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Statistical fluctuations of an ocean surface inferred from shoes and ships

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Abstract

This paper shows that it is possible to roughly estimate some ocean properties using simple time-dependent statistical models of ocean fluctuations. Based on a real incident, the loss by a vessel of a Nike shoes container in the North Pacific Ocean, a statistical model was tested on data sets consisting of the Nike shoes found by beachcombers a few months later. This statistical treatment of the shoes' motion allows one to infer velocity trends of the Pacific Ocean, together with their fluctuation strengths. The idea is to suppose that there is a mean bulk flow speed that can depend on location on the ocean surface and time. The fluctuations of the surface flow speed are then treated as statistically random. The distribution of shoes is described in space and time using Markov probability processes related to the mean and fluctuating ocean properties. The aim of the exercise is to provide some of the properties of the Pacific Ocean that are otherwise calculated using a sophisticated numerical model, OSCURS, where numerous data are needed. Relevant quantities are sharply estimated, which can be useful to (1) constrain output results from OSCURS computations, and (2) elucidate the behavior patterns of ocean flow characteristics on long time scales.

1. Introduction

Ebbesmeyer and Ingraham (1992) have recently provided a remarkably detailed study of the arrival distribution of Nike brand shoes, lost overboard from the container vessel 'Ilansa Carrier' in the North Pacific Ocean on 27 May 1990 at about 48°N, 161°W, as a function of position along the coasts of Oregon, Washing-

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ton State and Canada. Not only is the description by Ebbesmeyer and Ingraham (1992) very clear concerning the circumstances of the loss, beachcomber collection efficiencies, and reporting of arrival times, but a sophisticated computer code, known as the Ocean Surface Current Simulations (OSCURS) numerical model, was used to hindcast the motion of the shoes in relation to long-term mean surface geostrophic currents with wind-generated, surface-mixed-layer currents, and using copious data from daily sea-level pressure data to estimate wind-driven currents.

It seemed that it might be possible to provide a simplified statistical model of shoe motion, so that quick estimates could be made of statistical properties of the ocean during the time between shoe release and collection. The idea here is to use the observed spatial distribution and arrival time distribution of collected shoes to infer the statistical properties of the ocean. Although not a substitute at the same level of detail as the OSCURS code is capable of providing, nevertheless a statistical treatment of motion of the shoes can be used to infer some of the velocity trend properties, and their fluctuation strengths, of the Pacific Ocean surface. In this paper, we illuminate how statistically sharp estimates of relevant quantities can be made.

2. A statistical model

The essence of the statistical model shoe motion is to suppose that there is a mean bulk surface flow speed, V, which can depend on both location $\boldsymbol{x}$ on the ocean surface, and time t. In addition, it is assumed that there are also fluctuations of the surface flow speed, δV, which are sufficiently difficult to unravel in a deterministic sense that they can be treated as statistically random over the time scale of interest (of the order of 6 months to a year for the time between shoe release and collection). Thus the shoes are regarded as test particles, tracking the flow, and, on a long time basis, following the direction of mean bulk flow (on average). However, at any particular instant of time the shoes follow the instantaneous flow direction, so that a diffusion-like spreading of the shoes will occur away from their initial point of insertion.

The idea is to describe the spatial distribution of shoes at any location in response to both the bulk flow and appropriate statistical properties of the surface velocity fluctuations of the ocean.

3. Mathematical description

3.1. Statistical fluctuations

Bold characters notify a vector representation throughout this section. Suppose that the two-dimensional surface velocity field, $\boldsymbol{V}(\boldsymbol{x}, t)$, is responsible for the motion of the shoes, where $\boldsymbol{x}$ is a two-dimensional cartesian vector characterizing location on the ocean surface, and t is time. Denote $\boldsymbol{i}$ and $\boldsymbol{j}$ as the unit vectors in

x- and y-directions respectively, so that one has $\boldsymbol{x} = x\boldsymbol{i} + y\boldsymbol{j}$. Let the velocity field have a systematic ordered component, $V(t)$, in the x-direction, and let there be a velocity $\boldsymbol{w}(\boldsymbol{x}, t)$, relative to the ordered velocity; $\boldsymbol{w}(\boldsymbol{x}, t)$ is treated as a randomly varying vector field. Then the displacement of a test particle with coordinates x, y at time t, will have a displacement characterized by

$$\frac{dx}{dt} = V(t) + w_x(\boldsymbol{x}, t) \tag{1a}$$

$$\frac{dy}{dt} = w_y(\boldsymbol{x}, t) \tag{1b}$$

where w_x and w_y are the components of the random velocity field with zero mean values:

$$\langle w_x \rangle = \langle w_y \rangle = 0 \tag{1c}$$

Throughout the remainder of this paper, we use angular brackets to denote an ensemble average.

It is convenient to introduce a random velocity potential $\phi(x, t)$, with

$$w_x = -\frac{\partial \phi}{\partial y} \tag{2a}$$

$$w_y = \frac{\partial \phi}{\partial x} \tag{2b}$$

so that the random component of ocean velocity is regarded as being incompressible in the surface dimensions (i.e. little vertical mixing); this is in agreement with the basic tenets of the OSCURS model, which 'combines long-term mean geostrophic currents with wind-generated, surface-mixed-layer currents to form a resultant current vector each grid point' (Ebbesmeyer and Ingraham, 1992). A referee has noted that 'upwelling and downwelling can cause non-zero divergence' of the random component of the flow, which is, of course, correct. However, for those shoes that float, and so can reach the shore-line, their positions remain close to the oceanic surface at all times and so those shoes are dominantly influenced only by oceanic flow close to the surface. It then is appropriate to treat the random component of the surface ocean flow influencing the floating shoes as a two-dimensional random variable. Then

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0 \tag{3}$$

is satisfied automatically by the velocity potential representation provided $\langle \phi \rangle = 0$.

