A (DIS)ORIENTED TOUR AROUND SCALING, DIMENSIONS, REVERSIBILITY AND EQUILIBRIUM IN TURBULENCE

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AV60September 22/24, 2014 Rome -Strolling on Chaos, Turbulence and Statistical Mechanics.

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WITH:
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F. TOSCHI (TUE-NL)
S. MUSACCHIO (CNRS-FR)
E. TITI (WEIZMANN-IL)

A (DIS)ORIENTED TOUR AROUND SCALING, DIMENSIONS, REVERSIBILITY AND EQUILIBRIUM IN TURBULENCE

-INVISCID CONSERVED QUANTITIES AND FLUXES IN 2D-3D FLOWS
-EQUILIBRIUM AND OUT-OF-EQUILIBRIUM FLOWS
-TUNING/REVERSING THE ENERGY FLUX IN TURBULENCE BY PLAYING WITH HELICITY-WHAT DO WE LEARN?

PHYSICAL REVIEW A VOLUME 43, NUMBER 2 15 JANUARY 1991

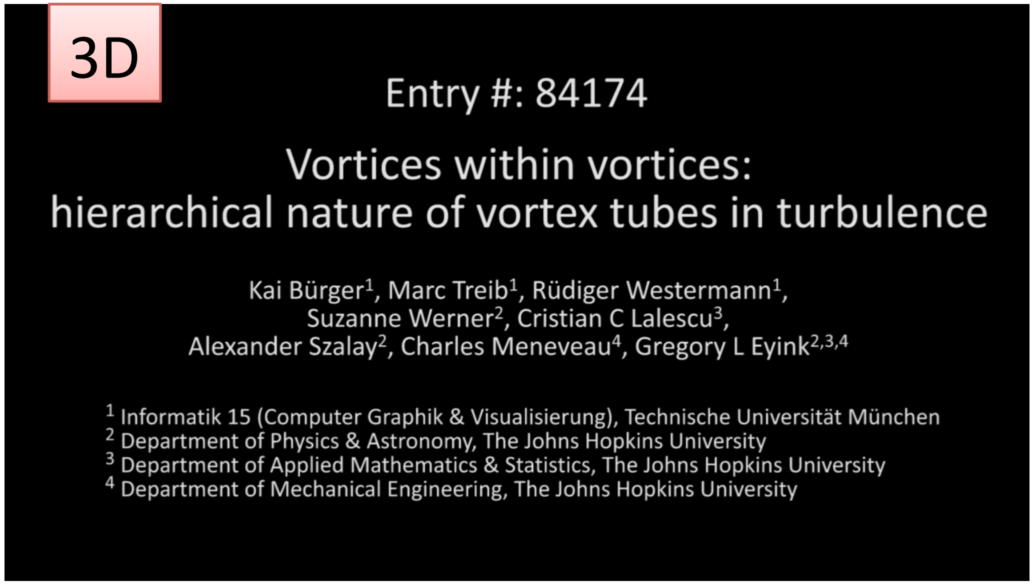
Intermittency in a cascade model for three-dimensional turbulence

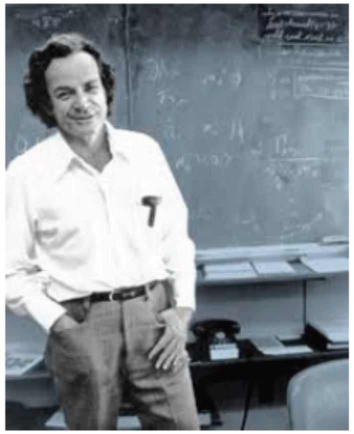
M. H. Jensen, G. Paladin,* and A. Vulpiani*
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
(Received 6 July 1990)

We discuss a possible mechanism for intermittency of the energy dissipation in a model for three-dimensional fully developed turbulence. We compute the structure functions for the velocity field and show that their behavior can be described in the context of a multifractal approach. We also compute the instantaneous maximum Lyapunov exponent and the corresponding (stability) eigenvector. Violent bursts of energy dissipation are related to a sudden increase of the instantaneous Lyapunov exponent, and simultaneous localization of its eigenvector on the high wave numbers at the end of the inertial range. In particular, we relate the correction to the $k^{-5/3}$ Kolmogorov law for the energy spectrum to the fractal dimension extracted by temporal sequences both of the instantaneous Lyapunov exponent and of the eigenvector.

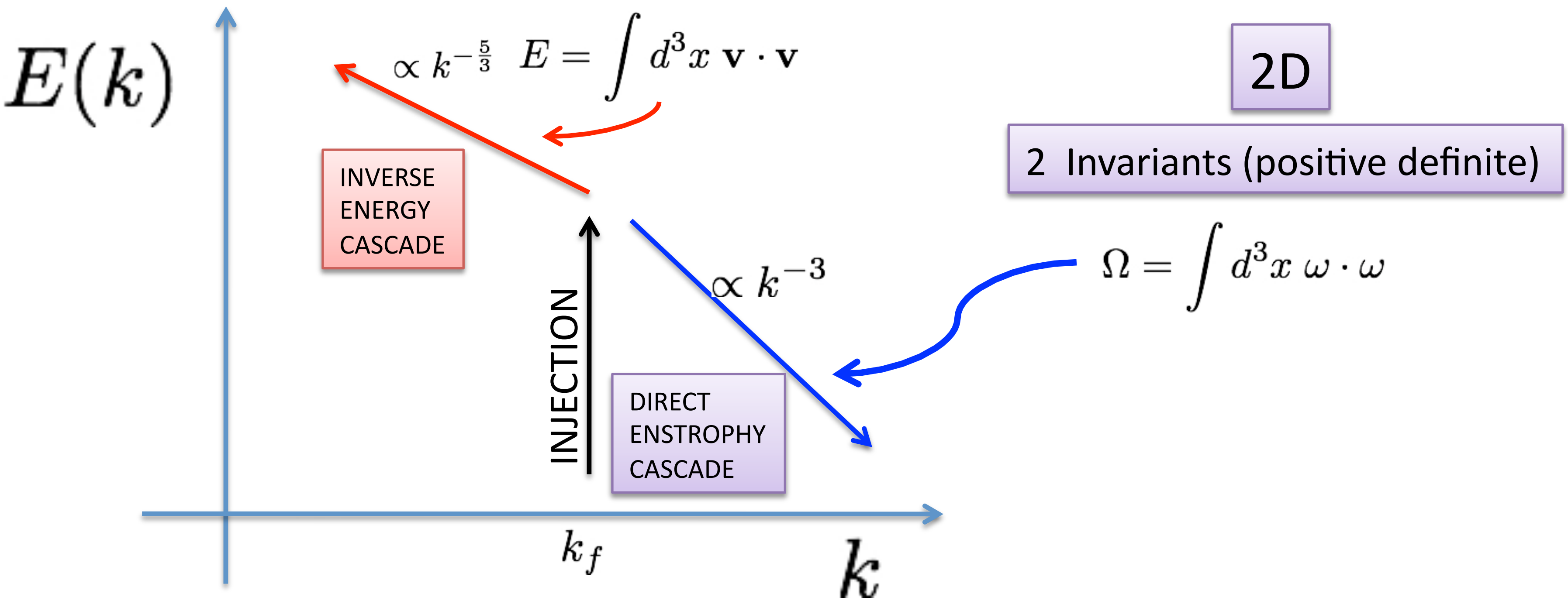
NAVIER-STOKES 3D-2D

(NASA/Goddard Space Flight Center Scientific Visualization Studio)

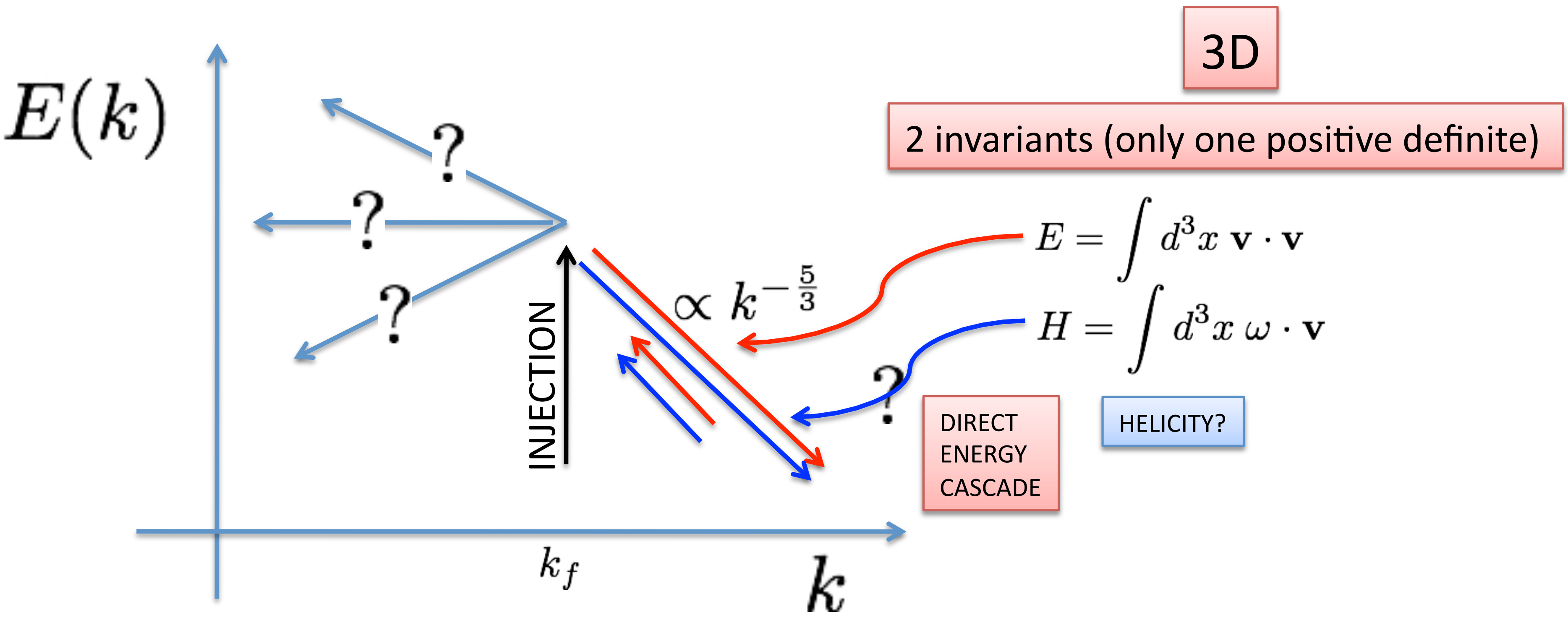




"With turbulence, it's not just a case of physical theory being able to handle only simple cases—we can't do any. We have no good fundamental theory at all." (Feynman, 1979, Omni Magazine, Vol. 1, No.8).



