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Dynamics of a massive intruder in a homogeneously driven granular fluid

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Abstract A massive intruder in a homogeneously driven granular fluid, in dilute configurations, performs a memory-less Brownian motion with drag and temperature simply related to the average density and temperature of the fluid. At volume fraction ∼10–50% the intruder's velocity correlates with the local fluid velocity field: such situation is approximately described by a system of coupled linear Langevin equations equivalent to a generalized Brownian motion with memory. Here one may verify the breakdown of the Fluctuation-Dissipation relation and the presence of a net entropy flux - from the fluid to the intruder—whose fluctuations satisfy the Fluctuation Relation.

Keywords Granular materials · Non-equilibrium fluctuations

Granular fluids represent a valid benchmark for modern theories of non-equilibrium statistical mechanics [9]. Due to dissipative interactions among the microscopic constituents, energy is not conserved and an external source is necessary to maintain a stationary state. The consequence is a breakdown of time reversal invariance and the failure of properties such as the Equilibrium Fluctuation-Dissipation relation (EFDR) [6]. In recent years, a systematic theory for the dilute limit has been developed, in good agreement with numerical simulations [1,2], while a general understanding of dense granular fluids is still lacking. A common approach is the so-called Enskog correction [2,3], which reduces the breakdown of Molecular Chaos to a renormalization of the collision frequency. In cooling regimes, the Enskog theory may describe strong non-equilibrium effects, due to the explicit cooling time-dependence [21]. Nevertheless it cannot describe dynamical effects in stationary regimes, such as multiple characteristic times or different decays of response and autocorrelation [5,17].

Here we review a recent model [23] for the dynamics of a massive tracer moving in a gas of smaller granular particles, both coupled to an external bath. Taking as reference point the dilute limit, where the system has a closed analytical description [22], a Langevin equation linearly coupled to a fluctuating local velocity field is proposed as first approximation capable of describing the dense case. Its main features are: (i) the decay of correlation and response functions is not simply exponential and shows backscattering [14,4] and (ii) the EFDR [10,13] of the first and second kind do not hold. In such a model, detailed balance is not necessarily satisfied, and a fluctuating entropy production [24] can be measured, which fairly verifies the Fluctuation Relation [11–13].

The model reviewed here is the following: an "intruder" disc of mass $m_0 = M$ and radius R, moving in a gas of N granular discs with mass $m_i = m$ ($i > 0$) and radius r, in a two dimensional box of area $A = L^2$. We denote by $n = N/A$ the number density of the gas and by ϕ the occupied volume fraction, i.e. $\phi = \pi(Nr^2 + R^2)/A$ and we denote by $\boldsymbol{V}$ (or $\boldsymbol{v}_0$) and $\boldsymbol{v}$ (or $\boldsymbol{v}_i$ with $i > 0$) the velocity vector of the tracer and of the gas particles, respectively. Interactions among the particles are hard-core binary instantaneous inelastic collisions, such that particle i, after a

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collision with particle j, comes out with a velocity $\boldsymbol{v}_i' = \boldsymbol{v}_i - (1+\alpha)\frac{m_j}{m_i+m_j}[(\boldsymbol{v}_i - \boldsymbol{v}_j)\cdot\hat{\boldsymbol{n}}]\,\hat{\boldsymbol{n}}$ where $\hat{\boldsymbol{n}}$ is the unit vector joining the particles' centers of mass and $\alpha \in [0, 1]$ is the restitution coefficient ($\alpha = 1$ is the elastic case). The mean free path of the intruder is proportional to $l_0 = 1/(n(r+R))$ and we denote by τ_c its mean collision time. Two kinetic temperatures can be introduced for the two species: the gas temperature $T_g = m\langle \boldsymbol{v}^2\rangle/2$ and the tracer temperature $T_{tr} = M\langle \boldsymbol{V}^2\rangle/2$.

The equation of motion of the ith particle reads: $m_i\dot{v}_i(t) = -\gamma_b v_i(t) + f_i(t) + \xi_b(t)$. Here $f_i(t)$ is the force taking into account the collisions of particle i with other particles, and $\xi_b(t)$ is a white noise (different for all particles), with $\langle\xi_b(t)\rangle = 0$ and $\langle\xi_b(t)\xi_b(t')\rangle = 2T_b\gamma_b\delta(t-t')$. The effect of the external energy source balances the energy lost in the collisions and a stationary state is attained with $m_i\langle v_i^2\rangle \leq T_b$ [8,25,15,18,27].