It follows from Eqs. (1) that, in accord with the random statistical distribution of velocity, the displacement of a particle will also have a random character. We then write

$$x(t) = \langle x(t) \rangle + \delta x(t) \tag{4a}$$

$$y(t) = \delta y(t) \tag{4b}$$

where $\delta x(t)$ and $\delta y(t)$ are the random components of displacement in the direction of mean flow and in the transverse direction, respectively. Then

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = V(t) \tag{5a}$$

$$\frac{\mathrm{d}(\delta x)}{\mathrm{d}t} = w_x[\boldsymbol{x}(t), t] \tag{5b}$$

and the transverse displacement δy is random with

$$\frac{\mathrm{d}(\delta y)}{\mathrm{d}t} \equiv \frac{\mathrm{d}y}{\mathrm{d}t} = w_y[\boldsymbol{x}(t), t] \tag{5c}$$

Let a test particle be inserted into the ocean at statistically sharp coordinates $x = x_0$ and $y = y_0$ at time $t = 0$. It follows that

$$\langle x(t)\rangle = x_0 + \int_0^t V(t')\,\mathrm{d}t' \tag{6a}$$

$$y(t) = y + \int_0^t w_y[\boldsymbol{x}(t'), t']\,\mathrm{d}t' \equiv y_0 + \delta y(t) \tag{6b}$$

with

$$\delta x(t) = \int_0^t w_x[\boldsymbol{x}(t'), t']\,\mathrm{d}t' \tag{7a}$$

$$\delta y(t) = \int_0^t w_y[\boldsymbol{x}(t'), t']\,\mathrm{d}t' \tag{7b}$$

Although the transverse coordinate, $y(t)$, has the mean value y_0 for all time, nevertheless, there is a root mean square (r.m.s.) dispersion of test particles around the direction of mean flow with $\langle y(t)^2\rangle$ characterizing the transverse diffusion. From Eqs. (6b) and (7b), it follows that

$$\langle y(t)^2\rangle = \langle y_0^2\rangle + \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \langle w_y[\boldsymbol{x}(t_1), t_1] w_y[\boldsymbol{x}(t_2), t_2]\rangle \tag{8}$$

Following a conventional format (Lumley, 1970), we take the autocorrelation behavior to be dominantly (a) homogeneous and (b) stationary, and so write

$$R_{yy}[\boldsymbol{x}(t_1) - \boldsymbol{x}(t_2), t_1 - t_2] = \langle w_y[\boldsymbol{x}(t_1), t_1] w_y[\boldsymbol{x}(t_2), t_2]\rangle \tag{9}$$

where $R_{yy}(\boldsymbol{x}, t)$ is the autocorrelation function of the transverse (to the bulk flow direction) fluctuations in the surface ocean velocity.

The behavior of the lateral spatial dispersion, $\langle \delta y(t)^2\rangle \equiv \langle y(t)^2\rangle - y_0^2$, then depends on the autocorrelation function of the transverse velocity field. The lateral dispersion is provided through

$$\langle \delta y(t)^2\rangle = \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 R_{yy}[\boldsymbol{x}(t_1) - \boldsymbol{x}(t_2), t_1 - t_2] \tag{10}$$

The conventional approximations (Lumley, 1970) are now made that (a) the lateral distance is much smaller than the distance of transport parallel to the bulk flow direction, and (b) the bulk velocity $V(t)$ can be replaced by a constant, V_0, during the study interval. Then

$$R_{yy}[\boldsymbol{x}(t_1)-\boldsymbol{x}(t_2),\, t_1-t_2]=R_{yy}[V_0(t_1-t_2)\boldsymbol{i},\, t_1-t_2] \tag{11}$$

Eq. (10) can then be written

$$\langle \delta y(t)^2\rangle=\int_0^t \mathrm{d}t_1\int_0^t \mathrm{d}t_2 R_{yy}[V_0(t_1-t_2)\boldsymbol{i},\, t_1-t_2] \tag{12}$$

In the direction of mean flow, under the same conditions, one obtains

$$\langle \delta x(t)^2\rangle=\int_0^t \mathrm{d}t_1\int_0^t \mathrm{d}t_2 R_{xx}[V_0(t_1-t_2)\boldsymbol{i},\, t_1-t_2] \tag{13}$$

Expressions (12) and (13) can be simplified to yield

$$\langle \delta x(t)^2\rangle=t\int_0^t R_{xx}(V_0 u\boldsymbol{i},\, u)\,\mathrm{d}u \tag{14a}$$

$$\langle \delta y(t)^2\rangle=t\int_0^t R_{yy}(V_0 u\boldsymbol{i},\, u)\,\mathrm{d}u \tag{14b}$$

It should be noted that $R_{jj}(V_0 u\boldsymbol{i}, u)$ is dependent on spacing in x and time only to lowest order.

For t large, both integrals tend to finite values (respectively A and B), with

$$\langle \delta x(t)^2\rangle\to t\int_0^\infty R_{xx}(V_0 u\boldsymbol{i},\, u)\,\mathrm{d}u\equiv tA, \text{ at } t\to\infty \tag{15a}$$

$$\langle \delta y(t)^2\rangle\to t\int_0^\infty R_{yy}(V_0 u\boldsymbol{i},\, u)\,\mathrm{d}u\equiv tB, \text{ at } t\to\infty \tag{15b}$$

3.2. Gaussian probabilities

At the time ($t=0$) of dumping of the shoes (centered at $x=x_0$ and $y=y_0$) it is taken that the distribution of the shoes can be described by the conventional (Lumley, 1970) Gaussian probability

$$P(\boldsymbol{x},\, t=0)\propto \exp\left[-(x-x_0)^2/\sigma_x^2-(y-y_0)^2/\sigma_y^2\right](\sigma_x\sigma_y)^{-1} \tag{16}$$

where σ_x and σ_y respectively measure the r.m.s. spread in positions around $x=x_0$ and $y=y_0$ over which the shoes are dumped (presumably of the order of the size of the container).

At a later time, the probability can then be written

$$\begin{aligned}P(\boldsymbol{x},\, t)\propto &\left[\sigma_x^2+\langle\delta x(t)^2\rangle\right]^{-1/2}\left[\sigma_y^2+\langle\delta y(t)^2\rangle\right]^{-1/2} x\\ &\times\exp\Big\{-(x-x_0-V_0t)^2\left[\sigma_x^2+\langle\delta x(t)^2\rangle\right]^{-1}\\ &-(y-y_0)^2\left[\sigma_y^2+\langle\delta y(t)^2\rangle\right]^{-1}\Big\}\end{aligned} \tag{17}$$

For $\sigma_x^2 \ll \langle \delta x(t)^2 \rangle$, $\sigma_y^2 \ll \langle \delta y(t)^2 \rangle$, and time sufficiently long that $\langle \delta x(t)^2 \rangle$ and $\langle \delta y(t)^2 \rangle$ can be written by their asymptotic representations through Eqs. (15a) and (15b) as tA and tB respectively, then

$$P(x, t) \propto t^{-1}(AB)^{-1/2} \exp\left[-(x - x_0 - V_0 t)^2 (tA)^{-1} - (y - y_0)^2 (tB)^{-1}\right] \tag{18}$$

