Turbulence development and Lagrangian scale of 2D turbulece

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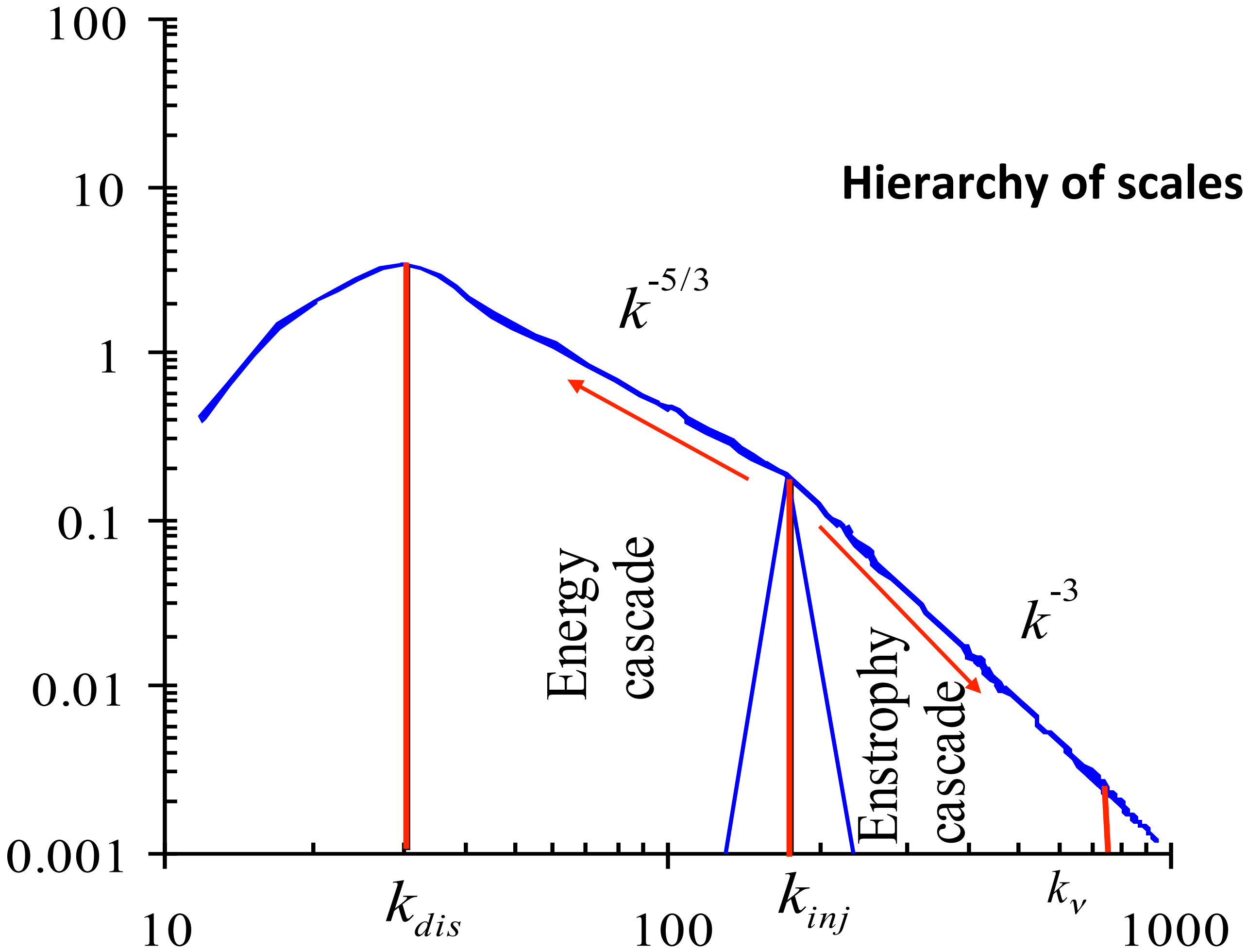
2D turbulence

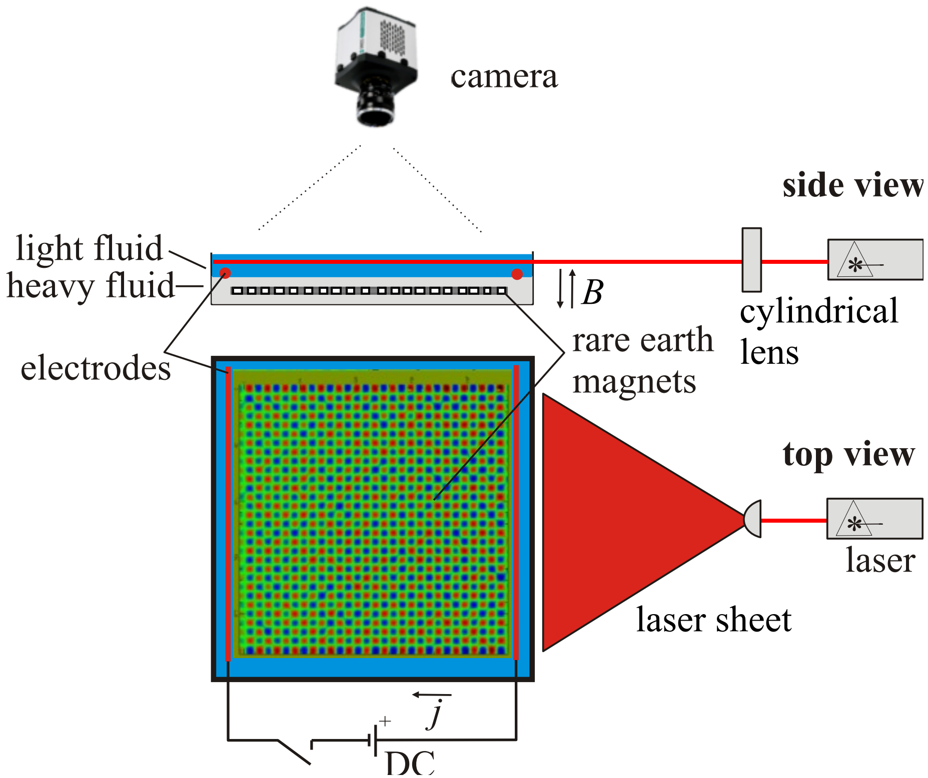


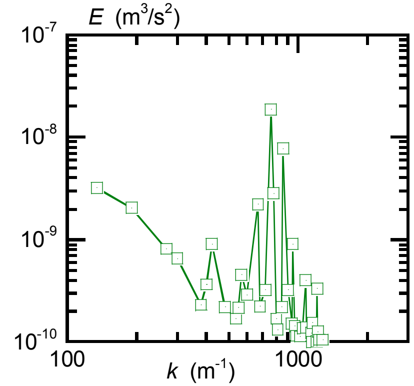
Hierarchy of scales

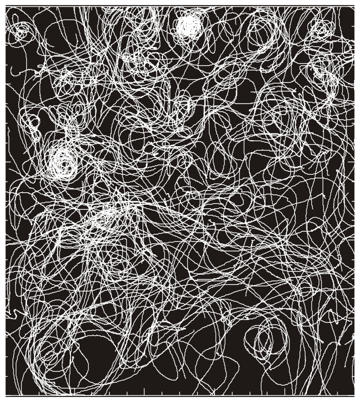
Which scales determine particle dispersion?