$E(k)$
$\propto k^{-\frac{5}{3}}$
$E = \int d^3x \, \mathbf{v} \cdot \mathbf{v}$
INVERSE ENERGY CASCADE
INJECTION
$\propto k^{-3}$
DIRECT ENSTROPHY CASCADE
k_f
k
2D
2 Invariants (positive definite)
$\Omega = \int d^3x \, \omega \cdot \omega$



$E(k)$
?
?
?
INJECTION
$\propto k^{-\frac{5}{3}}$
k_f
k
3D
2 invariants (only one positive definite)
$E = \int d^3x \, \mathbf{v} \cdot \mathbf{v}$
$H = \int d^3x \, \omega \cdot \mathbf{v}$
?
DIRECT ENERGY CASCADE
HELICITY?

FULLY OUT-OF-EQUILIBRIUM (SINKS AND SOURCES AT DIFFERENT SCALES)

2D: ENERGY GOES UP (IN SCALES)
3D: ENERGY GOES DOWN

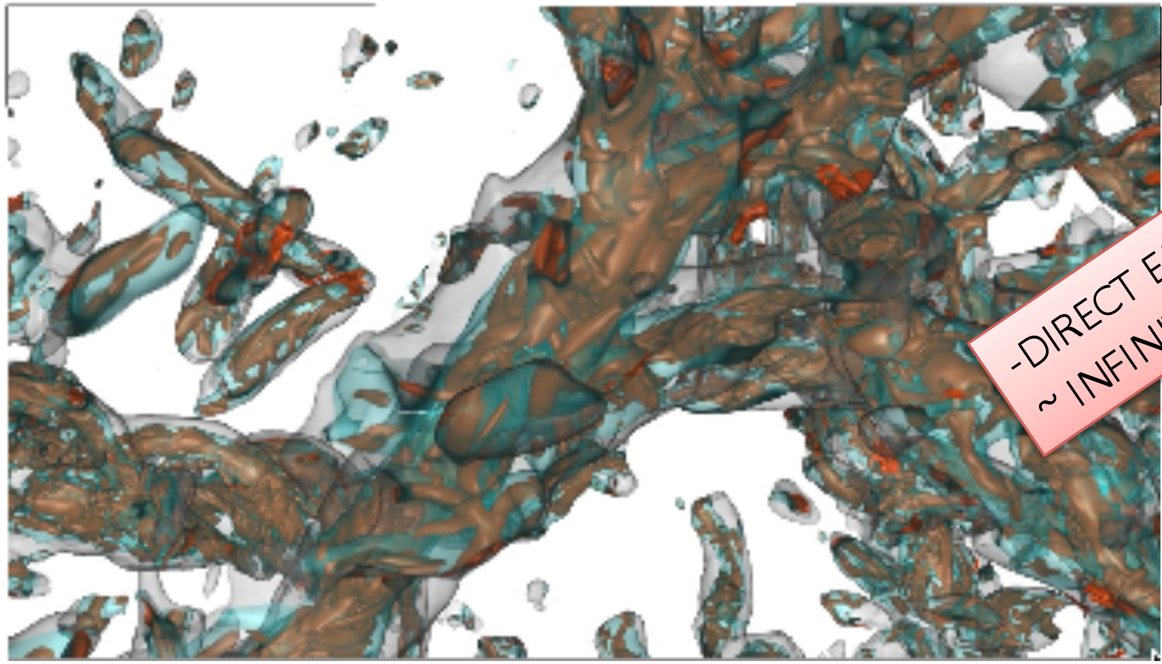
$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \; Boundary \; Conditions \end{cases}$$

DIRECT <-> INVERSE ENERGY TRANSFER

3D + ROTATION + HELICITY INJECTION (Mininni & Pouquet 2013)
THICK LAYER + ROTATION (Smith et al 1996)
SQUEZED DOMAINS (Celani et al 2010, Xia et al 2012)
STRONG SHEAR (Herbert et 2012)
SMALL SCALES HELICITY INJECTION (Sulem et al 1986)

1. NAVIER-STOKES 3D

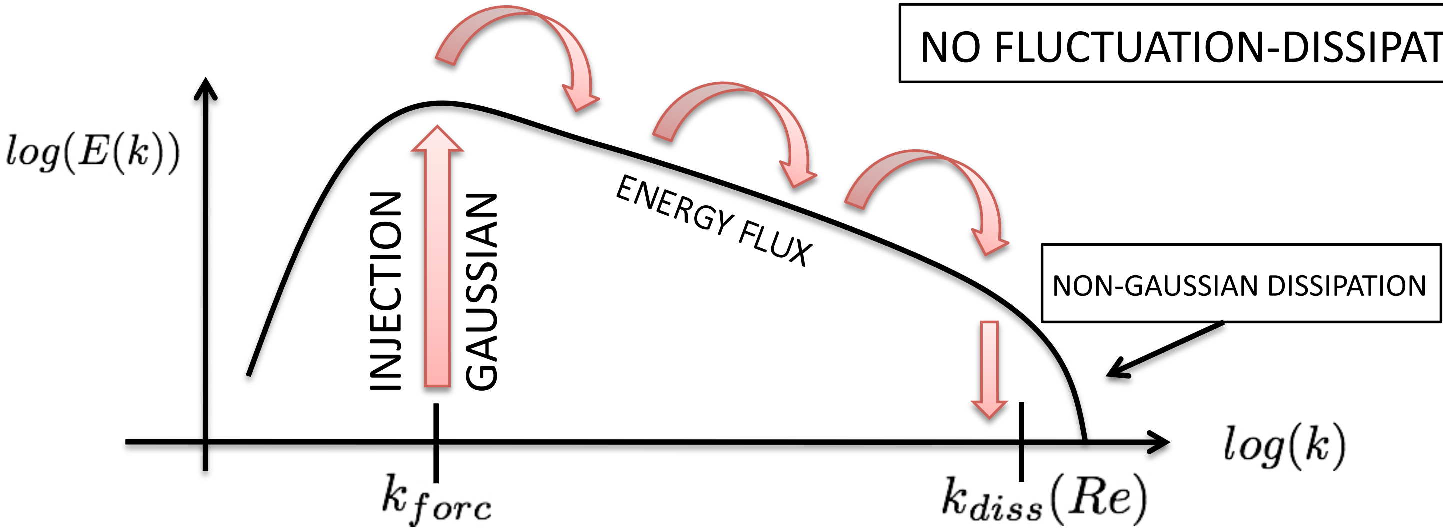
$$\begin{cases} Re \to \infty \\ \nu \to 0 \end{cases}$$



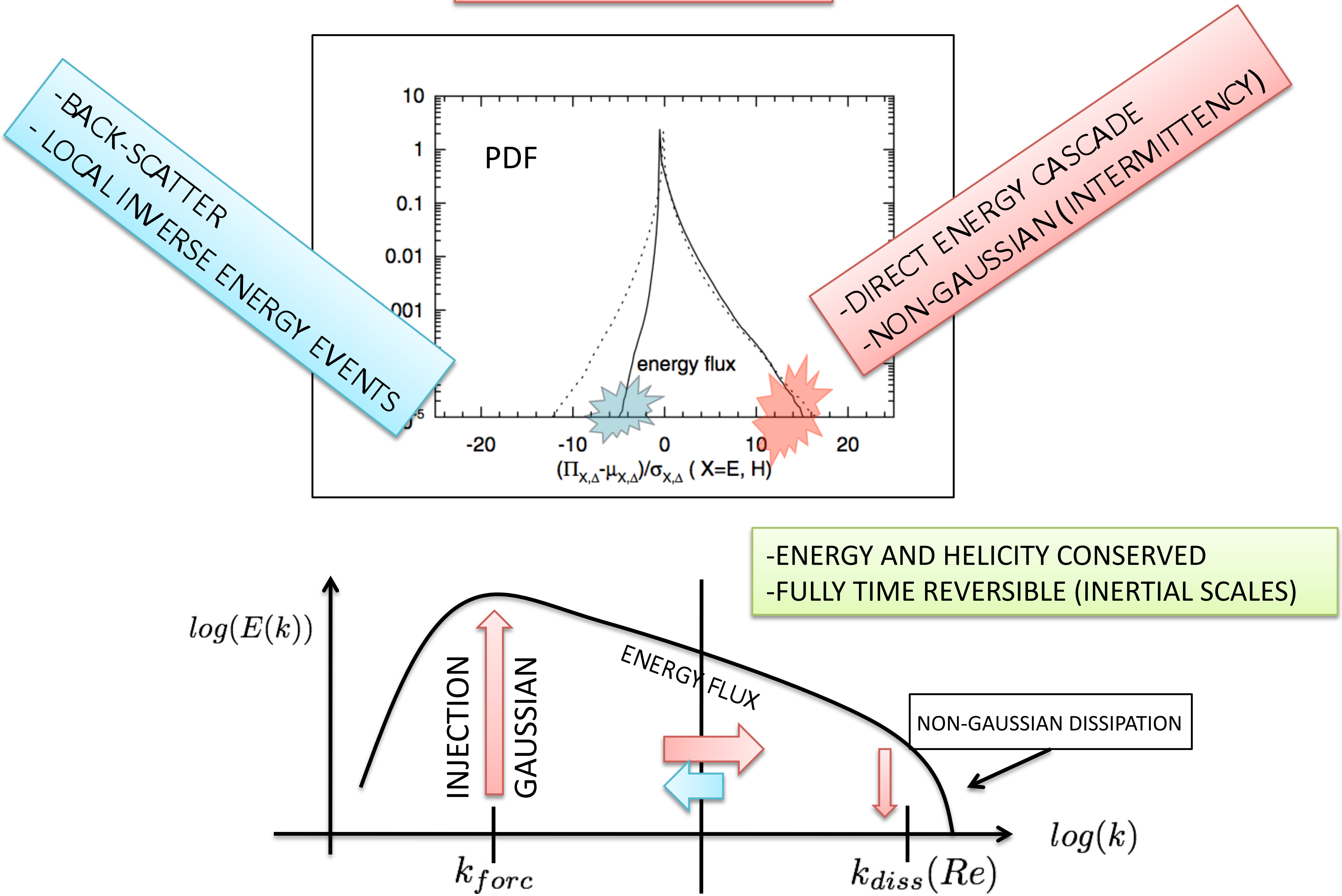
-DIRECT ENERGY CASCADE
~ INFINITE $\#_{dof}$

