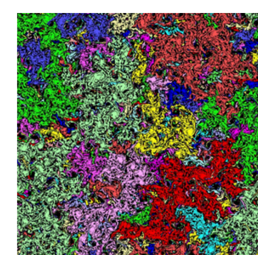
FloMat2015， Rome
(3.24)

Helicity/chirality in 2D passive-scalar/2D3C turbulence

Zhu, Jian-Zhou

Su-Cheng Centre for Fundamental and Interdisciplinary Sciences, Gaochun, Nanjing, China (@SCCFIS.org)



Reflection on statistical/probability description

- Covariance/correlation – (not) linear dependent – dynamical (in)dependence
- Passive correlation/dependence
- ……


hydrochirality

$$\partial_t \boldsymbol{u} = \boldsymbol{u} \times (\nabla \times \boldsymbol{u}) - \nabla P + \nabla^2 \boldsymbol{u}/Re \text{ and } \nabla \cdot \boldsymbol{u} = 0$$

3D with \partial_z=0, \kappa=\nu and \theta=u_z

$$\partial_t \theta + \boldsymbol{v} \cdot \nabla \theta = \kappa \nabla^2 \theta + f_\theta,$$

2-Dimension-**3-C**omponent

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla P + \nu \nabla^2 \boldsymbol{v} + f_{\boldsymbol{v}}, \ \nabla \cdot \boldsymbol{v} = 0$$

helicity 3D→2D3C under appropriate b.c. $C = 2\langle \omega \theta \rangle = \frac{2}{\mathcal{V}} \int \omega \theta d\boldsymbol{r}$

[reminiscent of the "vertical helicity" used in the studies of atmosphere sciences (all kinds of "cyclones" and "sand/duststorms")]

Schematic diagram of 2D3C flow: spinning skaters with \partial_z=0
(relevant to rotating flows: Ro→0)

2D3Craichnan chiral absolute equilibria

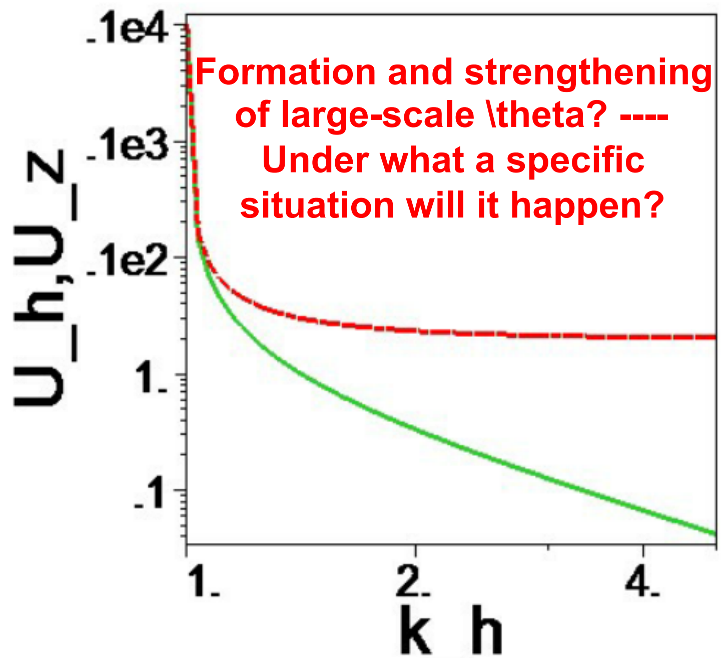
Kraichnan 67' :

In two dimensions, the absolute equilibrium has a more complicated structure because there are two linearly independent quadratic constants of motion. The general form of the equilibrium spectrum is

$$U(k) = 1/(\beta k^2 + \alpha), \tag{3.1}$$

where β and α are constants. This is an equipartition distribution[12] for the constant of motion

$$\sum_{\mathbf{k}} (\beta k^2 + \alpha)\ |\mathbf{u}(\mathbf{k})|^2.$$



pure 2D→2D3C

Introducing corresponding Lagrange multipliers or the (inverse) temperature parameters $\Gamma_\bullet$, we now apply the Gibbs distribution $\sim \exp\{-(\Gamma_{\mathcal{C}}\mathcal{C} + \Gamma_{\mathcal{E}}\mathcal{E} + \Gamma_{\mathcal{W}}\mathcal{W} + \Gamma_{\mathcal{Z}}\mathcal{Z})\}$ to obtain the modal spectral densities, U_h of $\mathcal{E}$, W of $\mathcal{W}$, $Q_{\mathcal{C}}$ of $\mathcal{C}$ and U_z of $\mathcal{Z}$:

with caveats: ergodicity, 'physical' measure, back reaction, equivalent ensembles, 'counter example' …But the **Potential** might present in some specific situations

