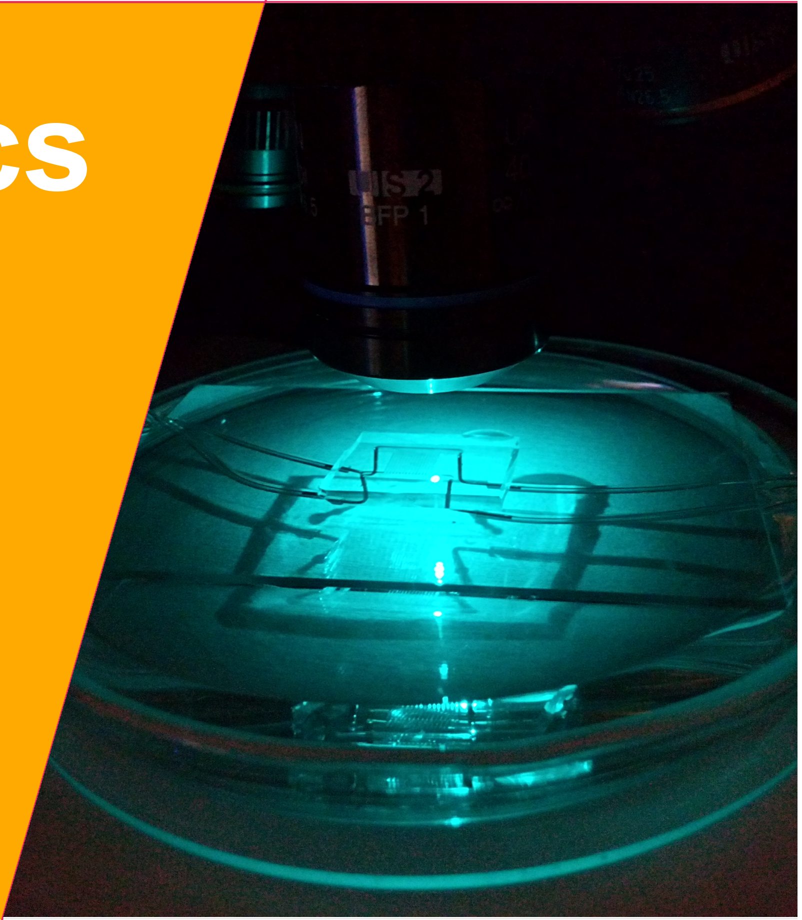
Population dynamics in flowing environments

Federico Toschi

Flowing Matter Across the Scales

23 March 2015



Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

Outline of the talk

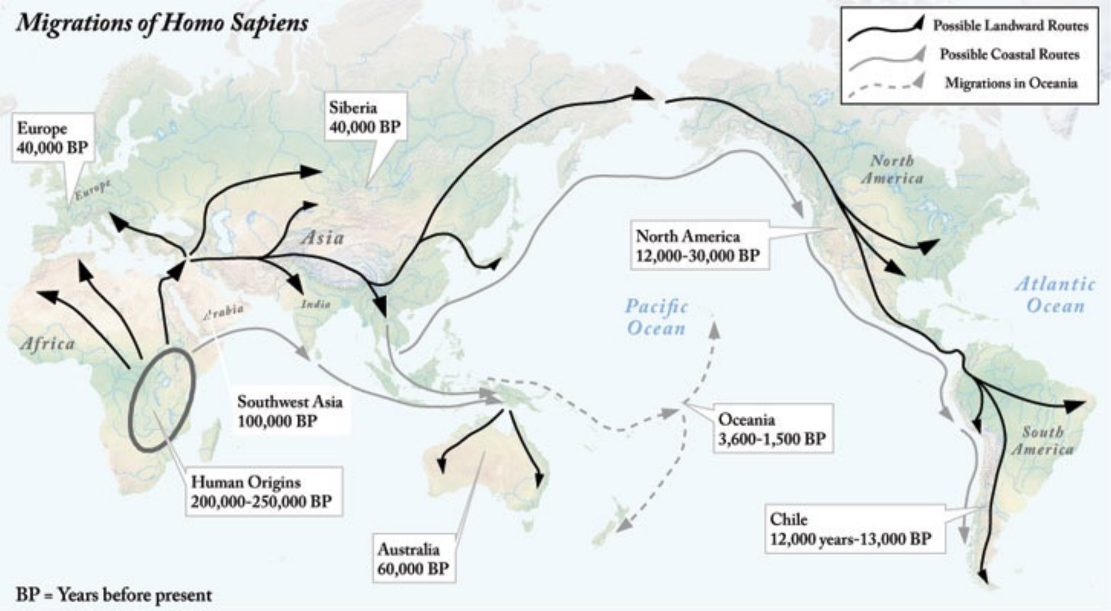
- **Motivation, scope and previous works**
    - Population expansion and genetics (on a Petri dish)
    - Marine populations (phytoplankton layers)

- **Experiments with populations growth under flow**

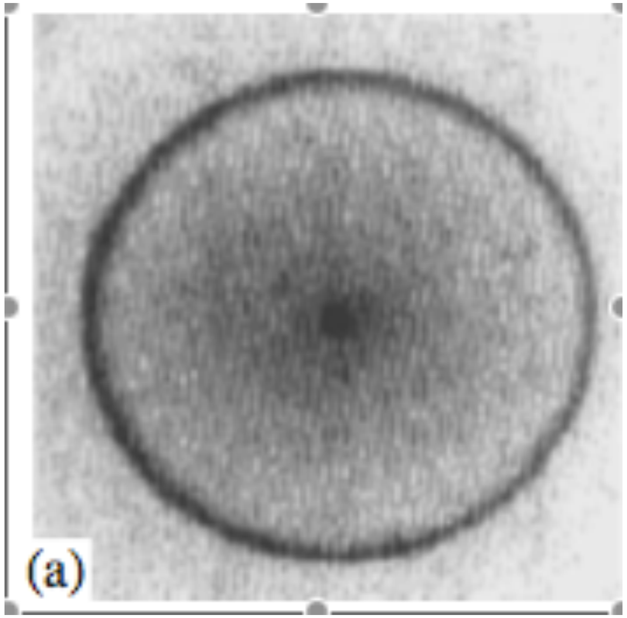
- **Numerical modelling for populations**
    - Effect of (**turbulent**) flow advection in populations
    - Population expansion in **non homogeneous** landscapes

- Conclusions

Population dynamics and range expansion



http://wiznerdan.weebly.com/from-flinstones-to-mill-stones-beginning-of-human-societies.html



In 500 generations....

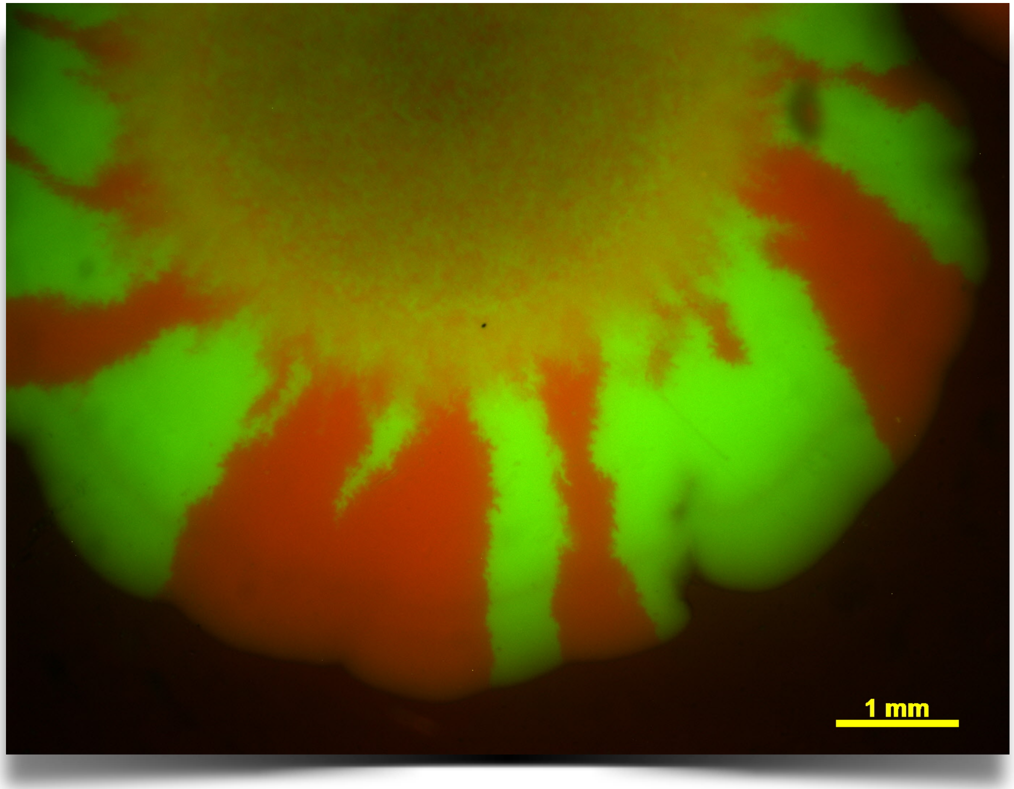
Large mammals expand over ~**10^4 km**

E. Coli (without flagella) expand ~ **1 cm**

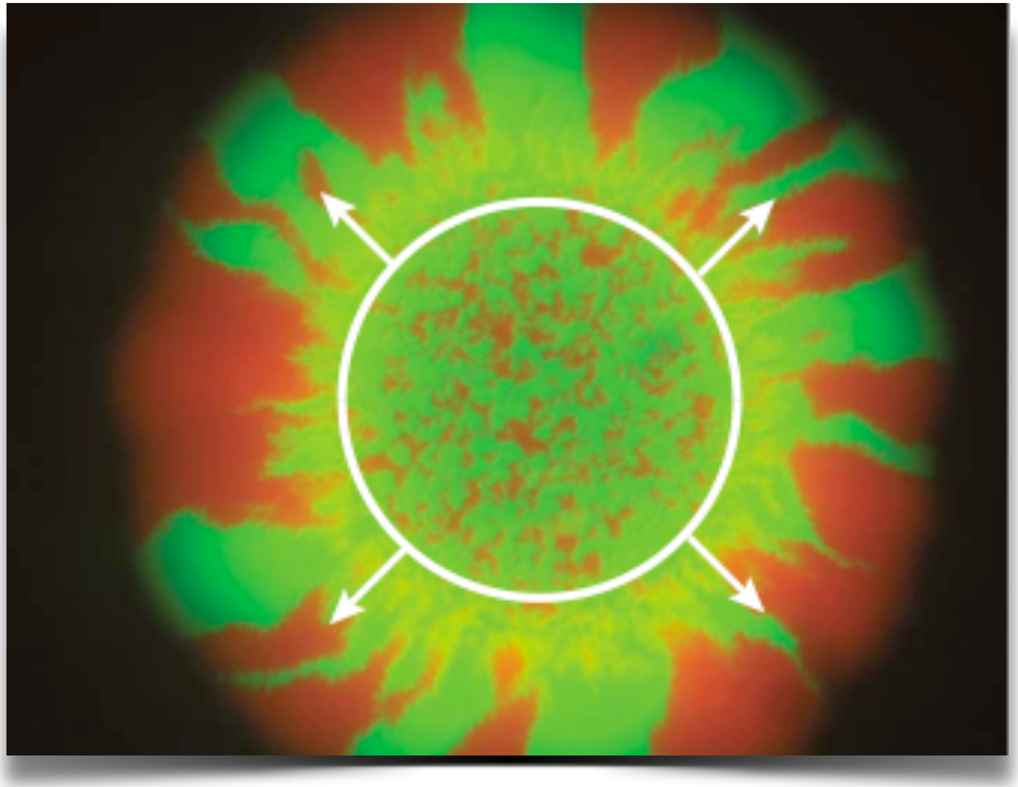
Acknowledgment: D.R. Nelson

Population dynamics experiments (no flow)