At low packing fractions, $\phi < 0.1$, and in the large mass limit, $m/M \ll 1$, using the Enskog approximation it has been shown [22] that the dynamics of the intruder is described by a linear Langevin equation:

$$M\dot{V} = -\Gamma_E V + \mathcal{E}_E, \tag{1}$$

with $\mathcal{E}_E$ a white noise with $\langle\mathcal{E}_E\rangle = 0, \langle\mathcal{E}_E(t)\mathcal{E}_E(t')\rangle = 2\delta(t-t')\Gamma_E T_{tr}^E$ and $T_{tr}^E = (\gamma_b T_b + \gamma_g^E\frac{1+\alpha}{2}T_g)/\Gamma_E$ is the tracer's temperature. In this limit the velocity autocorrelation function shows a simple exponential decay, with characteristic time M/Γ_E, where $\Gamma_E = \gamma_b + \gamma_g^E$ and $\gamma_g^E = \frac{g_2(r+R)}{l_0}\sqrt{2\pi m T_g}(1+\alpha)$ where $g_2(r+R)$ is the pair correlation function for a gas particle and the intruder at contact. Time-reversal and the EFDR, weakly modified for uniform dilute granular gases [5,16,20], become perfectly satisfied for a massive intruder.

As the packing fraction is increased, the Enskog approximation fails in predicting dynamical properties. In particular, velocity autocorrelation $C(t) = \langle V(t)V(0)\rangle/\langle V^2\rangle$ and linear response function $R(t) = \overline{\delta V(t)}/\delta V(0)$ show an exponential decay modulated by oscillating functions [4,23]. Moreover violations of the EFDR $C(t) = R(t)$ are observed for $\alpha < 1$ [17,26]. The Enskog approximation is unable to explain the observed functional forms, because it only modifies by a constant factor the collision frequency [2,22]: a model with more than one characteristic time is needed. A first approximation is given by an auxiliary field coupled to the intruder's velocity:

$$\begin{aligned} M\dot{V} &= -\Gamma_E(V-U) + \sqrt{2\Gamma_E T_g}\mathcal{E}_V \\ M'\dot{U} &= -\Gamma' U - \Gamma_E V + \sqrt{2\Gamma' T_b}\mathcal{E}_U, \end{aligned} \tag{2}$$

where $\mathcal{E}_V$ and $\mathcal{E}_U$ are white noises of unitary variance. Two new parameters appear: the mass of the local field M' and its drag coefficient Γ'. The dilute limit here is obtained for $\Gamma' \sim M' \to \infty$. In such a limit indeed $U \to 0$ and the equation for V comes back in the form discussed above [22]. In such a form (2), the dynamics of the tracer is remarkably simple: indeed V follows a Langevin equation in a *Lagrangian frame* with respect to a field U, which is the *local average velocity field* of the gas particles colliding with the tracer. A first justification of this model comes from realizing [23] that it is equivalent to a Generalized Langevin Equation with exponential memory, which is consistent with a typical approximation done for Brownian Motion when, at high densities, the coupling of the intruder with fluid hydrodynamic modes, decaying exponentially in time (see [28], Cap. 8.6 and 9.1), must be taken into account. Such a coupling, which in principle involves a continuum of modes, is reduced here to a single dominant mode: this is sufficient to introduce a new non-trivial timescale. The full coupling would reproduce finer features which become relevant at larger densities or larger times, such as long-time power-law tails. The fact that the "temperature" of the local velocity field U is equal to the bath temperature T_b comes as a consequence of the conservation of momentum in collisions, implying that the average velocity of a group of particles is not changed by collisions among themselves and is only affected by the external bath and a (small) number of collisions with outside particles. This scenario is fully consistent with recent study of hydrodynamic fluctuations for the velocity field of the same fluid model [7,8].

A stronger justification comes, however, from its effectiveness in reproducing the numerical results, as detailed in [23]. From the simulations it is seen that the relaxation time of the local field $\tau_U = M'/\Gamma'$, rescaled by the mean collision time, increases with the packing fraction and with the inelasticity, as expected. At high densities it appears that $\Gamma' \sim 1/\phi$, and $T_{tr} \sim T_g \sim T_g^E$, likely due to stronger correlations among particles. At large ϕ we observe $T_{tr} > T_{tr}^E$, consistent with a *smaller* dissipation for correlated collisions. Model (2) predicts $C = f_C(t)$ and $R = f_R(t)$ with

$$f_{C(R)} = e^{-gt}[\cos(\omega t) + a_{C(R)}\sin(\omega t)]. \tag{3}$$

The variables g, ω, a_C and a_R are known algebraic functions of $\Gamma_E, T_g, \Gamma', M'$ and T_b. In particular, the ratio $a_C/a_R = [T_g - \Omega(T_b - T_g)]/[T_g + \Omega(T_b - T_g)]$, with $\Omega = \Gamma_E/((\Gamma' + \Gamma_E)(\Gamma_E M'/M - \Gamma'))$. Hence, in the elastic ($T_g \to T_b$) as well as in the dilute limit ($\Gamma' \to \infty$), one gets $a_C = a_R$ and recovers the EFDR $C(t) = R(t)$. Such predictions are all verified in numerical simulations [23]. In particular Fig. 1 depicts correlation and response functions in a dense case (elastic and inelastic): symbols correspond to numerical data and continuous lines the analytical curves. In the inelastic case, deviations from EFDR $R(t) = C(t)$ are observed. In the inset of Fig. 1 the ratio $R(t)/C(t)$ is also reported. It is important to notice that the main