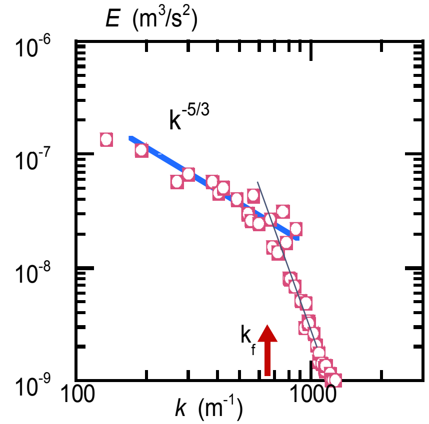
Turbulence generation

Electromagnetically driven turbulence (EMT)







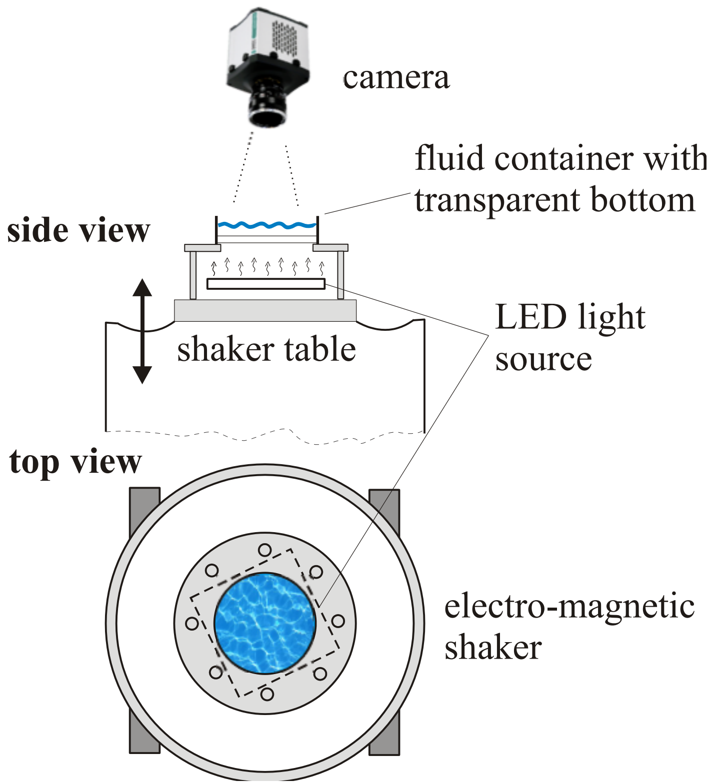


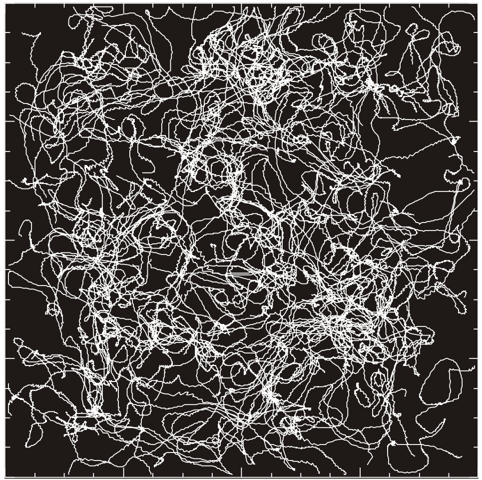
Forcing: current density

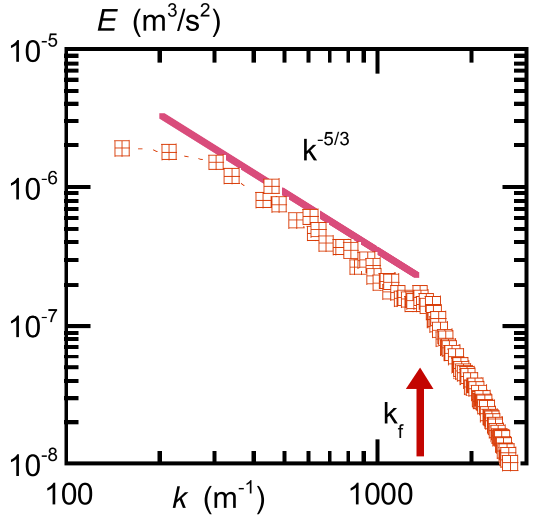
[Shats et al. PRE(2005), Xia et al. PRL(2008), Xia et al Phys. Fluids (2009)]

Turbulence generation

Faraday wave driven turbulence (FWT)







Forcing => Vertical acceleration

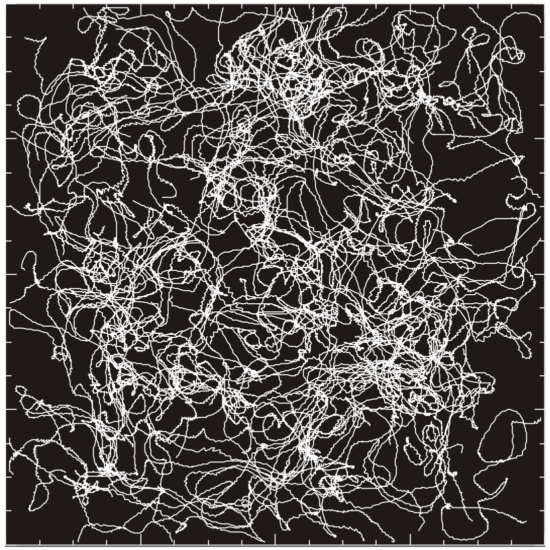
Forcing scale=> Shaker frequency

[Von Kameke et al., PRL(2011), Francois et al. , PRL(2013)]

Single particle dispersion

Particle tracking:

- Direct tracking, nearest neighbour
- Long trajectories (t > 10 T_L)
- Large number of trajectories

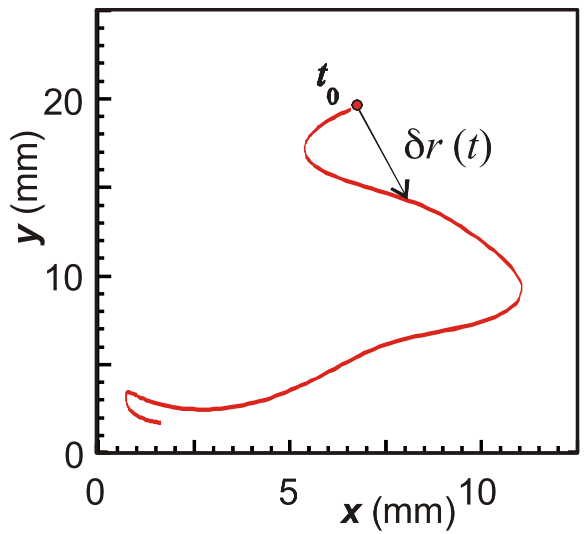


Taylor (1921):