For a coast-line making an angle θ with respect to the direction of bulk flow, so that a coordinate along the coast-line is Y and a coordinate perpendicular to the coast-line is X, one has

$$x = X \cos\theta - Y \sin\theta \tag{19a}$$

$$y = Y \cos\theta + X \sin\theta \tag{19b}$$

The time-dependent probability distribution of shoes measured in coordinates parallel and perpendicular to the coast-line is

$$P(x, t) \propto t^{-1}(AB)^{-1/2} \exp\left[-(X\cos\theta - Y\sin\theta - x_0 - V_0 t)^2 (tA)^{-1} - (Y\cos\theta + X\sin\theta - y_0)^2 (tB)^{-1}\right] \tag{20}$$

Without loss of generality one can take the origin of axes of both the coast-line coordinates and the (x, y) coordinate system to be centered at the dumping position, and so set $x_0 = 0 = y_0$. The distance from the dumping position to the impact point of the mean ocean flow (directed along x) on the true coast-line is then $X\cos\theta - Y\sin\theta = x_{\text{impact}}$. In this origin-centered coordinate system (Fig. 3) one has

$$P(x, t) \propto t^{-1}(AB)^{-1/2} \exp\left[-(x_{\text{impact}} - V_0 t)^2 (tA)^{-1} - (Y\sec\theta + x_{\text{impact}}\tan\theta)^2 (tB)^{-1}\right] \tag{21}$$

3.3. Non-Gaussian arrival time distribution

The arrival time distribution at the mean flow impact point (i.e. $Y = 0$) is then

$$P(x_{\text{impact}}\, t) \propto t^{-1}(AB)^{-1/2} \exp\left[-(x_{\text{impact}} - V_0 t)^2 (tA)^{-1} - x_{\text{impact}}^2 \tan^2\theta (tB)^{-1}\right] \tag{22}$$

which has a peak at $t = t_{\text{peak}}$, where

$$2t_{\text{peak}} = \left\{-(A/V_0^2) + \left[(A/V_0^2)^2 + 4x_{\text{impact}}^2 V_0^{-2}(1 + A\tan^2\theta/B)\right]^{1/2}\right\} \tag{23}$$

For times $t \ll t_{\text{peak}}$, the shape of $P(x_{\text{impact}}, t)$ is roughly proportional to

$$P(x_{\text{impact}}, t) \propto t^{-1} \exp\left[-t^{-1} x_{\text{impact}}^2 (1/A + \tan^2\theta/B)\right] \equiv t^{-1}\exp(-t_{\text{rise}}/t) \tag{24a}$$

whereas for $t \gg t_{\text{peak}}$,

$$P(x_{\text{impact}}, t) \propto t^{-1} \exp\left[-\left(V_0^2 A^{-1}\right)t\right]$$
$$\equiv t^{-1} \exp(-t/t_{\text{decay}}) \tag{24b}$$

Thus the arrival time distribution at the mean flow impact point has a very sharp rise over a time scale controlled by

$$t_{\text{rise}} \approx x_{\text{impact}}^2 \left(1/A + \tan^2 \theta/B\right) \tag{25a}$$

before peaking at $t = t_{\text{peak}}$, followed by a slow decline at later times in a roughly exponential manner over a time scale

$$t_{\text{decay}} \approx A/V_0^2 \tag{25b}$$

Thus, the arrival-time distribution is non-Gaussian with respect to time.

3.4. Lateral coast-line distribution

At lateral distances ($Y \neq 0$) along the coast-line one has

$$P(x, t) = P(x_{\text{impact}}, t) \exp\left[-(tB)^{-1} Y \sec^2 \theta (Y + 2x_{\text{impact}} \sin \theta)\right] \tag{26}$$

Thus the spatial distribution along the coast is asymmetrically spread with position (relative to lateral distances measured from $Y = 0$). Centered at

$$Y_{\text{center}} = -x_{\text{impact}} \sin \theta \tag{27a}$$

the distribution is spread laterally symmetrically with half-width

$$\Delta Y \approx \cos \theta (tB)^{1/2} \tag{27b}$$

which systematically increases in time. Thus the lateral distribution of shoes along the coast is modulated by the mean flow impact point arrival distribution in time, but contains (at any instant of time) its own response to randomly diffusive component of motion of the shoes in their journey across the ocean.

It should be noted that for small angles of impact ($\theta \ll 1$) the influence of the lateral spreading diffusion factor, B, on the arrival time distribution at the mean flow impact point ($Y = 0$) is much less than the longitudinal diffusion factor A, because combinations involving B are always in the form $A^{-1} + \tan^2\theta/B$ so that effects of B are negligible compared with effects of A, provided only that

$$B \gtrsim \theta^2 A \tag{28}$$

For instance even at $\theta = 10°$ (approximately 1/6 radian), any B value greater than 3% of A will not be significant in affecting the arrival time distribution at $Y = 0$; and at $\theta = 5$ (approximately 1/12 radian) B has only to exceed A by about 2/3% to be ignorable.

Thus one anticipates that arrival time data at the mean flow impact point will lead to a fairly accurate determination for A but a fairly poor determination for B; conversely, the half-width distribution along the coast-line is sensitive

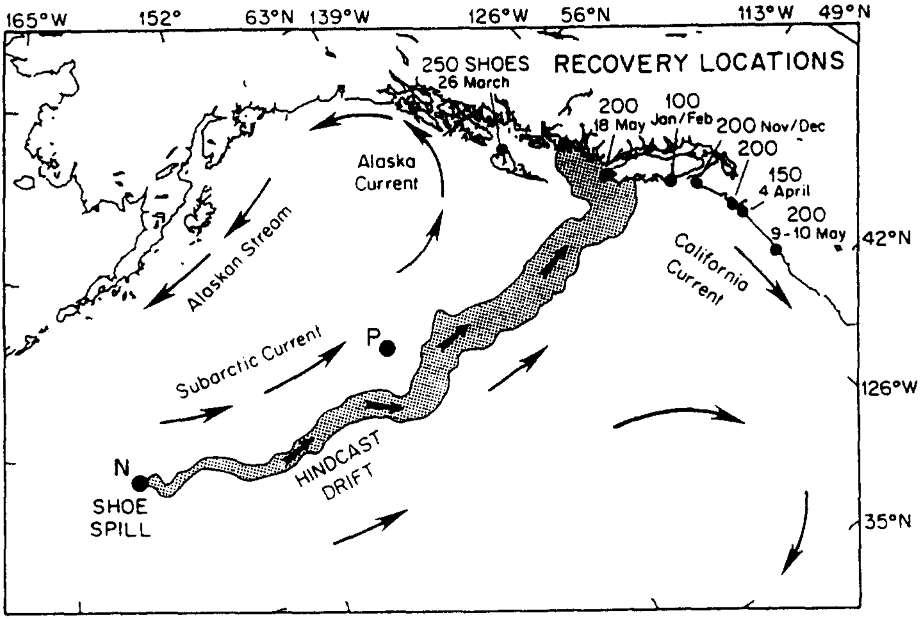

Fig. 1. Map locating where the 80000 shoes were lost overboard on 27 May 1990 (N: shoe spill), and dates (year 1991) and locations where 1.6% (1300 of 80000) were discovered by beachcombers (dots on the top right) (after Ebbesmeyer and Ingraham (1992)).