Life on a Petri dish



Francesca Tesser (TU/e)

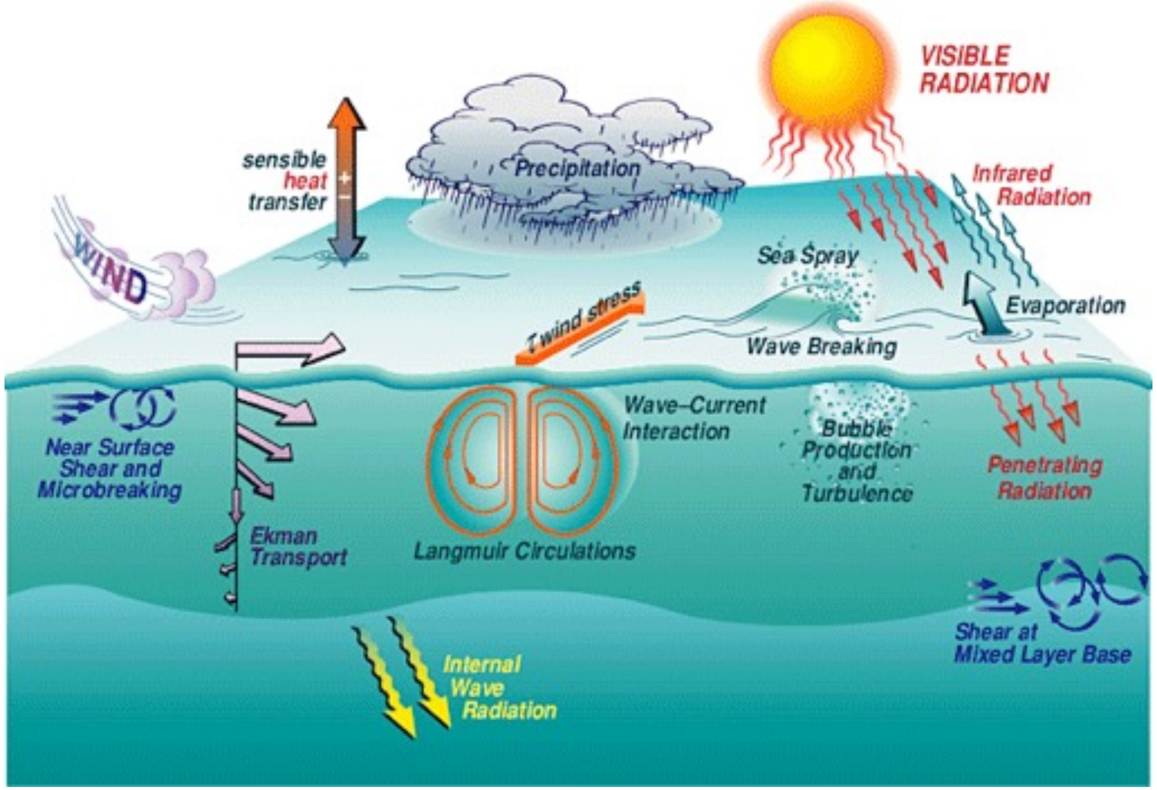


Korolev et al. Genetic demixing and evolution in linear stepping stone models. Rev. Mod. Phys. (2010) vol. 82 (2) pp. 1691

Life in water…

- Why ***population dynamics*** and ***fluid transport*** ?

- ~2-3 billion years ago, **growth and evolution of microorganisms** took place in the **oceans**. Water covered most of the earth.
- Fossilized, oxygen-producing **cyanobacteria** have been dated at **~2.8-3.5 billion years ago**.
- **Cyanobacteria transformed the atmosphere** via oxygenic photosynthesis, and may have been the ancestor of chloroplasts in plants and eukaryotic algae.
- **Spatial growth, competition and fixation** between different photosynthetic bacterial variants presumably took place at **high Reynolds number in upper layers of ocean**.
- **Such competition continues to this day**, with **organisms controlling their buoyancy** to resist down welling currents and stay close to the ocean surface.



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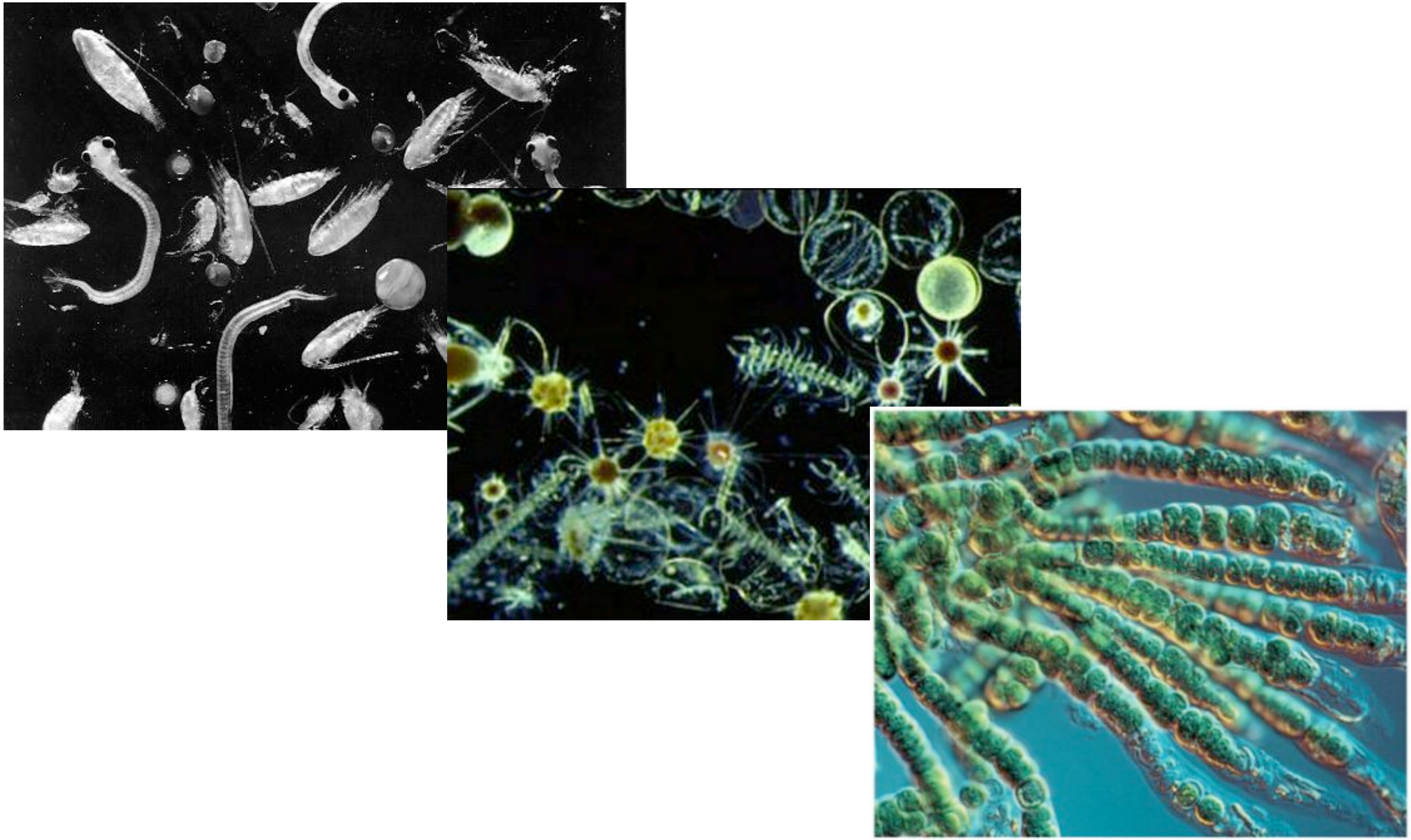
Bacteria and plankton: small scale view



Cyanobacteria

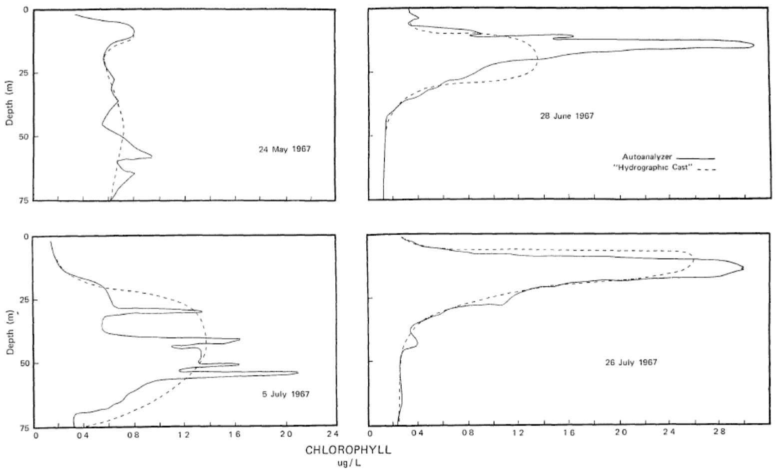
Bacteria and plankton: large scale view

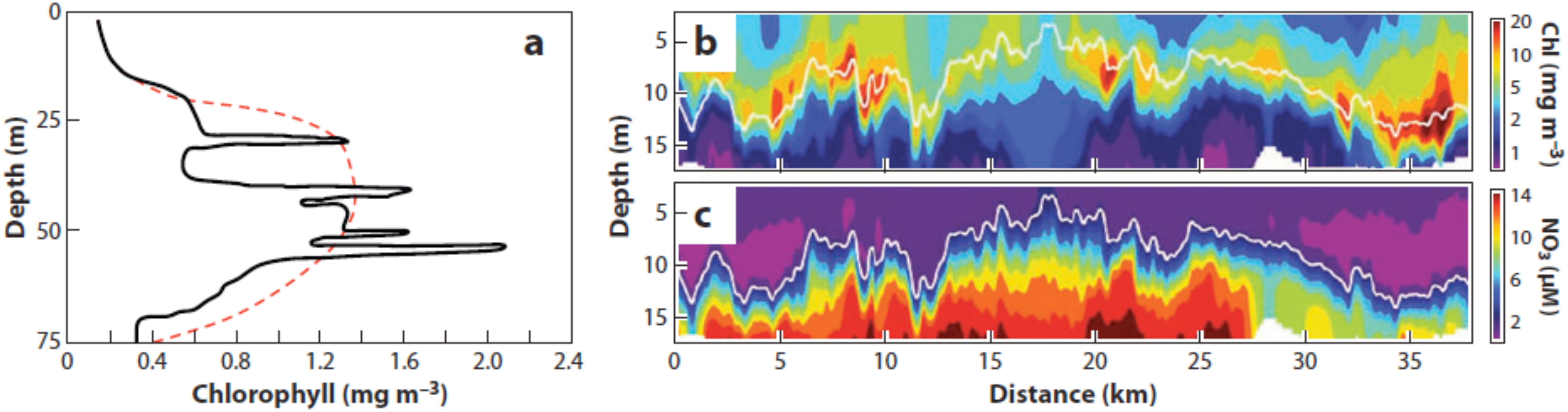
Plankton bloom off Waiheke Island

http://earthobservatory.nasa.gov/IOTD/view.php?id=41385

Phytoplankton Layers

Strickland, J. D. H. (1968). A Comparison of Profiles of Nutrient and Chlorophyll Concentrations Taken from Discrete Depths and by Continuous Recording. Limnology and Oceanography, 13(2), 388–391.



