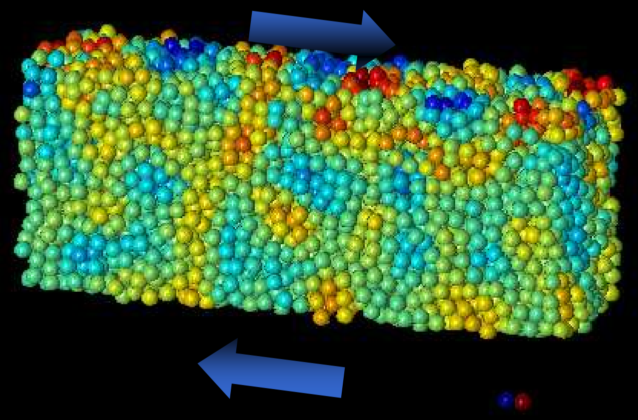
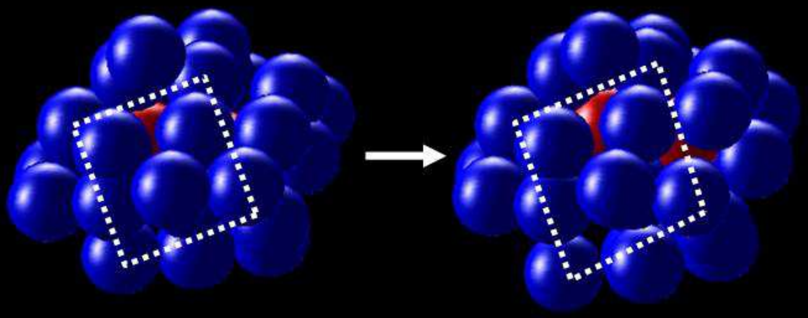


Flow of Glasses

2. Scaling & Multiscale phenomena

Peter Schall

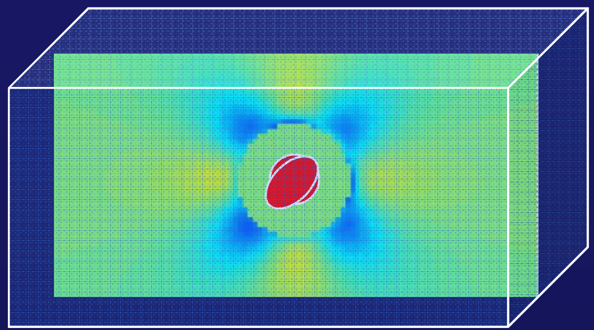
University of Amsterdam



Flow of glasses

Lecture 1

- Glass Phenomenology
- Basic concepts: Free volume, elastic fields



Elastic continuum

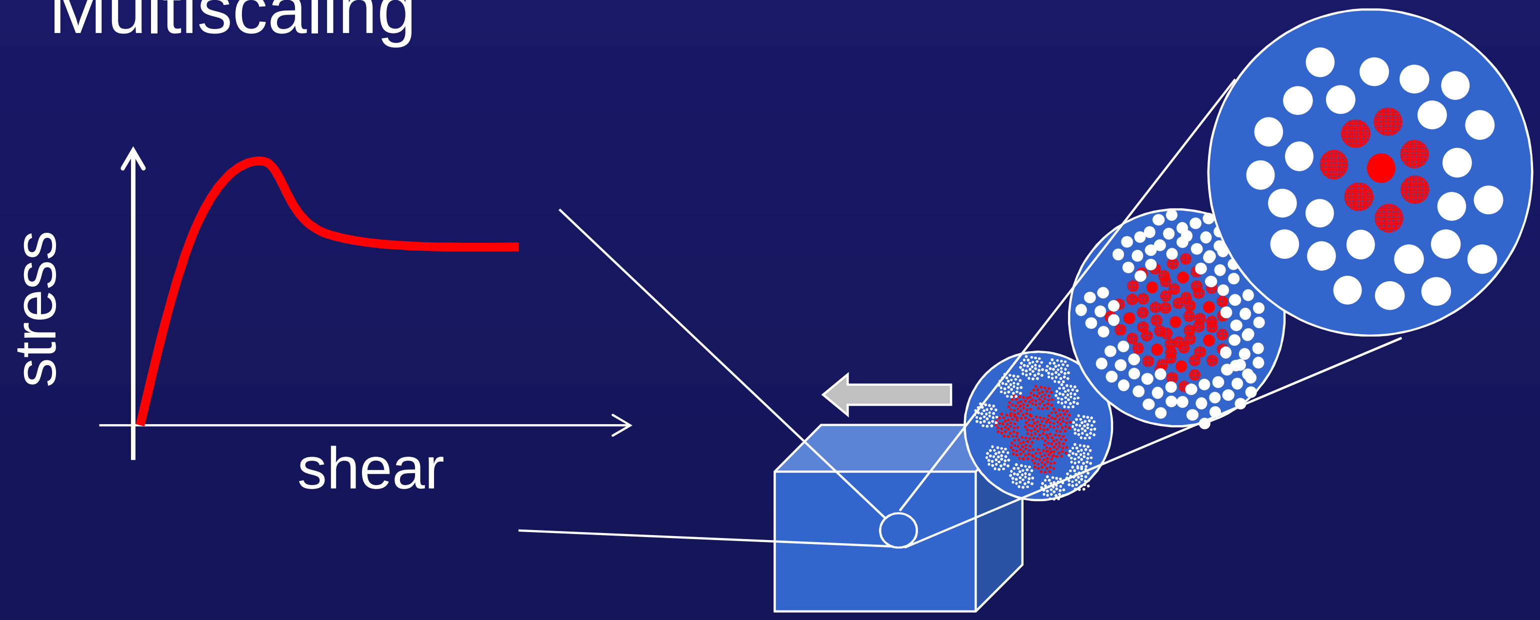
Strain Field

$$\epsilon_{xz} \propto \frac{1}{r^3}$$

Flow of glasses

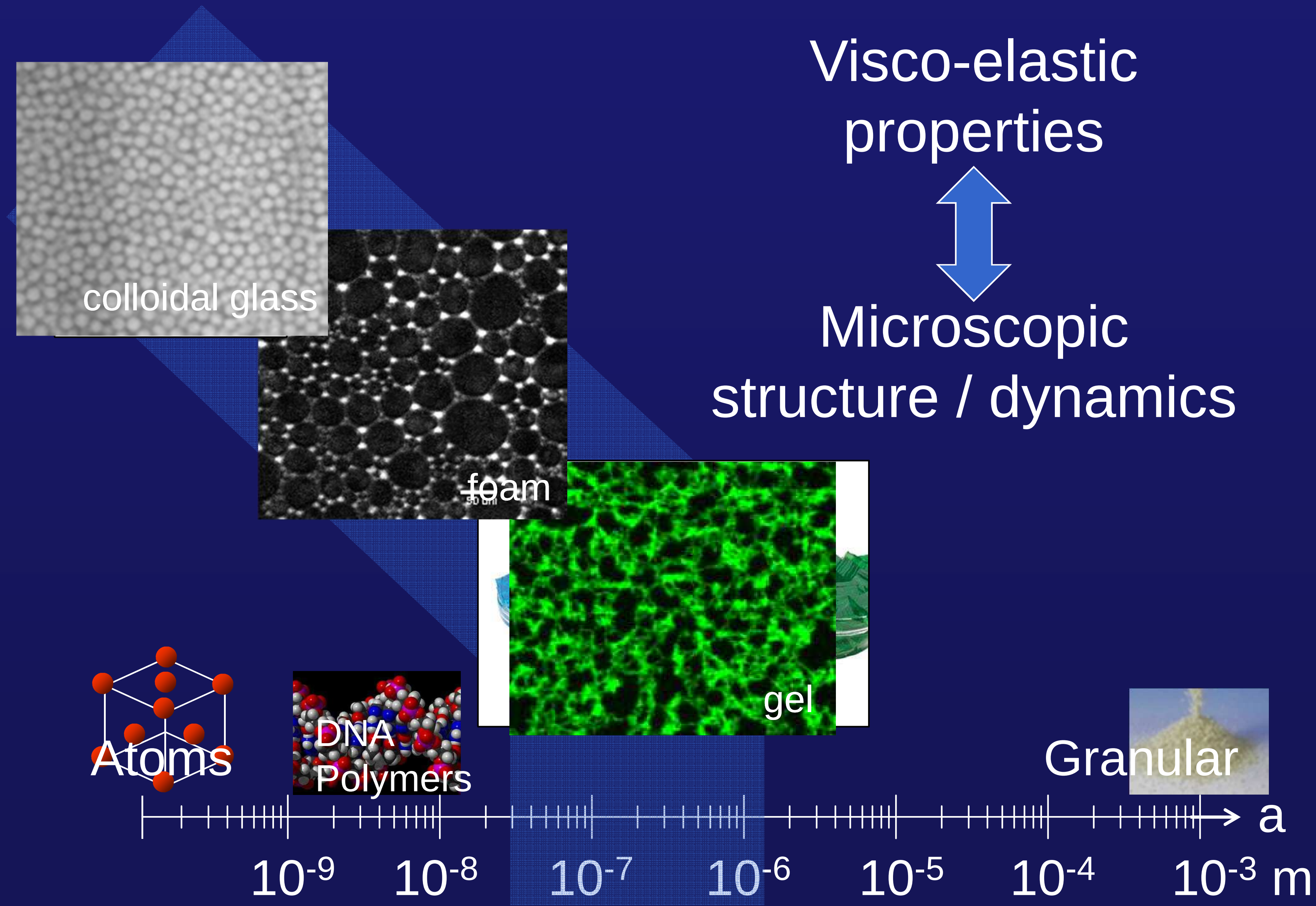
Lecture 2

- Correlations in slow flow
- Material stability → Nonequilibrium phase transitions
- Multiscaling