predominantly to the lateral spreading diffusion factor B. As a consequence, measures of the half-width along the coast should provide a more accurate measure of B. As we shall see in the next section these expectations are substantiated by the observations.

4. Application

The position of the shoe spill (48°N, 161°W) has been recorded, and 1300 of the 80 000 shoes lost overboard were known to be found by beachcombers. All these records have been put together by Ebbersmeyer and Ingraham (1992), providing us with Fig. 1, showing the distribution of the Nike shoes in time and space (along the coast-line). From Fig. 1, and measuring along the coast relative to the mean flow impact point shown in Fig. 2, the relevant data can be collected as shown in Table 1.

The half-width spread, ΔY, along the coast is of the order of a few hundred kilometers, whereas the mean flow impact distance from the ship spill is of order 2500 km. Thus $\Delta Y^2/x^2_{\text{impact}} \ll 1$. Accordingly, to estimate the fluctuation parameters of the ocean during the journey of the shoes to the coast-line, two simplifying approximations can be made: (1) for the arrival time distribution at the mean flow impact point ($Y = 0$) it is an accurate enough approximation to consider that all

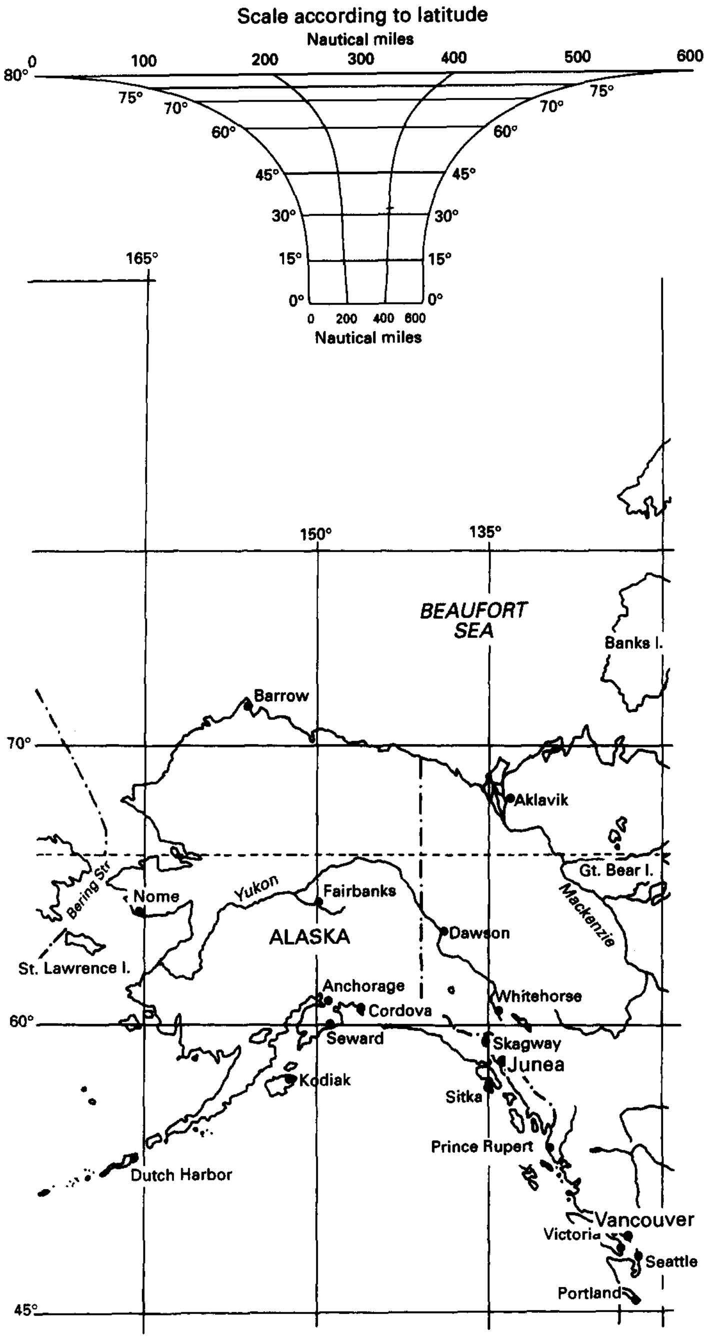

Fig. 2. General map of the region allowing for distance estimation so that physical scales are available for use in text.

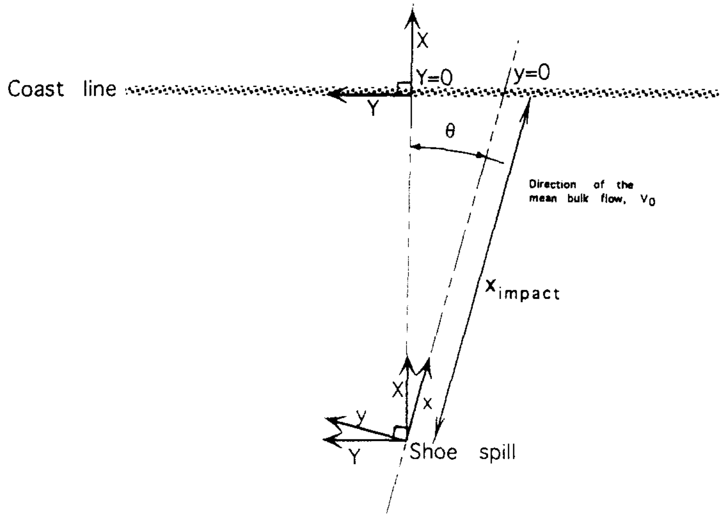

Fig. 3. Sketch of the coordinate system with the coast-line making an angle θ with the mean flow direction coordinates.

shoes found (1.6% of the total lost) are located on $Y = 0$ but arrive at their posted collection times as in Table 1; (2) for the spatial distribution along the coast-line, it is an accurate enough approximation to consider that all shoes arrived at about the same time of $t_{\text{impact}} \equiv x_{\text{impact}}/V_0$ and are spread along the coast, centered at $|Y| = |x_{\text{impact}} \sin\theta|$, and with an approximately Gaussian distribution of half-width $\Delta Y \approx \cos\theta\,(t_{\text{impact}}\,B)^{1/2}$ around the centered location.

