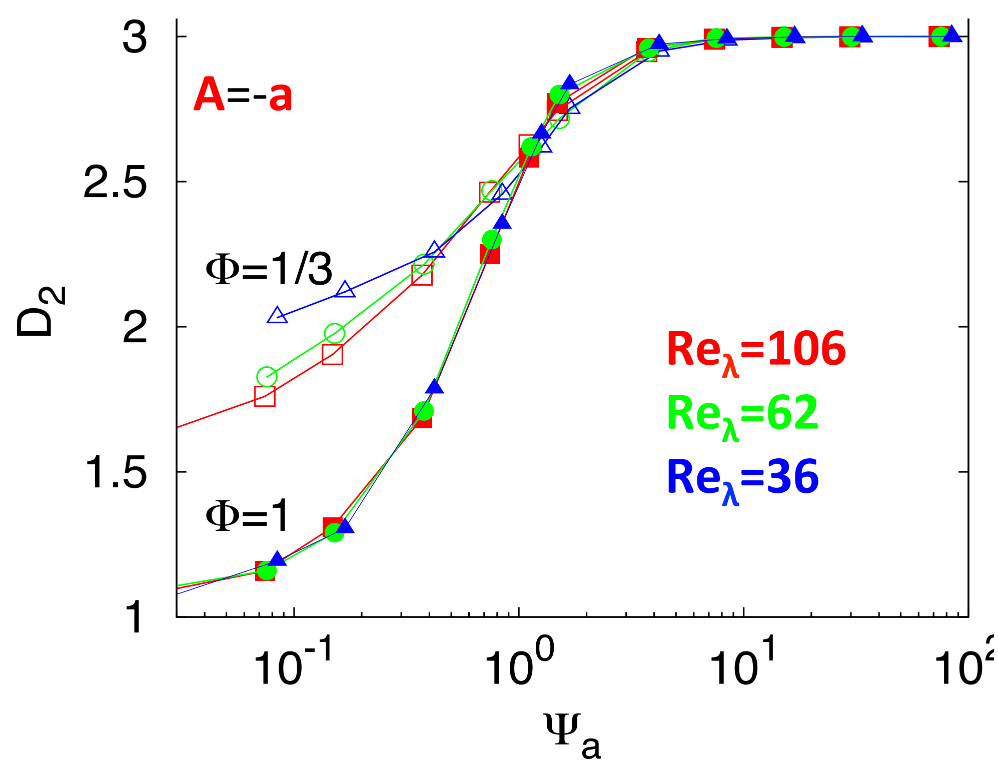
$$\frac{d\mathbf{p}}{dt} = \frac{1}{2\Psi}\left[\mathbf{a} - (\mathbf{a}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega \times \mathbf{p}$$

To understand the role of acceleration we consider the case $a_{rms} >> g$ and take $g=0$

clustering increases with cell stability i.e when $\Psi << 1$

clustering increases with swimming speed i.e with Φ

weak (if any) dependence on Re_λ



what is driving clustering? where do cells go?

Predictions for small Ψ

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x},t) + \Phi\mathbf{p}$$

$$\frac{d\mathbf{p}}{dt} = \frac{1}{2\Psi}\left[\mathbf{a} - (\mathbf{a}\cdot\mathbf{p})\mathbf{p}\right] + \frac{1}{2}\omega \times \mathbf{p}$$

$\Psi \ll 1$, **p** is mainly aligned with **a**.