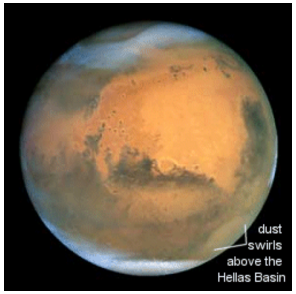
no effective constraint on u_z

$$\text{with } D = \Gamma_{\mathcal{E}}\Gamma_{\mathcal{Z}} + (\Gamma_{\mathcal{W}}\Gamma_{\mathcal{Z}} - \Gamma_{\mathcal{C}}^2)k_h^2 > 0,\ U_h \triangleq \langle|\hat{u}_h|^2\rangle = \frac{\Gamma_{\mathcal{Z}}}{D},\ W = k_h^2 U_h, \tag{7}$$

$$Q_{\mathcal{C}} \triangleq \langle \hat{i}k_h\hat{u}_h\hat{u}_z^*\rangle + c.c. = \langle\hat{\zeta}\hat{\theta}^*\rangle + c.c. = \frac{-2\Gamma_{\mathcal{C}}k_h^2}{D},\ U_z \triangleq \langle|\hat{\theta}|^2\rangle = \frac{1}{\Gamma_{\mathcal{Z}}} + \frac{\Gamma_{\mathcal{C}}^2 k_h^2}{\Gamma_{\mathcal{Z}} D}. \tag{8}$$

Possible relevance of 2D3C chiral turbulence with hazardous weather

- Various cyclones (above the rotating planet – 转球之上)



- Sand and/or dust storms

- smog/haze (under the dome – 穹顶之下)

(Wild) speculation: 2D3C dominated helical cyclogenesis/storm or smog formation?

Modified chiral Kraichnan model: trying for systematic analyses

$\boldsymbol{v}(\boldsymbol{x}, t)$ is Gaussian white in time

. . .

stirring of the velocity through the latter, so $\langle f_\omega \theta \rangle$ is not easily controllable (though might be small); but, we may still take the velocity be a synthetic delta-correlated Gaussian field and let f_θ be linearly correlated to ω, thus still Gaussian, to inject C - though how $\langle f_\omega \theta \rangle$ is changed by such a particular forcing is unknown, we might assume the change should be little by the independence between the pumping and forcing. Anyhow, among various uncertainties, it still appears that we may try to control

. . .

$$\partial_t \theta_1 \theta_2 + (\boldsymbol{v}_1 \cdot \nabla_1 + \boldsymbol{v}_2 \cdot \nabla_2) \theta_1 \theta_2 = \kappa (\nabla_1^2 + \nabla_2^2) \theta_1 \theta_2 + f_1 \theta_2 + f_2 \theta_1$$

Among other things, to obtain $\langle \theta_1 \theta_2 \rangle$ from the above equation, we need to evaluate in the r.h.s. $\langle f_1 \theta_2 + f_2 \theta_1 \rangle$, by Gaussian integration by parts.

: undefined ***functional differentiation***

and

undefined ***integration***:

$$C_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t) = \langle \theta_1 \theta_2 \rangle = \left\langle \int_{-\infty}^{t} f[\boldsymbol{R}(s_1), s_1; \boldsymbol{r_1}, t] ds_1 \int_{-\infty}^{t} f[\boldsymbol{R}(s_2), s_2; \boldsymbol{r_2}, t] ds_2 \right\rangle$$

(possible) non-universality depending on the properties

of the gauge field $\boldsymbol{g}(\boldsymbol{x}, t)$ with $\nabla \times \boldsymbol{g} = 0$.