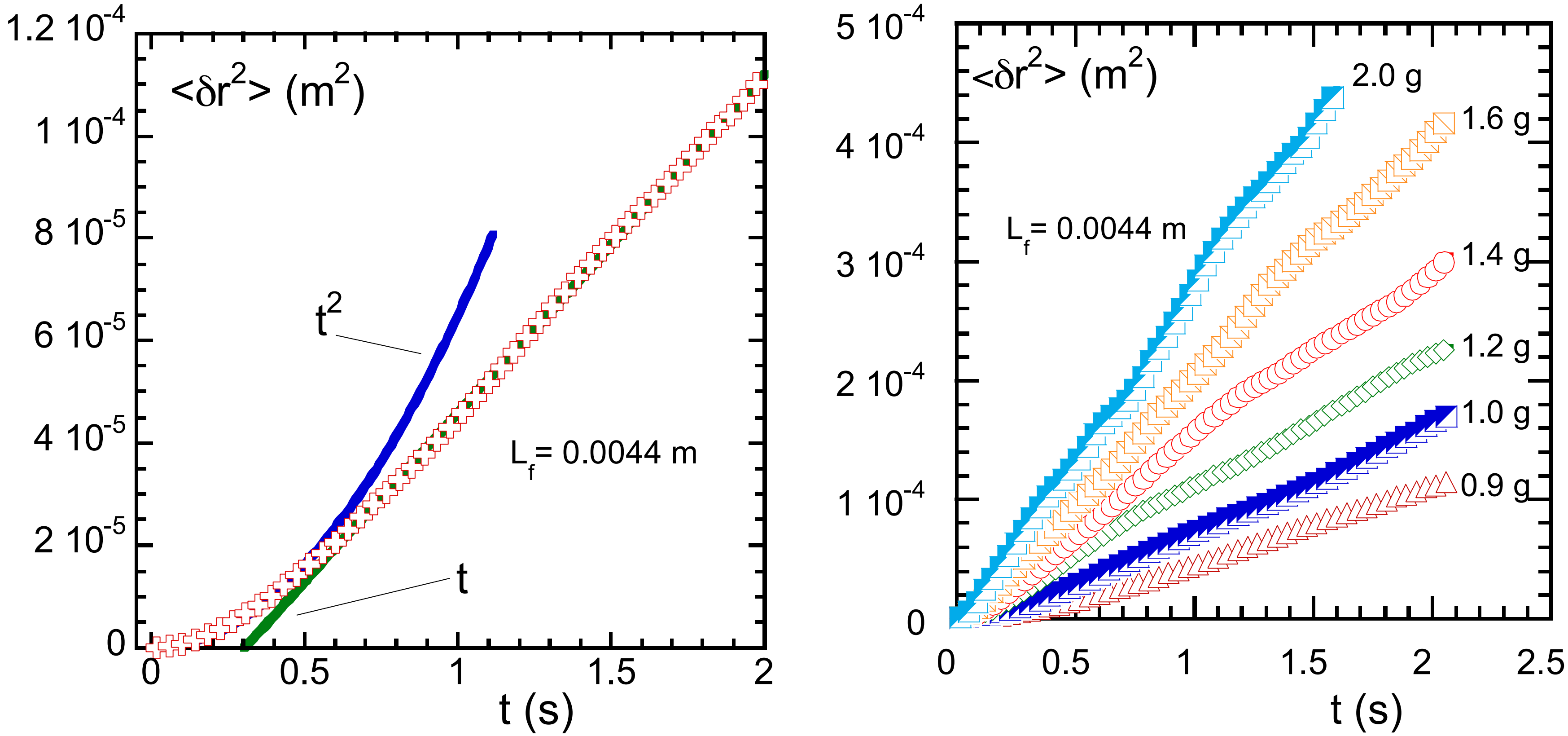
$$\langle \delta r^2 \rangle \approx \langle u^2 \rangle t^2 \qquad t << T_L$$
$$\langle \delta r^2 \rangle \approx 2 \langle u^2 \rangle T_L t \qquad t >> T_L$$

$$\rho(t) = \langle u[\vec{r}(t_0 + t)] u[\vec{r}(t_0)] \rangle / \sigma^2$$

$$T_L = \int_0^\infty \rho(t) dt$$

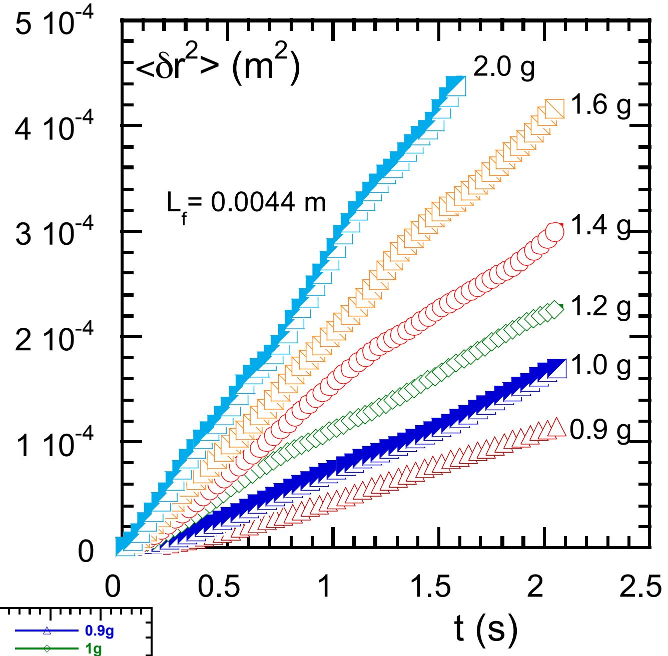


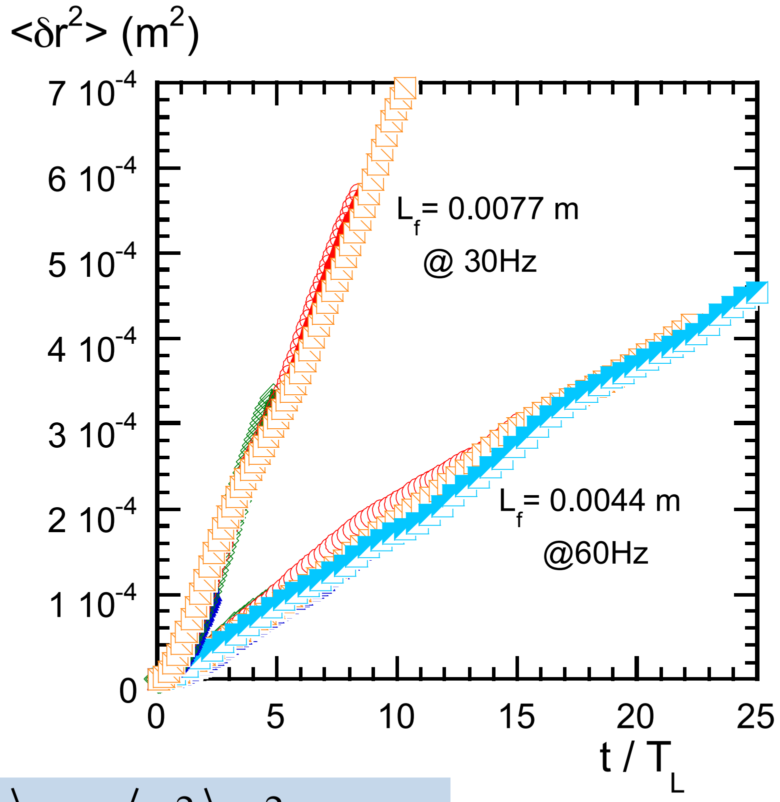
Mean square displacement (MSD)

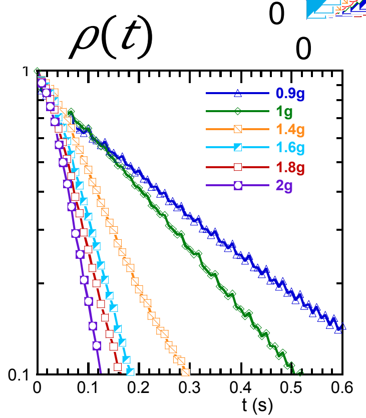


Taylor's single particle dispersion observed in a broad range of experimental conditions.