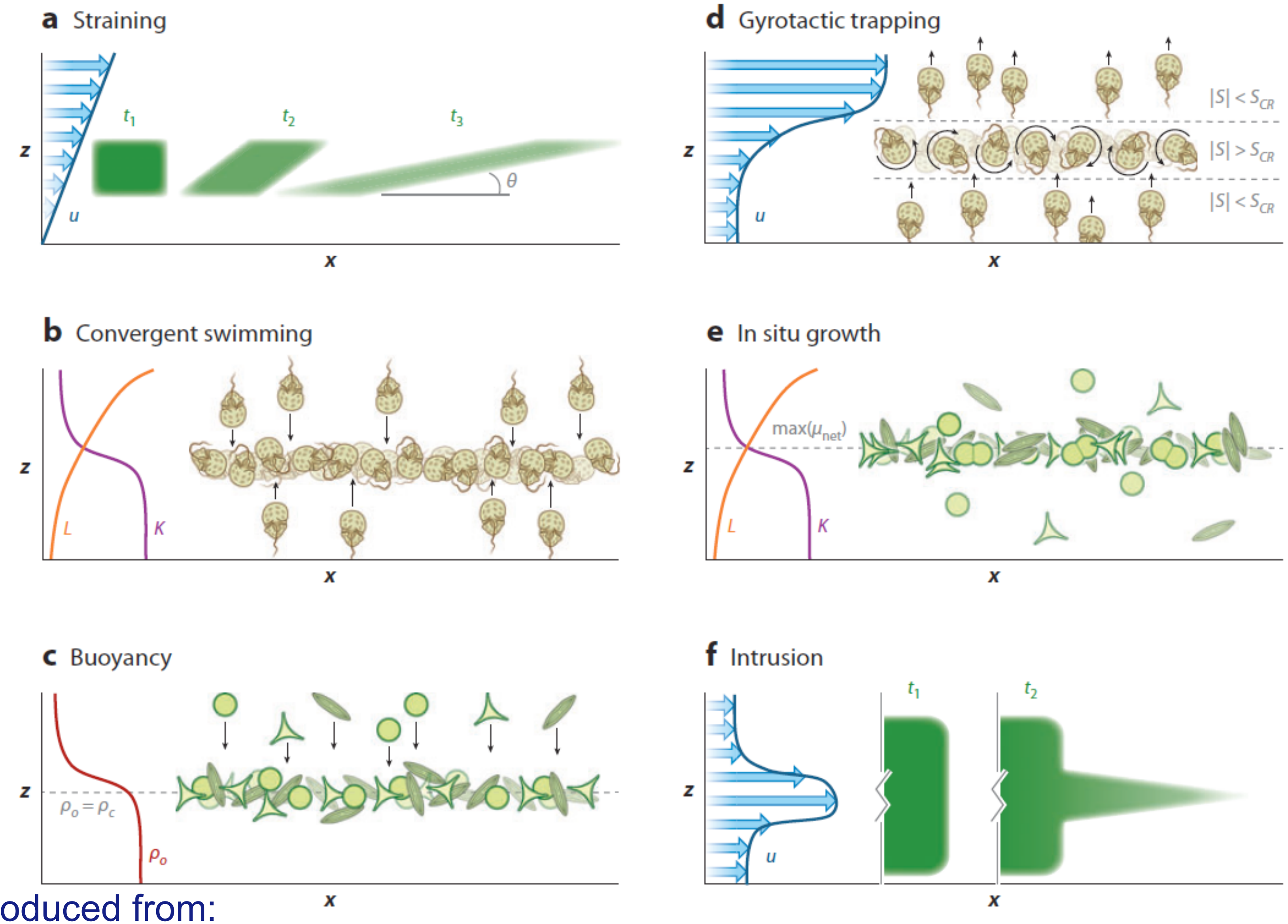


Thin layers observed in 1967 off La Jolla, California

reproduced from:

Durham, W. M., & Stocker, R. (2012). Thin Phytoplankton Layers: Characteristics, Mechanisms, and Consequences. Annu. Rev. Marine. Sci., 4(1), 177–207.

Mechanisms for Phytoplankton Layers formation



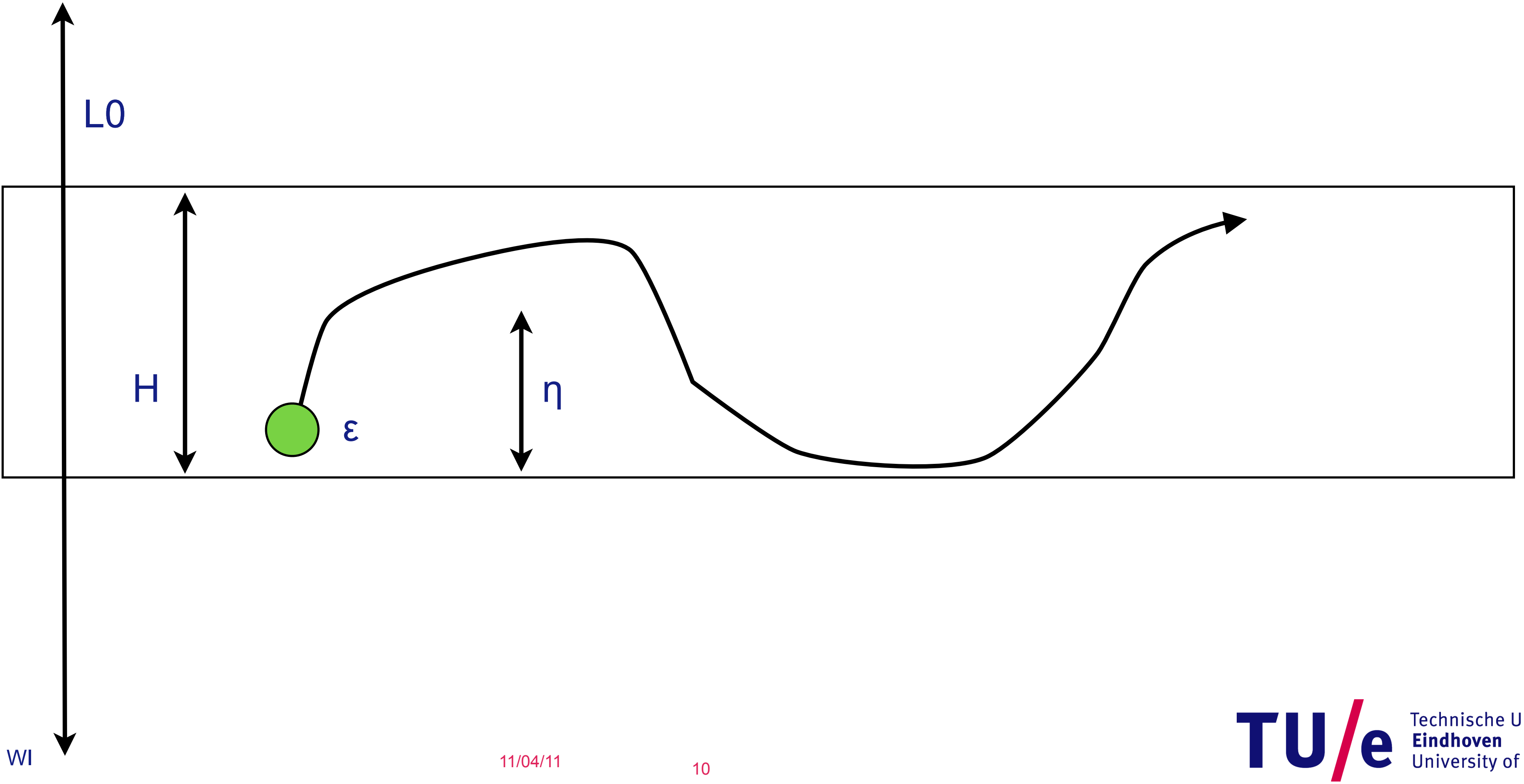
reproduced from:

Durham, W. M., & Stocker, R. (2012). Thin Phytoplankton Layers: Characteristics, Mechanisms, and Consequences. Annu. Rev. Marine. Sci., 4(1), 177–207.

How “thin” are phytoplankton thin layers ?

- Several causes for formation of non homogeneities

- But: does plankton feel a compressible flow ?