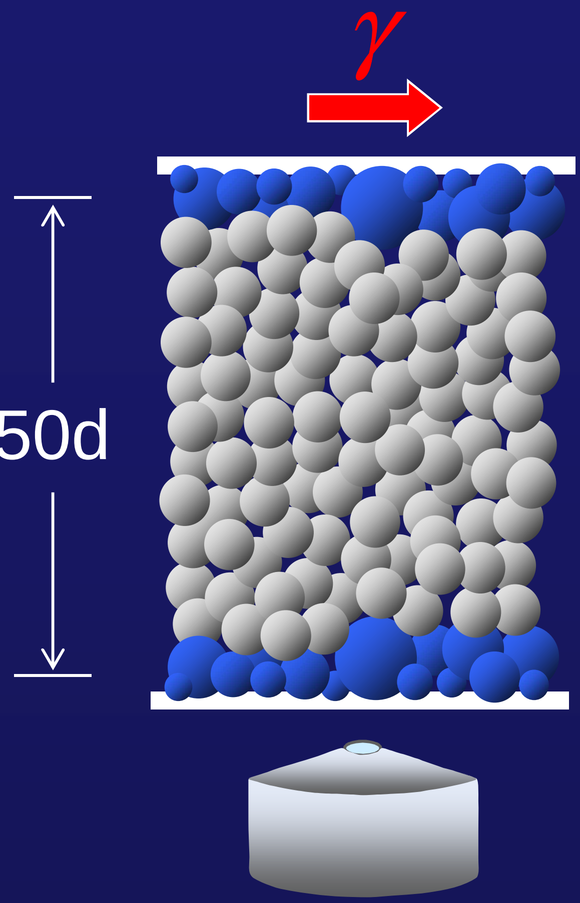


Mechanical response ⟺ Correlations

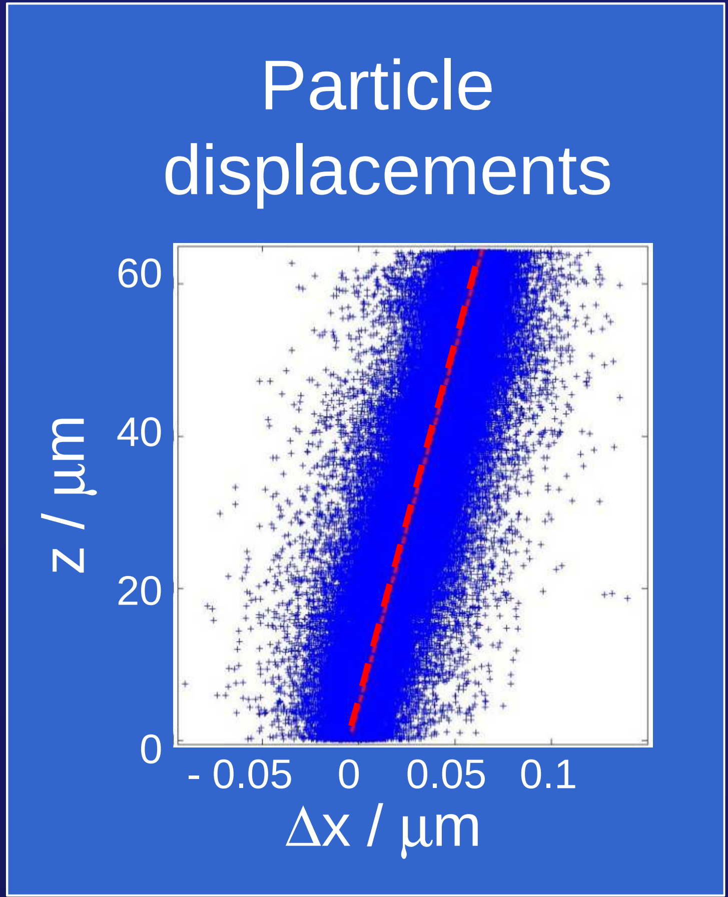
Soft Glasses



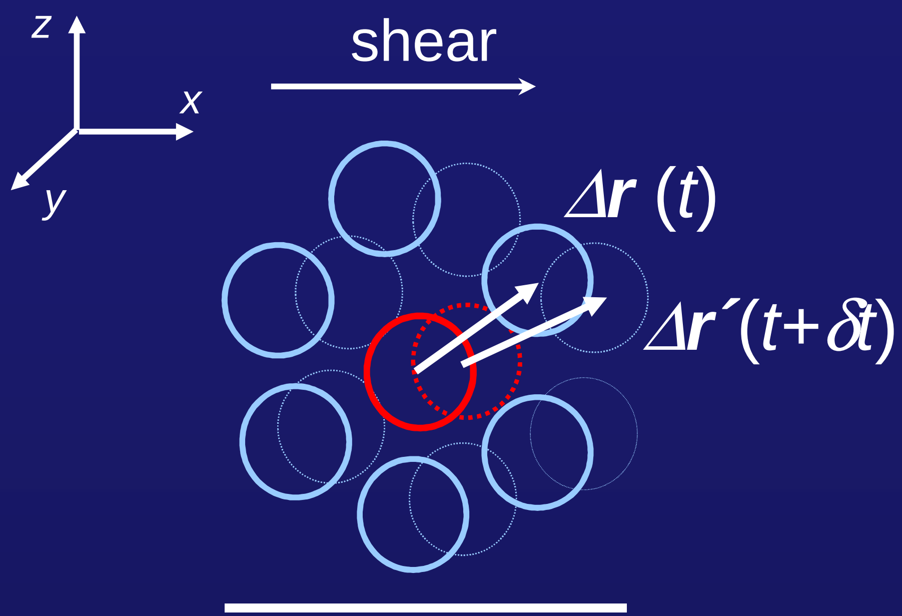
Application of stress



Confocal microscopy



Strain and non-affine displacements

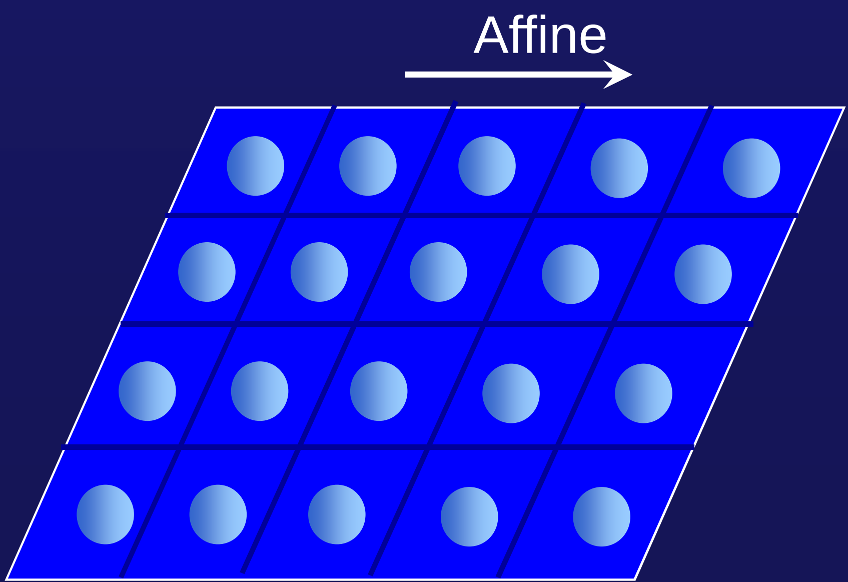


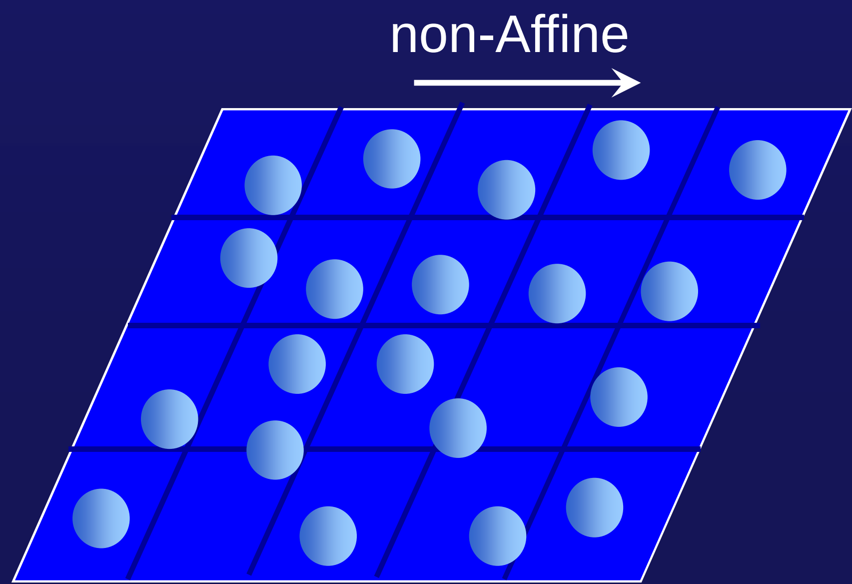
Affine transformation : γ

$$\Delta \mathbf{r}' = \Delta \mathbf{r} + \gamma \Delta \mathbf{r}$$

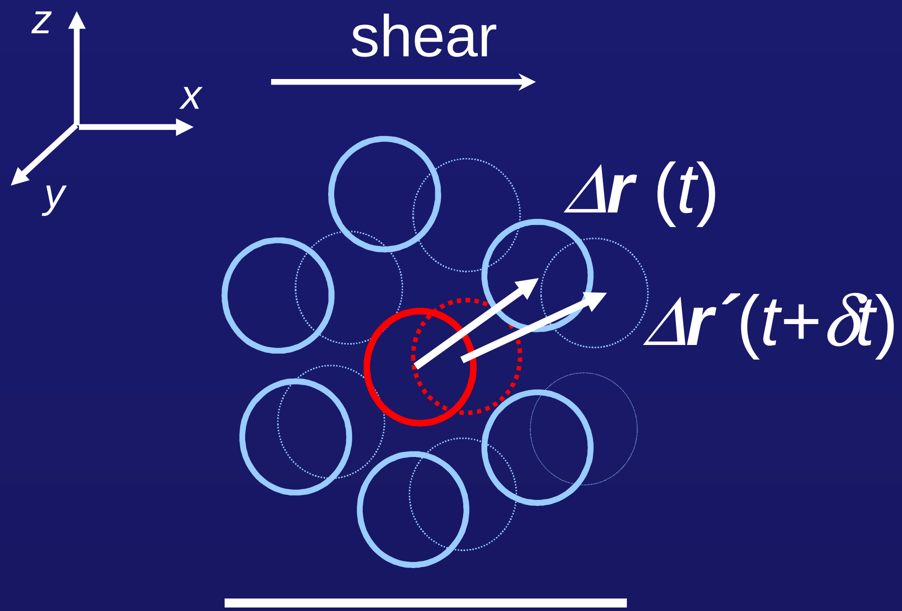
$$D^2_{min} = \sum^{\text{neighbors}} (\Delta \mathbf{r}' - \gamma \Delta \mathbf{r})^2$$

Falk and Langer, *PRE* 1998.





Strain and non-affine displacements



Affine transformation : γ

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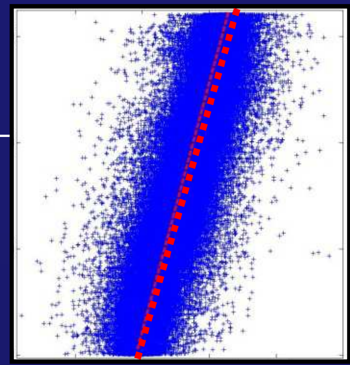
$$D^2_{min} = \sum^{neighbors} (\Delta \mathbf{r}' - \gamma \Delta \mathbf{r})^2$$

Falk and Langer, *PRE* 1998.

Symmetric part of affine component...

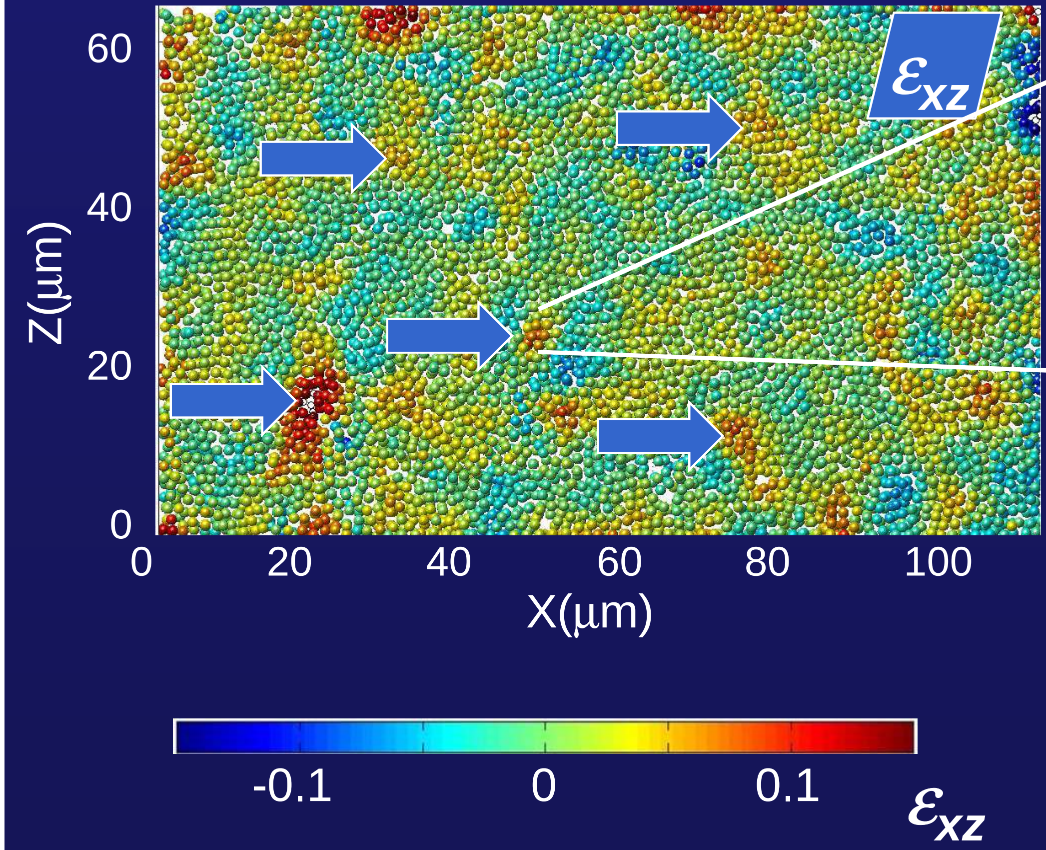
$$\text{Strain tensor } \varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

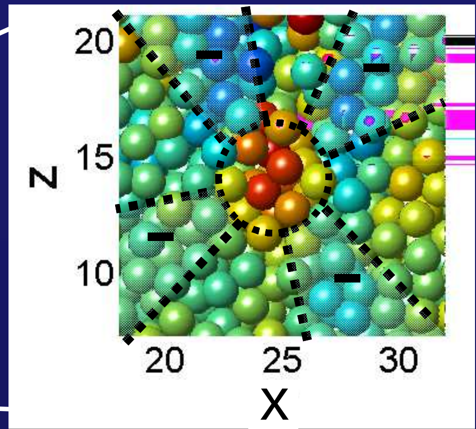
Homogeneous flow



$\dot{\gamma}\tau \sim 0.2$

Affine part





Induce
Local Quadrupole $\varepsilon \sim 1/r^3$

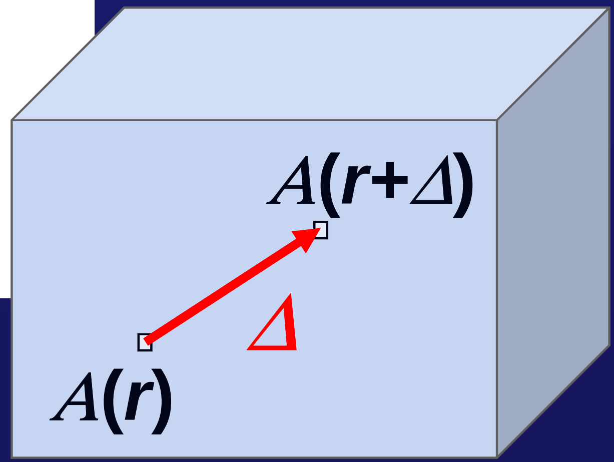
Schall, Weitz, Spaepen, *Science* (2007)

Spatial Correlations

$$C_A(\Delta) = \frac{\langle A(\bar{r})A(\bar{r}+\Delta)\rangle - (\langle A\rangle)^2}{\langle (A)^2\rangle - (\langle A\rangle)^2}$$