$$\#_{dof} \propto Re^{9/4}$$

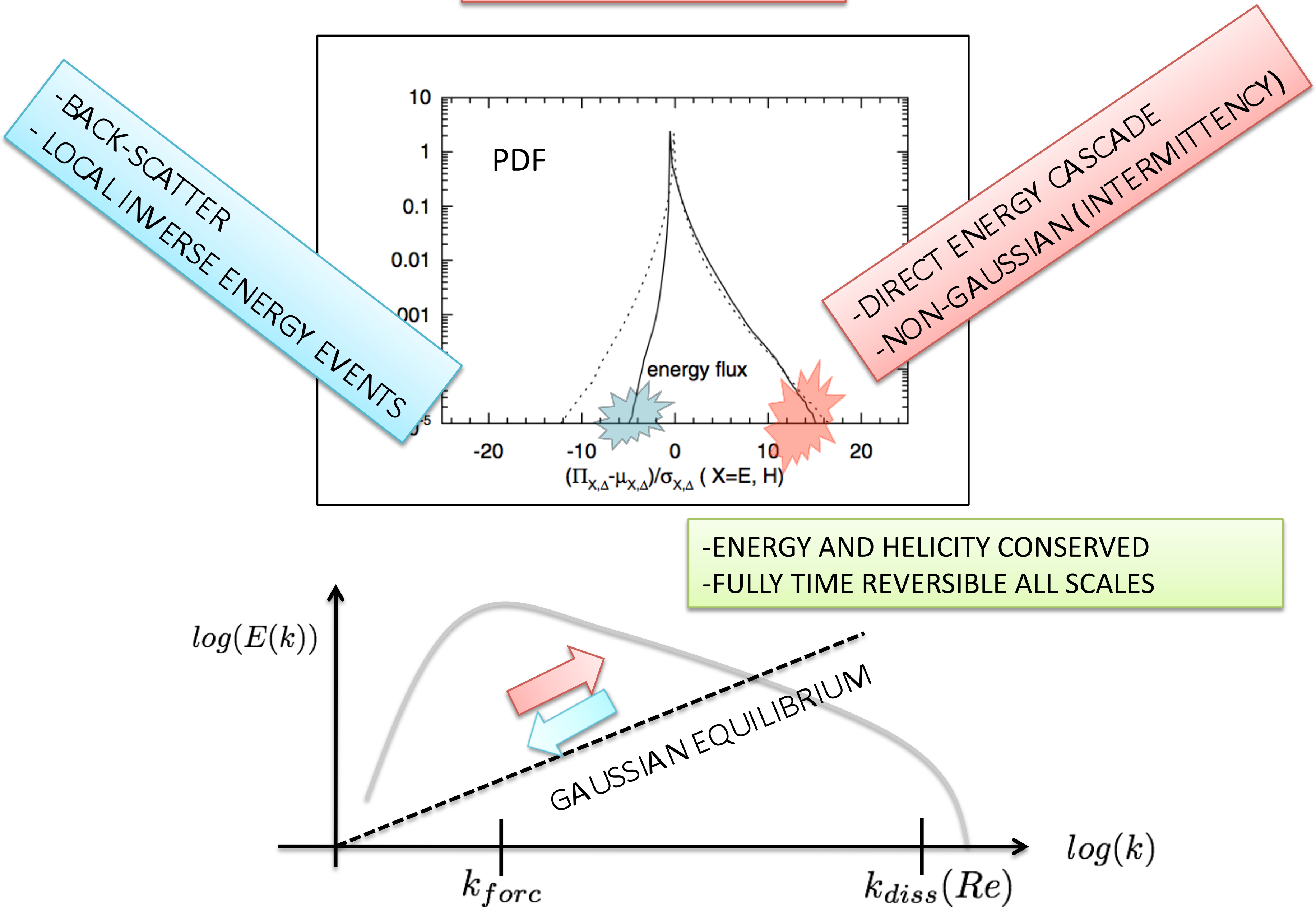
NO FLUCTUATION-DISSIPATION

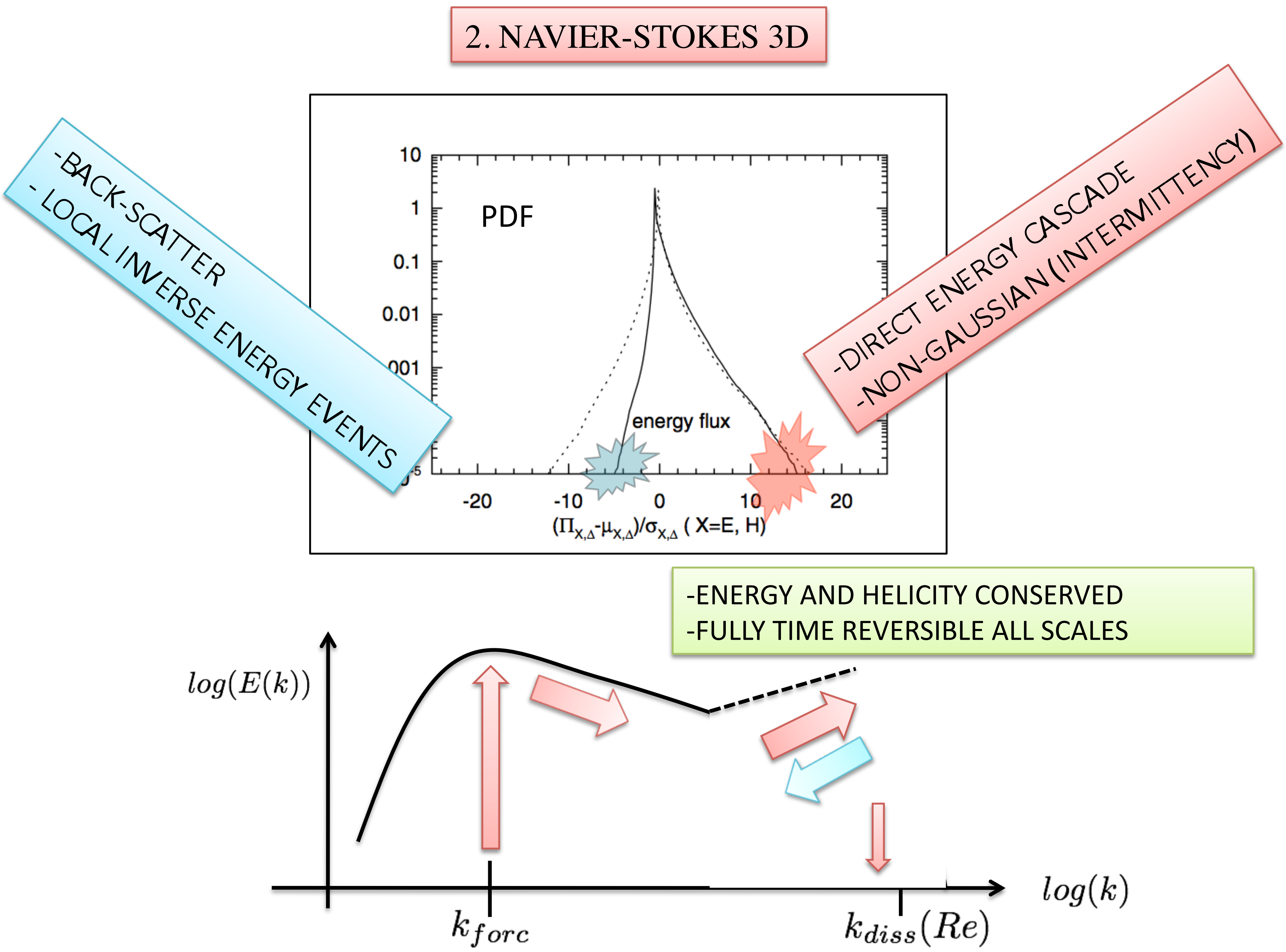


2. NAVIER-STOKES 3D



2. NAVIER-STOKES 3D





2. NAVIER-STOKES 3D
PDF
energy flux
10
1
0.1
0.01
001
-5
-20
-10
0
10
20
(Π_{X,Δ}-μ_{X,Δ})/σ_{X,Δ} (X=E, H)
-BACK-SCATTER
- LOCAL INVERSE ENERGY EVENTS
-DIRECT ENERGY CASCADE
-NON-GAUSSIAN (INTERMITTENCY)
-ENERGY AND HELICITY CONSERVED
-FULLY TIME REVERSIBLE ALL SCALES
log(E(k))
log(k)
k_forc
k_diss(Re)