Universal spatial scale







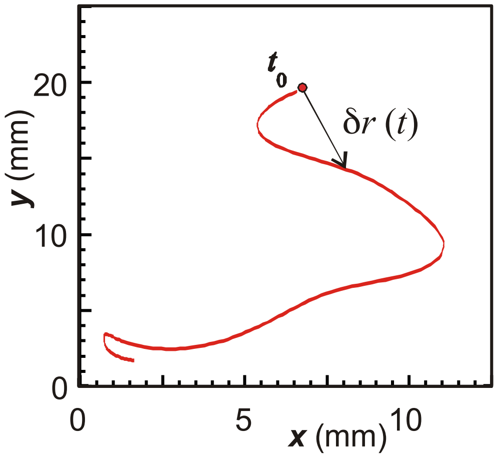
$$T_L = \int_0^\infty \rho(t)dt$$

$$\left\langle \delta r^2 \right\rangle \approx 2\left\langle u^2 \right\rangle T_L^2 (t / T_L)$$

$$\tilde{u}^2 T_L^2 \sim L^2$$

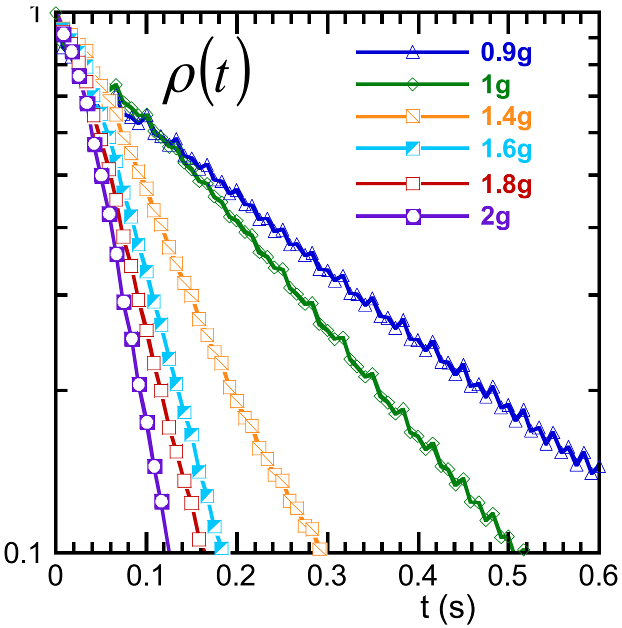
- **a universal spatial scale of particle dispersion**
- **related to the forcing scale.**

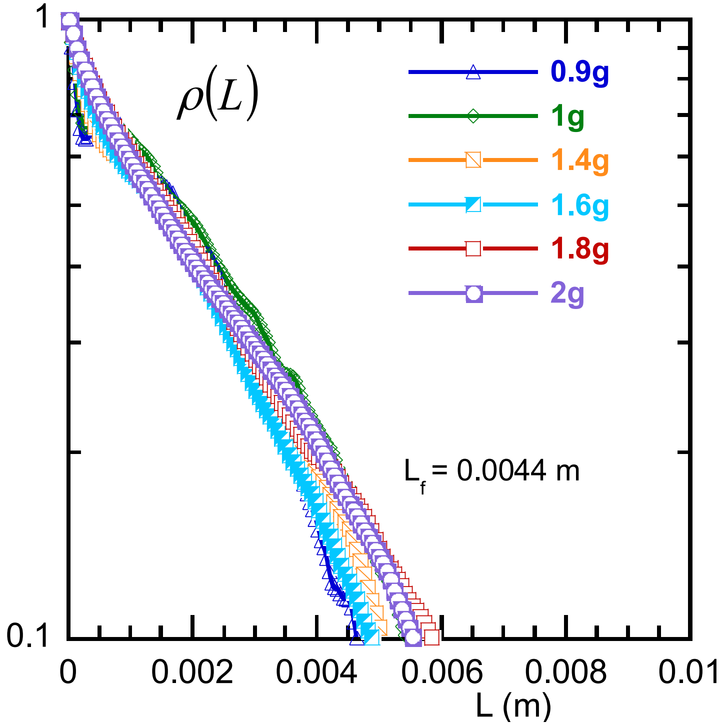
Computation of the Lagrangian scale



$$\left.\begin{matrix} u(t) \\ \delta r(t) \end{matrix}\right\} \Longrightarrow u(\delta r)$$

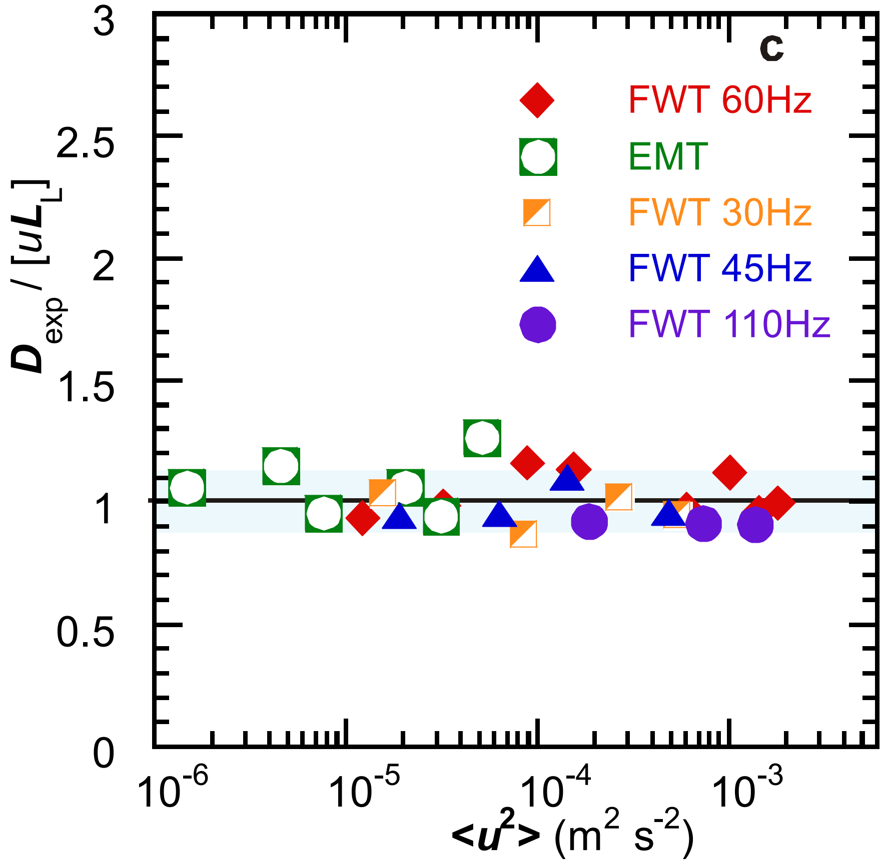
$$\rho(L) = \langle \vec{u}(r_0 + L)\vec{u}(r_0) \rangle / \sigma^2$$