Δ : *difference vector*

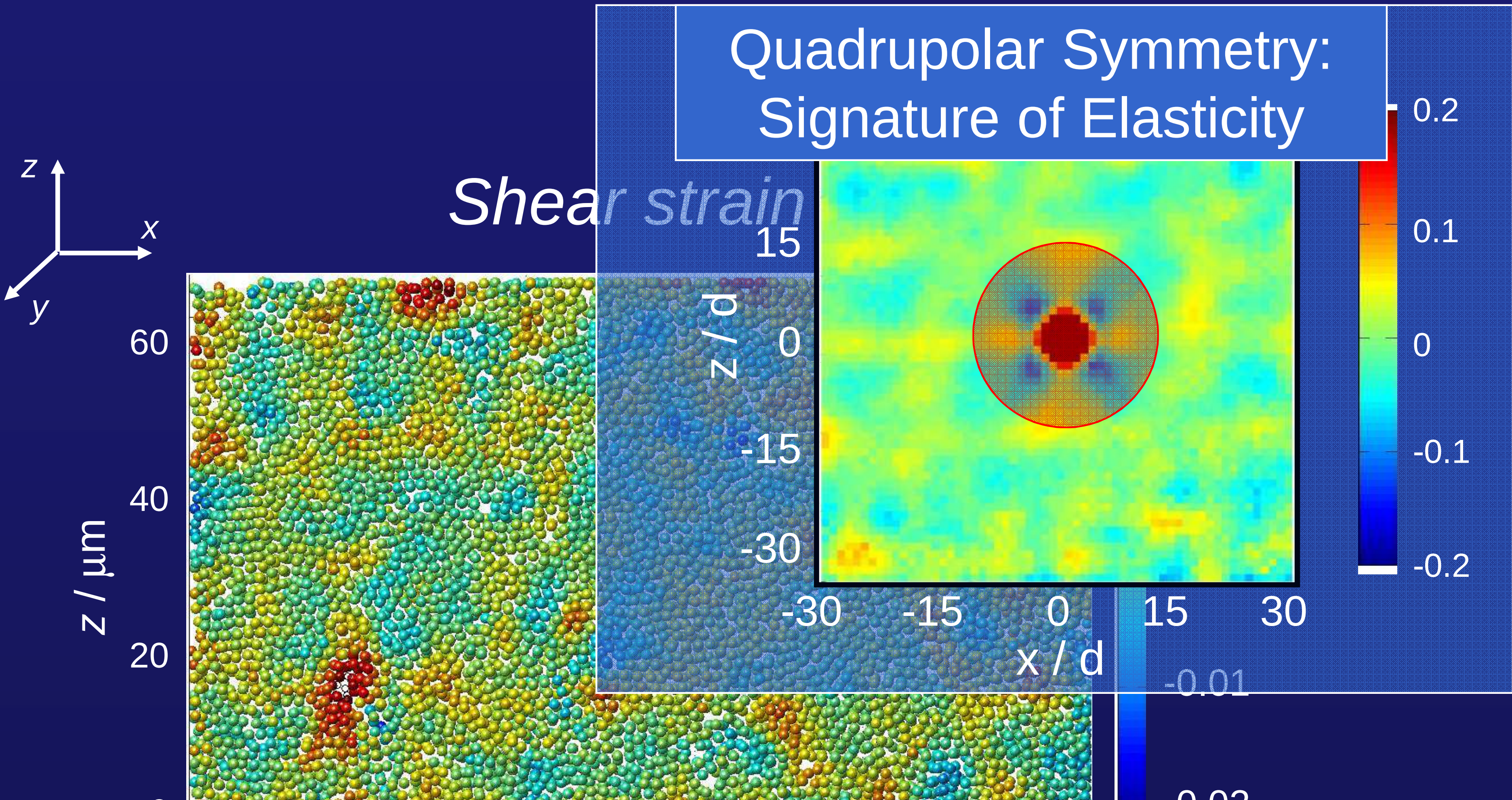
$\langle\ \rangle$: *spatial average*



Strain correlation : $A = \varepsilon_{xz}$

Non-affine correlation : $A = D^2_{min}$

Homogeneous flow

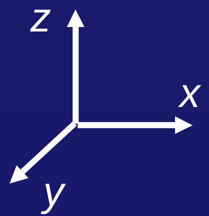


Elastic interactions
→ Self organization of STZ

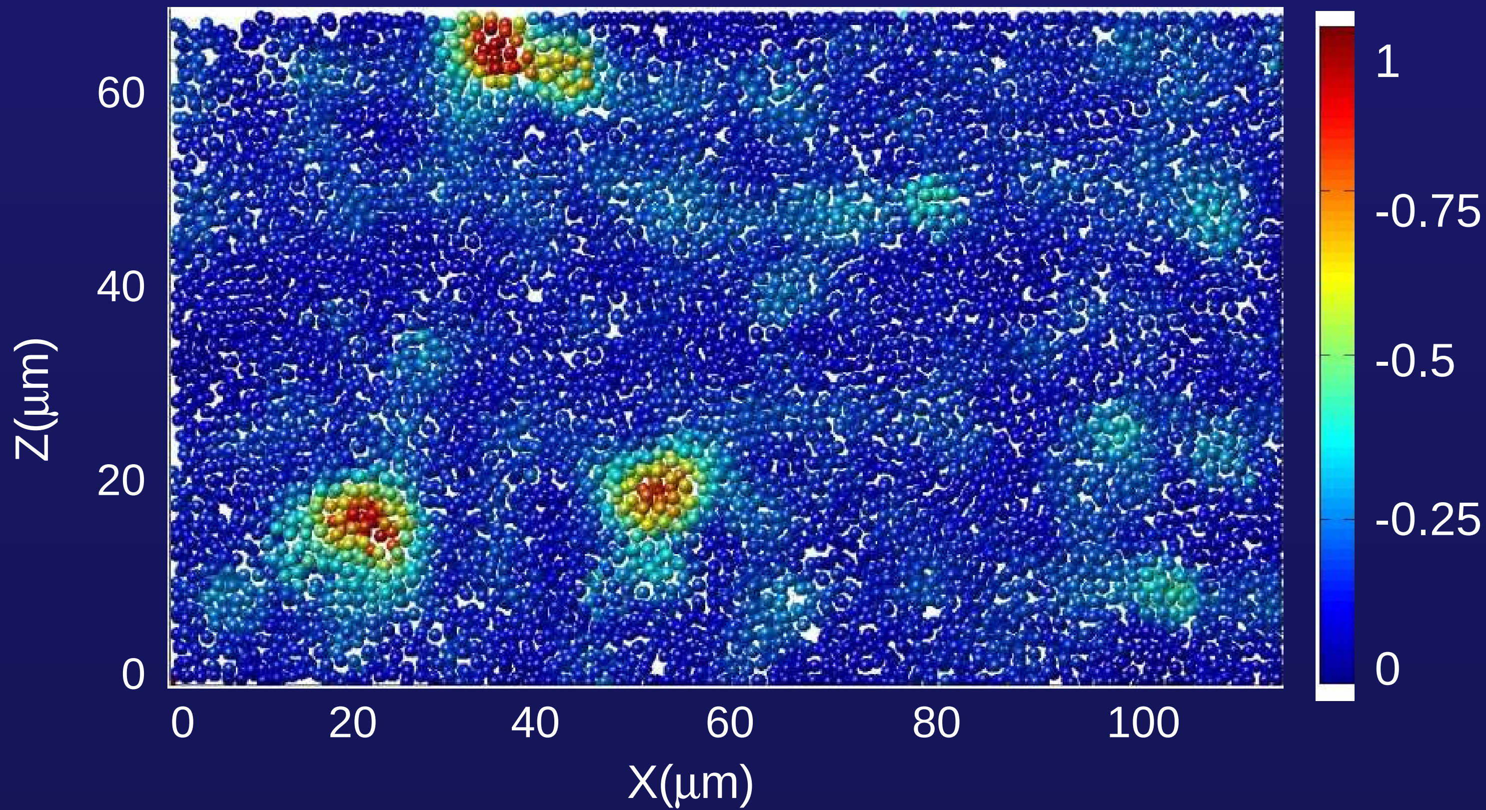
V. Chikka

V. Chikkadi, P.S., PRE 2012

Homogeneous flow



Non-affine part D^2

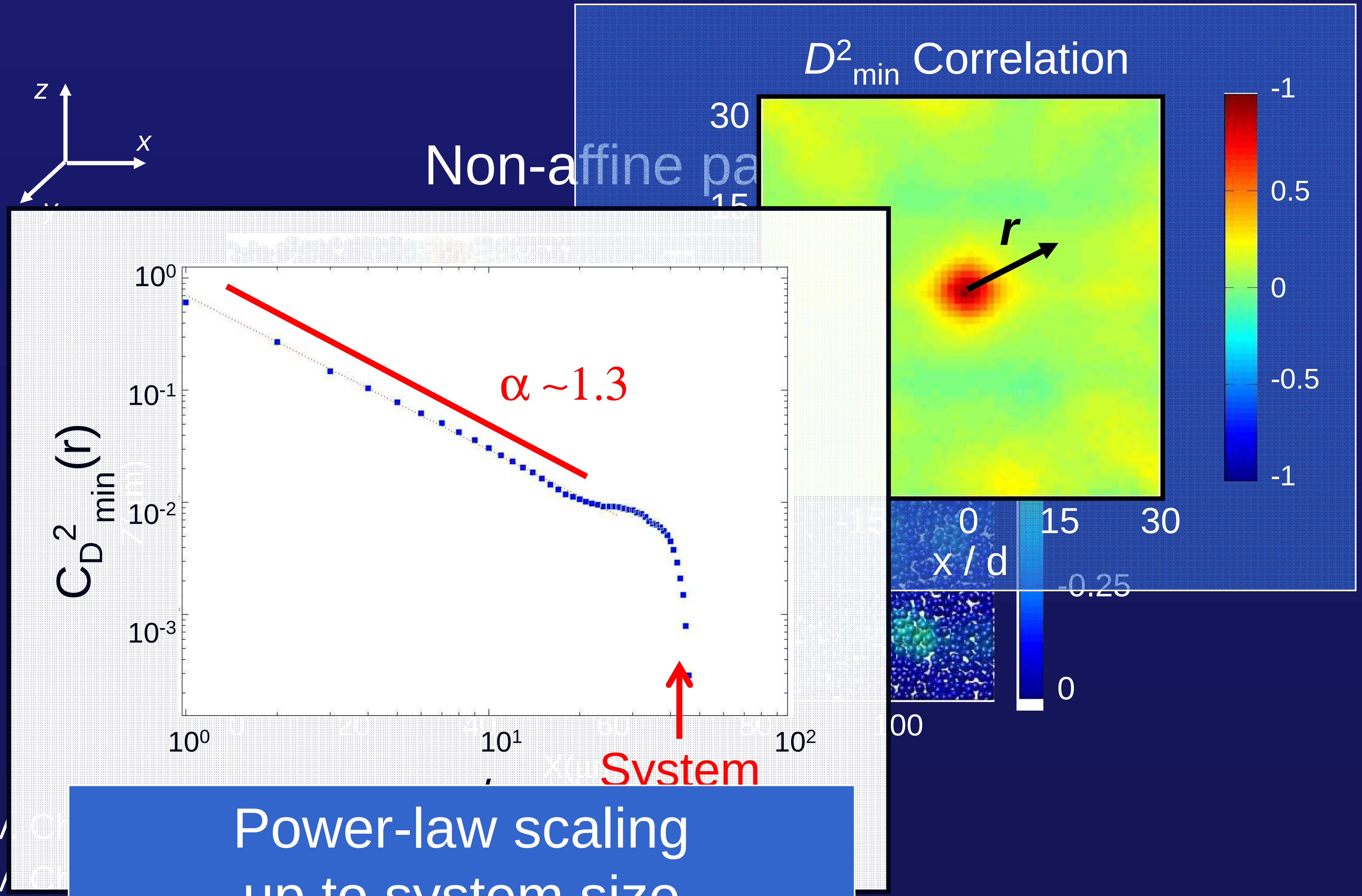


V. Chikkadi, G.Wegdam, B. Niehuis, P.S., PRL 2011

V. Chikkadi, P.S., PRE 2012

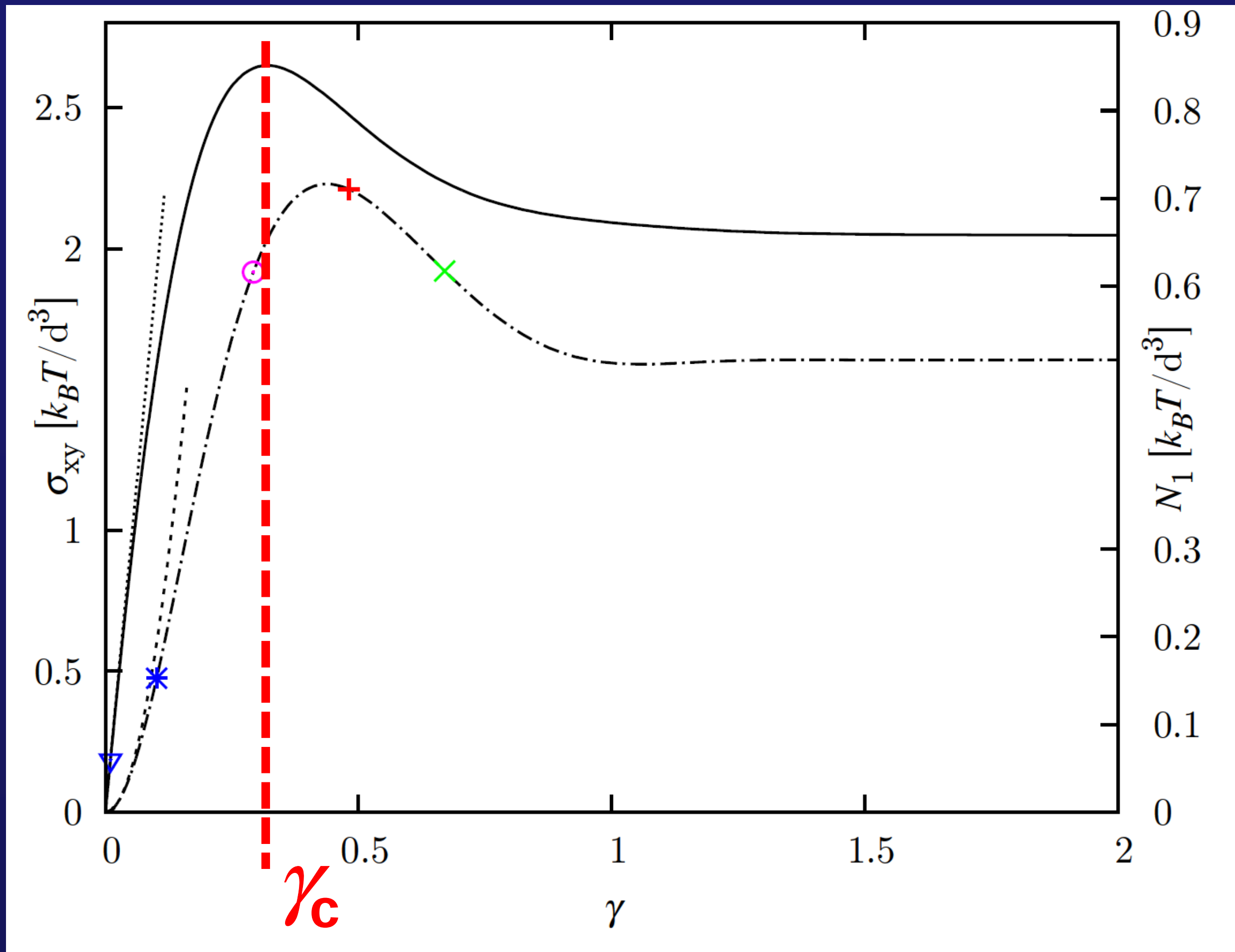
Homogeneous flow

Non-affine pa



Power-law scaling up to system size

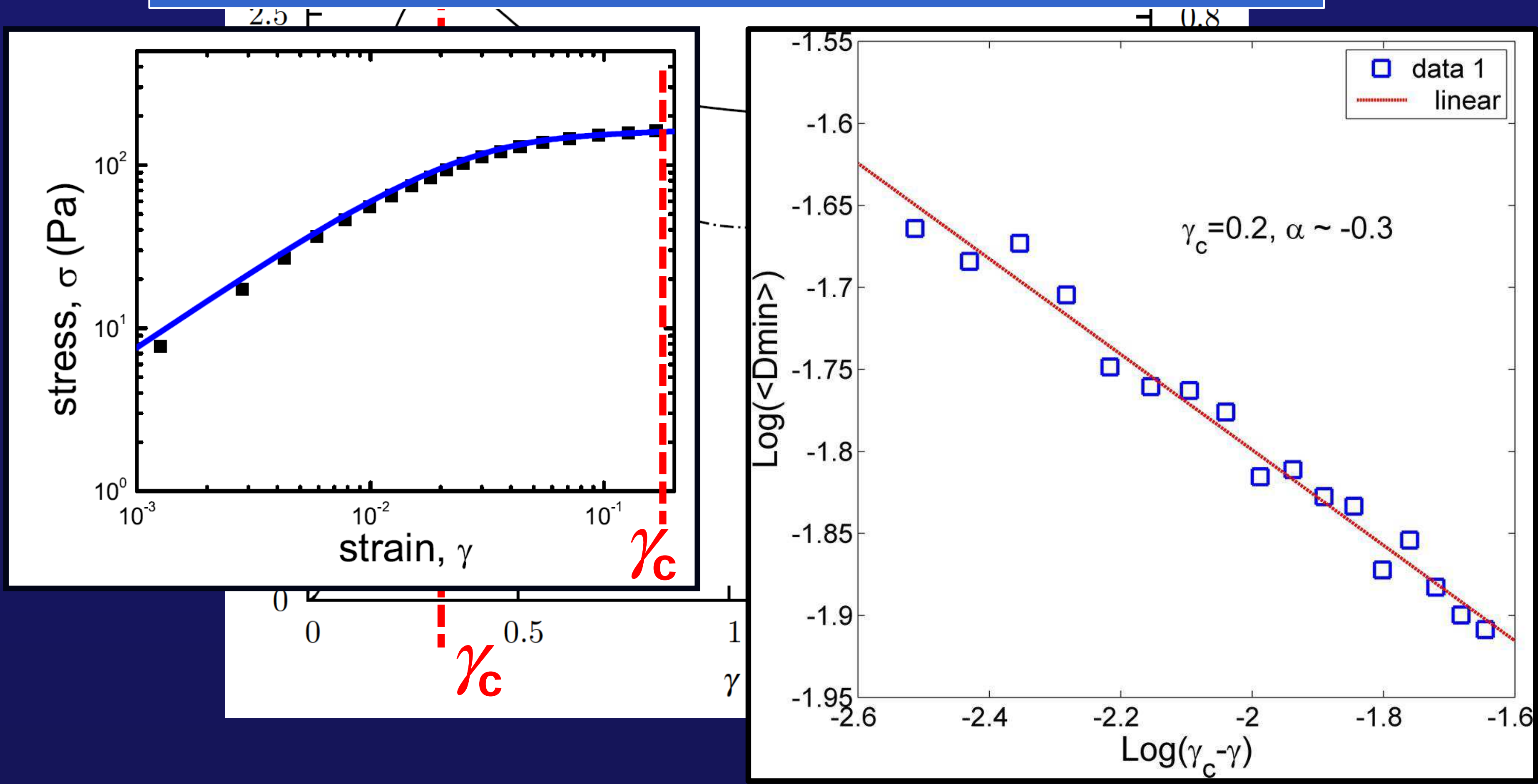
Yielding: 2nd order transition