Effective velocity for swimmers $\mathbf{v} \simeq \mathbf{u} + \Phi\hat{\mathbf{a}}$

a compressible field with

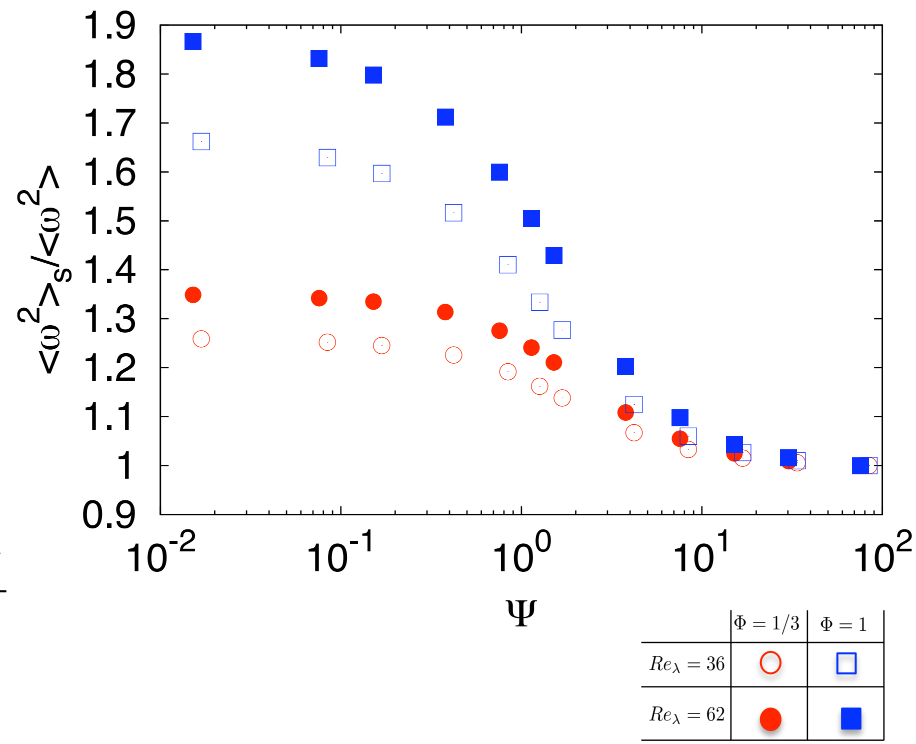
$$\nabla\cdot\mathbf{v} \simeq \Phi\nabla\cdot\hat{\mathbf{a}}$$

Clearly $\nabla\cdot\hat{\boldsymbol{a}} \not\propto \nabla\cdot\boldsymbol{a}$ however their sign is strongly correlated. Swimmers accumulate where

$$\nabla\cdot\mathbf{a} < 0$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{a} = \sum_{ij}(\hat{S}_{ij}^2 - \hat{\Omega}_{ij}^2)$$

$$\hat{S}_{ij} = \frac{\partial_j u_i + \partial_i u_j}{2} \qquad \hat{\Omega}_{ij} = \frac{\partial_j u_i - \partial_i u_j}{2}$$