A simple model for thin Plankton layer

$$\boldsymbol{v}(\boldsymbol{x},t) = \boldsymbol{u}(\boldsymbol{x},t) - k(z - z_0)\hat{\boldsymbol{z}}$$

u is a 3d turbulent field

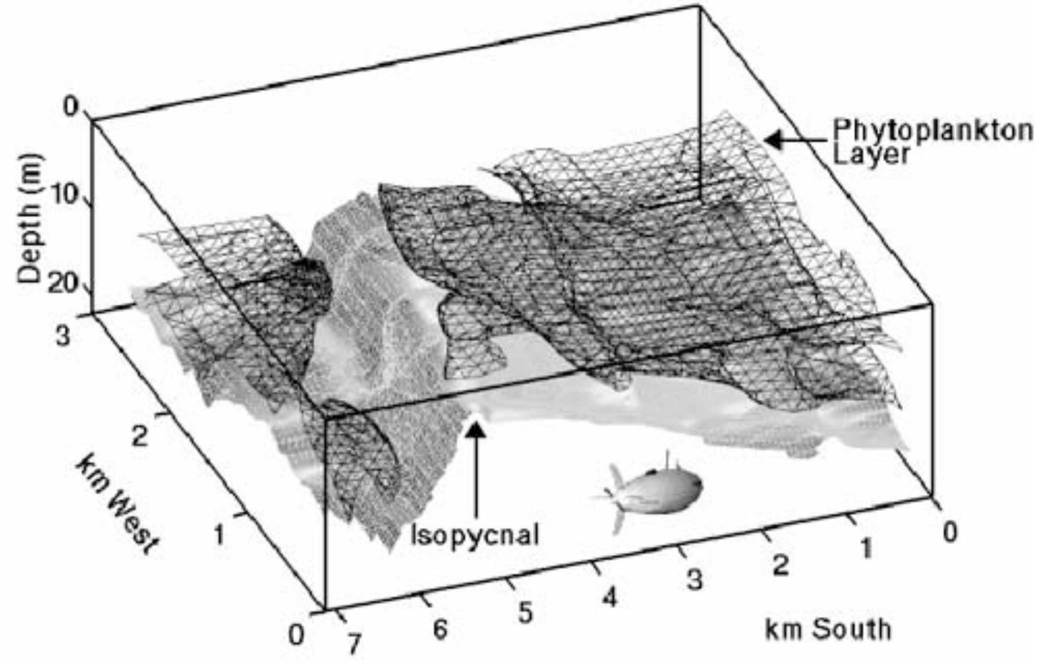
“elastic” contribution to model for density stratification

$$k \to \infty$$

perfect 2d stratification

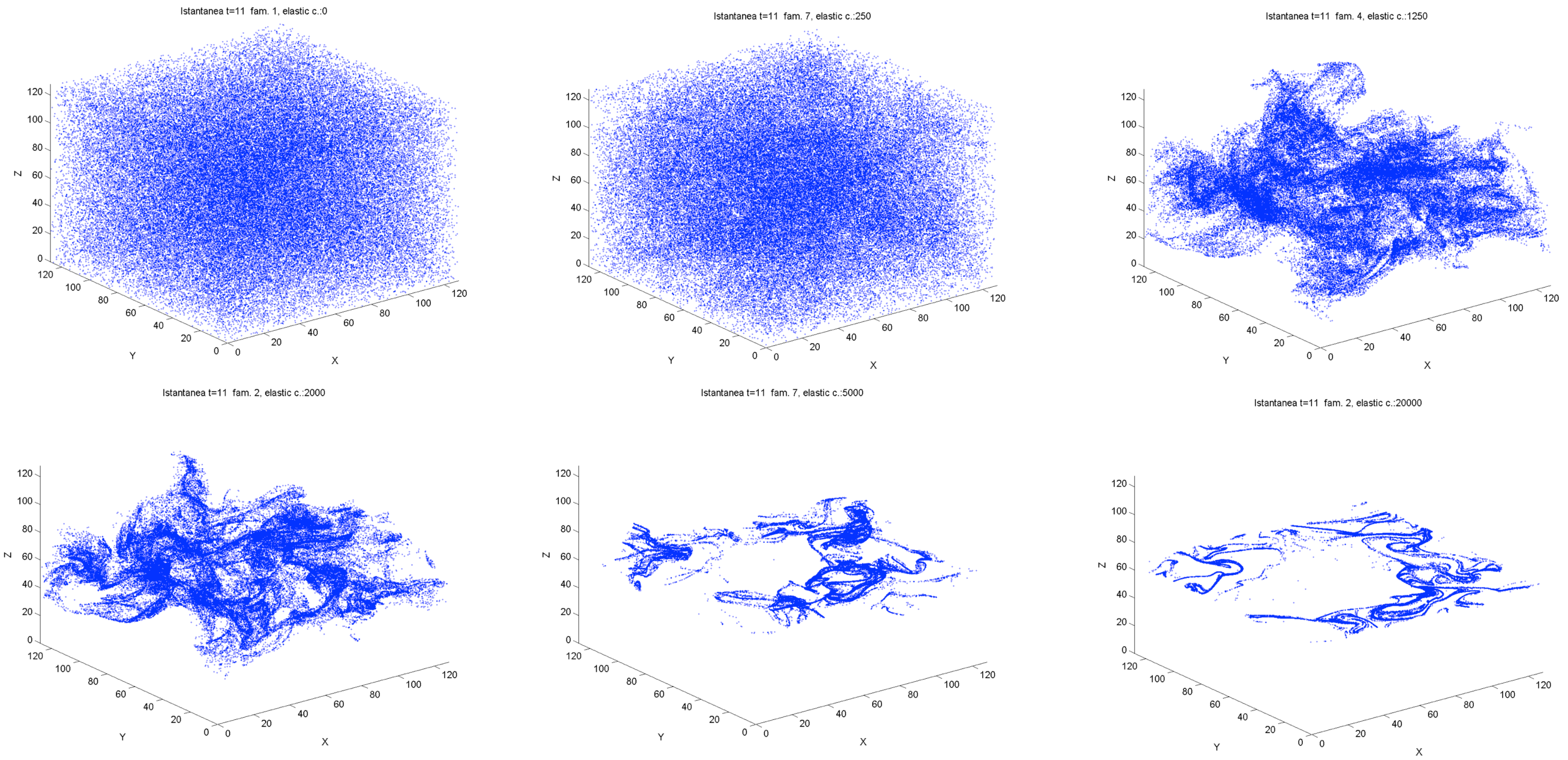
$$k \to 0$$

perfect 3d HI turbulence



Picture from: http://montereybay.noaa.gov/reports/2001/eco/openocean.html

Effect of confinement


Istantanea t=11 fam. 1, elastic c.:0

Istantanea t=11 fam. 7, elastic c.:250

Istantanea t=11 fam. 4, elastic c.:1250

Istantanea t=11 fam. 2, elastic c.:2000

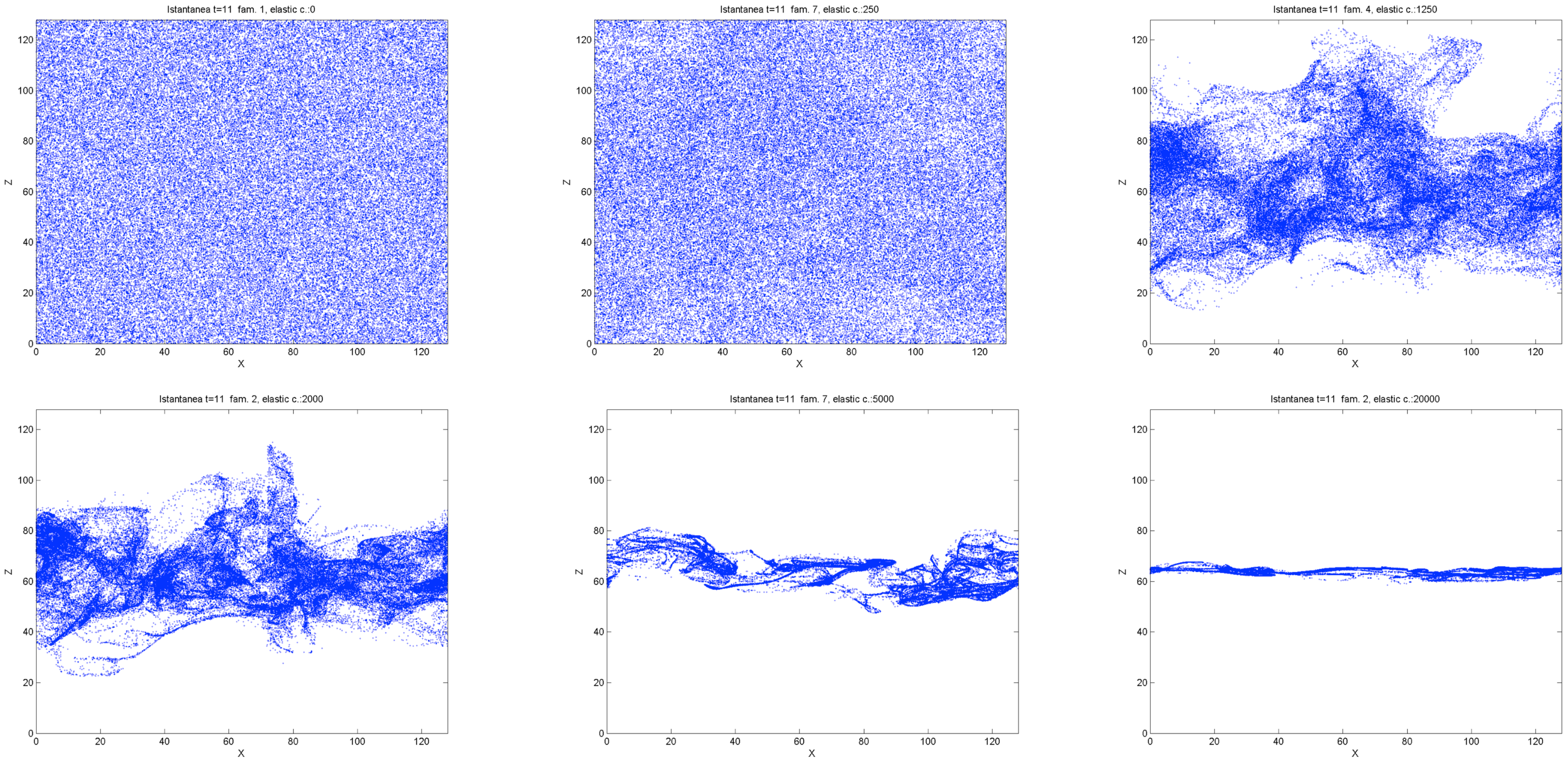
Istantanea t=11 fam. 7, elastic c.:5000

Istantanea t=11 fam. 2, elastic c.:20000

Confinement produces strong small scale non homogeneities !!

arXiv:1411.1950
On clustering of vertically constrained passive particles in homogeneous, isotropic turbulence
Massimo De Pietro, Michel A.T. van Hinsberg, Luca Biferale, Herman J.H. Clercx, Prasad Perlekar, Federico Toschi