A. Ghosh, V. Chikkadi, P. Schall (preprint 2015)

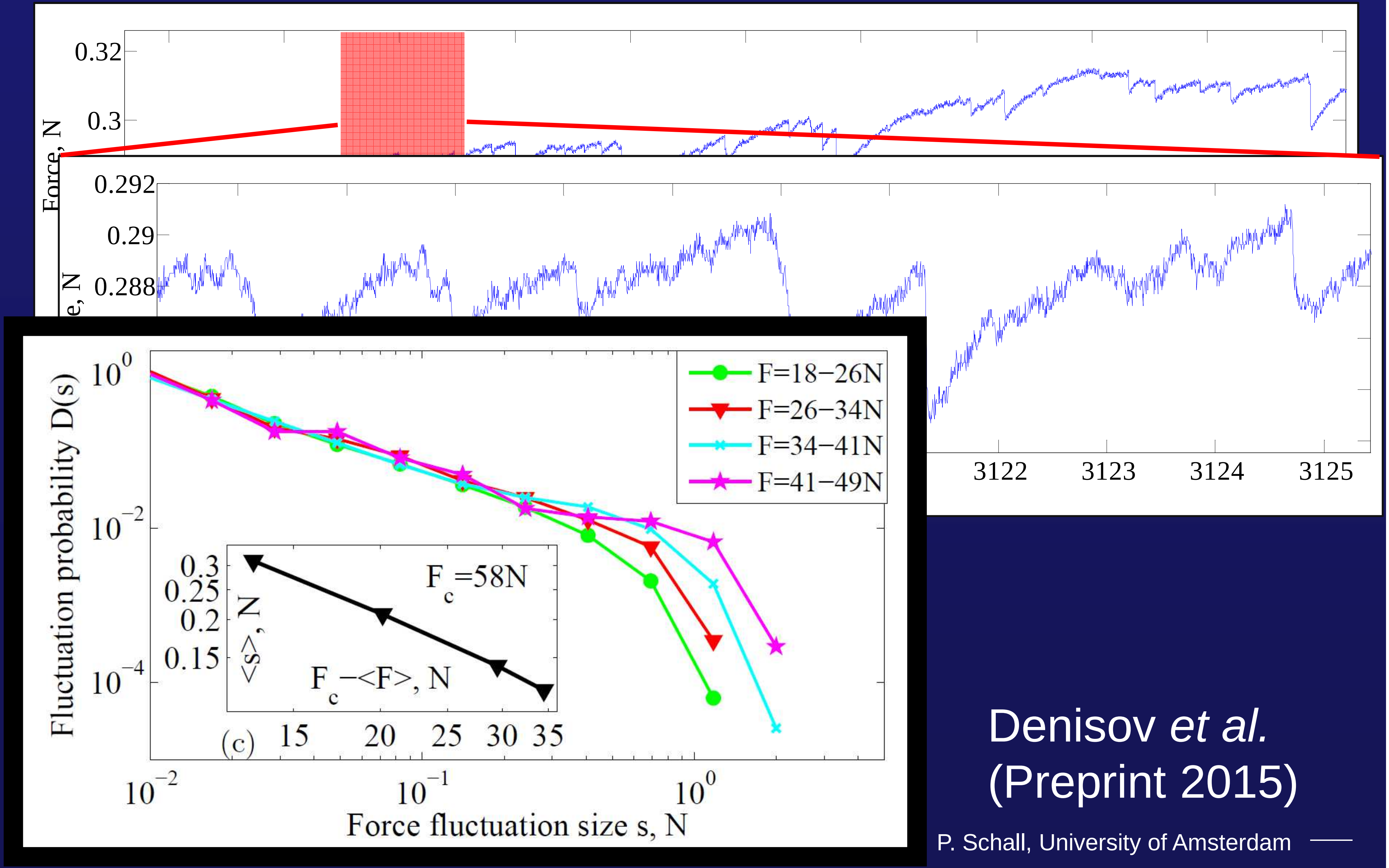
Yielding: 2nd order transition

Yield point as critical point



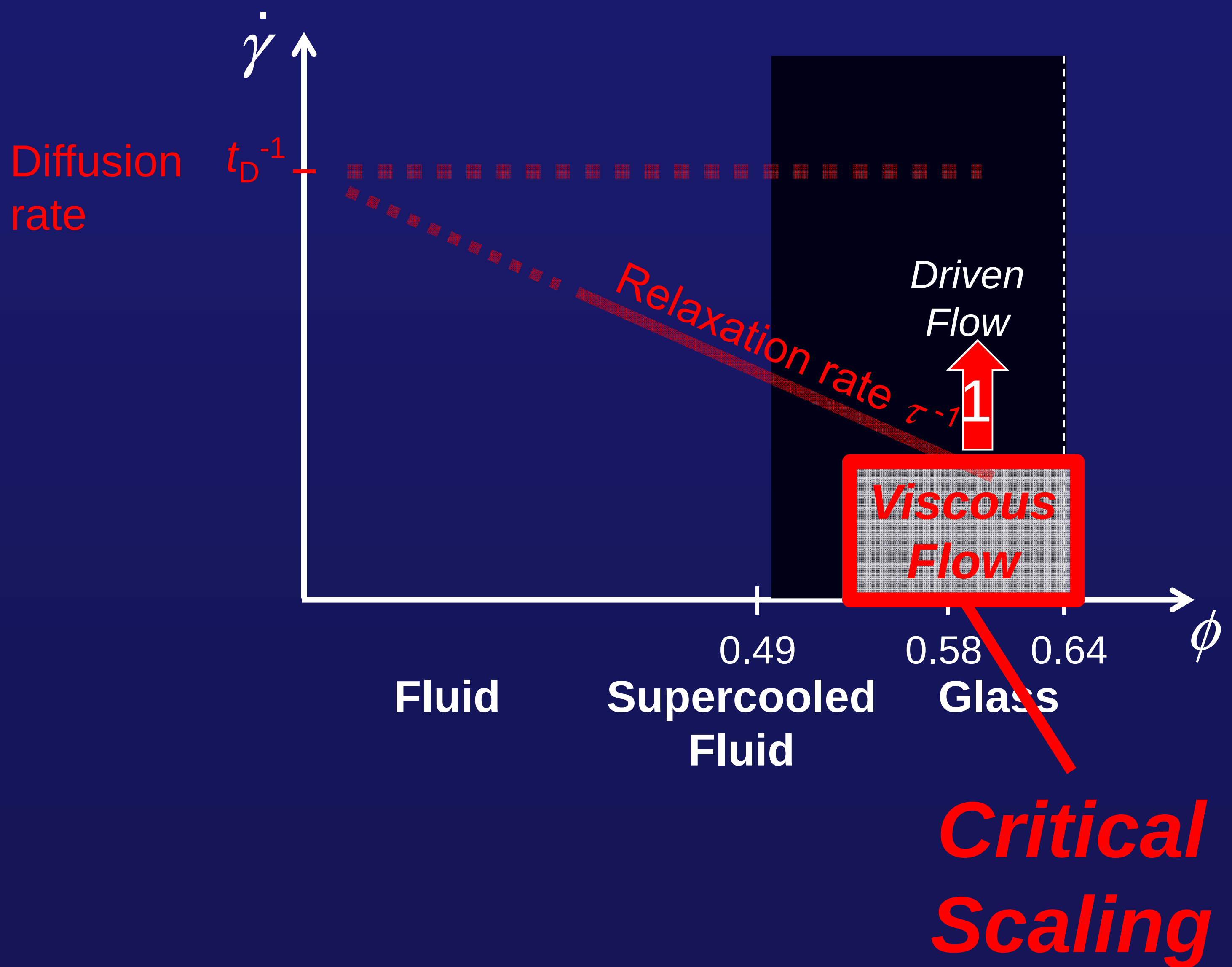
A. Ghosh, V. Chikkadi, P. Schall (preprint 2015)

Sheared Granulate – Force fluctuations



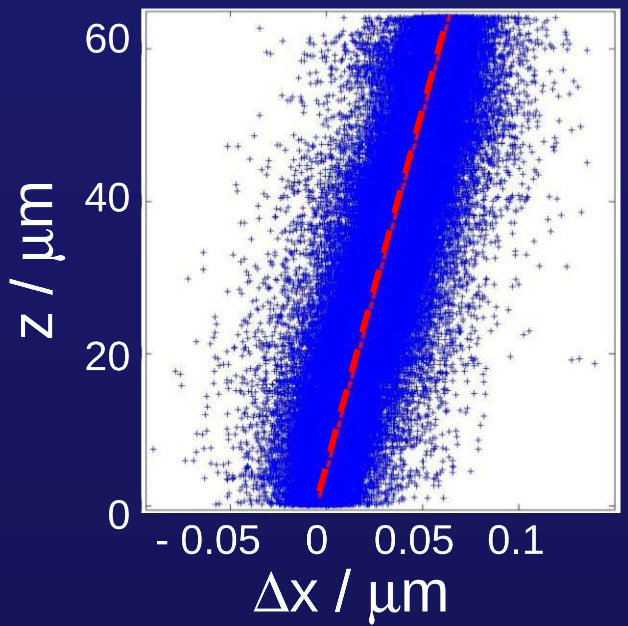
Denisov *et al.*
(Preprint 2015)

…towards faster flow

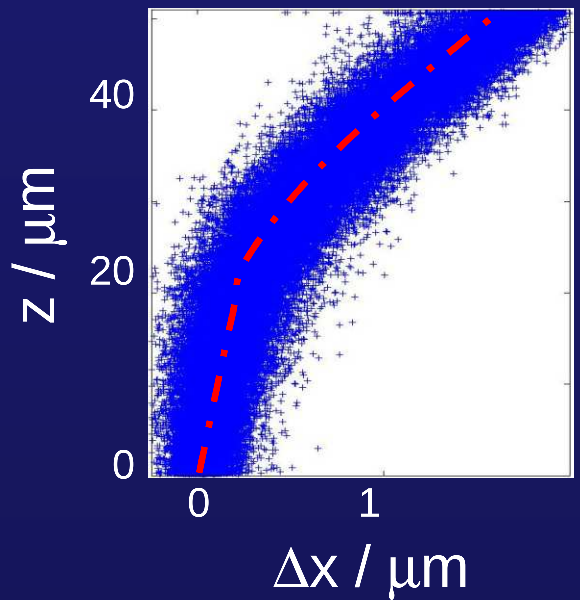


1. Transition to Driven flows

Homogeneous



Inhomogeneous

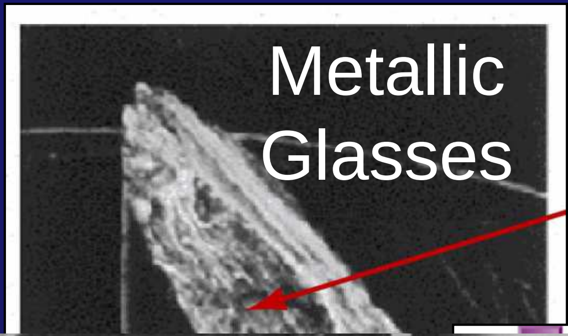


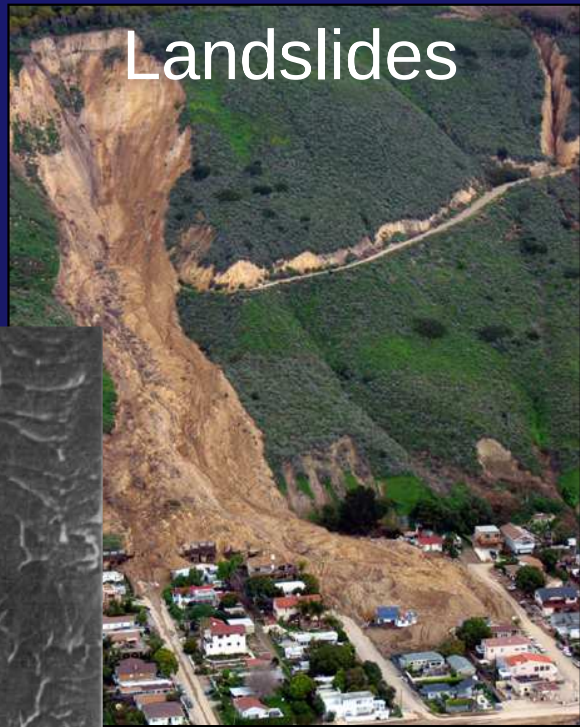
1 $\dot{\gamma}\tau$

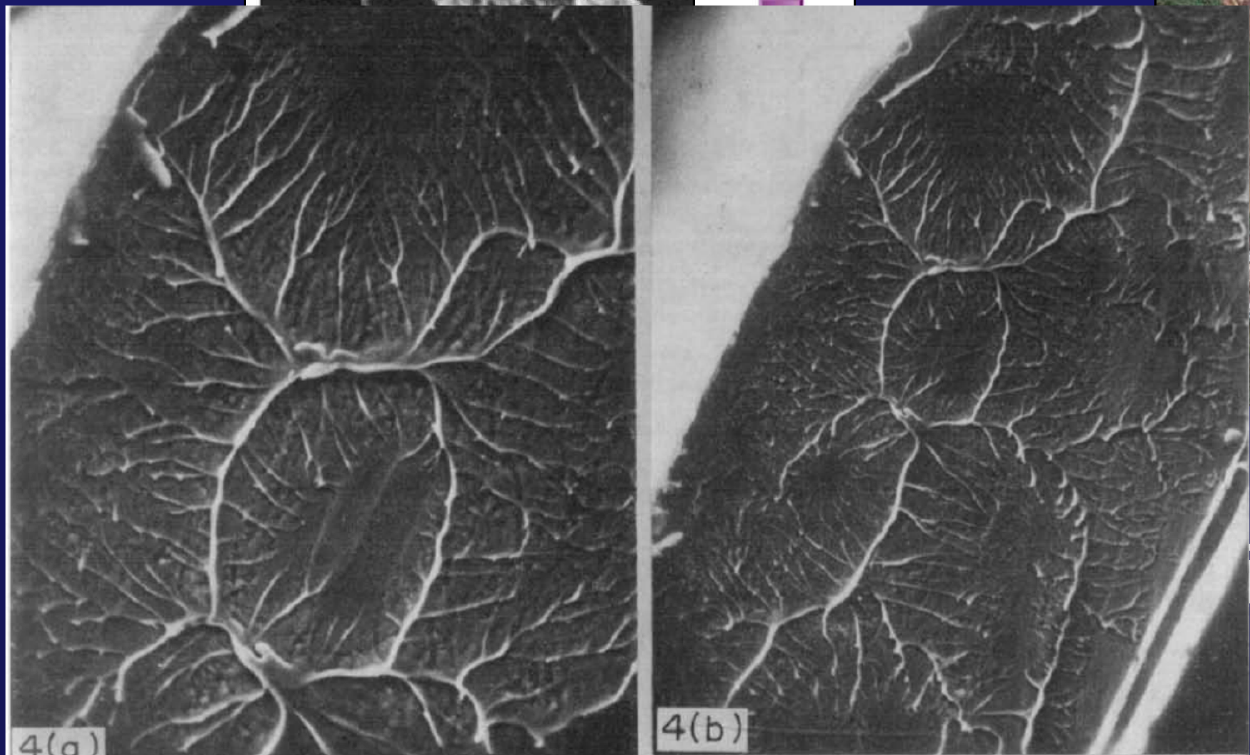
V. Chikkadi et al. *Phys. Rev. Lett.* (2011), *Phys. Rev. Lett.* (2014)

1. Transition to Driven flows

?







ion:

Failure

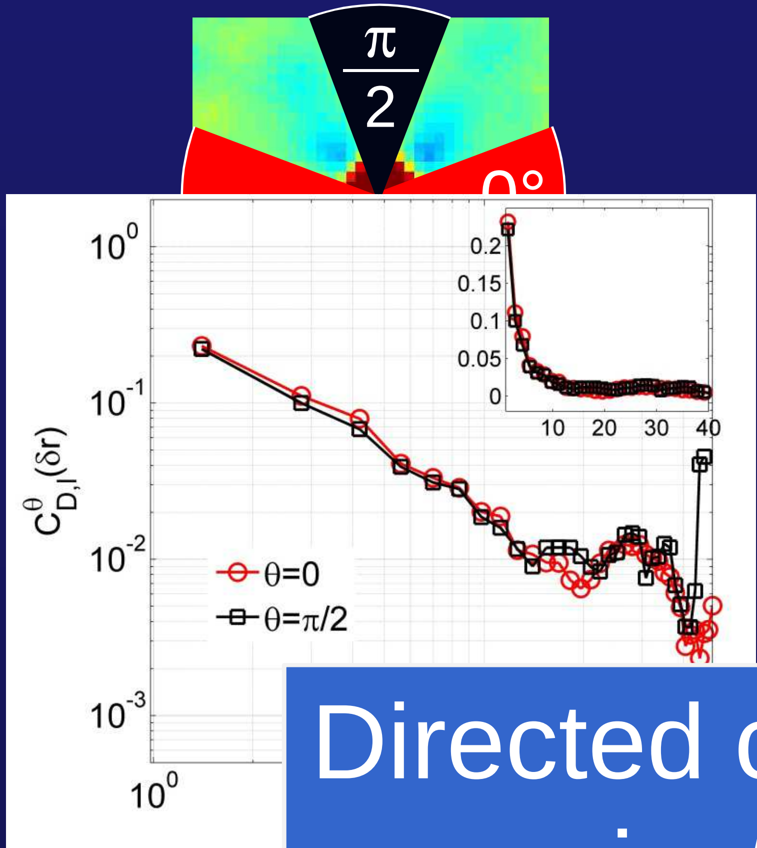
Anisotropic Strain correlations

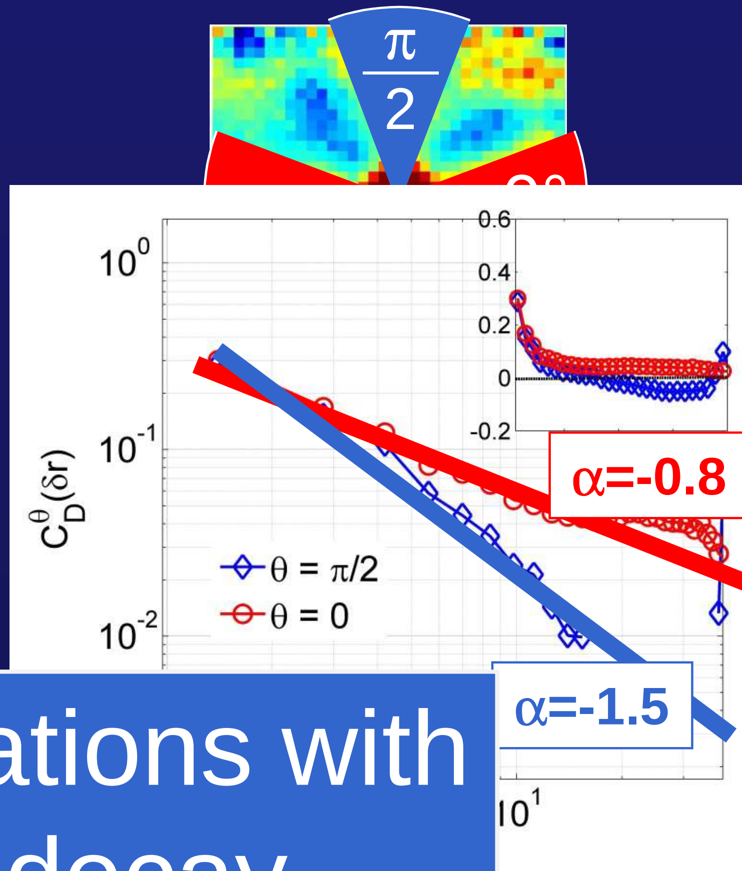
$\dot{\gamma} \cdot \tau_{\alpha} < 1$

Thermal regime

$\dot{\gamma} \cdot \tau_{\alpha} < 1$

Stress regime

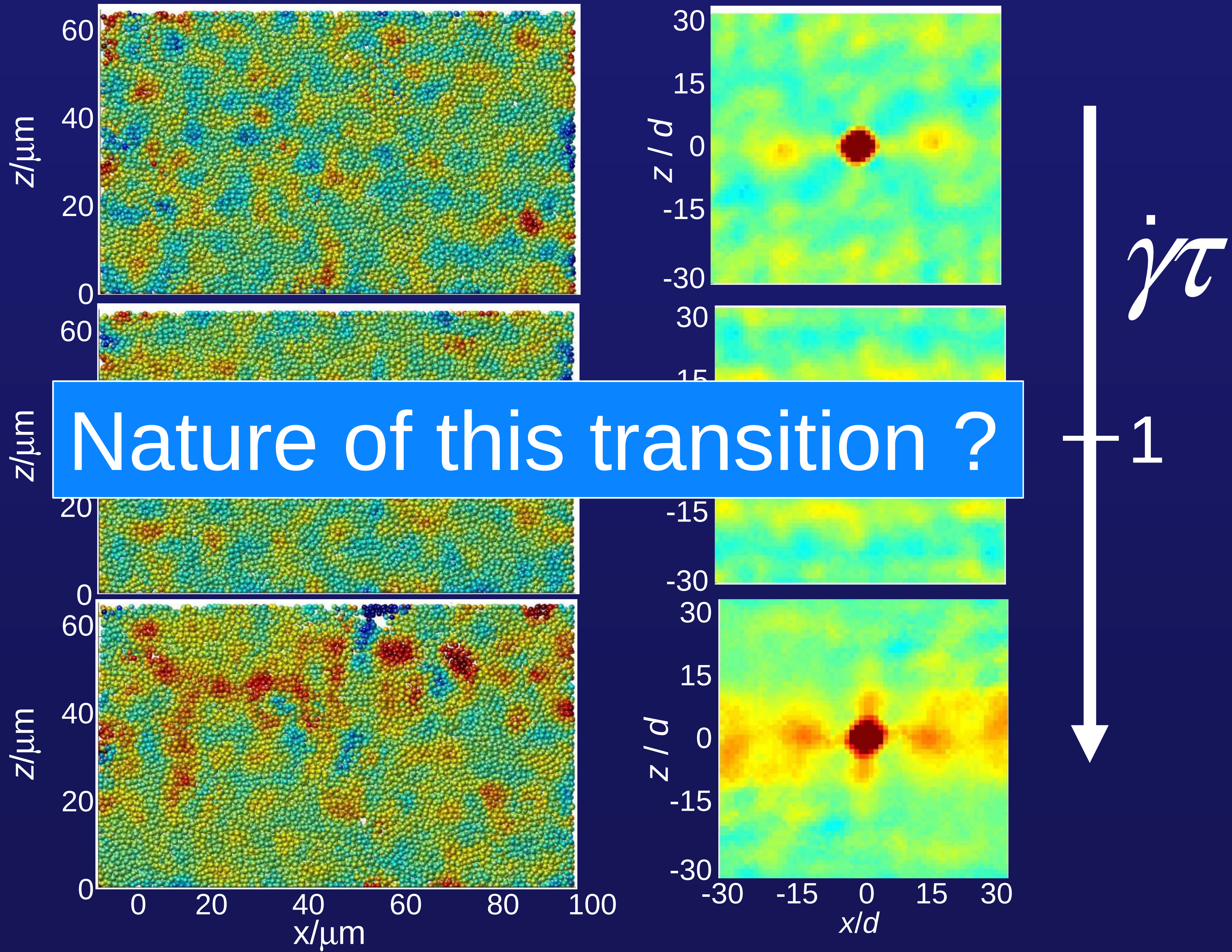




Directed correlations with anisotropic decay

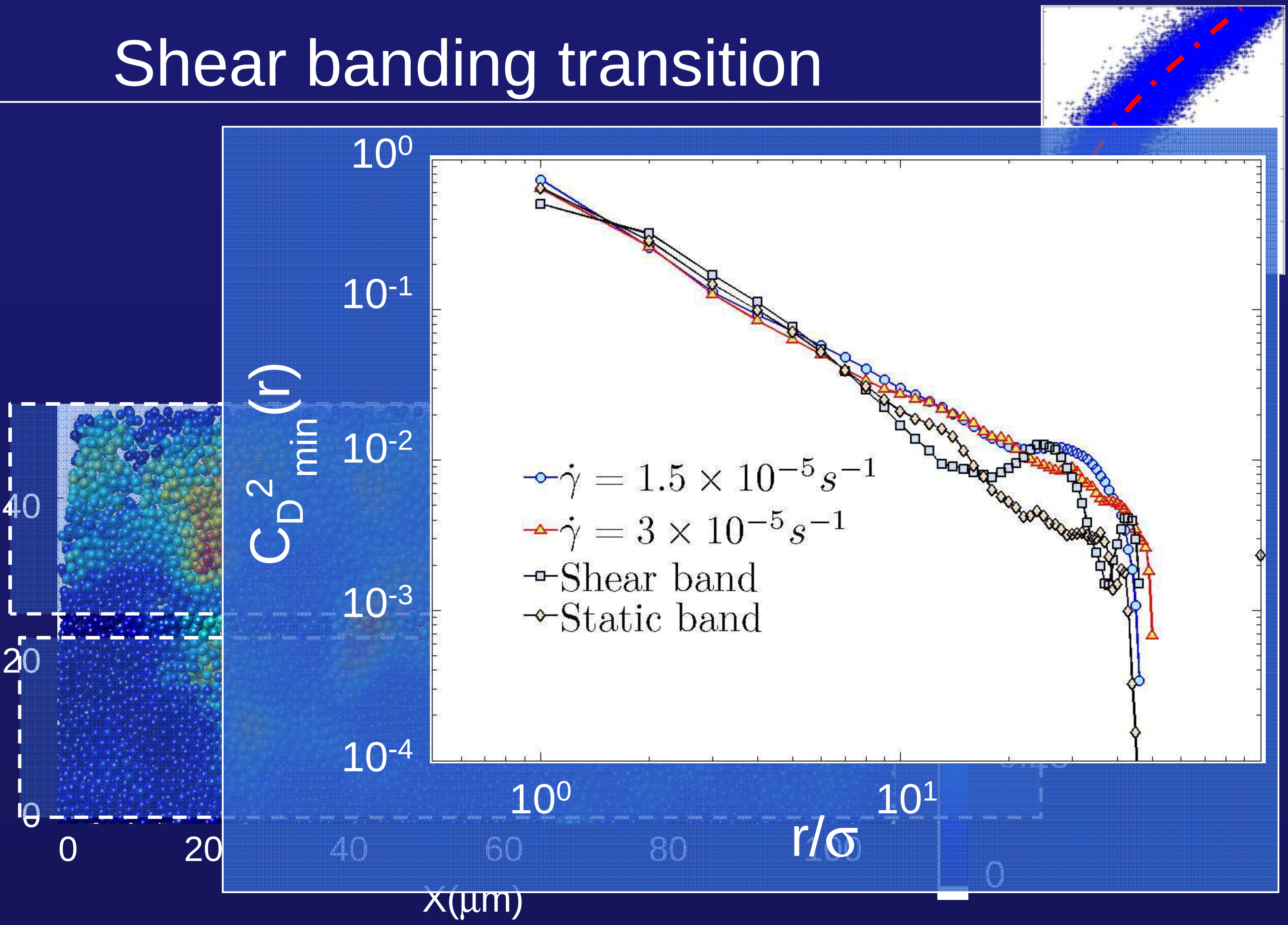
V. Cikkadi, S.Mandal, F.Varnik, P.S., *EPL* (2012)

Shear banding transition



Nature of this transition ?