4.1. Arrival time distribution at the mean impact point

Several factors control the arrival time distribution:

(1) the value of x_{impact}, which is measured from Fig. 1 at about 2500 km, but still contains some uncertainty.

Table 1
Data collection from Fig. 1

Y (distance along coast) (km)	Number of shoes found by beachcombers	Estimated time of recovery after shoe spill (days)
300	250	293
−600	200	355
−690	100	220
−750	200	200
−850	200	Not available
−900	150	311
−1060	200	346

Negative (positive) distances along the coast are south (north) of the mean flow impact point of Fig. 1.

(2) The value of the bulk flow speed V_0. We can make an initial rough estimate of V_0 from $x_{\text{impact}}/\langle t_d \rangle$, where $\langle t_d \rangle$ is the mean value of the times at which shoes were recovered (the range of times is from about 200 to 350 days so that V_0 ranges around 8–12 km day^{-1}).

(3) the angle θ between the coast-line and the mean flow direction, which can be measured from Fig. 1 as lying between about 4 and 8°.

(4) the values of the diffusion coefficients A and B. An arrival time spread of about 100 days suggests that A should be roughly of order 2000 km^2 day^{-1}, and the half-width spread along the coast provides the initial rough estimate of B of around 2600 km^2 day^{-1}.

The values of all five of these coefficients influence the arrival time distribution at the mean flow impact point. Taking the 1300 shoes recovered as representing 1.6% of the total shoes lost, a value of 200 shoes is then interpreted as a probability of $[(200/1300) \times 1.6]/100 \approx 2.5 \times 10^{-3}$.

The arrival time distribution at the mean flow impact point given by Eq. (22) has an overall normalization constant such that the total probability is normalized to be in agreement with the total fractional number of observed shoes, i.e.

$$\int P(x, t)\, \mathrm{d}x\, \mathrm{d}y = 1.6/100 \tag{29}$$

Thus the constant of proportionality, C, in Eq. (22) is $C \equiv (1.6)/(\pi 100)$ and is independent of any parameters in the shape distribution.

As the values for the parameters x_{impact}, V_0, θ, A and B are varied, the degree of agreement between Eq. (22) (with the normalization constant set as above) and the observed shoe distribution will change dramatically. For instance, the peak time, peak value of $P(x_{\text{impact}}, t)$, and rise and decay times (together with their associated values of $P(x_{\text{impact}}, t)$) depend on values assigned for the parameters.

Fig. 4(a) shows the result of a fit to the data using Eq. (22) when $x_{\text{impact}} = 2500$ km, $V_0 = 9$ km day^{-1}, $\theta = 5°$, $A = 2100$ km^2 day^{-1} and $B = 80$ km^2 day^{-1}. Fig. 4b shows the results of a similar fit to the data when it is assumed that the impact distance is only 2000 km, some 500 km closer to the coast than that nominally recorded in Fig. 1, and when all other parameters are held at the values given in Fig. 4(a). To be observed in Fig. 4(b) is the sharper rise of the probability and the shorter decay time, together with a higher peak value; all of these factors indicate less diffusion of the sharp initial 'spike' at dumping time owing to the shorter distance covered (2000 km). Conversely, Fig. 4(c) shows the effect of increasing the impact distance to 3000 km; a more 'rounded' rise is observed, a longer decay and less of a peak — all factors representing the greater effects of diffusive spreading over the larger space scale involved. The sensitivity of the results to variation of the mean flow impact distance is clear from Figs. 4(a)–4(c), suggesting that $x_{\text{impact}} \approx 2500$ km to within an uncertainty of less than about ± 300 km.

Equally, Figs. 4(d) and 4(e) show the effect of changing the bulk flow speed by ± 1 km day^{-1} around the 'canonical' value of 9 km day^{-1}. At a speed of 8 km day^{-1}, the predicted arrival time distribution is biased to later times compared with the observed data, whereas at 10 km day^{-1} the arrival time distribution is biased to earlier times. The predicted and observed probabilities are consistent in both cases, but are not as satisfactory fits to the data as Fig. 4(a) when $V_0 = 9$ km day^{-1}, suggesting that a restricted range of the bulk flow speed of 9 ± 1 km day^{-1} can be set.

The influence of varying the longitudinal diffusion coefficient, A, is exhibited in Figs. 4(f) ($A = 1500$ km^2 day^{-1}) and 4(g) ($A = 2700$ km^2 day^{-1}). At low longitudinal diffusion values, the predicted distribution is narrower, has a quicker rise and a faster decay than occurs in the case (Fig. 4(g)) of a larger longitudinal diffusion coefficient. A range of A values satisfactorily encompassing the data is provided by the rather generous range $A = 2100 \pm 600$ km^2 day^{-1}.

The variation of the average angle of impact between the mean flow direction and the coast-line is exhibited in Figs. 4(h) and 4(i) for $\theta = 0°$ and $\theta = 8°$, respectively. The high degree of sensitivity of the magnitude of the predicted probability to the observations is apparent by the considerable 'overshooting' of predictions vs. observations in the case of $\theta = 0°$ (Fig. 4(h)) and the considerable 'undershooting' of predictions vs. observations in the case of $\theta = 8°$ (Fig. 4(i)). This high degree of sensitivity allows the angle θ to be determined rather precisely. A value of $\theta = 5 \pm 2°$ covers rather well the range consistent with the data.