Vertical planes



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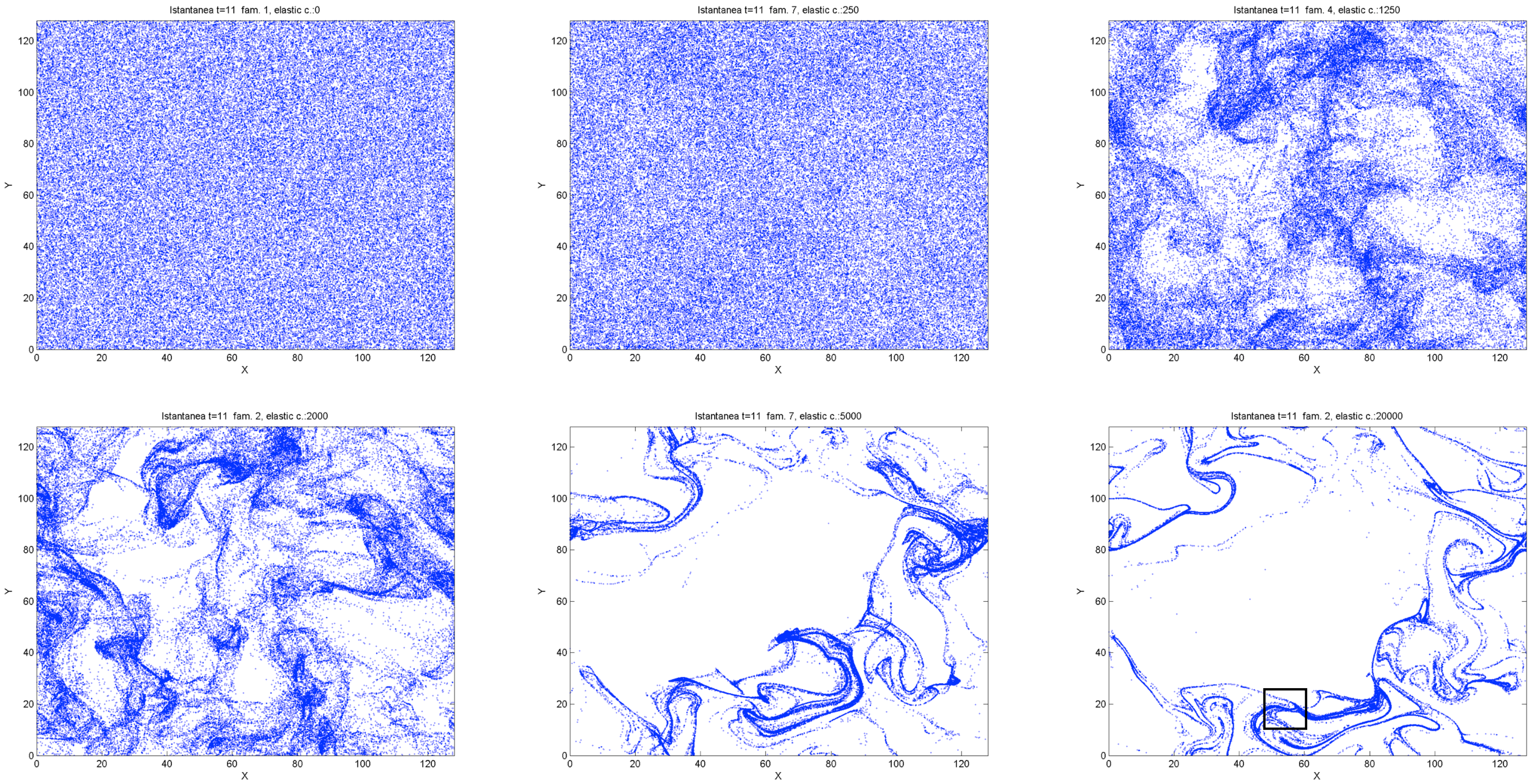
Horizontal planes



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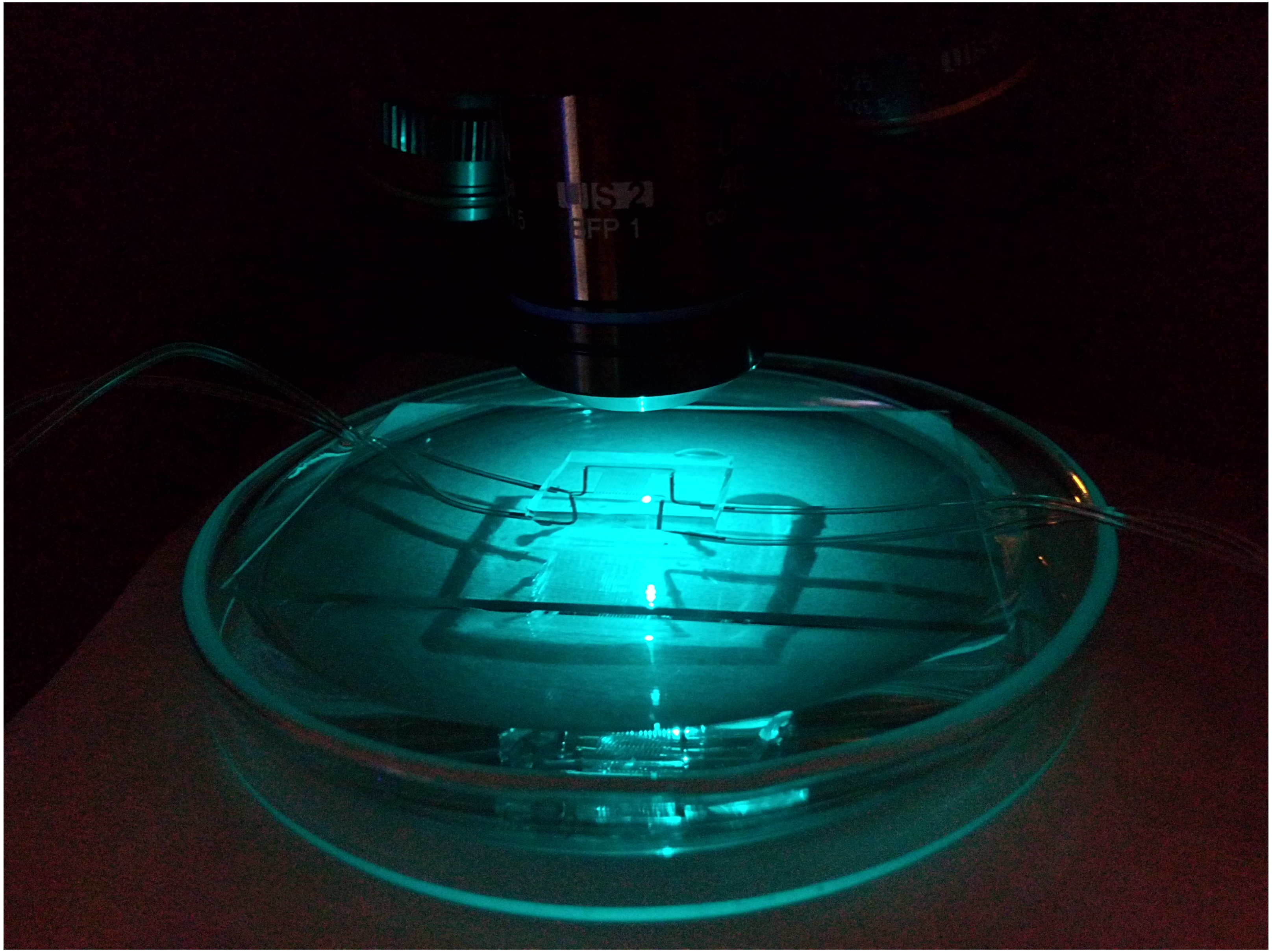
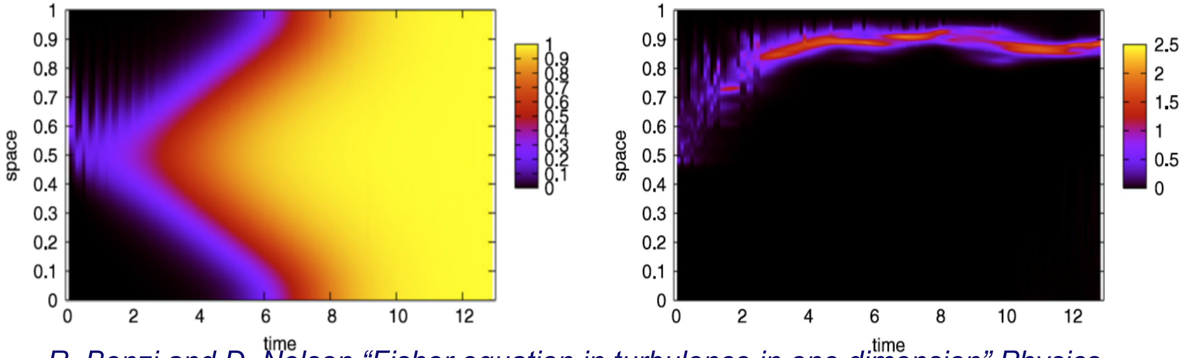
Experiments with growth under flow



F. Tesser

Numerical studies with flow

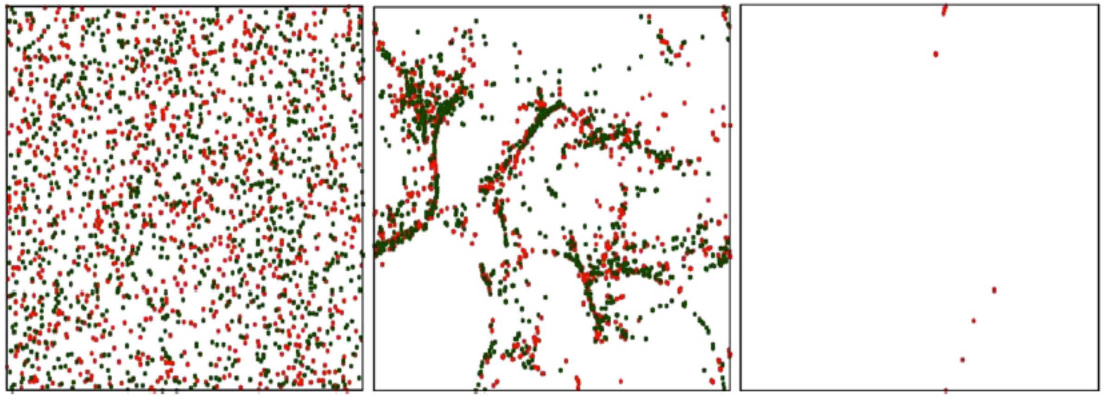
- Continuum Fisher equation in 1D with advection



R. Benzi and D. Nelson "Fisher equation in turbulence in one dimension" Physics D 2009

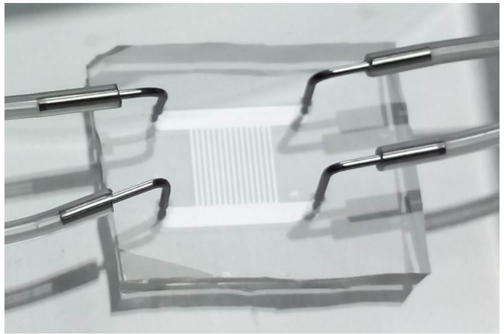
$$\frac{\partial b(\vec{x},t)}{\partial t} + \nabla\cdot\left(\vec{u}b\right) = D\nabla^2 b + mb(1-b)$$

- Discrete particle model in turbulence



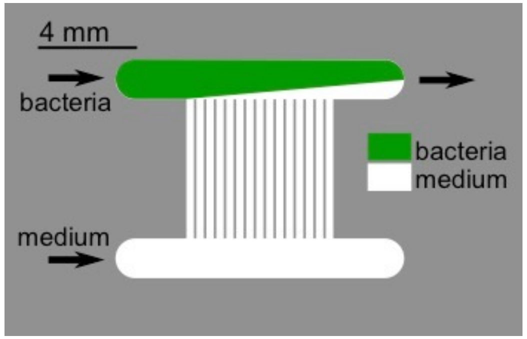
S. Pigolotti, R. Benzi, P. Perlekar, M.H. Jensen, F. Toschi and D.R. Nelson "Growth, competition and cooperation in spatial population genetics" Theoretical Population Biology 2013