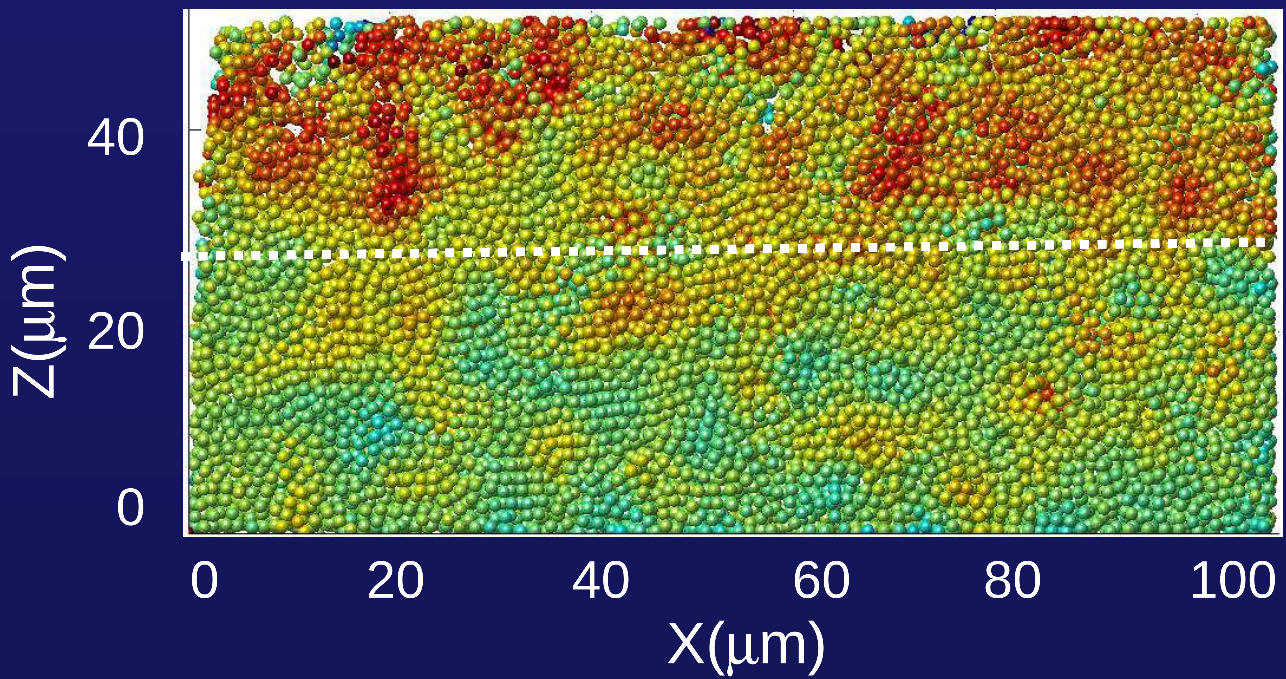
Shear banding transition



V. Chikkadi, G.Wegdam, B. Niehuis, P.S., PRL 2011

V. Chikkadi, P.S., PRE 2012

Shear banding transition

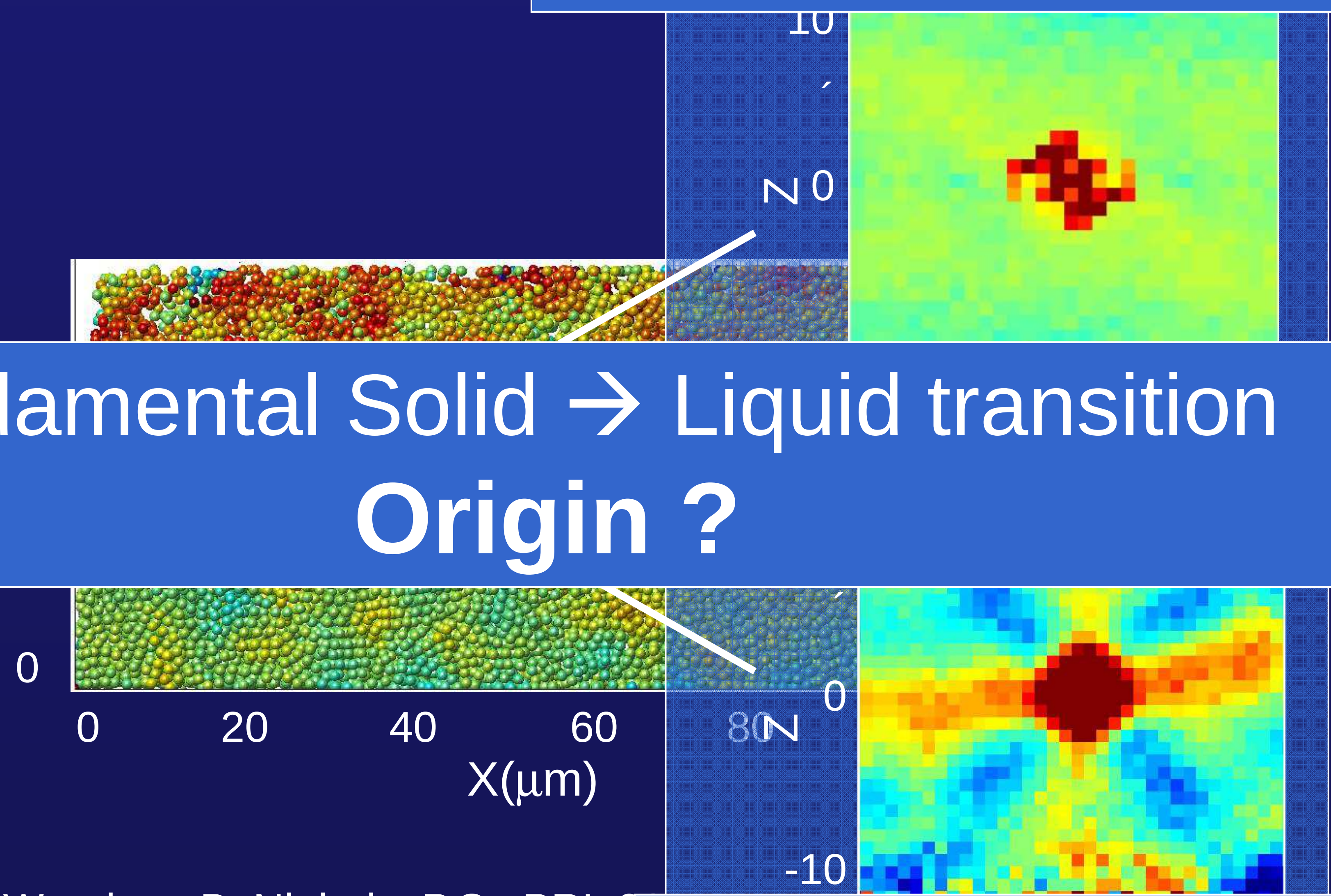


V. Chikkadi, G.Wegdam, B. Niehuis, P.S., PRL 2011

V. Chikkadi, P.S., PRE 2012

Shear bandin

No quadrupolar symmetry
liquid like

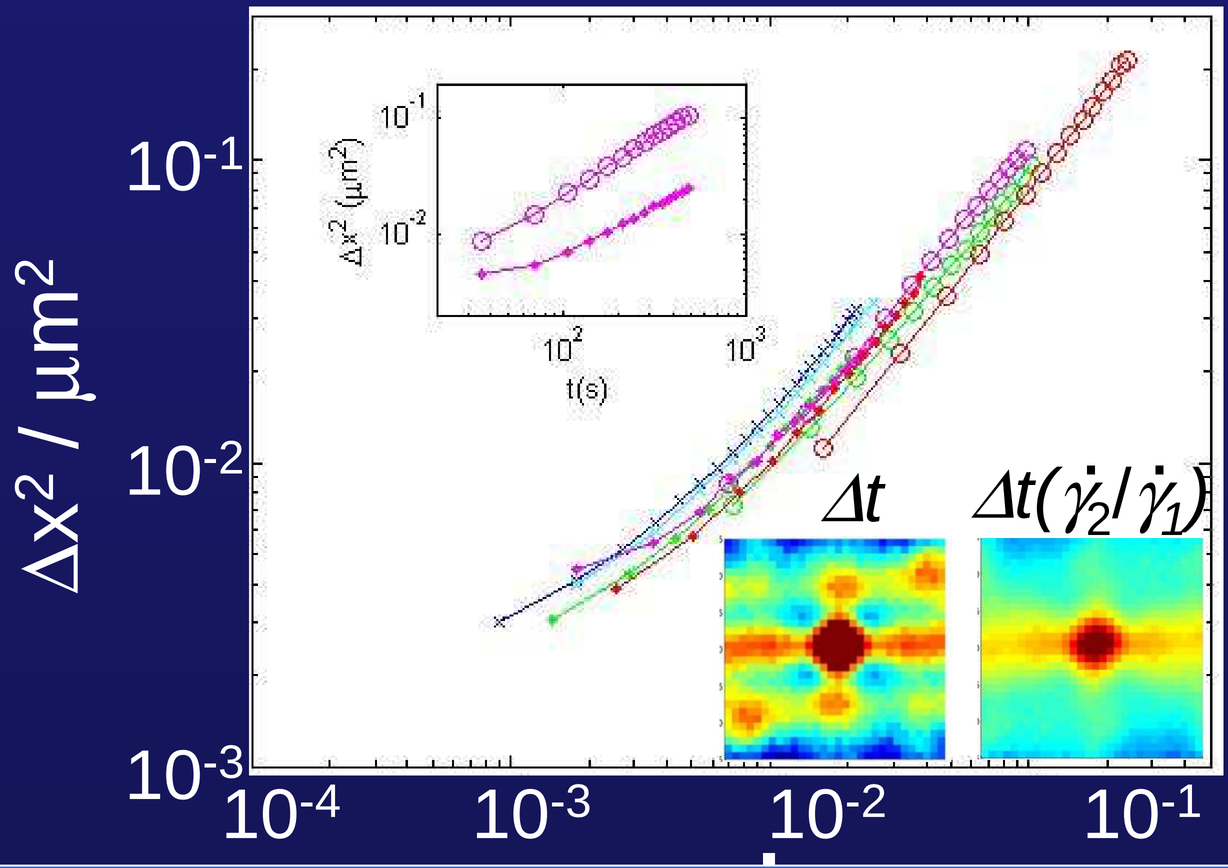


Fundamental Solid → Liquid transition
Origin ?

Quadrupolar symmetry
solid like

V. Chikkadi, G.Wegdam, B. Niehuis, P.S., PRL 2
V. Chikkadi, P.S., PRE 2012

First order transition – diffusion time scale

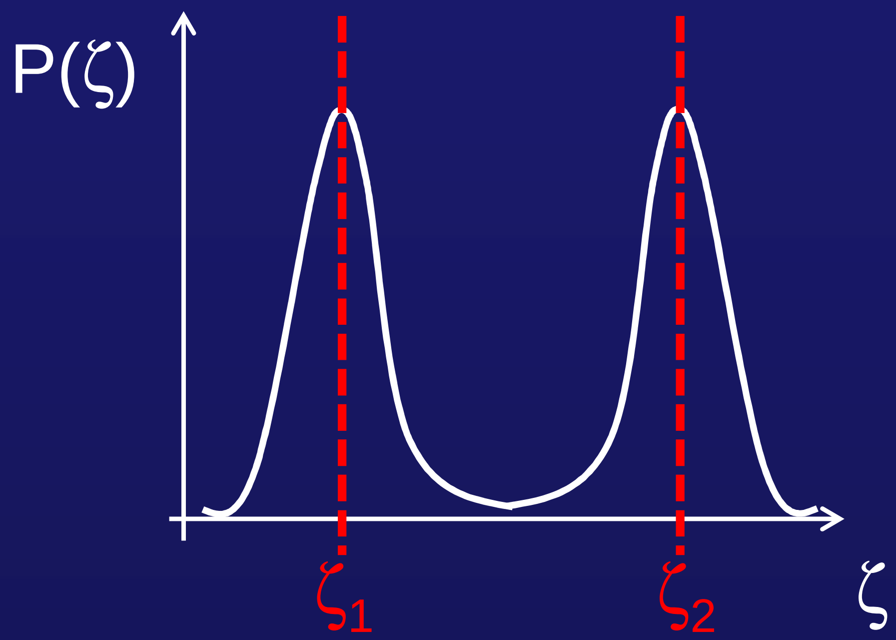


Discontinuity in diffusion time scale!

V. Chikkadi, et al. *Phys. Rev. Lett.* (2014)

First order transition ?

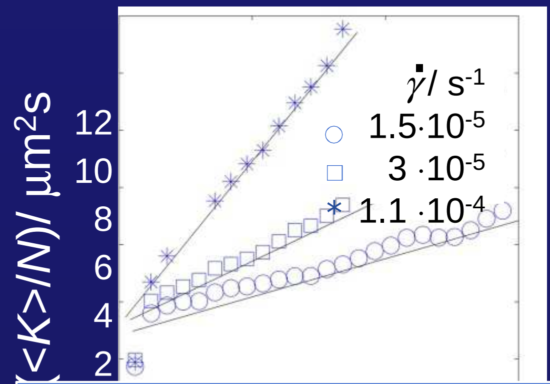
Order Parameter ζ



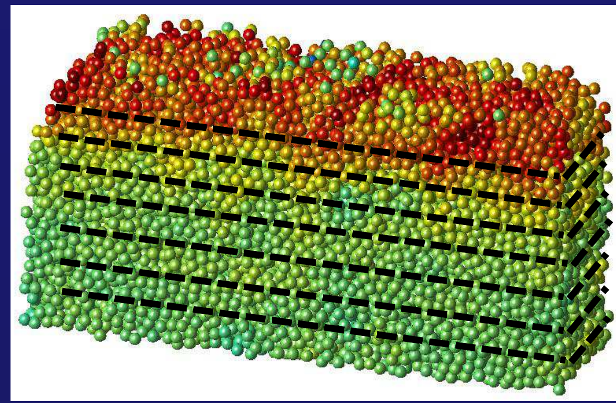
ζ : Time evolution

What is the right order parameter ?

First order transition ?



$\zeta = \langle K \rangle / t_{obs}$



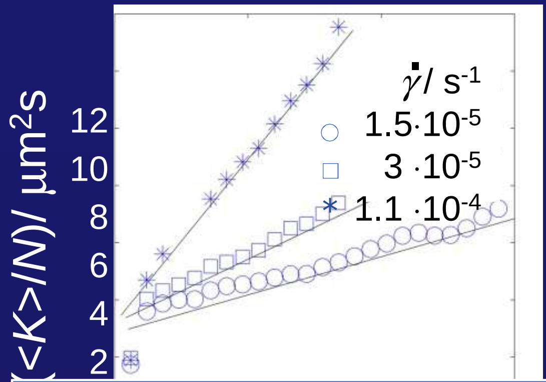
V. Chikkadi, et al.
Phys. Rev. Lett. (2014)

Dynamic Order Parameter (Garrahan, Chandler 2009)

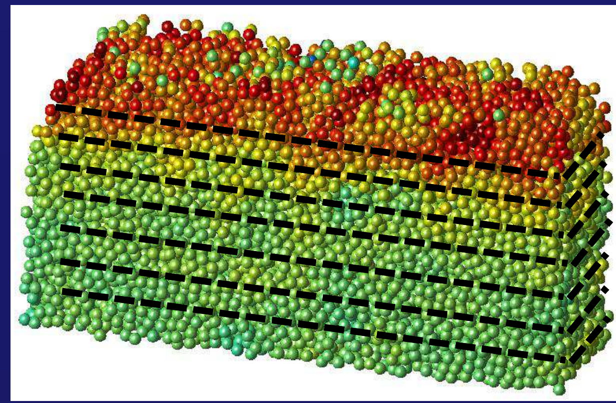
$$K = \sum_{i=1}^{N} \sum_{t=0}^{t_{obs}} |\delta \mathbf{r}_i|^2$$

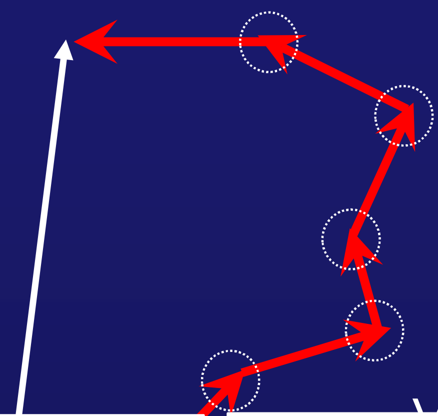
extensive in Space AND Time

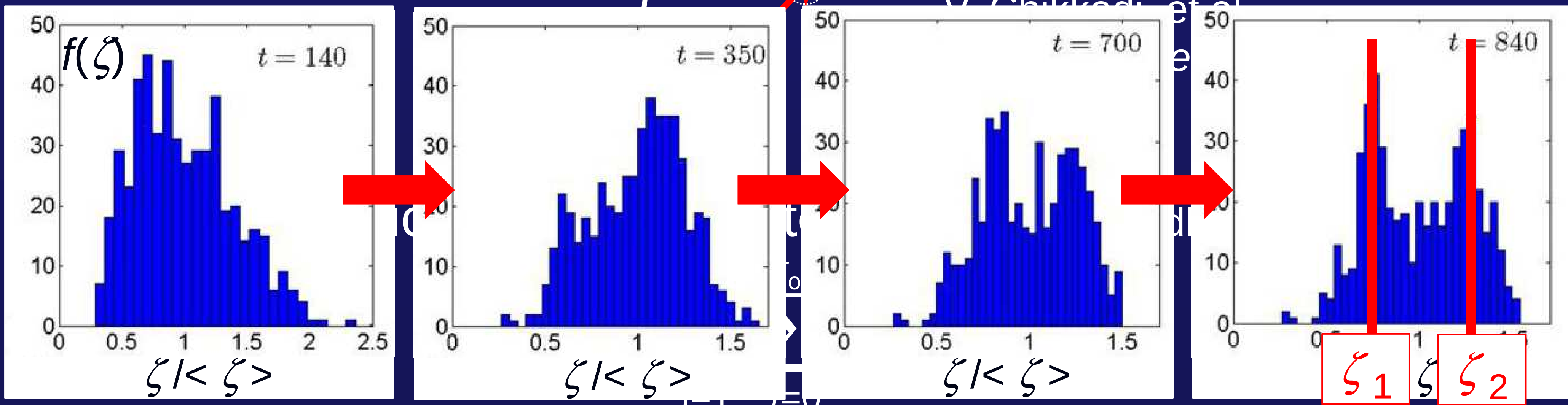
First order transition ?