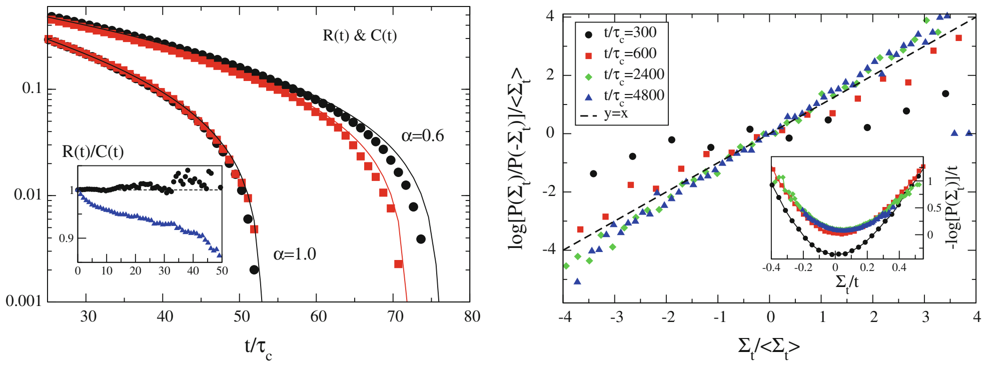

Fig. 1 (Color online) *Left* correlation function $C(t)$ (*black circles*) and response function $R(t)$ (*red squares*) for $\alpha = 1$ and $\alpha = 0.6$, at $\phi = 0.33$. *Continuous lines* show curves obtained with Eq. (3). *Inset* the ratio $R(t)/C(t)$ is reported in the same cases. *Right* check of the Fluctuation Relation (5) in the system with $\alpha = 0.6$ and $\phi = 0.33$. *Inset* collapse of the rescaled probability distributions of Σ_t at large times onto the large deviation function

responsibility for the breakdown of the EFDR is the coupling between V and U, indeed Eq. (3) can be expressed in a different way: $R(t) = aC(t) + b\langle V(t)U(0)\rangle$ with $a = [1 - (T_g - T_b)\Omega_a/\Gamma']$ and $b = (T_g - T_b)\Omega_b$, where Ω_a and Ω_b are known functions of the parameters. At equilibrium or in the dilute limit the EFDR is recovered.

An important independent assessment of model (2) comes from the study of the fluctuating entropy production [24] which quantifies the deviation from detailed balance in a trajectory. Given the trajectory in the time interval $[0, t]$, $\{V(s)\}_0^t$, and its time-reversed $\{\mathcal{I}V(s)\}_0^t \equiv \{-V(t-s)\}_0^t$, the entropy production for our model takes the form [19]

$$\Sigma_t = \log \frac{P(\{V(s)\}_0^t)}{P(\{\mathcal{I}V(s)\}_0^t)} \approx \Gamma_E \left(\frac{1}{T_g} - \frac{1}{T_b}\right) \int_0^t ds\ V(s)U(s). \tag{4}$$

This functional vanishes exactly in the elastic case, $\alpha = 1$, where equipartition holds, $T_g = T_b$, and is zero on average in the dilute limit, where $\langle VU\rangle = 0$. Formula (4) reveals that the leading source of entropy production is the energy transferred by the "force" $\Gamma_E U$ on the tracer, weighed by the difference between the inverse temperatures of the two "thermostats". Therefore, to measure entropy production, we need to measure the fluctuations of U: a possible choice is a local average of particles' velocities in a circle of radius $l + R$ centered on the tracer. Details on how to choose in a reliable way the correct l are given in [23]. Following such procedure, in the case $\phi = 0.33$ and $\alpha = 0.6$, we estimate for the correlation length $l \sim 9r \sim 6l_0$. Then, measuring the entropy production from Eq. (4) along many trajectories of length t, we computed the probability $P(\Sigma_t = x)$ and compared it to $P(\Sigma_t = -x)$, in order to verify the Fluctuation Relation [11–13]

$$\log \frac{P(\Sigma_t = x)}{P(\Sigma_t = -x)} = x. \tag{5}$$

In the right frame of Fig. 1 the results of this comparison are reported. The main frame confirms that at large times the Fluctuation Relation (5) is well verified within the statistical errors. The inset shows the collapse of $\log P(\Sigma_t)/t$ onto the large deviation rate function for large times. Notice that—in formula (4) - a wrong evaluation of the weighing factor $(1/T_g - 1/T_b)$ or of the "energy injection rate" $\Gamma_E U(t)V(t)$ in Eq. (4) could produce a completely different slope in Fig. 1 (right frame).

To conclude this paper, we stress that velocity correlations $\langle V(t)U(t')\rangle$ between the intruder and the surrounding velocity field are responsible for both the violations of the EFDR and the appearance of a non-zero entropy production, provided that the two fields are *at different temperatures*. We also mention that larger violations of EFDR can be observed using an intruder with a mass equal or similar to that of other particles [17], with the important difference that in such a case a simple "Langevin-like" model for the intruder's dynamics is not available.

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