Lack of experimental studies in controlled fluidic environment

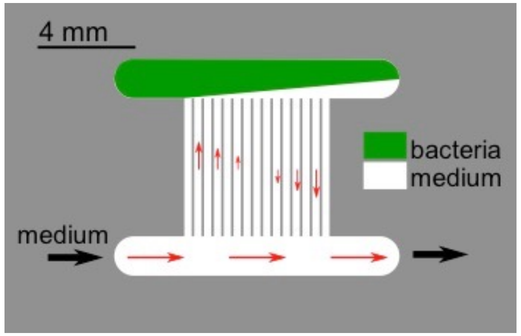


Bacteria growth in a microfluidic device

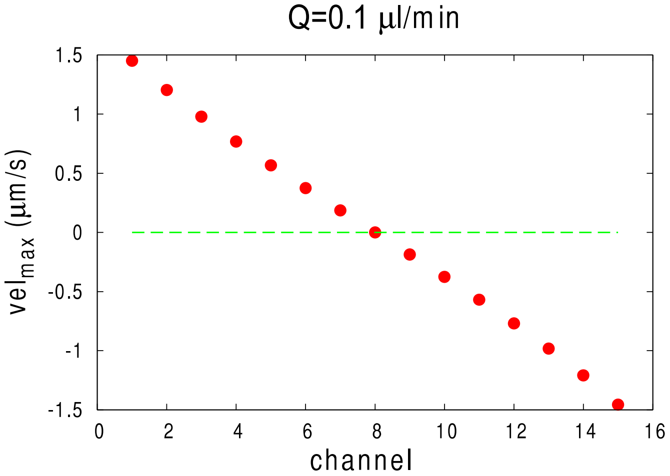
- **Fluorescent** E. coli **non-motile** bacteria injected in a PDMS device
- Series of identical parallel channels: 300μm x 280μm x 5.5mm
- Device at constant temperature 37°C.
- Duration of experiments: **24 - 72 hours**



Bacteria load protocol.

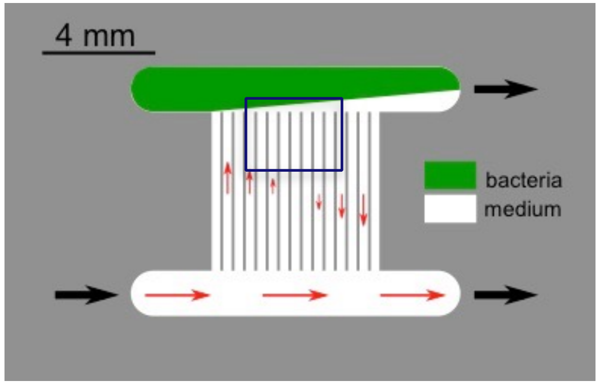


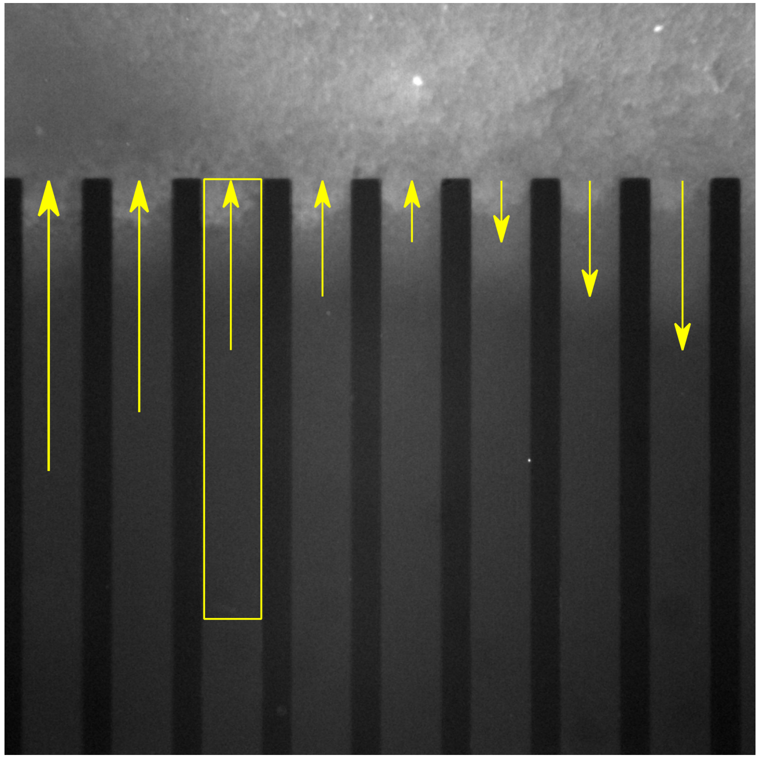
Initial condition for the experiment.

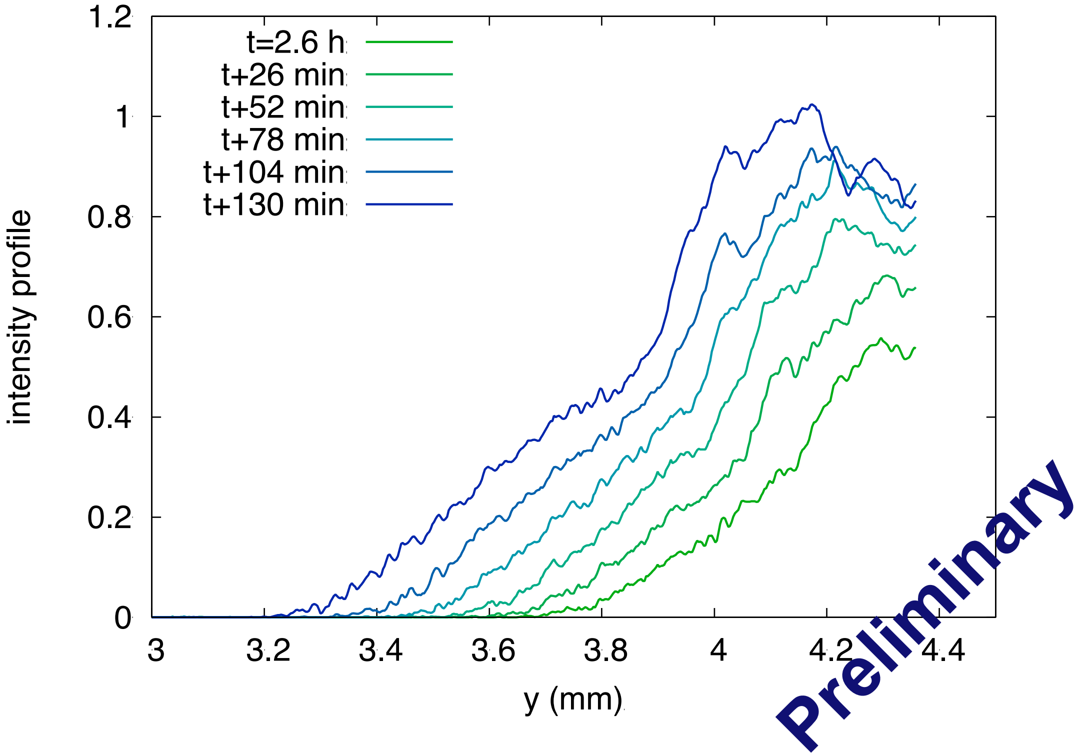


Key idea: growth along the channels at different flow conditions.