$$L_L = \int_0^\infty \rho(L)\,dL$$

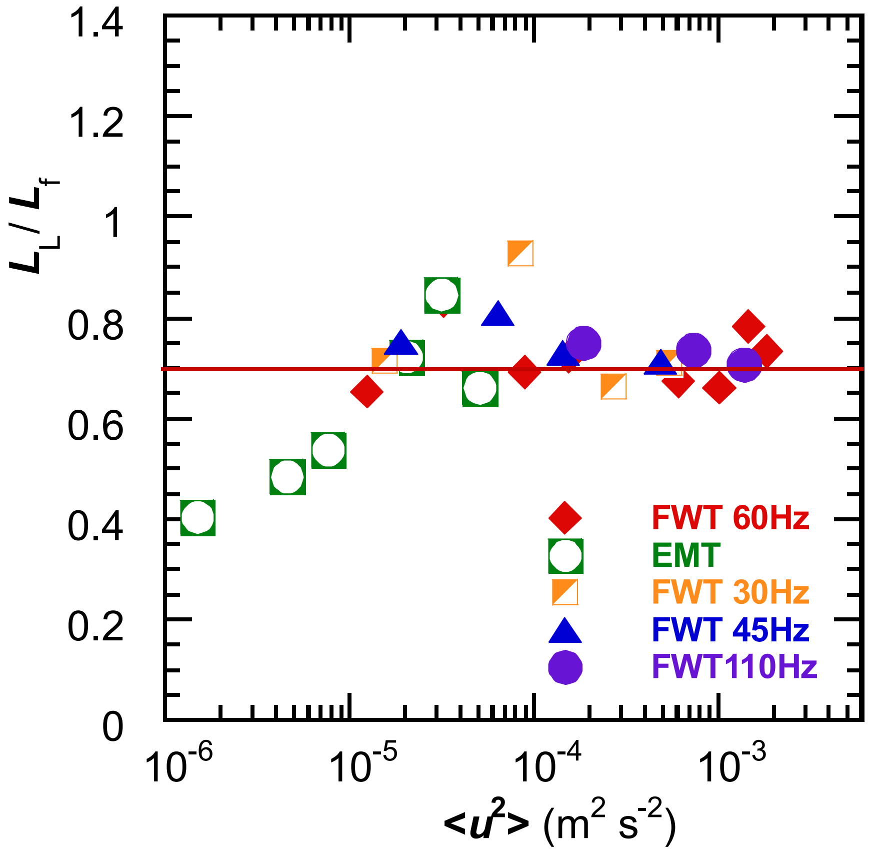
Lagrangian scale



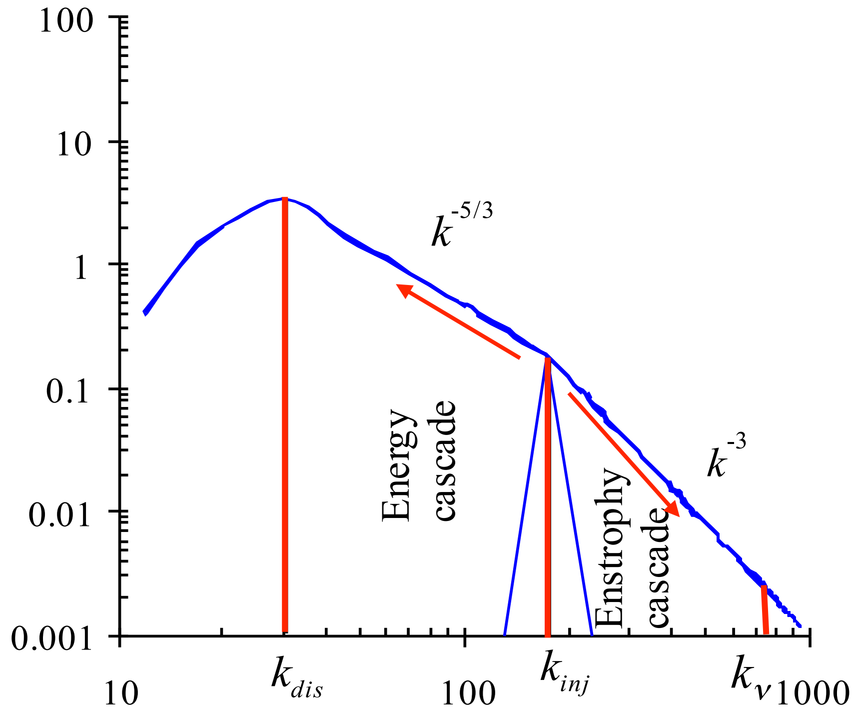
$$D_{\exp} = (\tilde{u}_{rms} L_L)$$

- Particle dispersion is determined by a single scale;
- Turbulence just affects u_{rms}