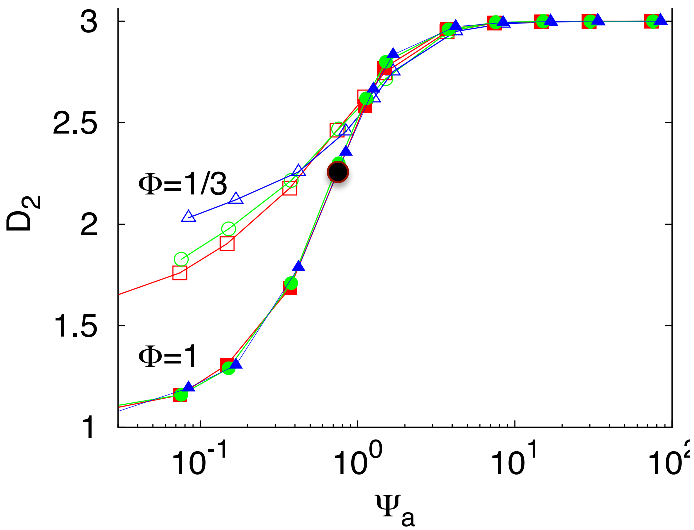


$\nabla\cdot\mathbf{a} < 0$ corresponds to large vorticity regions

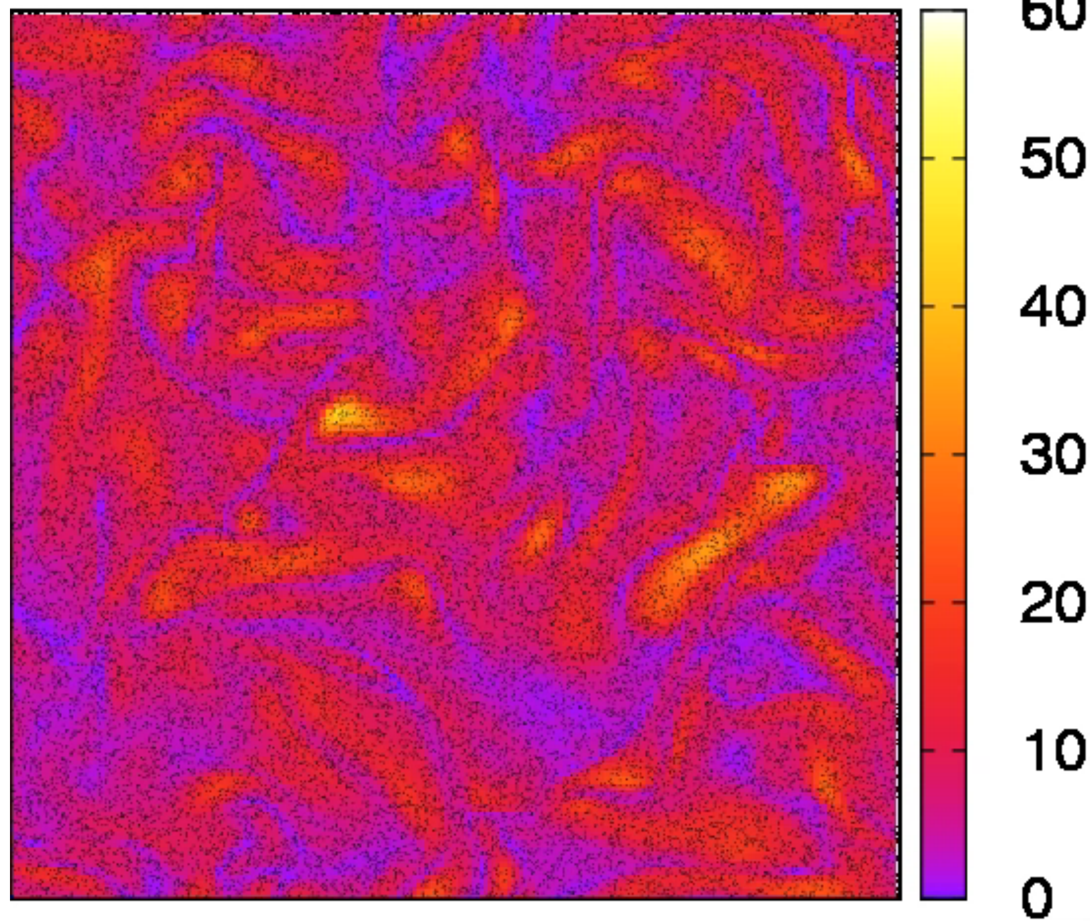
Swimmers concentrate in vortices (like light particles)