Bacteria growth at small velocities



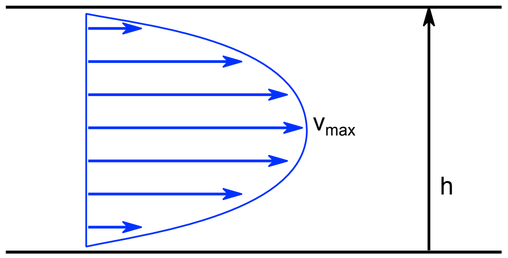




Front propagation speed

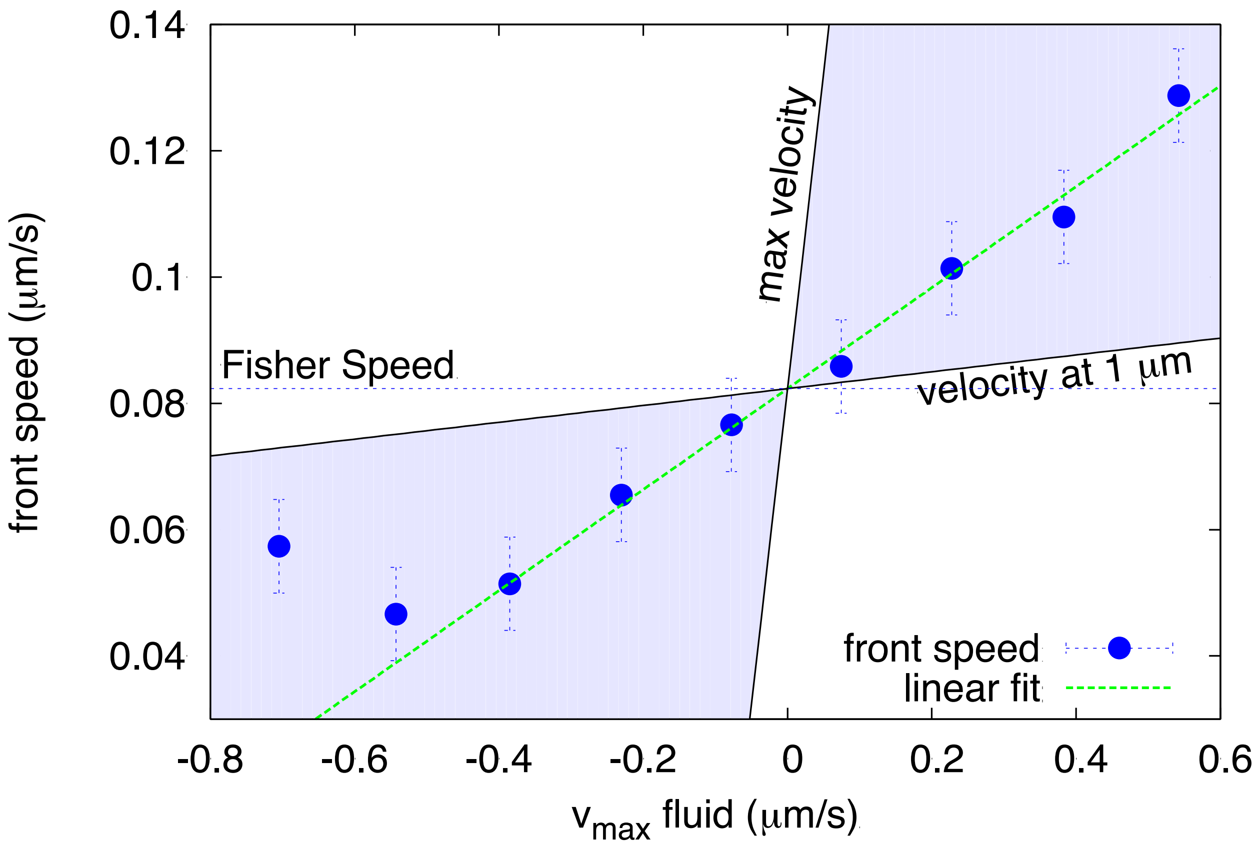
$$v_{front} = v_{Fisher} + v_{fluid}$$

$$v_{front} = v_{Fisher} + \alpha\, v_{\max}$$



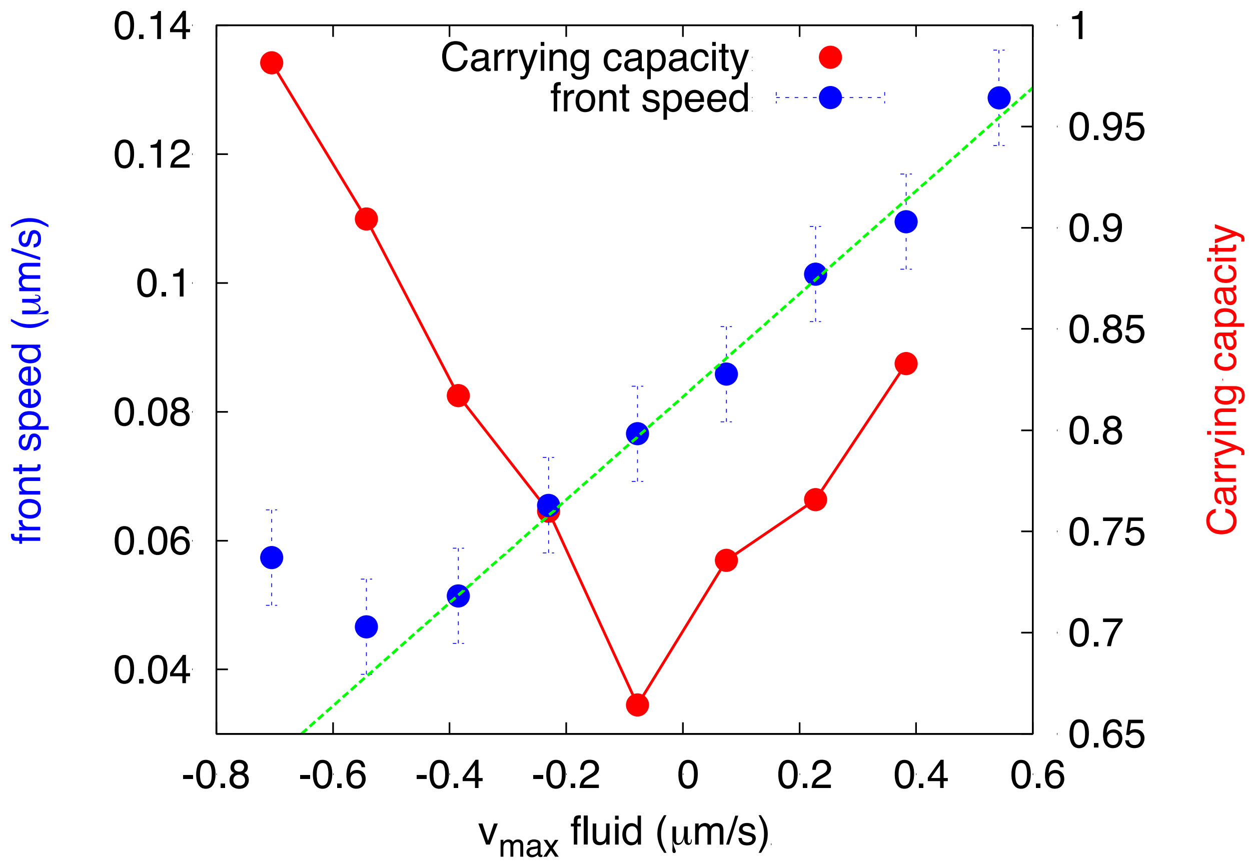
$$v_{Fisher} = 0.083\mu m/s$$

$$\alpha \rightarrow v_{fluid} = v(h = 6\mu m)$$



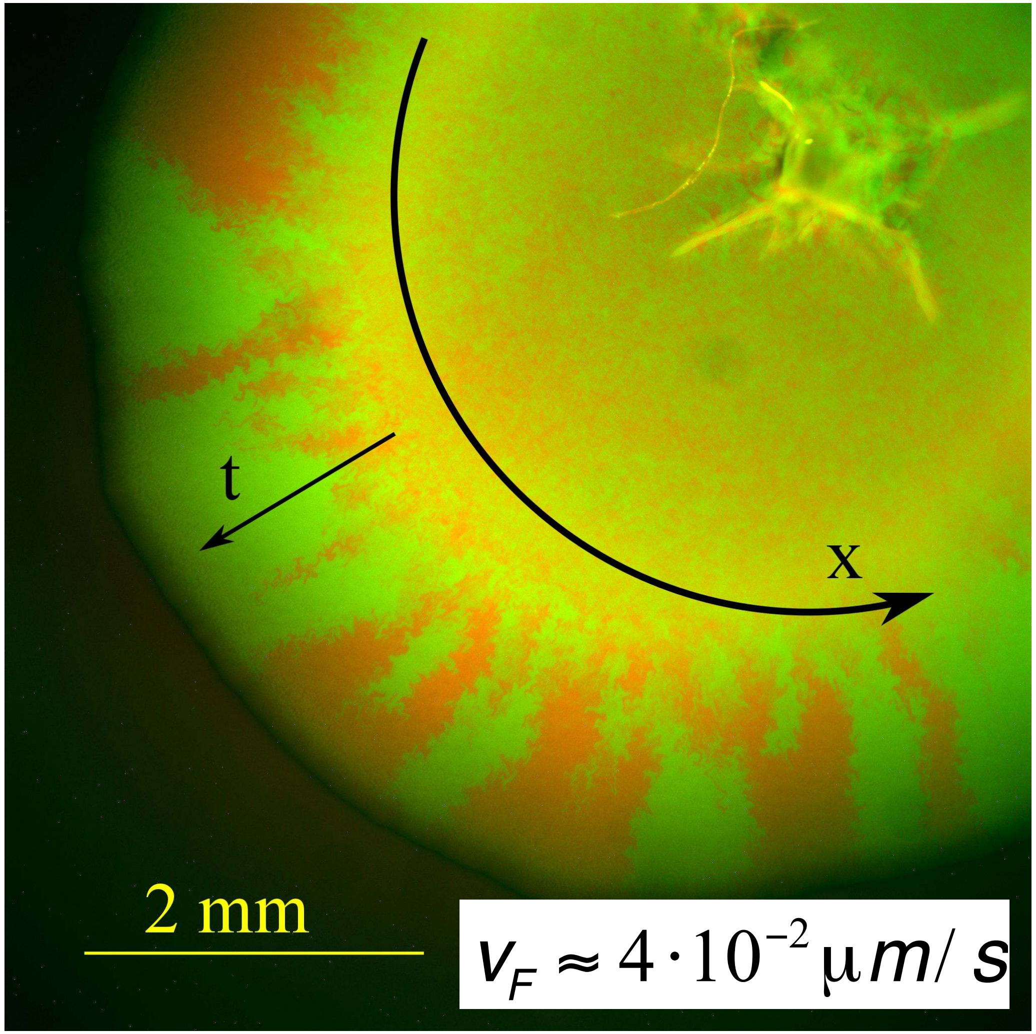
Preliminary

Front speed and carrying capacity



Preliminary

Numerical modeling for populations



$v_F \approx 4 \cdot 10^{-2} \mu m/s$

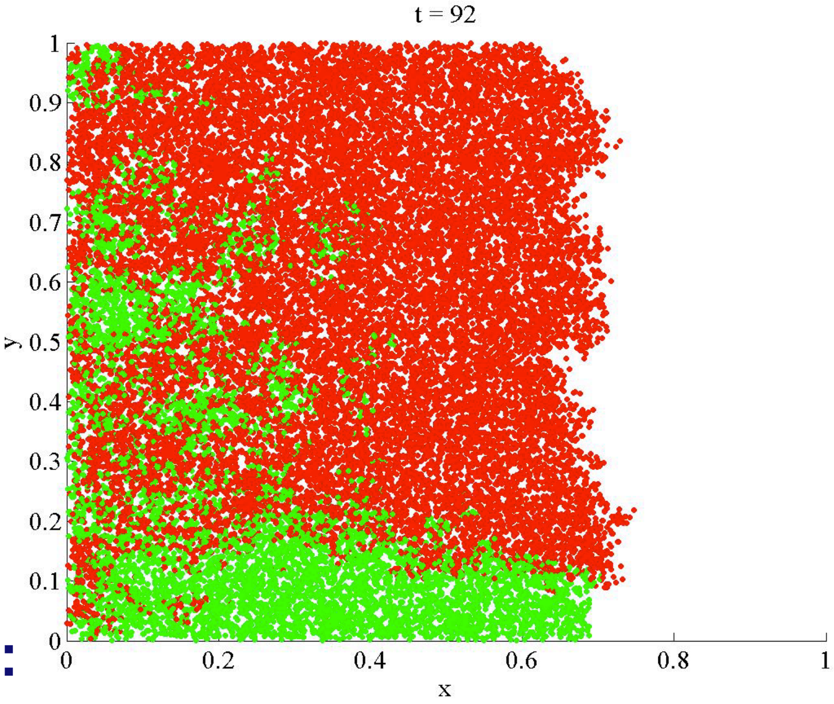
Bacteria growth on agar gel.
F. Tesser 2012