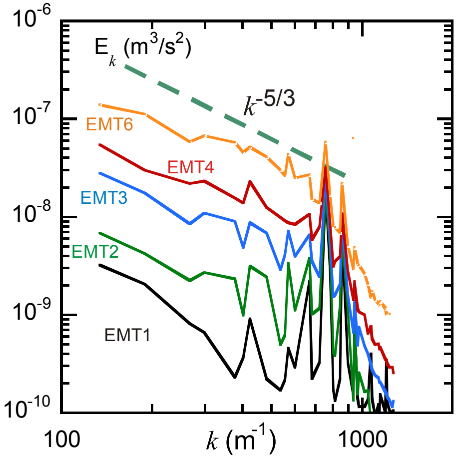
Lagrangian scale and forcing scale

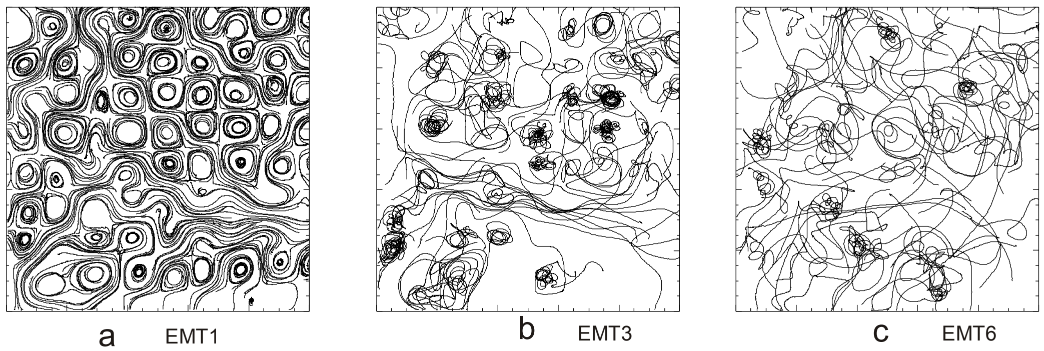


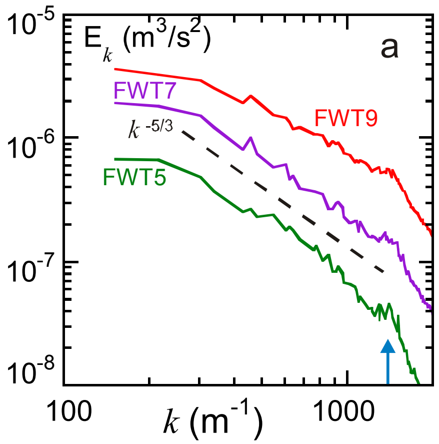
$$L_L \sim 0.7 L_f$$

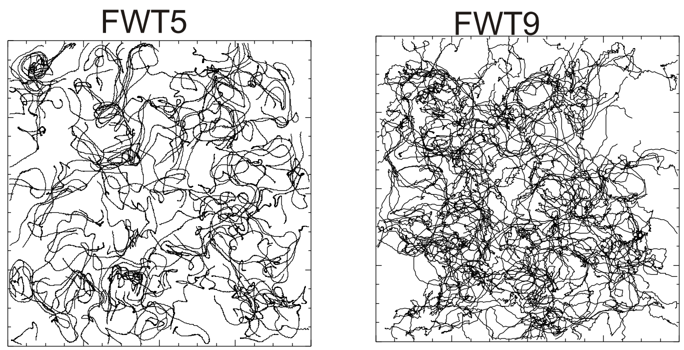


Turbulence development





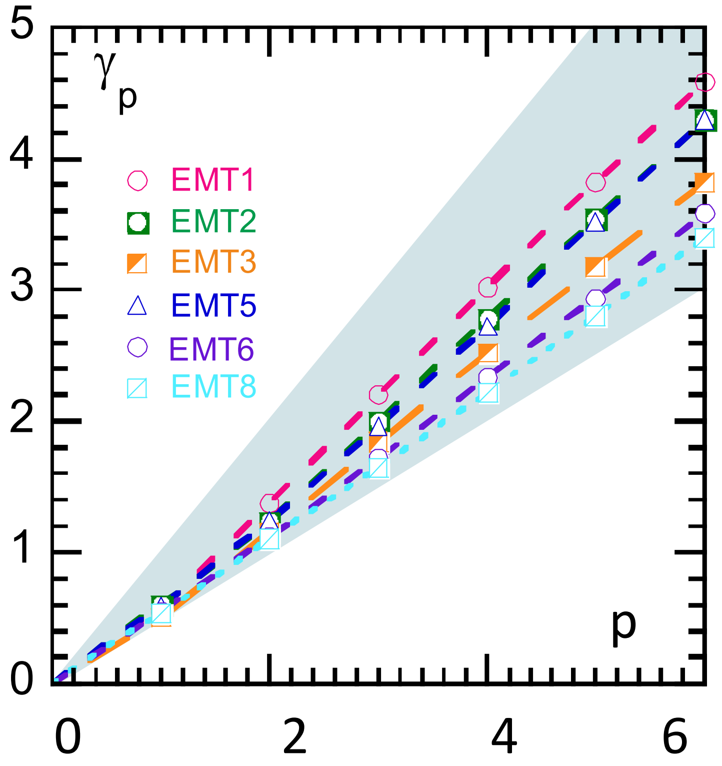




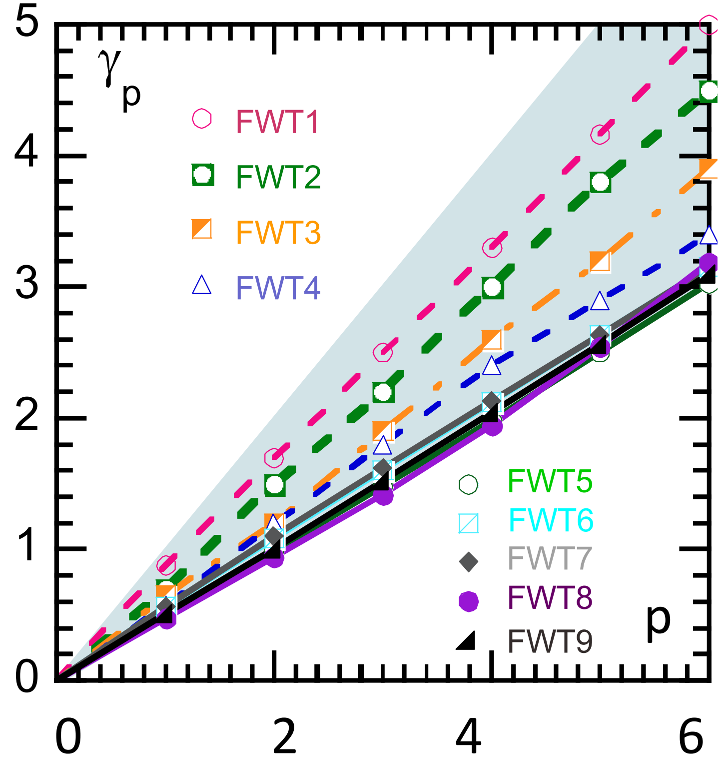
Higher order moments of δr

$$\left\langle \delta r^{p} \right\rangle \sim t^{\gamma_p}$$

EMT: Lower *Re*



FWT: low to High Re

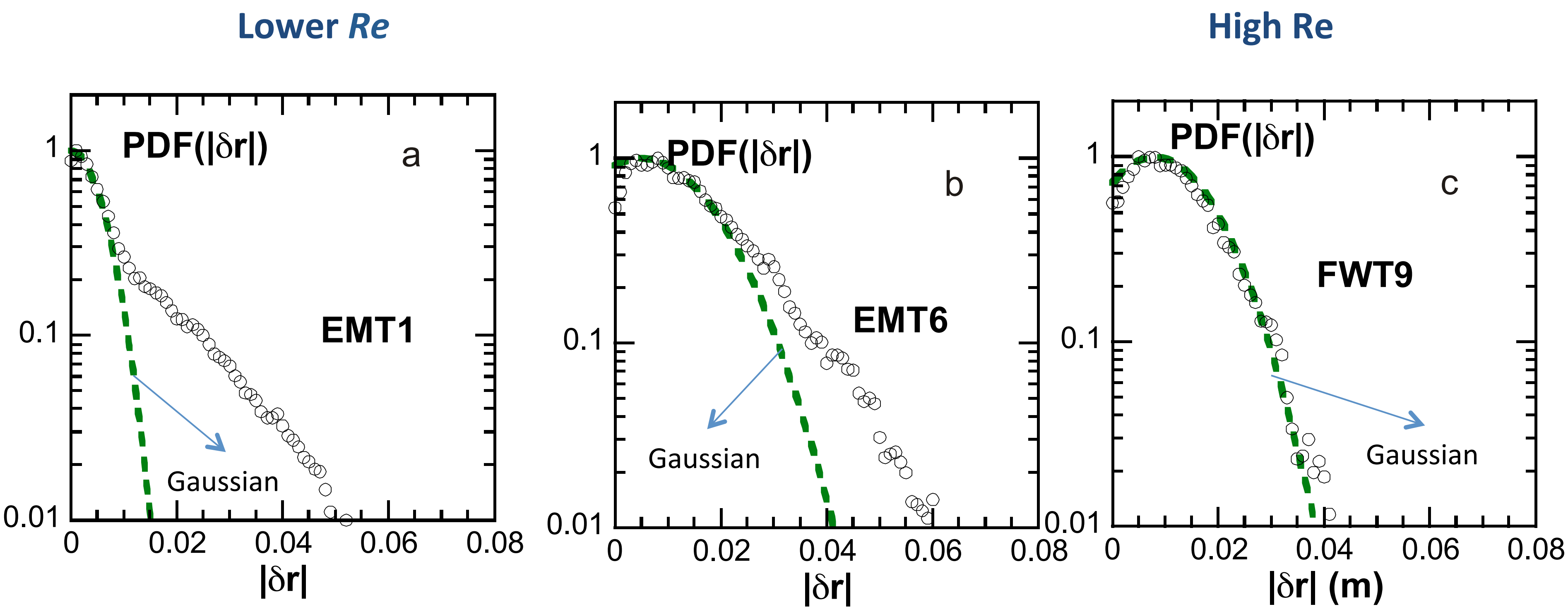


Non-self-similar $\gamma_p \sim nonlinear(p)$

$\gamma_p \sim p \neq p/2$

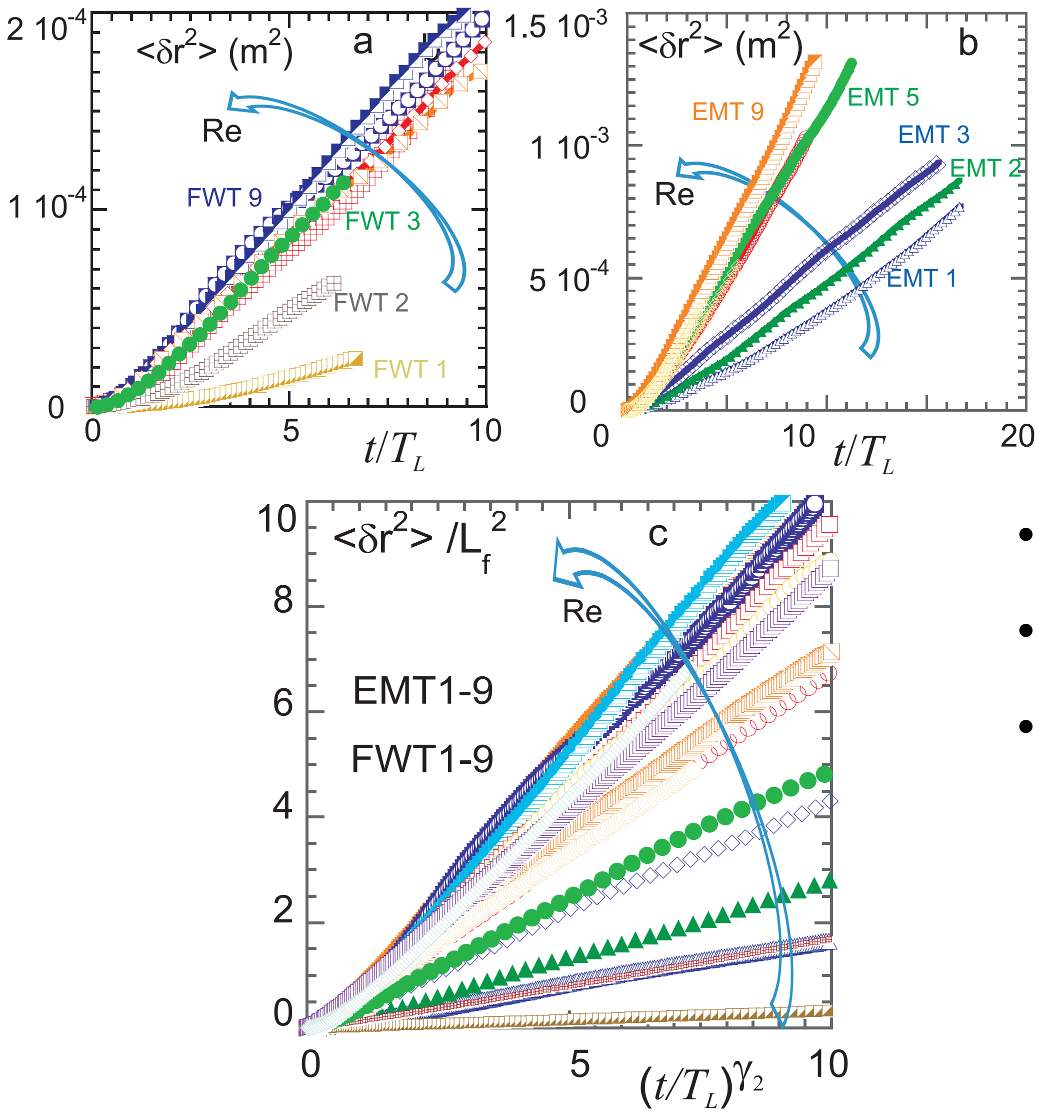
Diffusive, self-similar $\gamma_p = p/2$

PDF of δr



Lower *Re*
High Re
PDF(|δr|)
a
EMT1
Gaussian
1
0.1
0.01
0
0.02
0.04
0.06
0.08
|δr|
PDF(|δr|)
b
EMT6
Gaussian
|δr|
PDF(|δr|)
c
FWT9