$$\zeta = \langle K \rangle / t_{obs}$$

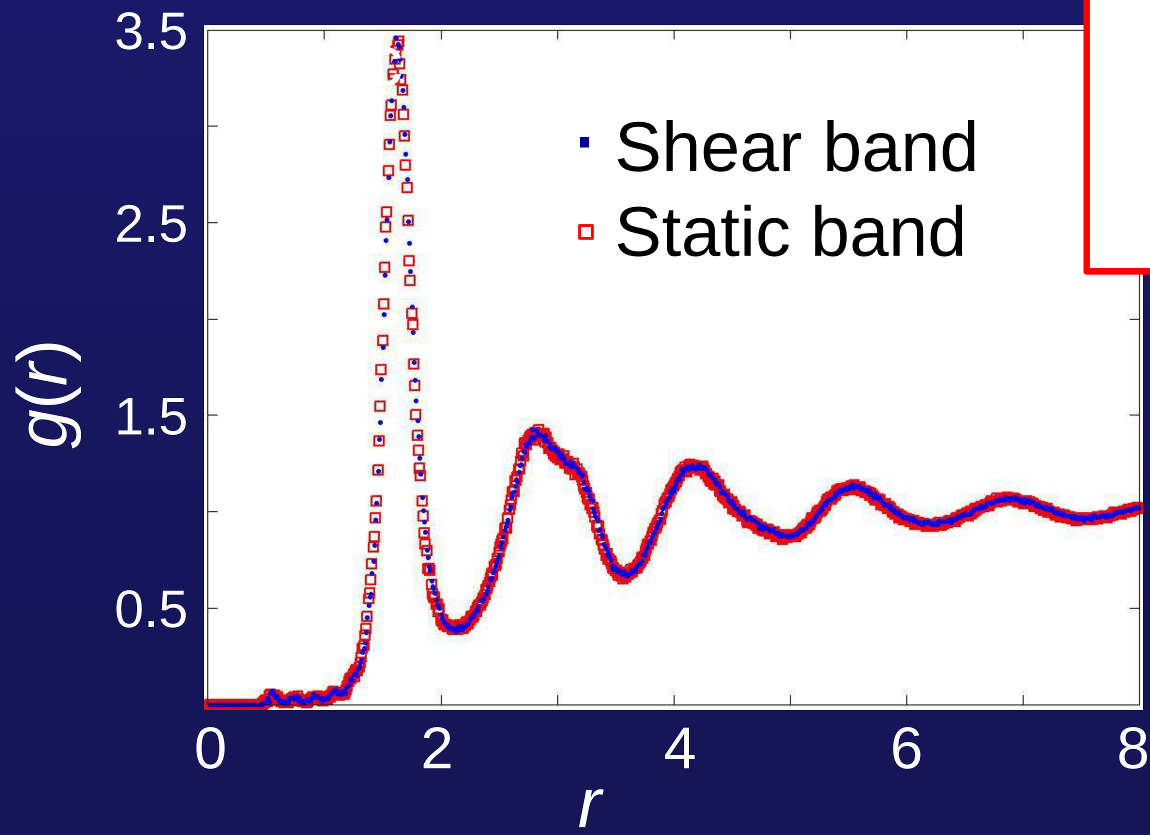


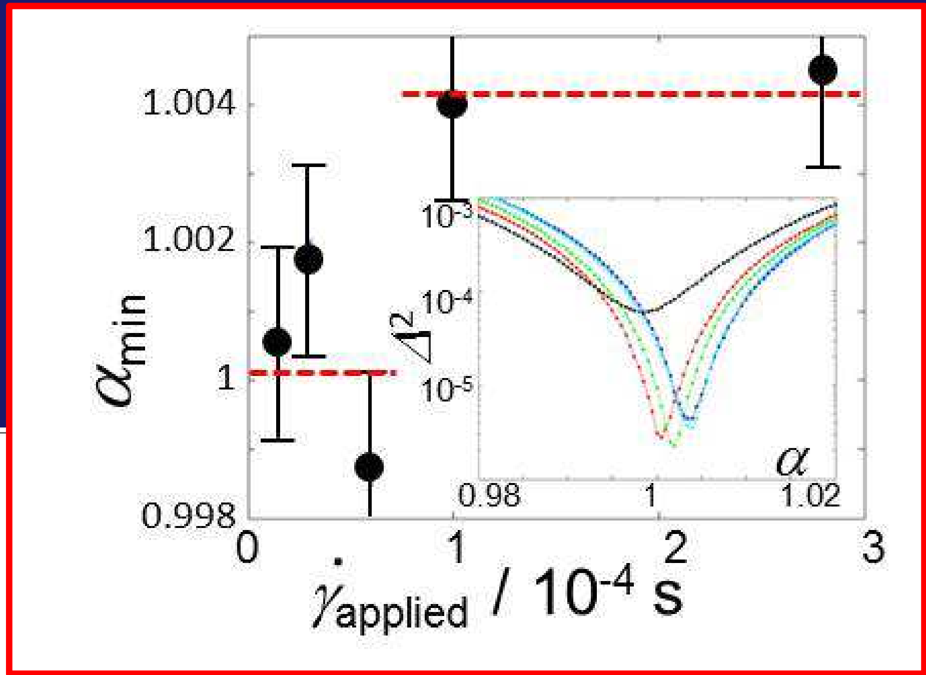




First order transition in 4D space-time

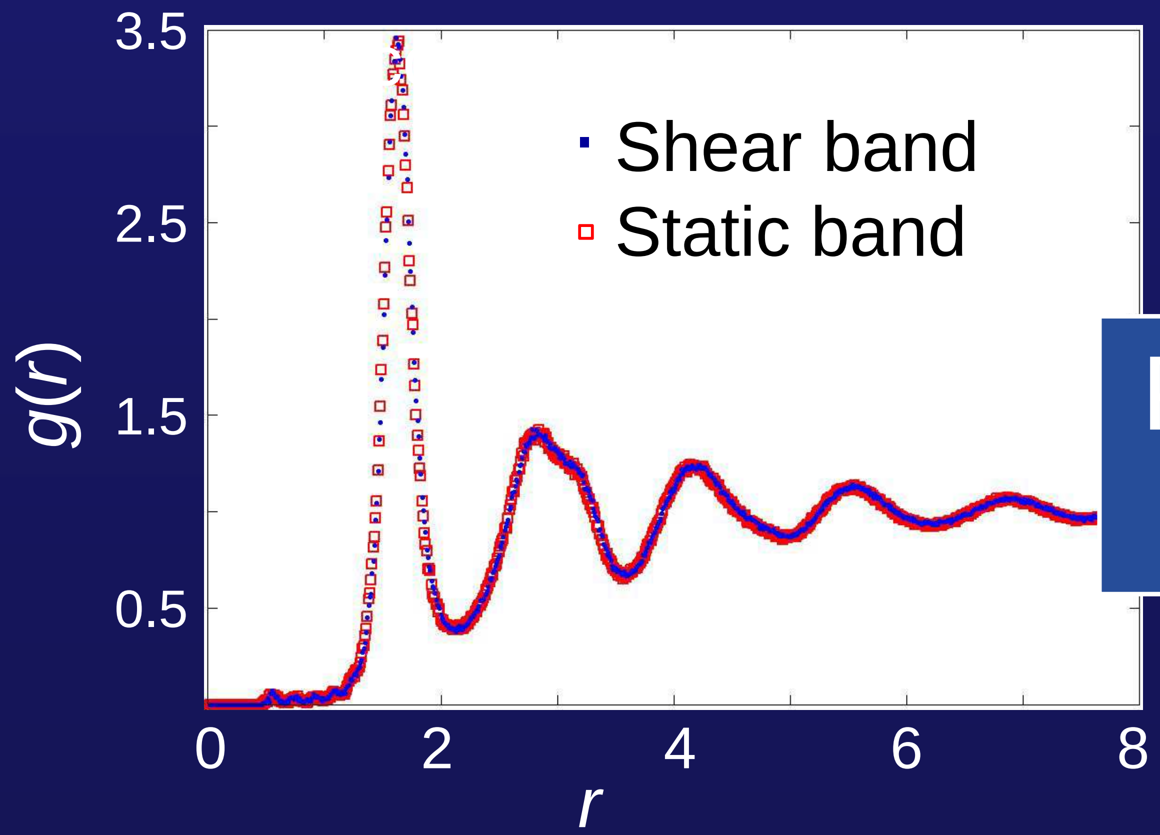
Structural transition ?

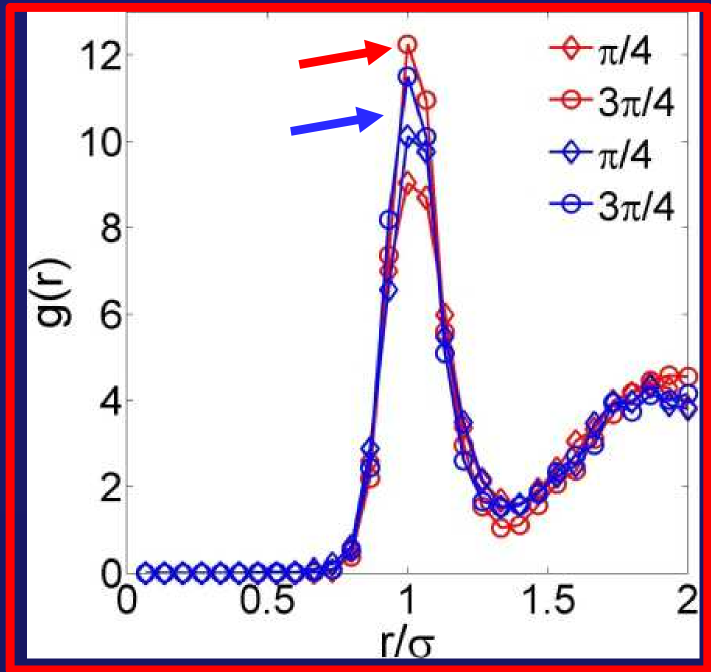




0.4% dilation

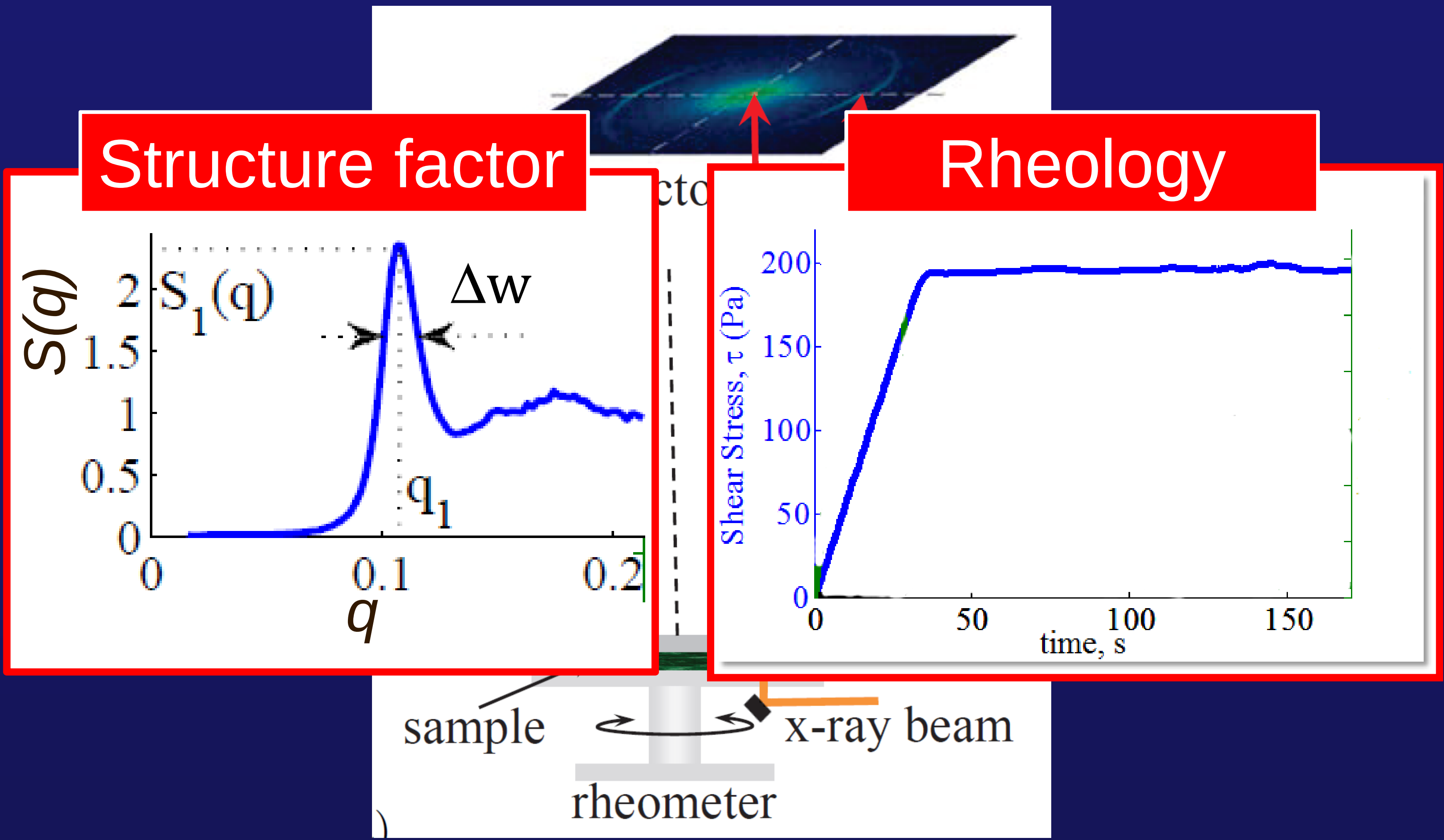
Structural transition ?





Particle connectivity changes

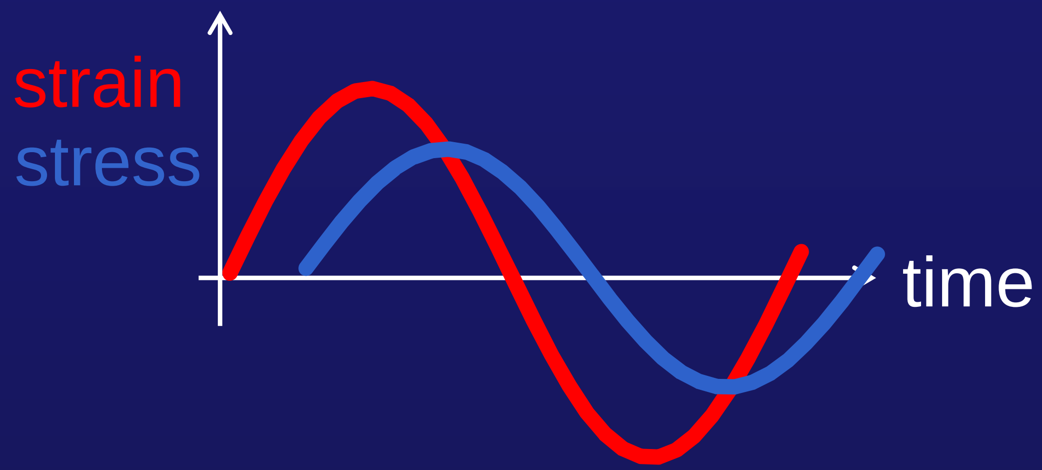
Simultaneous X-ray + Rheology



D. Denisov, T. Dang, B. Struth, P.S., *Sci. Rep.* (2013)

Origin of material failure?

“Mechanical Spectroscopy“



Linear Response:

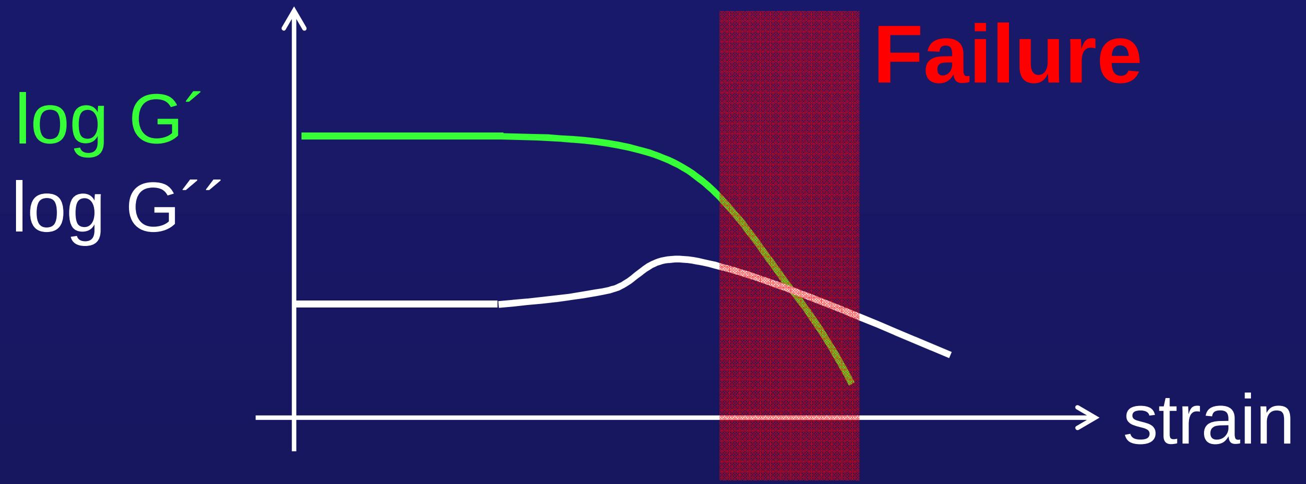
$$\text{stress} = G' \gamma_0 \sin(\omega t) + G'' \dot{\gamma}_0 \cos(\omega t)$$

storage modulus

loss modulus

Origin of yielding?