The value of the lateral diffusion coefficient, B, is not well determined by the arrival time distribution at the mean flow impact point, as expected. For $\theta \approx 7°$, and $A = 2100 \pm 600$ km^2 day^{-1}, values of B in excess of about $A \tan^2 \theta (\lesssim 50)$ should not provide a marked sensitivity in the distribution of arrival times. For instance, Fig. 4(j), drawn for $B \approx 5300$ km^2 day^{-1} provides an equally acceptable fit to the arrival time data as does the case of $B = 80$ km^2 day^{-1} shown in Fig. 4(a). Only at smaller values of B is there any sensitivity, as anticipated. Thus for $B = 50$ km^2 day^{-1}, as shown in Fig. 4(k), we note the mismatch to the data is outside the realm of acceptable fit. Thus the lateral diffusion coefficient, B, is not well determined by the estimate of the arrival time distribution at the mean flow impact point—except that it should be noted that B is probably greater than about 50 km^2 day^{-1}.

In short, from the mean arrival time distribution one can infer the values $x_{\text{impact}} \approx 2500 \pm 300$ km, $V_0 \approx 9 \pm 1$ km day^{-1}, $\theta \approx 5 \pm 2°$, $A \approx 2100 \pm 600$ km^2 day^{-1} and $B \gtrsim 50$ km^2 day^{-1}.

4.2. Spatial distribution along the coast-line

Here we use the approximation that all shoes arrive at approximately the same time (given by x_{impact}/V_0, i.e. approximately 280 days) after the spill and are spread along the coast-line in a roughly Gaussian manner. The estimate of the center of the lateral distribution is $Y_{\text{center}} = -x_{\text{impact}} \sin \theta \approx -200$ km, so that the lateral distribution is centered to the south of the mean flow impact point.

Presented in Fig. 5 is the spatial distribution of the collected shoes with three superposed Gaussian curves corresponding to values of B of 5300 km^2 day^{-1}, 2600 km^2 day^{-1}, and 1300 km^2 day^{-1}, respectively. The curve at $B = 5300$ km^2 day^{-1} is broader than the data distribution, the curve of $B = 1300$ km^2 day^{-1} is somewhat narrower than the observed distribution, and that at $B = 2600$ km^2 day^{-1} fits the observed distribution reasonably well, given the sparseness of the statistics. A representative estimate of the range of B satisfying the observed lateral distribution is encompassed by $B \approx 2600 \pm 1000$ km^2 day^{-1}, a value which is consistent with the fit to the longitudinal arrival time data.

Indeed, to within the error estimates given, the values of A and B are effectively the same, suggesting that ocean surface velocity fluctuations around the mean velocity are roughly isotropically distributed in two dimensions to within resolution.

In terms of a correlation length L for the velocity fluctuations, one has (Lumley, 1970)

$$A \approx \left(\frac{\langle \delta v_x^2 \rangle}{V_0^2} \right) V_0 L_x \approx B = \left(\frac{\langle \delta v_y^2 \rangle}{V_0^2} \right) V_0 L_y$$

so that $\langle \delta v_x^2 \rangle L_x \approx \langle \delta v_y^2 \rangle L_y$ with

$$\langle \delta v_x^2 \rangle L_x / V_0 \approx 2100\text{–}2600 \text{ km}^2 \text{ day}^{-1} \qquad (30)$$

to within an uncertainty of about ± 800 km^2 day^{-1}.

The correlation length, L, is, presumably, somewhat smaller than the impact distance $x_{\text{impact}} \approx 2500$ km; so that from Eq. (30) the r.m.s. velocity fluctuations are then greater than about ± 1 km day^{-1}. If the correlation length L_x is comparable with the lateral spread distance along the coast-line (approximately 500–1000 km), then, from Eq. (30), the r.m.s. ocean velocity fluctuations are of order $\pm$(3–7) km day^{-1}.

The point is that the r.m.s. velocity fluctuations are fairly insensitive to the correlation length and diffusion coefficients (varying only with the square root of both) so that a fairly accurate assessment of $\langle \delta v_x^2 \rangle^{1/2}$ can be made with just a rough estimate of the correlation length and of the diffusion coefficients.

It would appear that the data and model predictions support the notion of an ocean with about 30–60% random flow velocity components relative to the mean current, i.e. $|\delta v|/V_0 \approx 0.3\text{–}0.6$.

In reviewing this paper, both referees noted that some comment was needed on the consequences of open ocean diffusion effects vs. coastal currents and divergence of the ocean flow as it approaches the coast. In particular, one reviewer noted that prior estimates of open ocean diffusion coefficients are around 2×10^7 cm^2 s^{-1} (approximately 180 km^2 day^{-1}), whereas the estimates from fitting the simple diffusive model of this paper yield equivalent longitudinal and transverse diffusion coefficients of around 2300 ± 1000 km^2 day^{-1}, a factor of between 12 ± 6 larger. Thus estimates of r.m.s. random ocean velocity fluctuations with the 180 km^2 day^{-1} diffusion coefficient would be about a factor of three or four lower then obtained in the body of the paper, i.e. randomly fluctuating values of order $\pm$(1–2) km day^{-1} would then be more appropriate, rather than the $\pm$(3–7) km day^{-1} range estimated. Thus the range $|\delta v/V_0| \approx 0.1\text{–}0.6$ generously encompasses both situations.