$\langle\cdots\rangle_s$ average at swimmer positions

Swimmers concentrate in vortices



$$Re_\lambda = 62$$
$$\Psi = 1.5$$
$$\Phi = 1$$

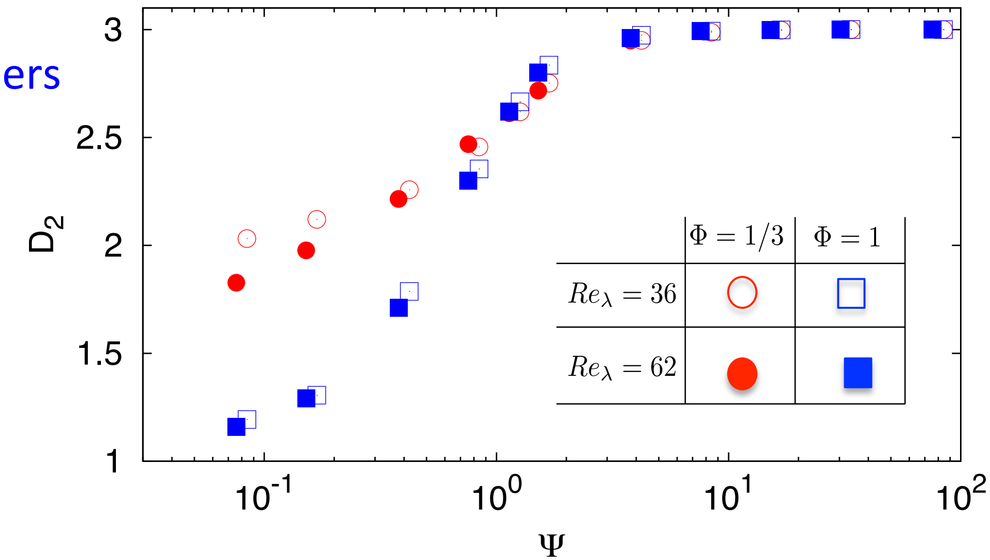


Section of $\ln \frac{|\omega|}{\omega_{rms}}$

A prediction for D_2

Clustering is more efficient for faster swimmers

How does D_2 depends on Φ ?

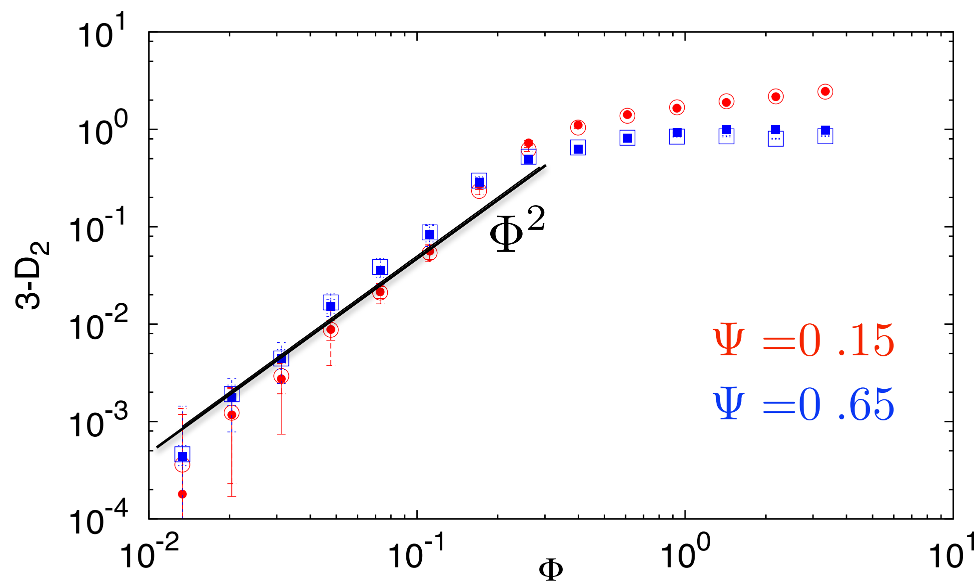


Effective velocity for swimmers

$$\mathbf{v} \simeq \mathbf{u} + \Phi\hat{\mathbf{a}}$$

with $\nabla \cdot \boldsymbol{v} \propto \Phi \nabla \cdot \hat{\boldsymbol{a}}$

$$3 - D_2 \simeq \Phi^2$$



Happy
Birthday
Angelo