ON THE ROLE OF INVISCID INVARIANTS (HELICITY & ENERGY) IN 3D FORWARD/ BACKWARD ENERGY CASCADES

$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$

$$i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k\boldsymbol{h}^\pm$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) = \sum_{s_p=\pm, s_q=\pm} g_{s_k,s_p.s_q} \sum_{p+q=k} u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q}) - \nu k^2 u^{s_k}(\mathbf{k})$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) = \sum_{s_p=\pm, s_q=\pm} g_{s_k,s_p.s_q} \sum_{p+q=k} u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q}) - \nu k^2 u^{s_k}(\mathbf{k})$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})$$

$$\rho(\{u_+(\mathbf{k}), u_-(\mathbf{k})\}_{\mathbf{k}\in\mathcal{B}}) = \frac{1}{\mathcal{Z}} e^{-\beta E - \alpha H}$$

$$\mathcal{Z} = \prod_{\mathbf{k}\in\mathcal{B}} \int_0^{+\infty} da_+ \int_0^{+\infty} da_- e^{-\frac{1}{2}(\beta+\alpha k)a_+^2 - \frac{1}{2}(\beta-\alpha k)a_-^2},$$

$$\mathcal{Z}_\pm = \prod_{\mathbf{k}\in\mathcal{B}} \sqrt{\frac{\pi}{2(\beta \pm \alpha k)}}. \longrightarrow \beta > |\alpha| k_{max}$$

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Printed in Great Britain

Helical turbulence and absolute equilibrium

By ROBERT H. KRAICHNAN
Dublin, New Hampshire

(Received 22 January 1973)

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) = \sum_{s_p=\pm,s_q=\pm} g_{s_k,s_p,s_q} \sum_{p+q=k} u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q}) - \nu k^2 u^{s_k}(\mathbf{k})$$

$$\langle E \rangle = -\frac{\partial \ln \mathcal{Z}}{\partial \beta} = \sum_{\mathbf{k}\in\mathcal{B}} \left(\frac{1}{\beta + \alpha k} + \frac{1}{\beta - \alpha k} \right)$$

$$\langle H(k) \rangle = \frac{4\pi\alpha k^4}{(\alpha^2 k^2 - \beta^2)} \qquad \langle E(k) \rangle = \frac{4\pi\beta k^2}{\beta^2 - \alpha^2 k^2}$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$

$\beta > |\alpha| k_{max}$

3D FULL NS EQUILIBRIUM WITH POSITIVE TEMPERATURE

3D HELICAL NS EQUILIBRIUM WITH NEGATIVE TEMPERATURE

$\beta > -\alpha k$

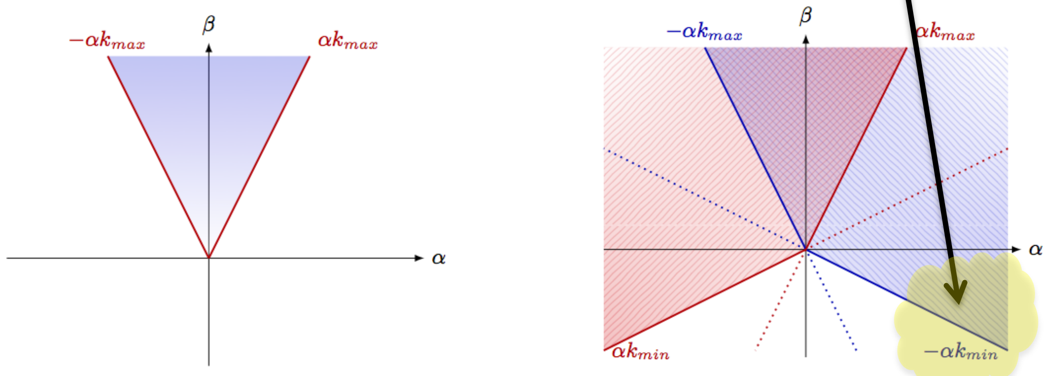


FIG. 1: (Color online). Accessible thermodynamic space for 3D helical flows (left: $\beta > |\alpha|k_{max}$) and for phase space reduced to maximal symmetry breaking (right: $\beta \pm \alpha k_{min} > 0$ and $\beta \pm \alpha k_{max} > 0$ for $\Lambda_\pm$, marked with slanted lines and shaded in blue (resp. red) — note that the intersection coincides with the full phase space condition $\beta > |\alpha|k_{max}$). Contrary to 2D turbulence, there is no accessible negative temperature for 3D flows, but they are recovered in the restricted phase space $\Lambda_\pm$.

3D

5

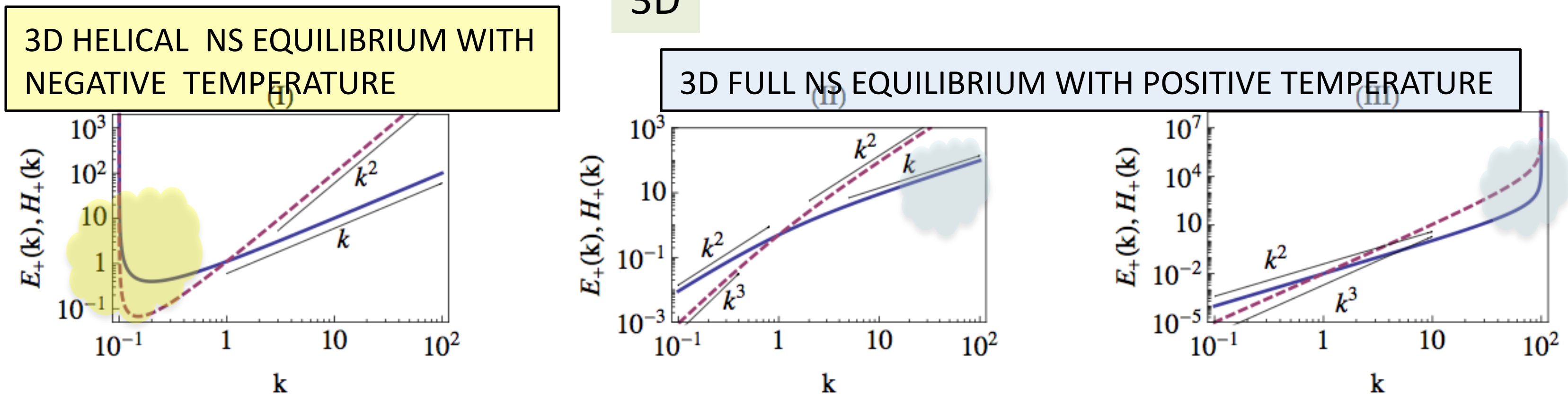


FIG. 3: (Color online). Equilibrium energy (solid blue lines) and helicity (dashed red lines) spectra $(E_+(k), H_+(k))$ for the different (α, β) regimes, for restricted phase space Λ_+. Left: $\alpha > 0, \beta < 0$ (regime I); the spectra have a well shape, with an infrared divergence and an increase at large k (the thin lines indicate the k and k^2 scaling). Middle: $\alpha > 0, \beta > 0$ (regime II); the spectra increase with k, with scalings (k^2, k^3) at low-k and (k, k^2) at large k. Right: $\alpha < 0, \beta > 0$ (regime III); the spectra increase as k increases, and there is an ultraviolet divergence (the thin lines indicate the (k^2, k^3) scalings at low k).

2D

6

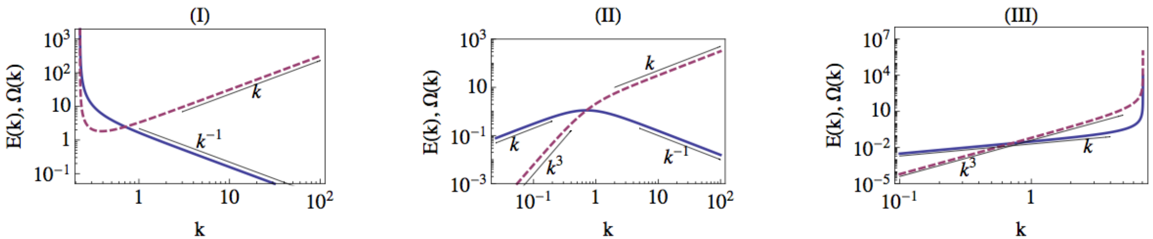
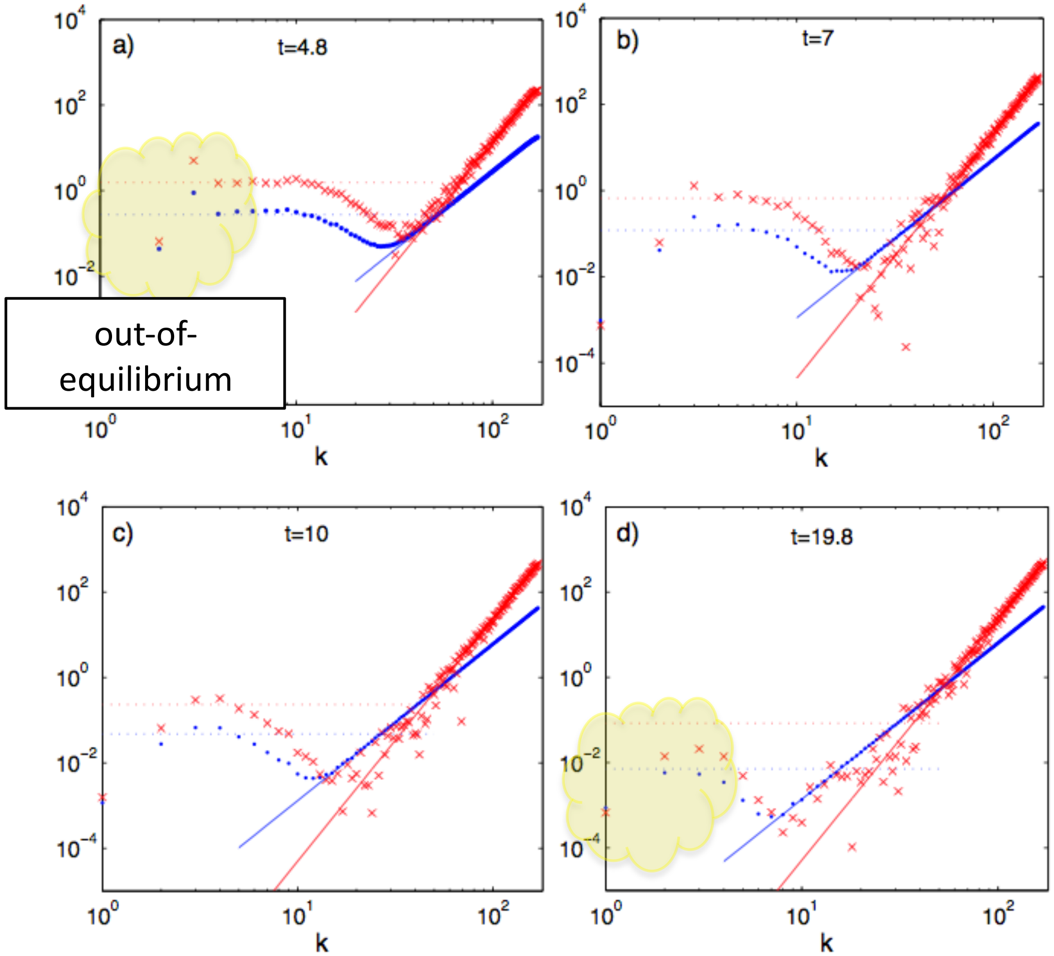