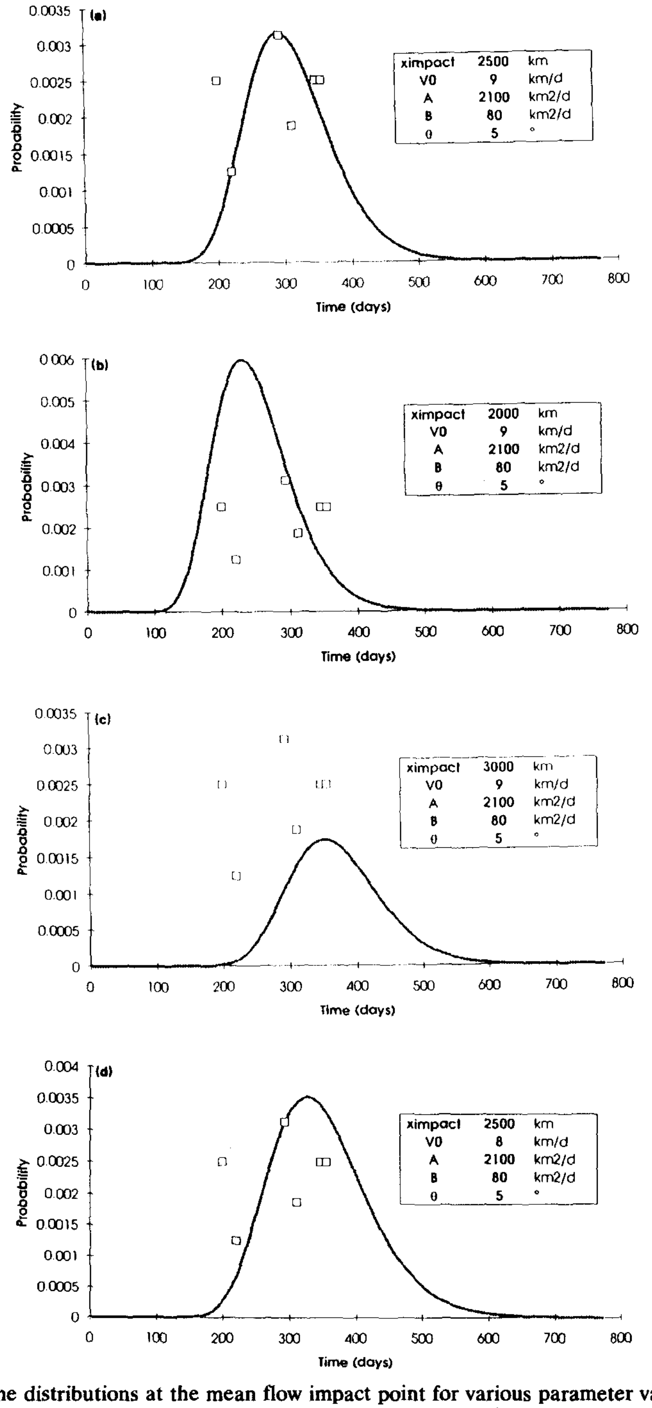

Fig. 4. Arrival time distributions at the mean flow impact point for various parameter values compared against observed shoe data. (a) $x_{\text{impact}} = 2500$ km, $V_0 = 9$ km day^{-1}, $\theta = 5°$, $A = 2100$ km^2 day^{-1}, $B \approx 80$ km^2 day^{-1} (b)–(k), Parameters as for (a) except (b) x_{impact} is lowered to 2000 km; (c) x_{impact} is increased to 3000 km; (d) V_0 is lowered to 8 km day^{-1}; (e) V_0 is increased to 10 km day^{-1}; (f) A is lowered to 1500 km^2 day^{-1}; (g) A is increased to 2700 km^2 day^{-1}; (h) θ is lowered to 0°; (i) θ is increased to 8°; (j) B is increased to 5300 km^2 day^{-1}; (k) B is decreased to 50 km^2 day^{-1}.

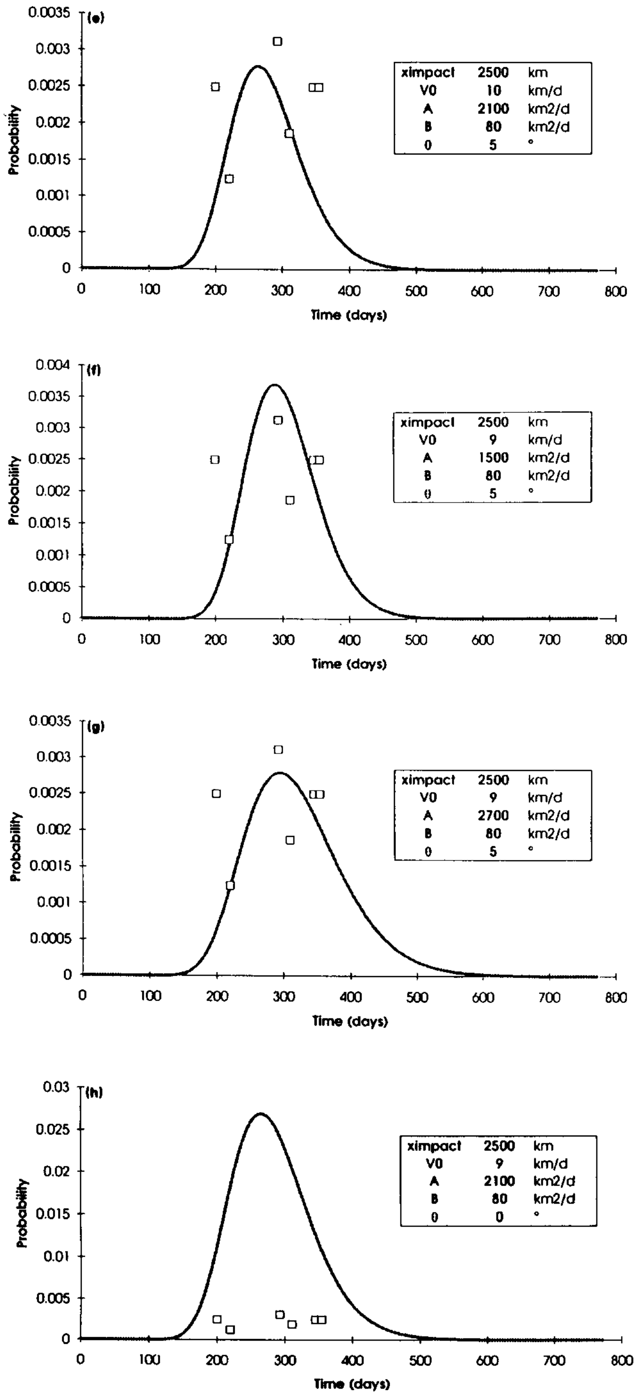

Fig. 4 (continued).

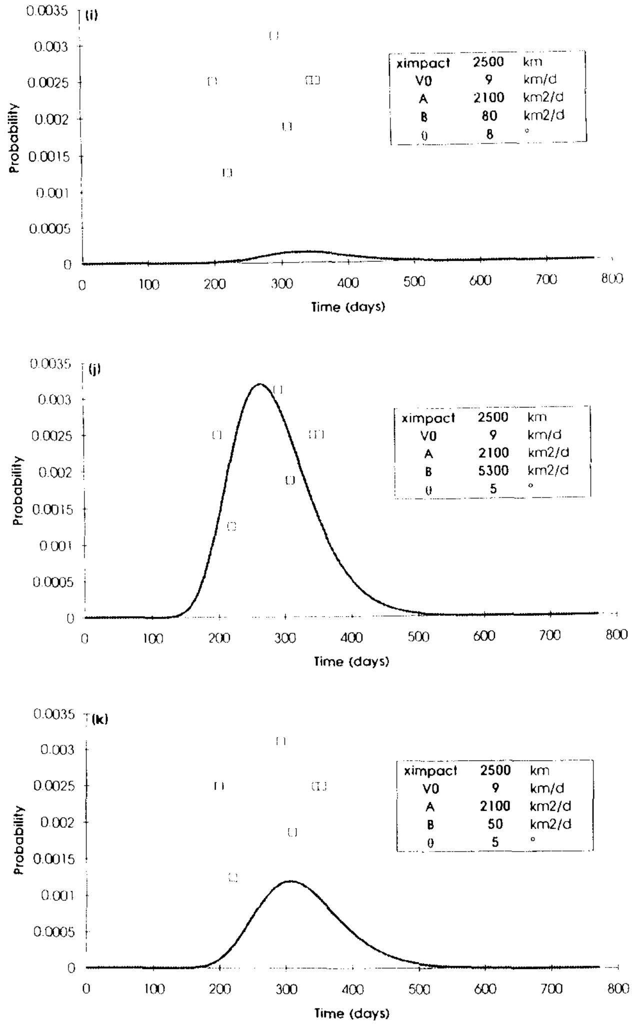

Fig. 4 (continued).