Gaussian
|δr| (m)

Dispersion: transition to turbulence



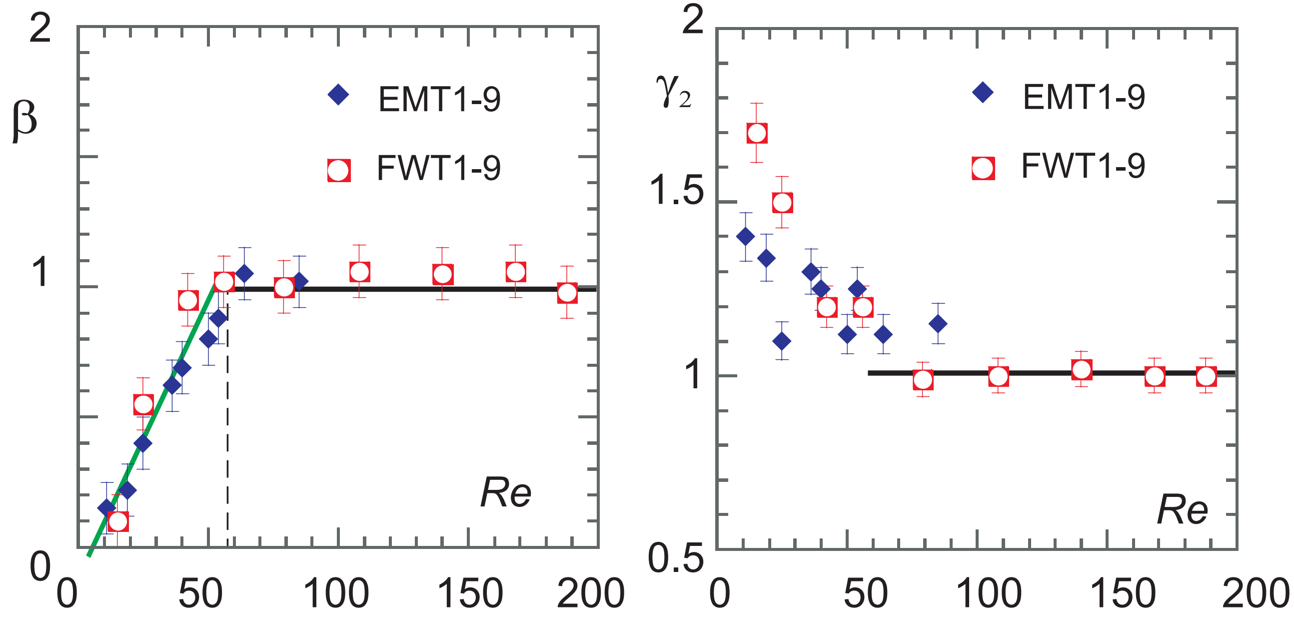
Transition from superdiffusion to diffusion with Re.

$$\left\langle \delta r^2 \right\rangle \sim t^{\gamma_2}$$

- **All curves are reasonably linear,**
- **Slopes increase with Re,**
- **Convergence at high *Re*.**

$$\frac{\left\langle \delta r^2 \right\rangle}{L_f^2} = \beta\left(\frac{t}{T_L}\right)^{\gamma_2}, \quad t > T_L$$

Taylor's particle dispersion



$$\frac{\langle \delta r^2 \rangle}{L_f^2} = \beta \left(\frac{t}{T_L} \right)^{\gamma_2}$$

$$\beta = \mathrm{Re}/\mathrm{Re}_t, \quad Re < \mathrm{Re}_t$$

$$= 1. \quad Re > \mathrm{Re}_t$$

$$\langle \delta r^2 \rangle \sim t^{\gamma_2}$$

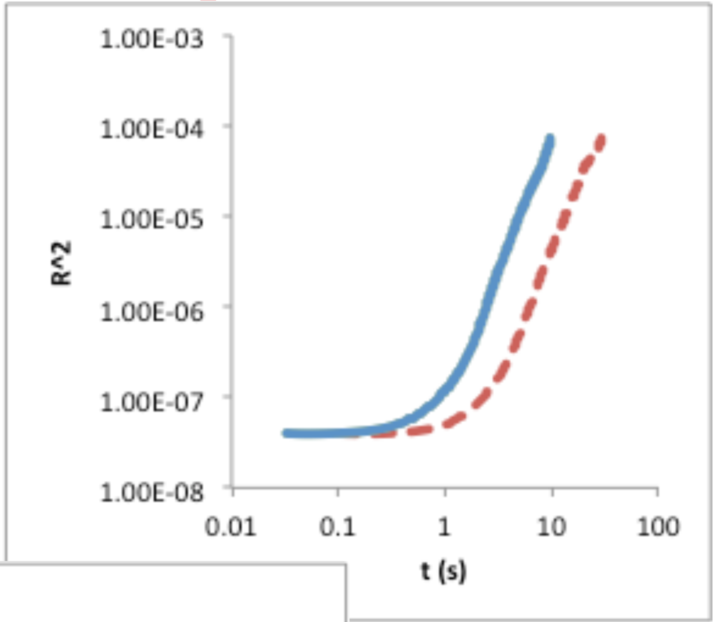
$$\langle \delta r^2 \rangle \approx 2 \langle u^2 \rangle T_L t$$