FIG. 4: (Color online). Equilibrium energy (solid blue lines) and enstrophy (dashed red lines) spectra $(E(k), \Omega(k))$ for the different (α, β) regimes in 2D Turbulence. Left: $\alpha > 0, \beta < 0$ (regime I); the spectra have an infrared divergence (the thin lines indicate the k and k^{-1} scaling). Middle: $\alpha > 0, \beta > 0$ (regime II); the energy spectrum has a bell shape, the enstrophy spectrum increases with k, with scalings (k, k^3) at low-k and (k^{-1}, k) at large k. Right: $\alpha < 0, \beta > 0$ (regime III); the spectra increase as k increases, and there is an ultraviolet divergence (the thin lines indicate the (k, k^3) scalings at low k).

Cascades, thermalization and eddy viscosity in helical Galerkin truncated Euler flows

G. Krstulovic[1], P.D. Mininni[2,3], M.E. Brachet[1,3], and A. Pouquet[3]



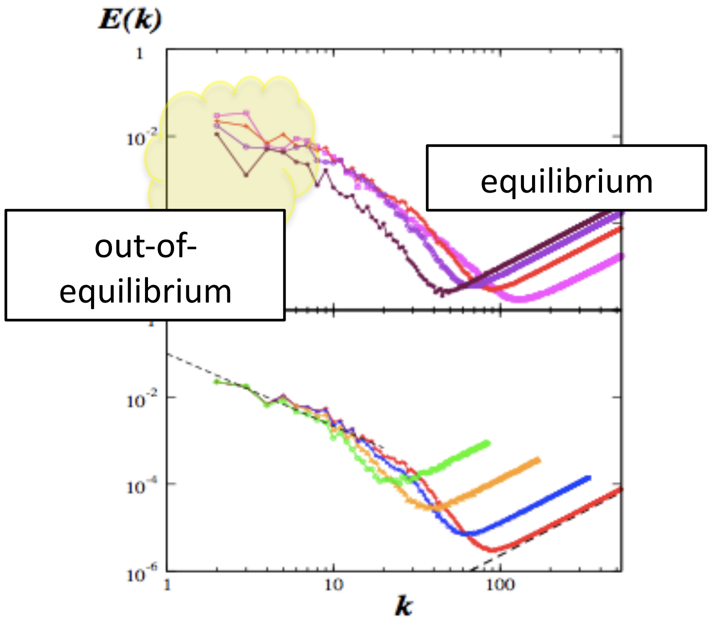


FIG. 1: Energy spectra, top: resolution 1600^3 at $t = (6.5, 8, 10, 14)$ ($\diamond$, $+$, $\circ$, $\star$); bottom: resolutions 256^3 (circle $\circ$), 512^3 (triangle $\triangle$), 1024^3 (cross $\times$) and 1600^3 (cross $+$) at $t = 8$. The dashed lines indicate $k^{-5/3}$ and k^2 scalings.

Ddynamical ensemble equivalence (Gallavotti)
reversible equilibrium ~ irreversible flux coexistency

$$\dot{\mathbf{u}} + (\mathbf{u}\cdot\boldsymbol{\partial})\mathbf{u} = -\boldsymbol{\partial} p + \mathbf{g} + \alpha(\mathbf{u})\Delta\mathbf{u}, \qquad \boldsymbol{\partial}\cdot\mathbf{u} = 0$$

$$\alpha(\mathbf{u}) \stackrel{def}{=} \frac{-\int \mathbf{g}\cdot\Delta\mathbf{u}\,d\mathbf{x} + \int \Delta\mathbf{u}\cdot\Big((\underset{\sim}{u}\ \cdot\underset{\sim}{\partial}\ )\mathbf{u}\Big)\,d\mathbf{x}}{\int(\Delta\mathbf{u})^2}$$

28 *L Biferale et al*

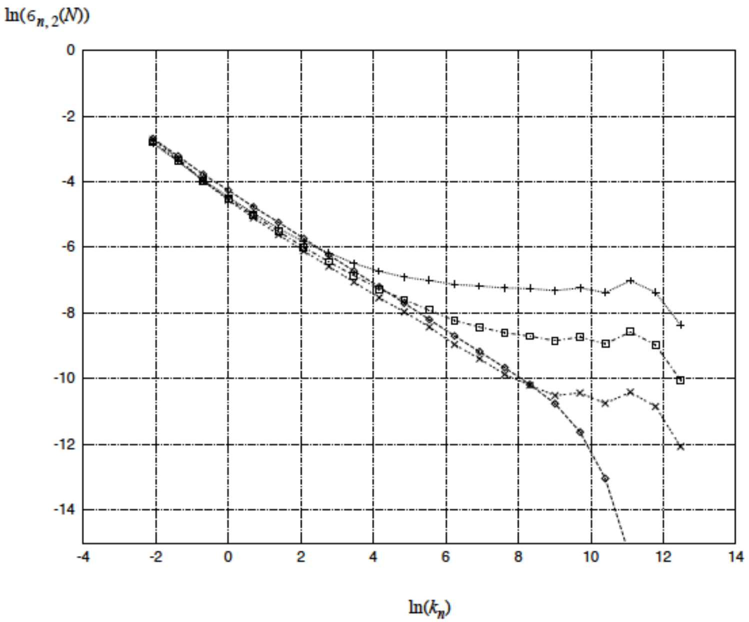


Figure 1. $\Sigma_{n,2}$ versus k_n in log–log scale obtained from an integration of the model with $N = 23$, $K_0 = 6.25\times10^{-2}$ and different values of the forcing: $f = 5\times10^{-3}(1+i)$ (plus), $f = 6\times10^{-3}(1+i)$ (squares), $f = 8\times10^{-3}(1+i)$ (cross). Diamonds represent the results obtained in the benchmark integration (i.e. the original GOY model).

J. Phys. A: Math. Gen. **31** (1998) 21–32. Printed in the UK   PII: S0305-4470(98)85879-4

Time-reversible dynamical systems for turbulence*

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‡ Dipartimento di Fisica, Università dell' Aquila, Via Vetoio 1, I-67010 Coppito, L'Aquila, Italy

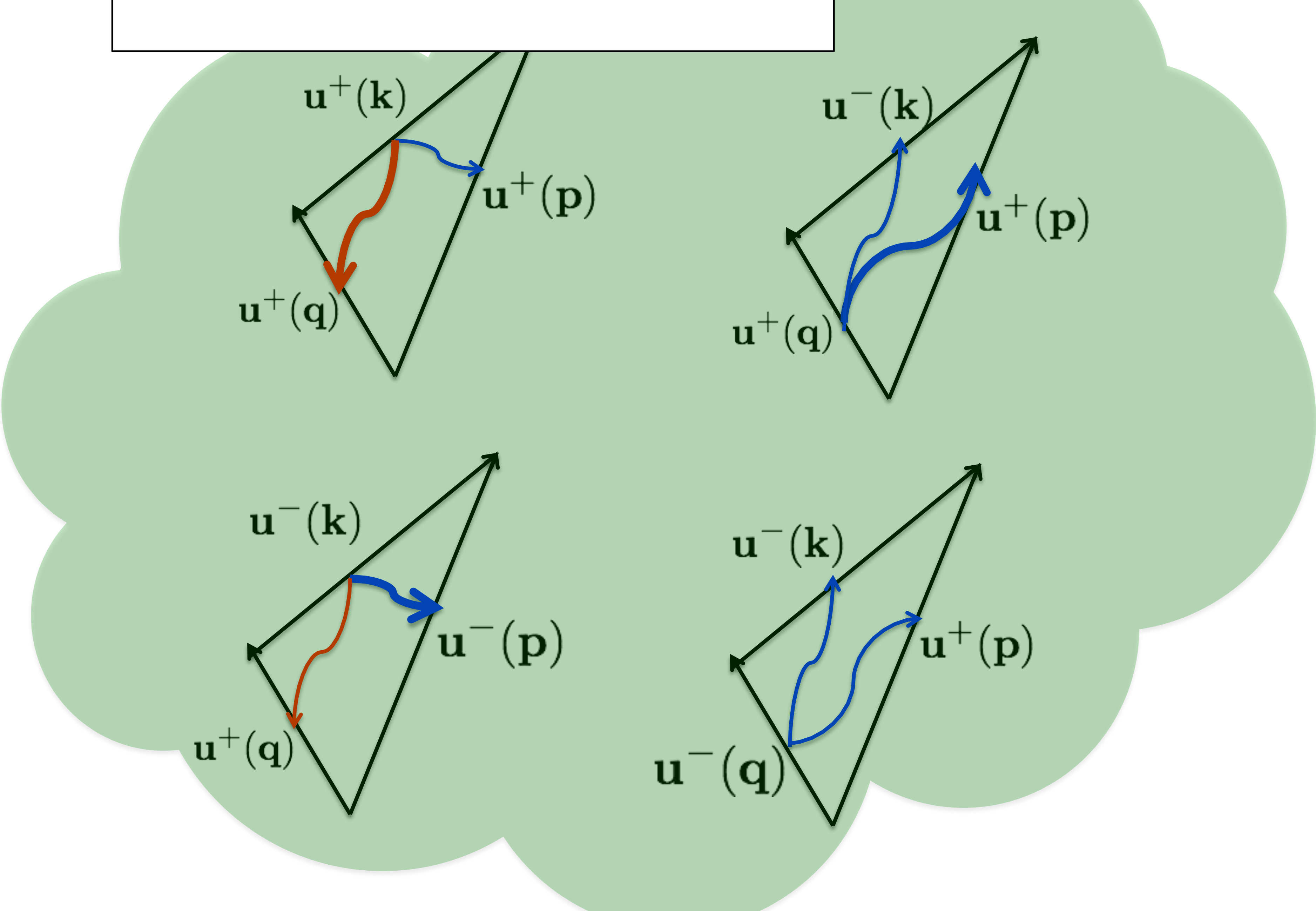
§ Dipartimento di Fisica, Università di Roma 'La Sapienza', Piazzale Aldo Moro 5, I-00185 Rome, Italy