Discrete particle model

- **Discrete individuals** X_i *of type* $i = A,B$
- **Birth** and **death** reactions

$$X_i \xrightarrow{\mu_i} 2X_i$$

$$X_i + X_j \xrightarrow{\lambda_{ij}} X_i$$



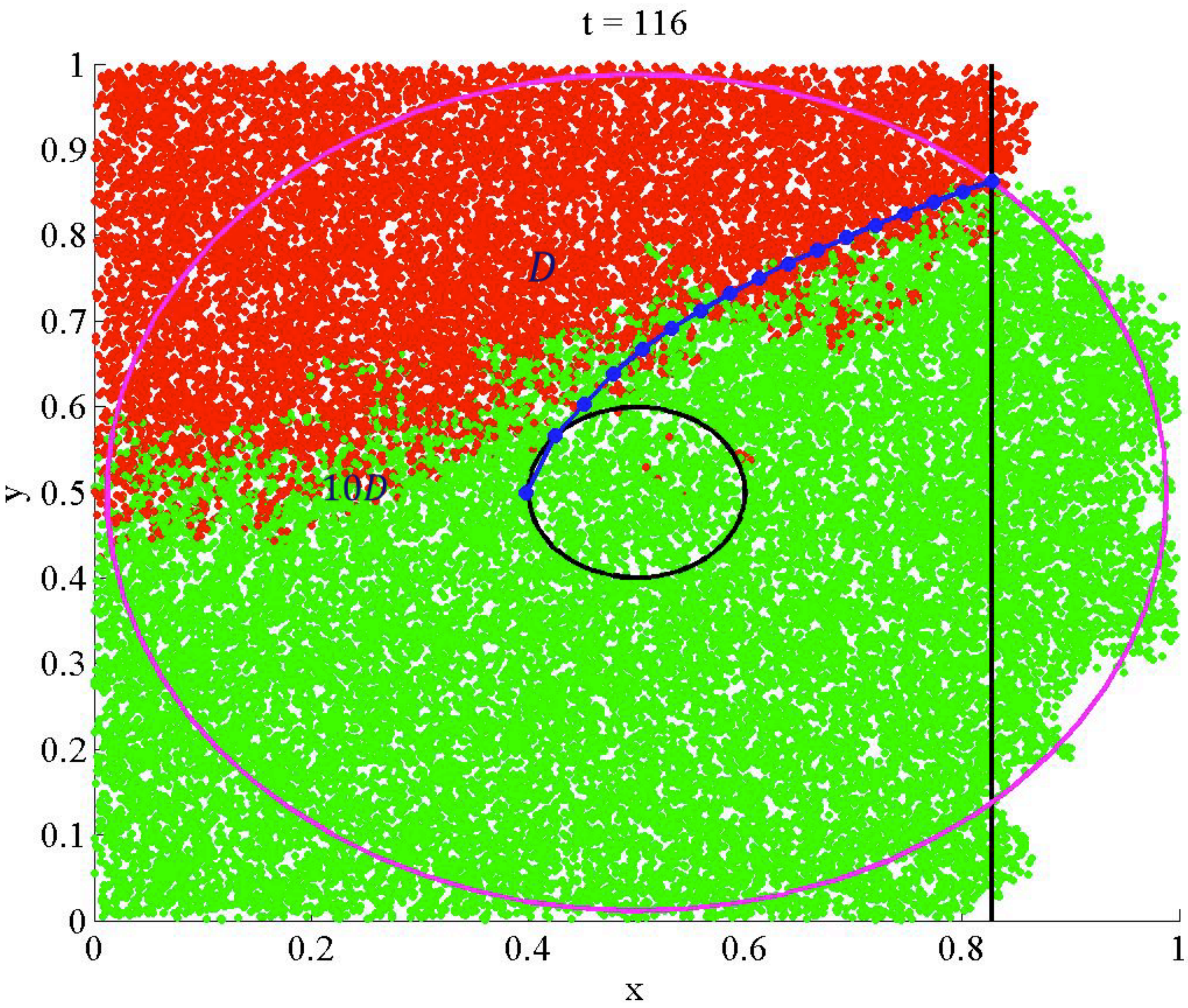
- Homogeneous diffusion in continuous space with diffusion coefficient D
- Front propagation with **Fisher velocity:**

$$u_f = 2\sqrt{\mu D}$$

- Local and global **fixation**

C.R. Doering et al., Physica A (2003)
S. Pigolotti et al., Theoretical population biology (2013)

Front in presence of a localised “defect”

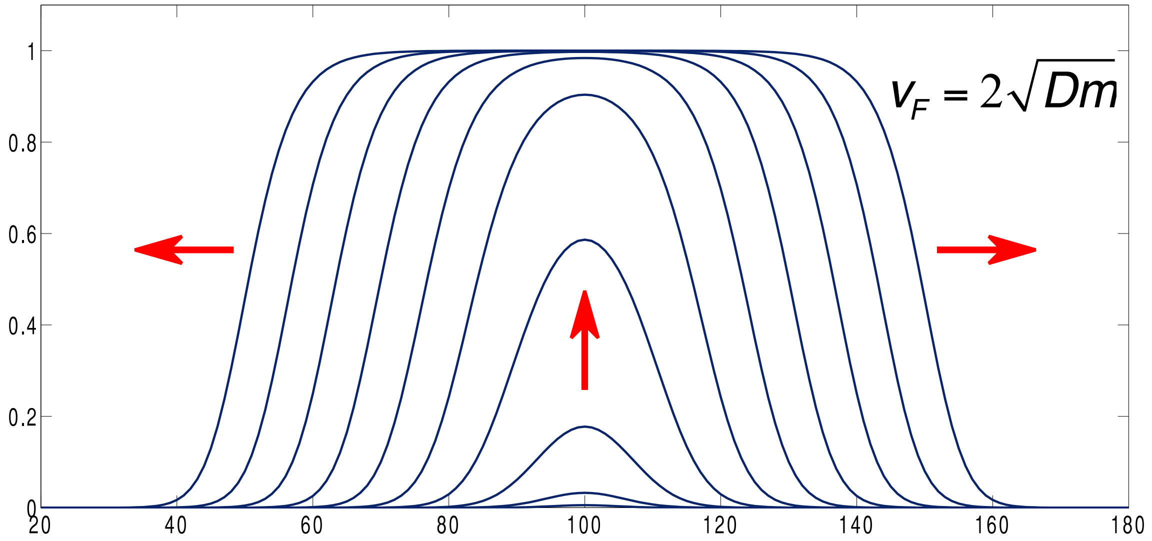


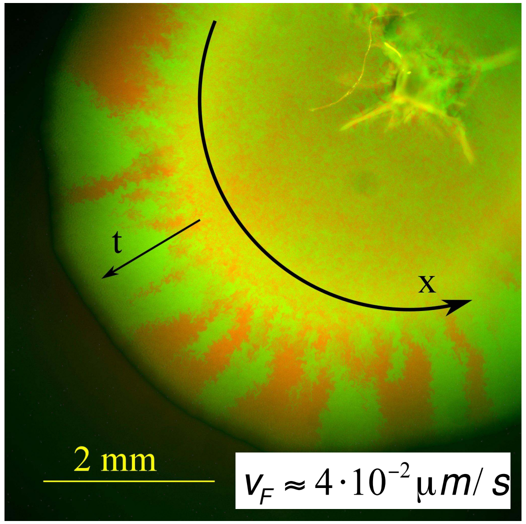
Continuum modeling: FKPP equation

Reaction diffusion dynamics: **Fisher-KPP equation**

- ***c***, density of bacteria
- ***μ***, growth rate of the population
- ***D***, diffusion coefficient →
- ***u***, advecting velocity

$$\frac{\partial c}{\partial t} + \boldsymbol{\nabla} \cdot (\boldsymbol{u} c) = D\nabla^2 c + \mu c\,(1 - c)$$

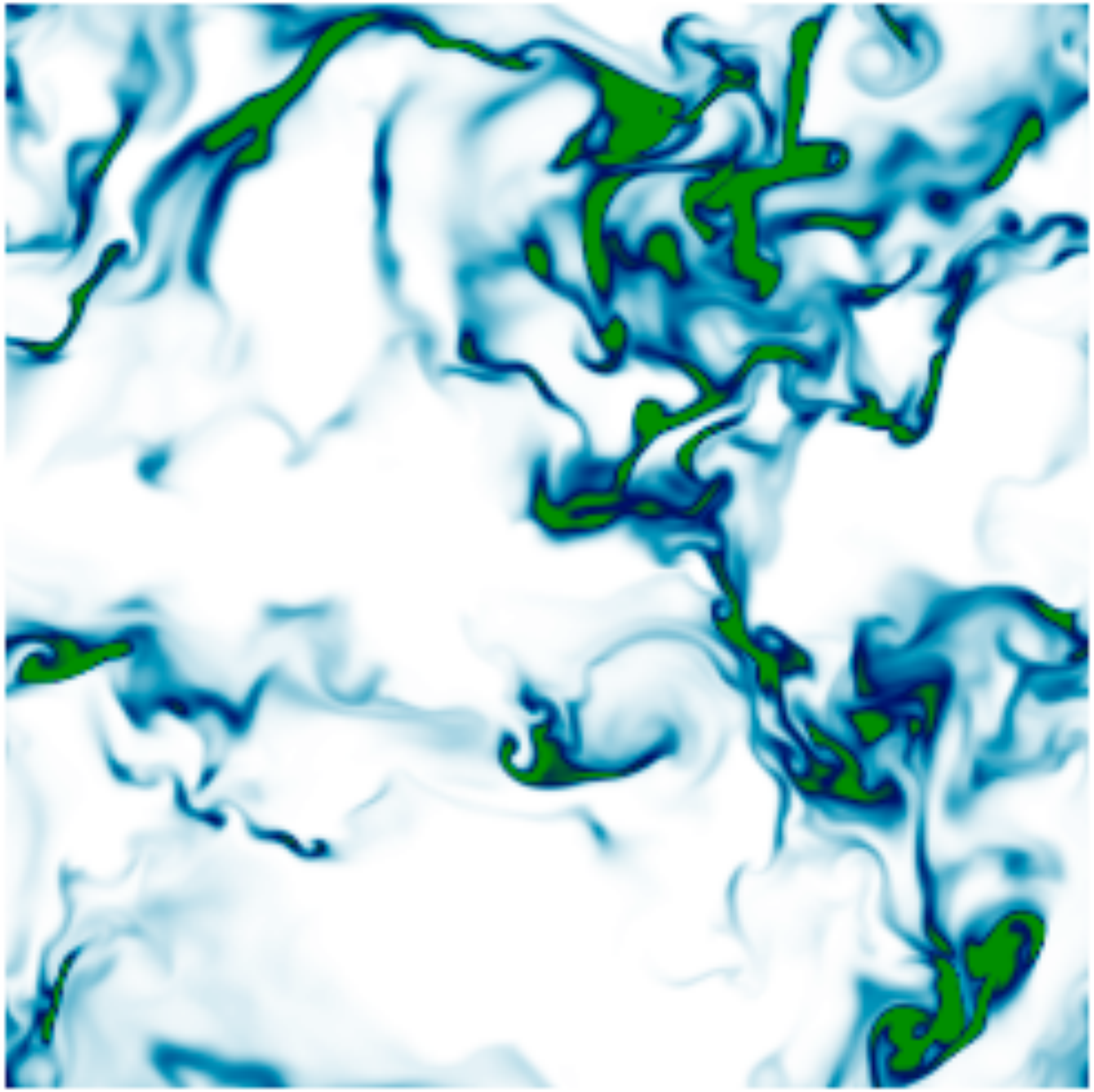




Bacteria growth on agar gel.
F. Tesser 2012

R.A. Fisher "The Wave of Advance of Advantageous Genes" 1937
A. Kolmogorov, I. Petrovsky, N.Piskunov Bull Univ. Moscow 1937
O. Hallatschek, P. Hersen, S. Ramanathan, and D.R. Nelson "PNAS" 2007

Life *in silico*



$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{f}$$

+

$$\frac{\partial c}{\partial t} + \boldsymbol{\nabla} \cdot (\boldsymbol{u} c) = D \nabla^2 c + \mu c (1 - c)$$

$$Re = \frac{u_{\mathrm{rms}} L}{\nu}$$

$$Sc = \frac{\nu}{D}$$

$$\mu \tau_\eta$$

Conclusions

- **Marine environments are non-homogeneous**
- **The role of non-homogeneities can be very important**
  - **At small scales:**
    - induced by turbulent fluctuations
  - **At large scales:**
    - non-homogeneous distribution of nutrients, temperature, turbulence intensities, etc.