Part of the difference is undoubtedly due to the simplifying assumptions made, which, as we remarked already in the text, were done so that rough, quick estimates of relevant quantities could be made simply without requiring access to sophisticated numerical model codes. As the referees have noted, part of the difference may also be due to the difference in open ocean vs. coastal regimes which disperse drifting shoes in different ways. Indeed, Ebbesmeyer and Ingraham (1992) had already noted that: 'As the shoes were found over distances of approximately 1000 km, processes other than oceanic dispersion would account for most of the lateral shoe dispersion.' It should be noted, however, that, as stated by Ebbesmeyer and Ingraham: 'At present, OSCURS does not resolve small scales in

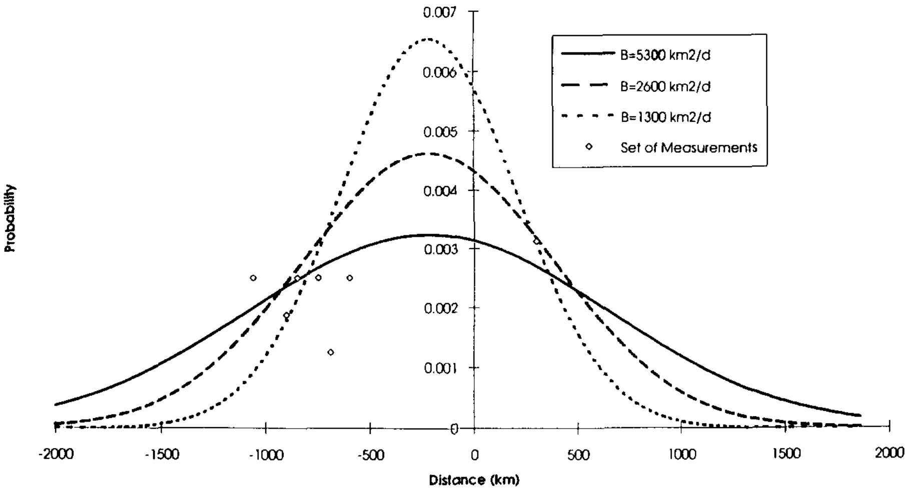

Fig. 5. Comparison of observed shoe data and predicted spatial distribution along the coast-line for various values of the parameter B while holding $x_{\text{impact}} = 2500$ km, $V_0 = 9$ km day^{-1}, $\theta = 5°$ and $A = 2100$ km^2 day^{-1}. Continuous line, $B = 5300$ km^2 day^{-1}; dashed line, $B = 1300$ km^2 day^{-1}; dot and dashed line, $B = 2600$ km^2 day^{-1}.

the near-shore, eastern coastal boundary currents. Since the shoes first reached the near-shore waters in winter, the northward-flowing Davidson Current may explain how the shoes got to the Queen Charlotte Islands. Many shoes were found along the Oregon coast through June when the local coastal currents are southerly. During the winter/spring, coastal currents apparently dispersed the shoes over a distance several-fold greater than estimated from available observations of ocean dispersion.' Taken at face value, the implication is that the observed lateral spread along the coast implies an effective lateral diffusion coefficient an order or magnitude or so greater than estimated from other observations of oceanic dispersion—as obtained in the text. However, the arrival time distribution at the coast depends also on the longitudinal diffusion coefficient as well as on both the mean flow and the mean flow extrapolated impact angle with the coast, which implies a component of bulk flow parallel to the coast. Thus, the arrival time distribution of the shoes at the coast-line would have to already be dispersed over a 100–300 day interval to allow lateral spreading to occur for about a year. Thus a high longitudinal diffusion coefficient is required.

For the values obtained (V_0 approximately 9 km day^{-1} and θ approximately $5 \pm 2°$) a steady longshore current speed of about 0.8 km day^{-1} is inferred, to within a factor of four to six of the estimated r.m.s. velocity fluctuations. Thus, it would appear that random oceanic flow effects, random coastal fluctuations and longshore coastal currents, either seasonally variable or otherwise, all provide comparable numbers to indicate that simple estimates from both arrival time and lateral coastal spread distributions are adequate to provide rough approximate values for those physical quantities which improve understanding of statistical fluctuations of the Pacific Ocean.

5. Discussion and conclusion

Although the use of a simplified statistical convective–diffusion transport model does not provide the same level of detailed information as extracted by Ebbesmeyer and Ingraham (1992) using the OSCURS code, nevertheless the model is of use in that it allows estimates to be made of velocity trend properties and fluctuation strengths of the Pacific Ocean surface without having to have access to the OSCURS code or having to have available the detailed data needed for the OSCURS code.

The simple procedures presented here allow one to infer estimates, with uncertainties, of (1) bulk flow speed $V_0 \approx 9 \pm 1$ km day^{-1}; (2) perpendicular and lateral diffusion coefficients of $A \approx B \approx 2100$–2600 km^2 day^{-1} to within about ± 800 km^2 day^{-1}, implying roughly isotropic surface velocity fluctuations and also implying $|\delta v|/V_0 \approx 0.1$–$0.6$; (3) the distance from the spill point to the mean flow impact point on the coast-line of $x_{\text{impact}} \approx 2500 \pm 300$ km; (4) the mean angle of intercept departure from perpendicular of the coast-line with the mean flow direction of $\theta \approx 5 \pm 2°$. Presumably these estimates of statistical properties of the Pacific Ocean surface are useful not only in constraining output results from the detailed OSCURS computations, but are also of use in attempts to elucidate the behavior patterns of ocean flow characteristics on the long time scale (of approximately 1 year) of the shoes' migration from the spill-point to the coast.

Acknowledgments

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