Received 10 July 1997

Abstract. *Dynamical ensemble equivalence* between hydrodynamic dissipative equations and suitable time-reversible dynamical systems has been investigated in a class of dynamical systems for turbulence. The reversible dynamics is obtained from the original dissipative equations by imposing a global constraint. We find that, by increasing the input energy, the system changes from an equilibrium state to a non-equilibrium stationary state in which an energy cascade, with the same statistical properties of the original system, is clearly detected.

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^* . \tag{15}$$

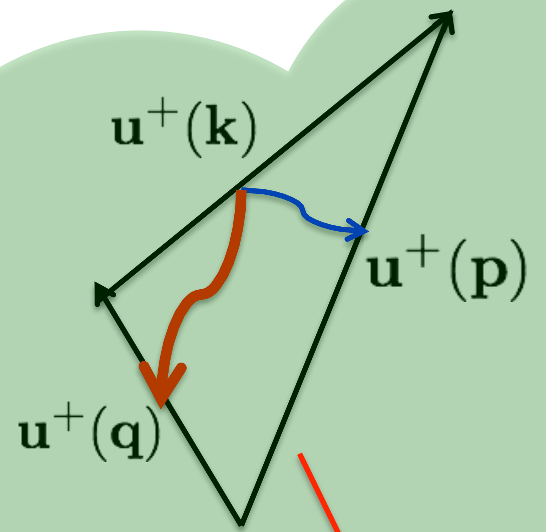
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

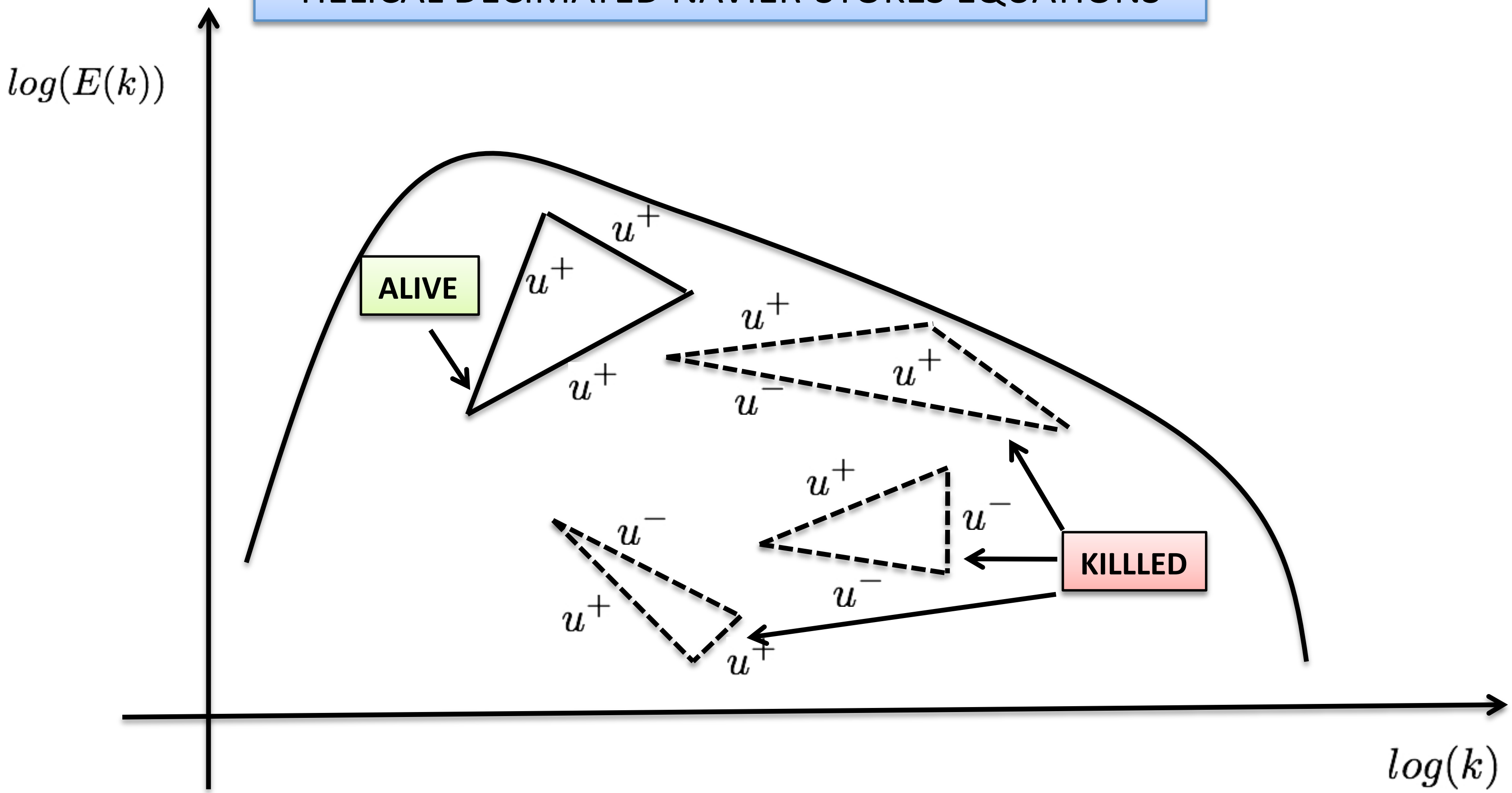
MILD SYMMETRY BREAKING

- HOMOGENEOUS OK
- ISOTROPY OK
- MIRROR SYMMETRY NO



$$\begin{cases} \partial_t \boldsymbol{v}^+ = \mathcal{P}^+(-\boldsymbol{v}^+ \cdot \boldsymbol{\nabla} \boldsymbol{v}^+ - \boldsymbol{\nabla} p^+) + \nu \Delta \boldsymbol{v}^+ + \boldsymbol{f}^+ \\ \nabla \cdot \boldsymbol{v}^+ = 0 \end{cases}$$

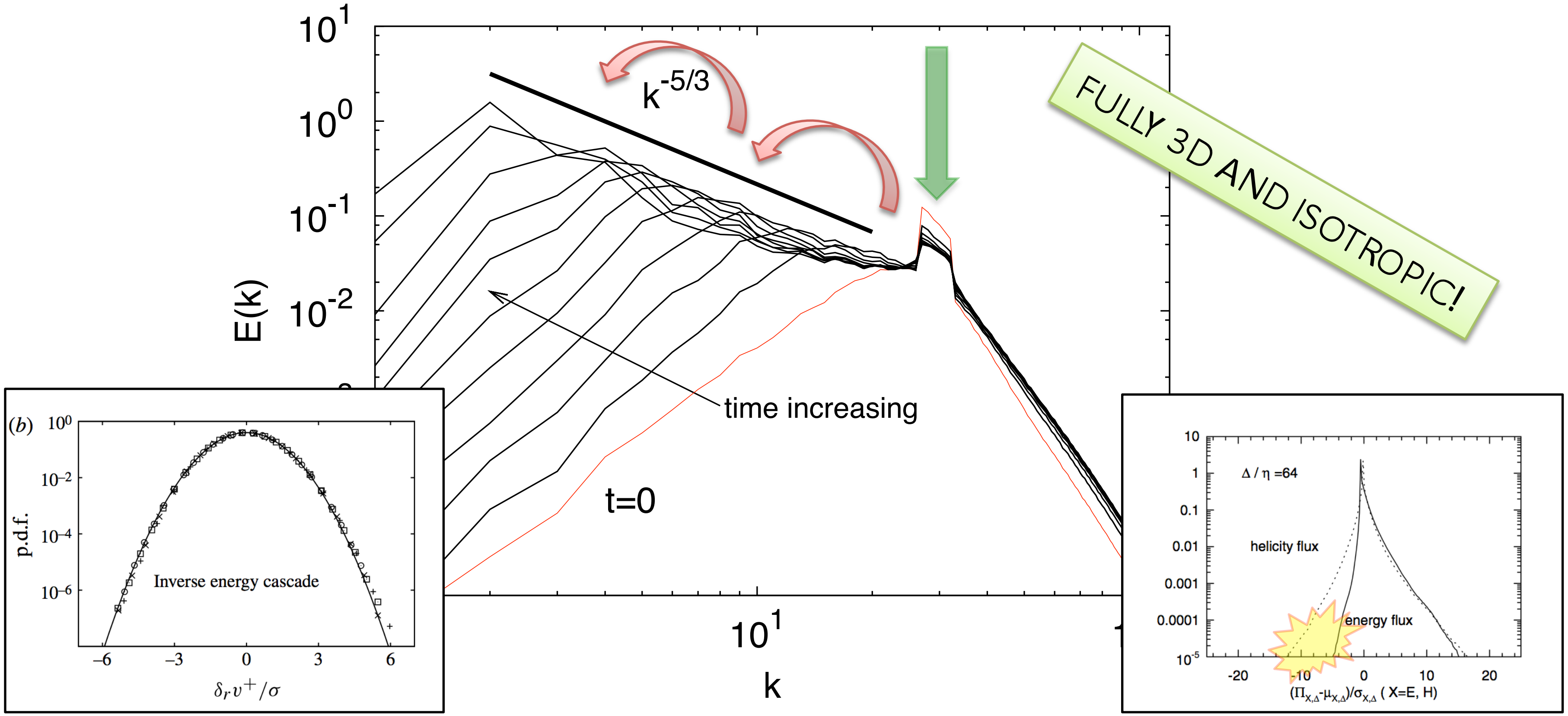
HELICAL DECIMATED NAVIER-STOKES EQUATIONS