Quantitative expression for particle dispersion in developed turbulence to chaotic flows

Pair dispersion: preliminary results

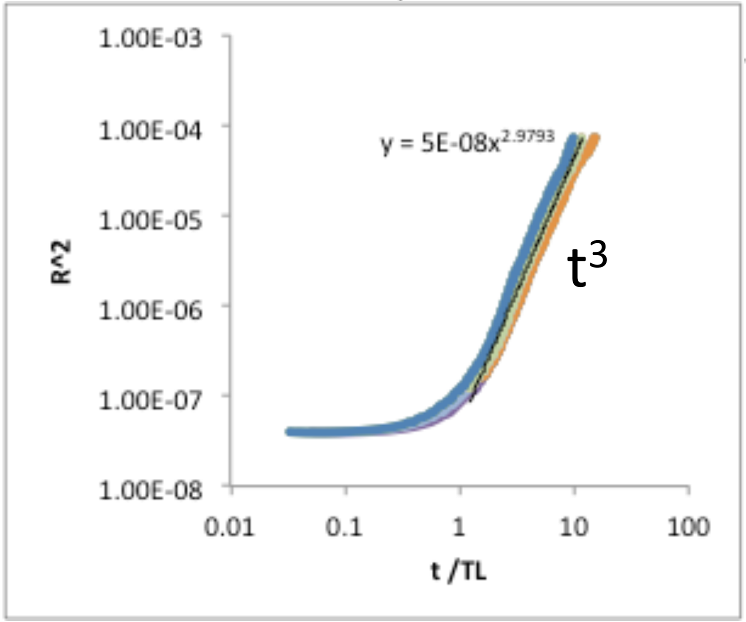
Particle tracking:

- Numeric integration using PIV
- Initial zero velocity

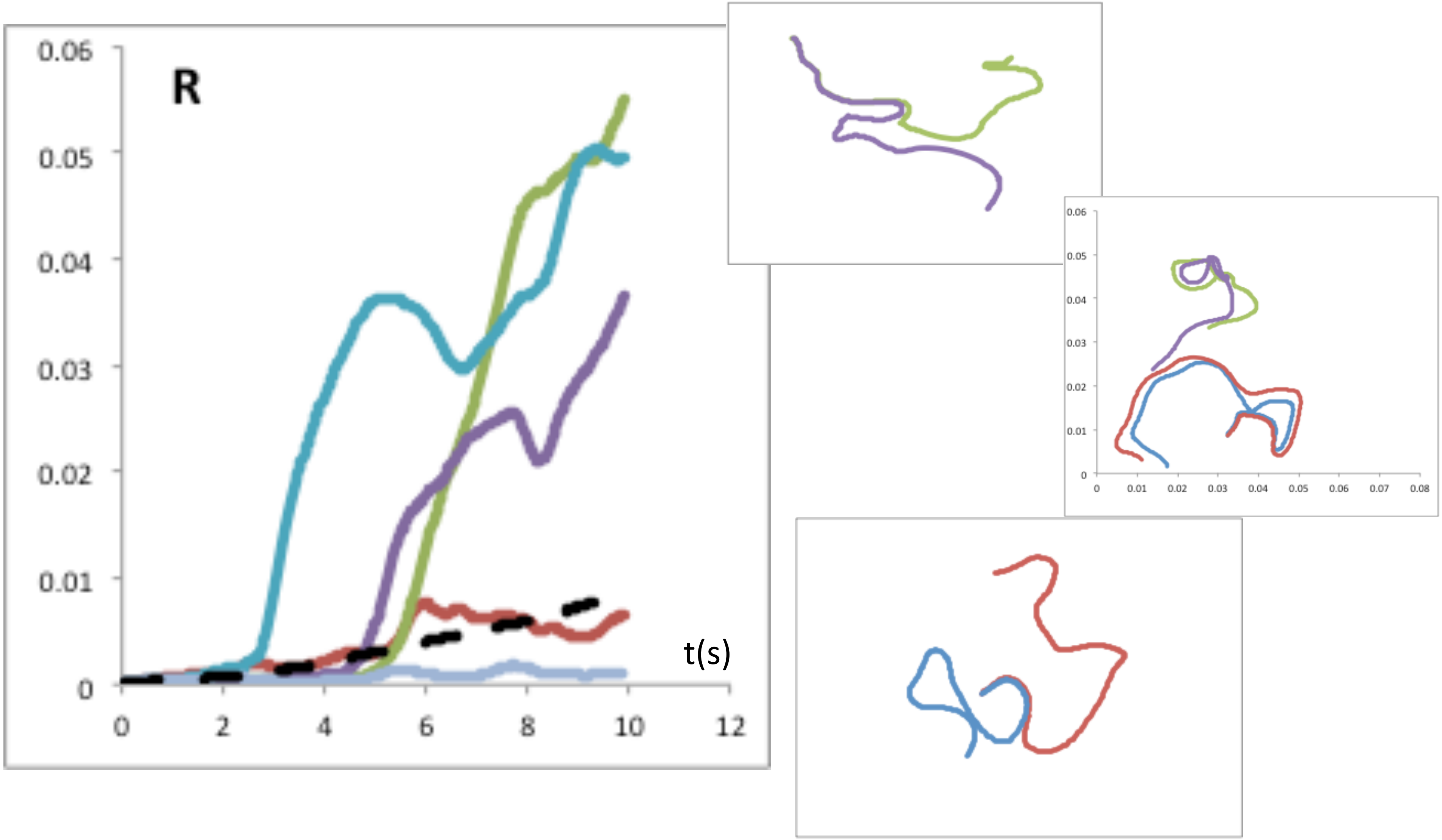


Richardson :

$$\left\langle \delta R^2 \right\rangle \approx g\varepsilon t^3 \quad t >> T_L$$



Pair dispersion: preliminary results



Conclusions

- **Single particle dispersion has been investigated in detail in 2D turbulence;**
- **A new statistical quantity: Lagrangian integral scale**
- **In fully developed turbulence, single particle dispersion is determined by the Lagrangian integral scale and the turbulence level.**
- **L_L is related to the forcing scale.**
- **Transition to turbulence: from superdiffusion to Brownian diffusion, development of self-similar diffusion, disappearance of the tails in the PDF**
- **Quantitative expression for particle dispersion: Taylor dispersion in developed turbulence to chaotic flows.**

Publications:

- Xia H., Francois N., Punzmann H., and Shats M. Lagrangian scale of particle dispersion in turbulence, Nature Communications, 4 , 2013(2013)
- Xia H., Francois N., Punzmann H., and Shats M. Taylor particle dispersion during transition to fully developed two-dimensional turbulence, Physical Review Letters, 112, 104501 (2014)