$$\frac{d}{dt}u^{s_k}(\mathbf{k}) = \sum_{s_p=\pm, s_q=\pm} g_{s_k,s_p.s_q} \sum_{p+q=k} u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q}) - \nu k^2 u^{s_k}(\mathbf{k})$$

INVERSE ENERGY FLUX: FROM SMALL TO LARGE SCALES in 3D!

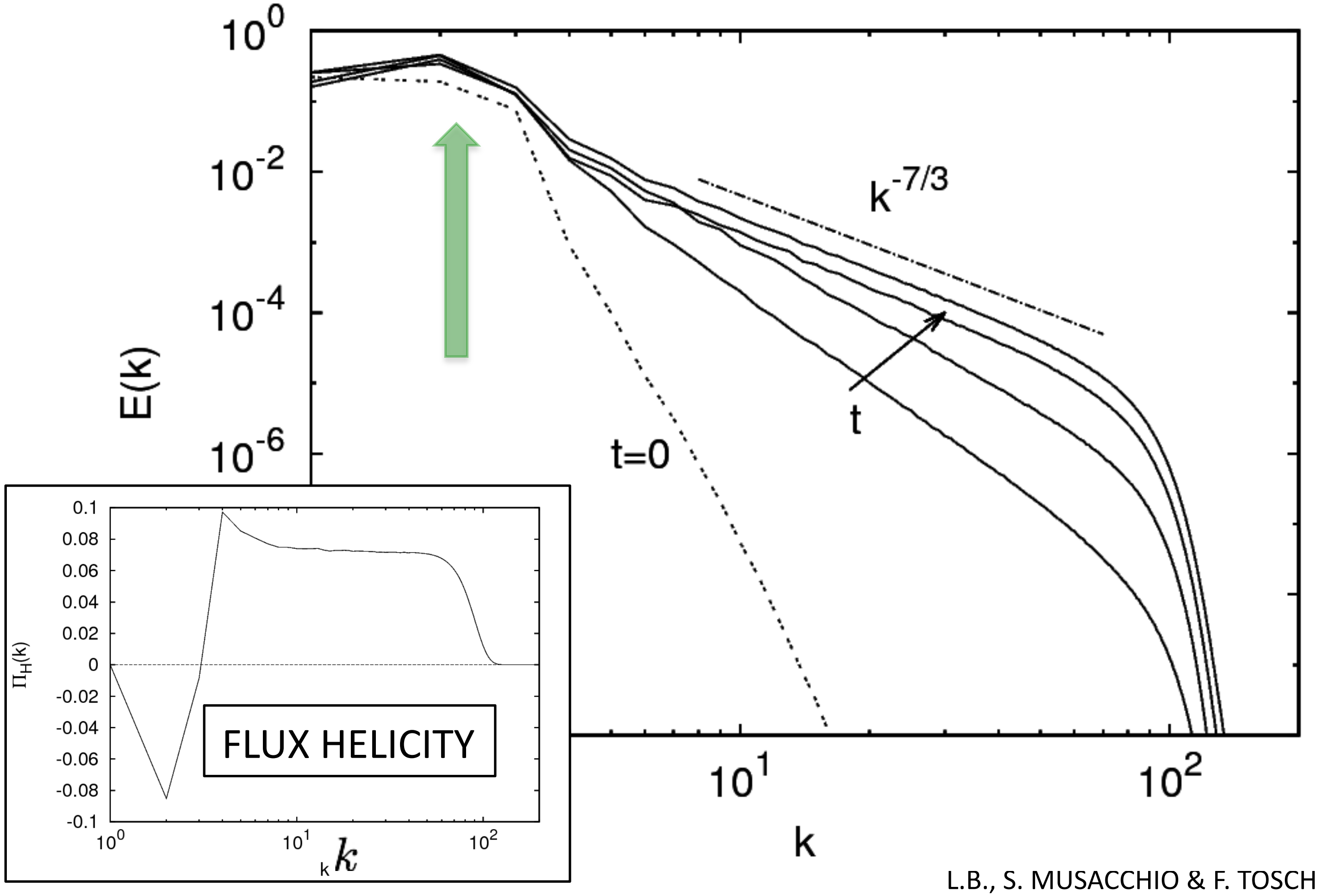
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. **108** 164501 (2012); JFM **730**, 309 (2013)

LARGE SCALES FORCING: DIRECT HELICITY CASCADE

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



L.B., S. MUSACCHIO & F. TOSCHI
730, 309 (2013) (2013)

SPLIT ENERGY-HELICITY CASCADES

P.D. Mininni and A. Pouquet. Phys. Rev. E **79,** 026304 (2009)

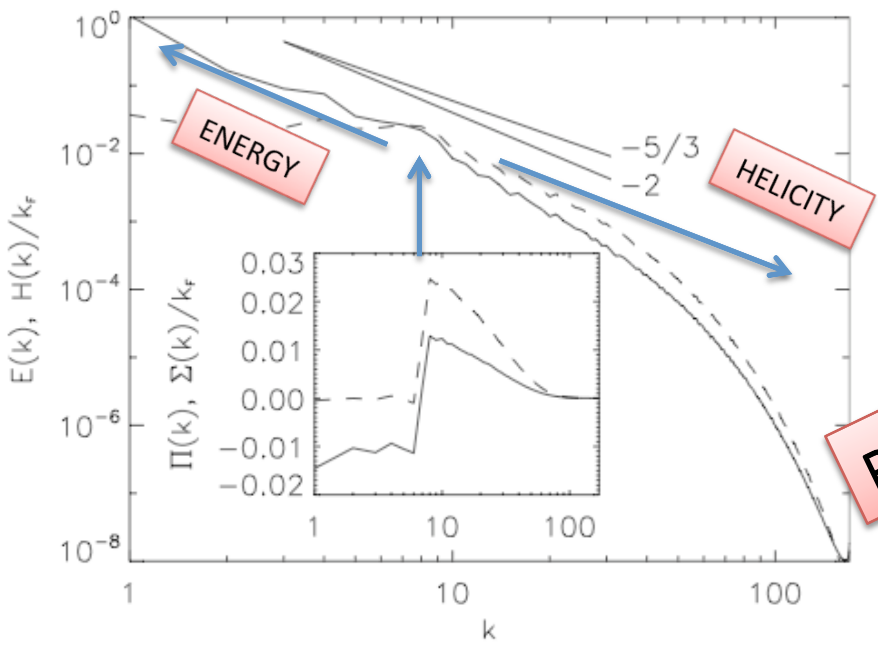


FIG. 2. Energy (solid) and helicity (dash) spectra in run A3 with the same forcing as run A2 but lower Rossby number. Different slopes are shown as a reference. The inset gives the energy and helicity fluxes and shows that there is both a direct and an inverse cascade of energy but only a direct cascade of helicity.

ROTATION 3D -> 2D

HELICITY ?
-> forward cascade

- ANISOTROPY
- HELICITY ≠ 0

PRL **104,** 184506 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

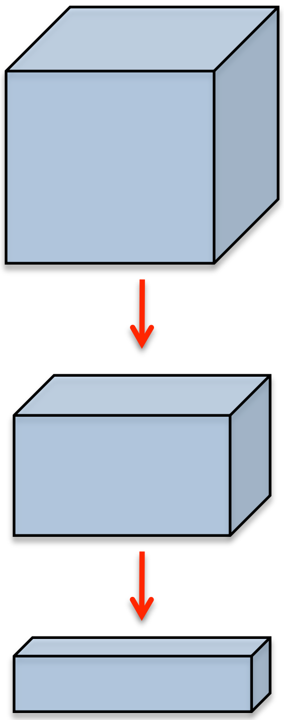
Turbulence in More than Two and Less than Three Dimensions

Antonio Celani,[1] Stefano Musacchio,[2,3] and Dario Vincenzi[3]

Upscale energy transfer in thick turbulent fluid layers

Nat Phys. 2011

H. Xia[1], D. Byrne[1], G. Falkovich[2] and M. Shats[1]*



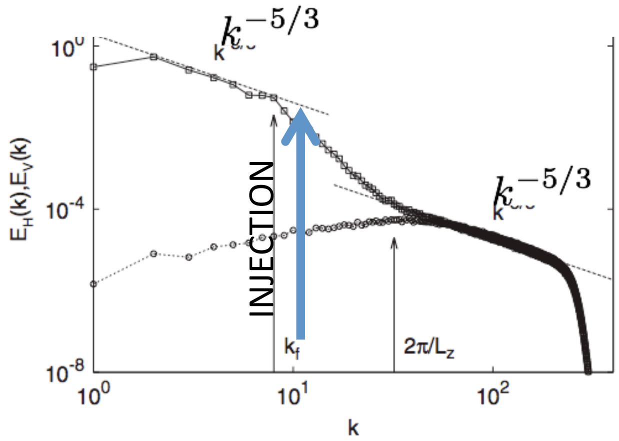
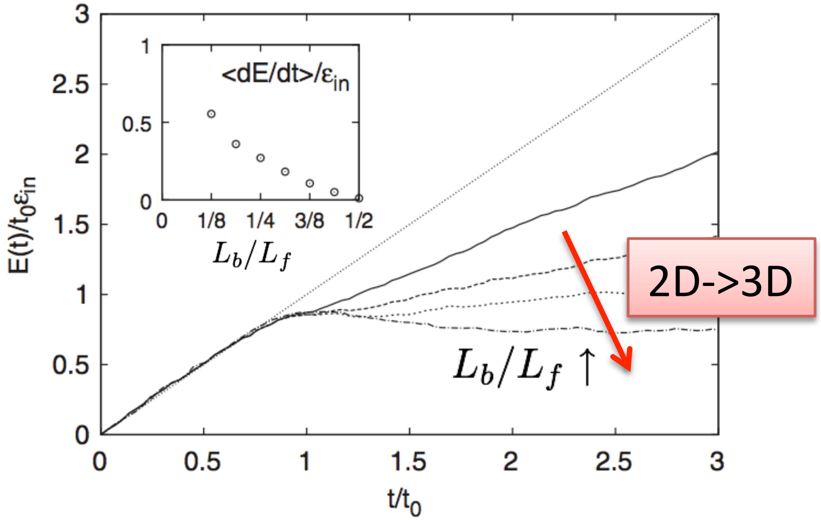


FIG. 3. Kinetic energy spectrum of horizontal (squares) and vertical (circles) velocities. Dashed lines represents Kolmogorov scaling. Parameters of the simulation: $L_x = 2\pi$, $\ell_f/L_x = 1/8$,

CONFINEMENT 3D → 2D



PHASE TRANSITION?

- ANISOTROPY
- HELICITY == 0

$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

ESISTENCE AND UNIQUENESS OF WEAK SOLUTIONS OF THE HELICAL-DECIMATED NSE

$$\begin{cases} \partial_t \boldsymbol{v}^+ = \mathcal{P}^+(-\boldsymbol{v}^+ \cdot \boldsymbol{\nabla} \boldsymbol{v}^+ - \boldsymbol{\nabla} p^+) + \nu \Delta \boldsymbol{v}^+ + \boldsymbol{f}^+ \\ \nabla \cdot \boldsymbol{v}^+ = 0 \end{cases}$$

HILBERT-NORM COINCIDES WITH THE SIGN-DEFINITE HELICTY

$$||g||_{H^{1/2}} = \sum_{\boldsymbol{k}} k|g(k)|^2$$

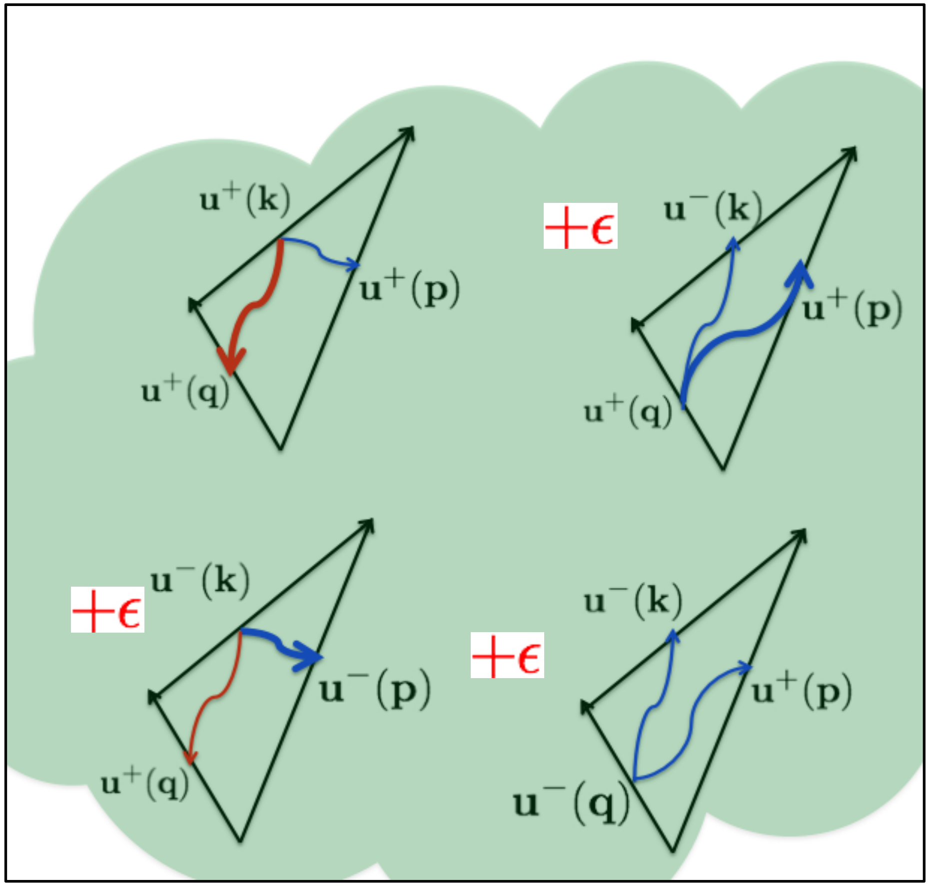
CONSERVATION HELICITY: NEW APRIORI BOUND ON THE VELOCITY

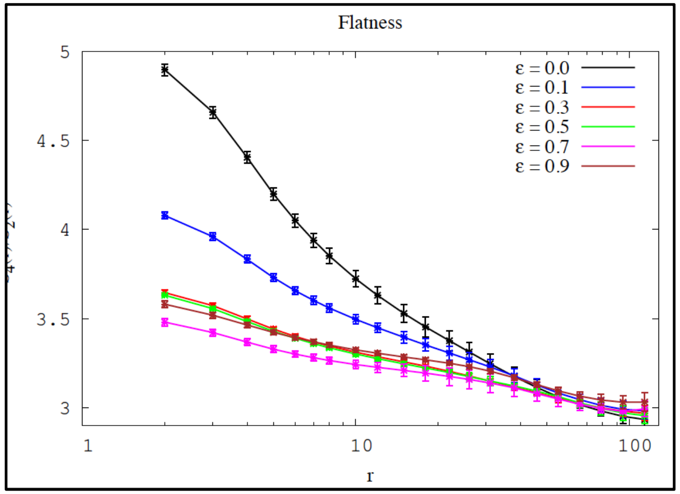
$$\frac{1}{2}\partial_t \sum_{\boldsymbol{k}} k|u^+(\boldsymbol{k},t)|^2 + \frac{\nu}{2}\sum_{\boldsymbol{k}} k^3|u^+(\boldsymbol{k},t)|^2 \leq \frac{1}{2\nu}\sum_{\boldsymbol{k}} |f^+(\boldsymbol{k})|^2 k^{-1}.$$

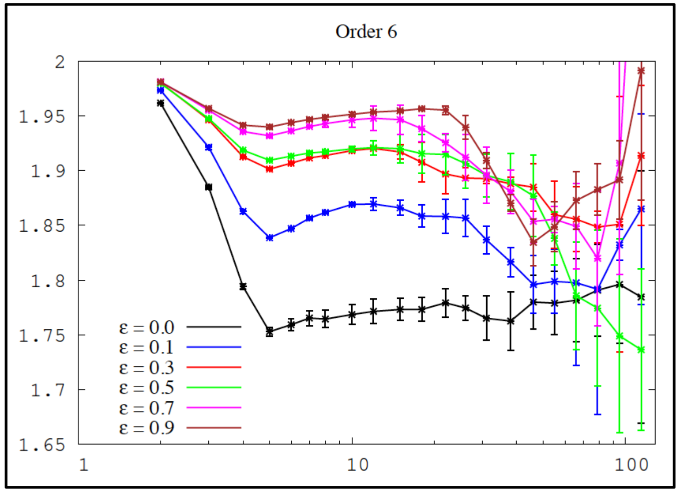
$$\frac{1}{2}\partial_t ||v^+||^2_{H^{\frac{1}{2}}} + \frac{\nu}{2}||v^+||^2_{H^{\frac{3}{2}}} \leq \frac{1}{2\nu}\sum_{\boldsymbol{k}} |f^+(\boldsymbol{k})|^2 k^{-1}.$$

$$v^+ \in L^\infty_t H^{\frac{1}{2}}_x; \qquad \sqrt{\nu} v^+ \in L^2_t H^{\frac{3}{2}}_x$$

L.B. & E. TITI J. Stat. Phys. **151**, 1089 (2013)







$$\mathbf{u}(\mathbf{k}) = u^{+}(\mathbf{k})\mathbf{h}^{+}(\mathbf{k}) + \epsilon(\mathbf{k})u^{-}(\mathbf{k})\mathbf{h}^{-}(\mathbf{k})$$

ON THE ROLE OF HELICITY IN 3D FORWARD/BACKWARD TURBULENT ENERGY TRANSFER

Q: Can we dissect (and reconstruct) NS equations to extract interesting information from its elementary constituents?

A: Yes, we can!

1) **We showed that ALL flows in nature posses a class of nonlinear interactions characterized by a backward energy transfer (inverse energy cascade), triggered by the dynamics of Helicity, and that this happens even in fully isotropic, homogeneous 3D turbulence**
2) **Connections to small-scales intermittency ?**
3) **Connections to regularity of NS equations in 3D ?**
4) **Extensions to Magnetohydrodynamics ?**
5) **Connections to energy reversal in rotating turbulence?**
6) **How does energy really flow in Fourier space?**

$$\mathbf{u}(\mathbf{k}) = u^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + \epsilon(\mathbf{k})u^-(\mathbf{k})\mathbf{h}^-(\mathbf{k})$$