“Mechanical Spectroscopy“



Linear Response:

$$\text{stress} = G' \gamma_0 \sin(\omega t) + G'' \dot{\gamma}_0 \cos(\omega t)$$

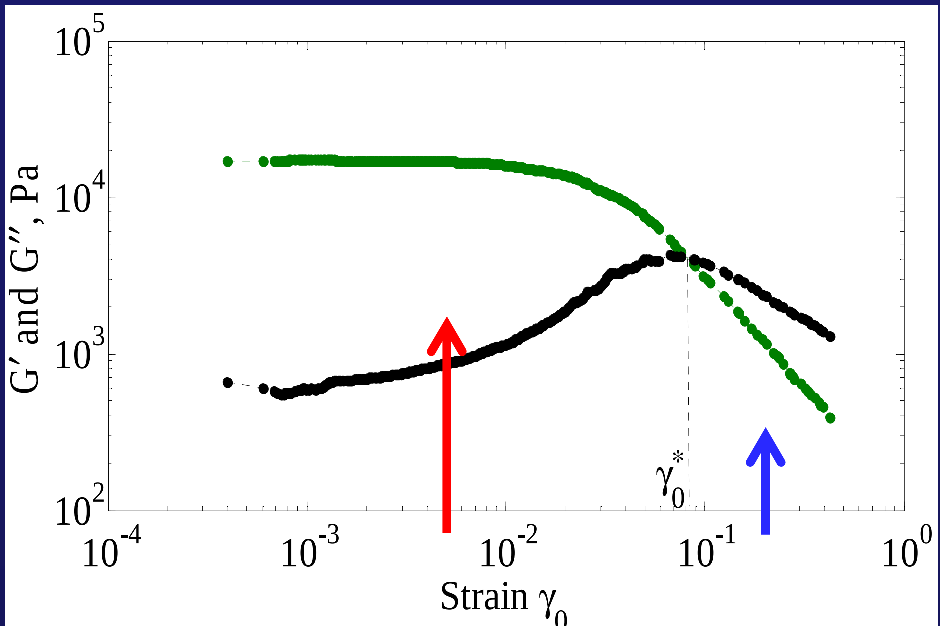
storage modulus

loss modulus

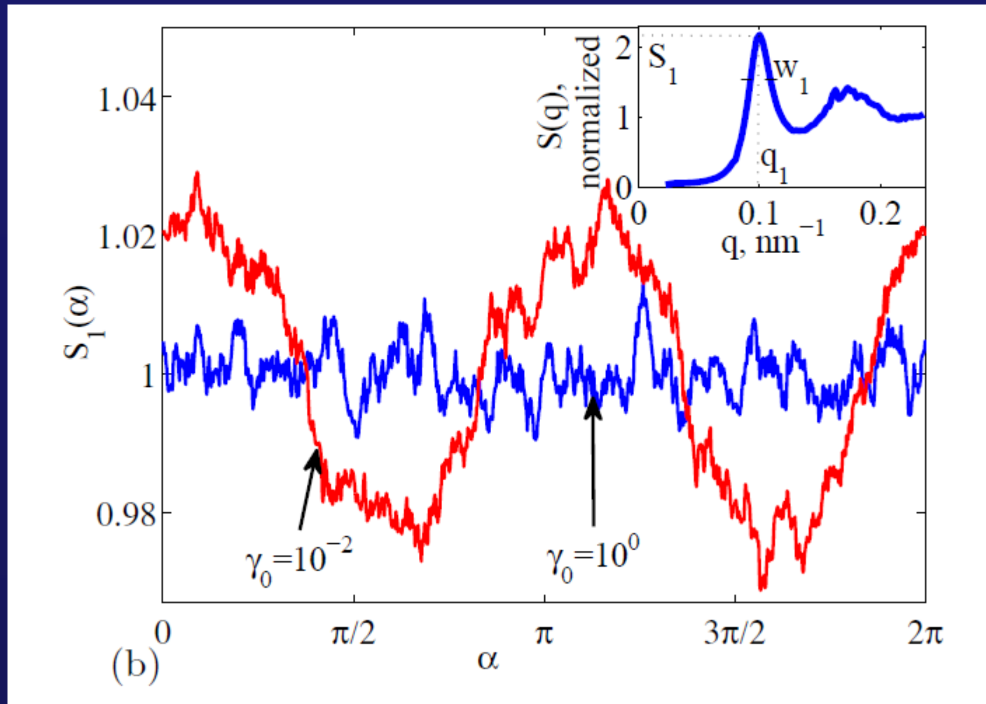
Simultaneous X-ray + Rheology

Yielding: Oscillatory Shear

Rheology



Structure



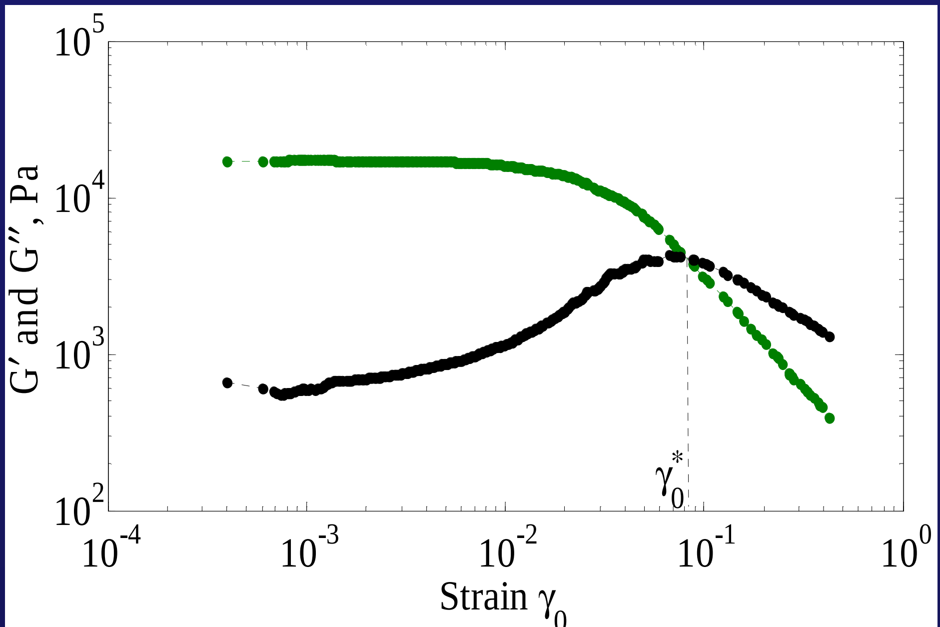
Solid and liquid-like structure factor

D. Denisov, T. Dang, B. Struth, P.S., *arXive* (2013)

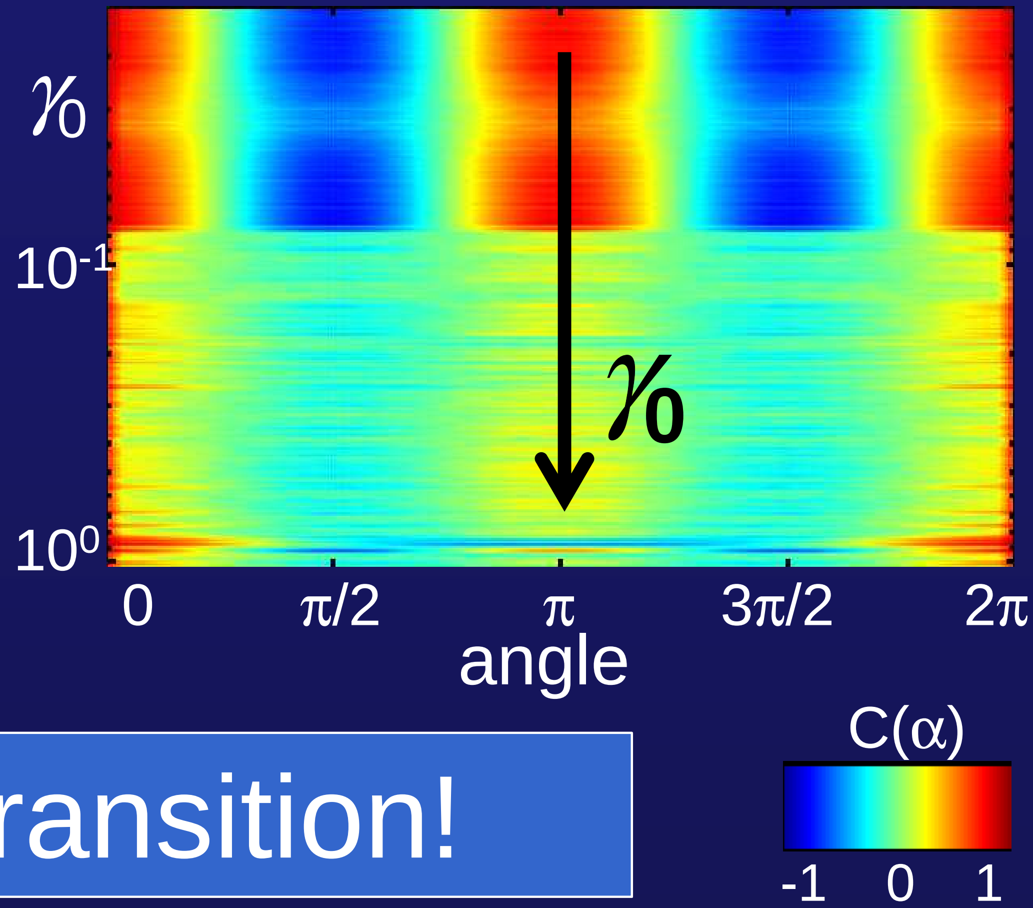
Simultaneous X-ray + Rheology

Yielding: Oscillatory Shear

Rheology



Structure

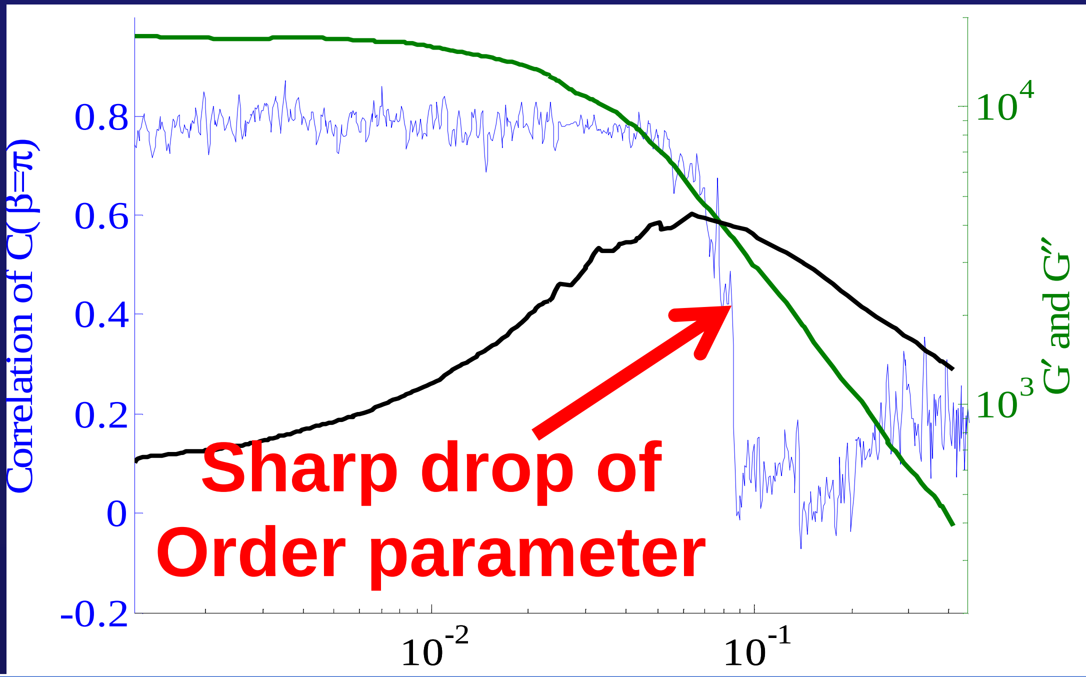


Sharp transition!

D. Denisov, T. Dang, B. Struth, P.S., *arXive* (2013)

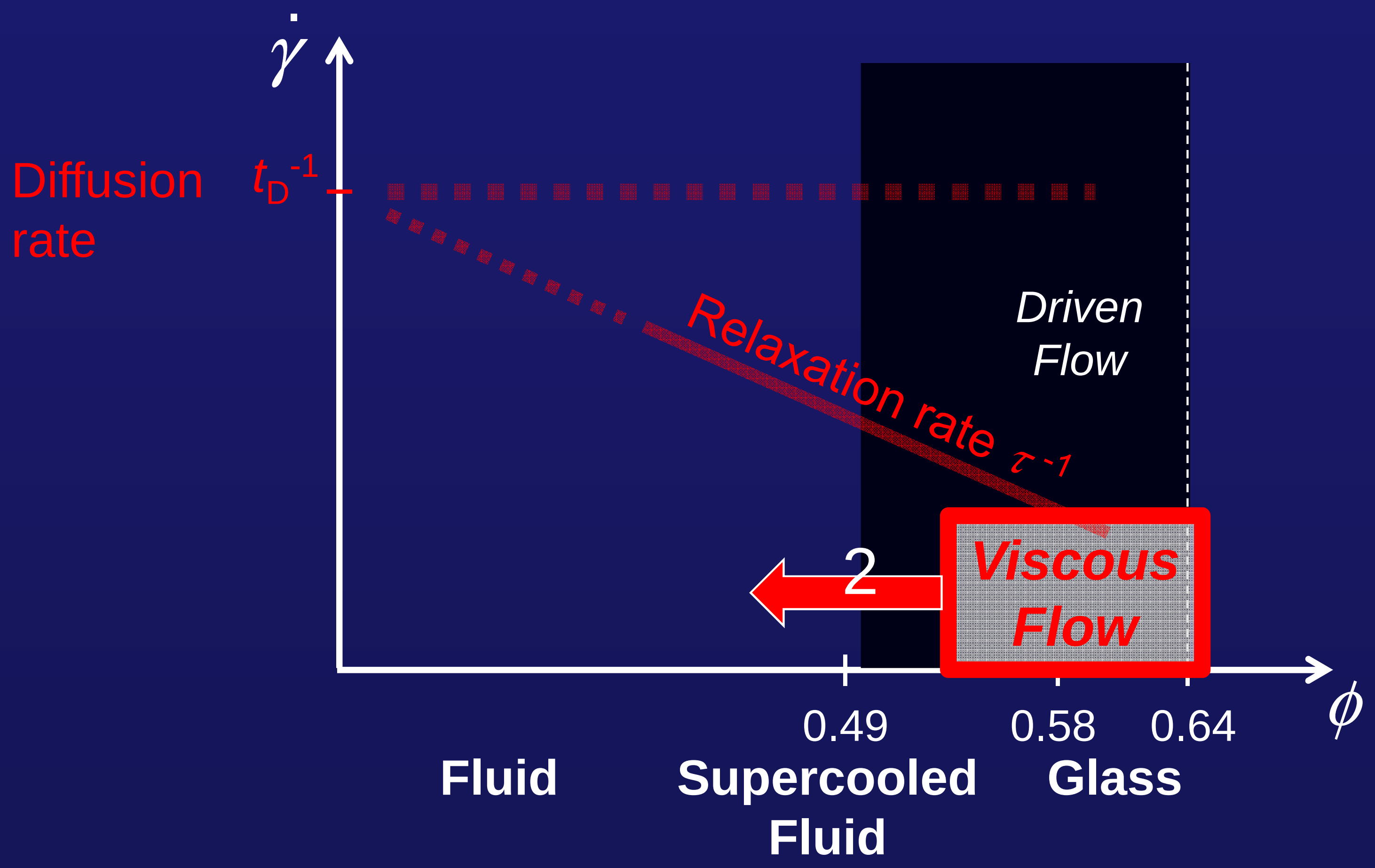
Yielding: Oscillatory Shear

Rheology + Struct. Order parameter



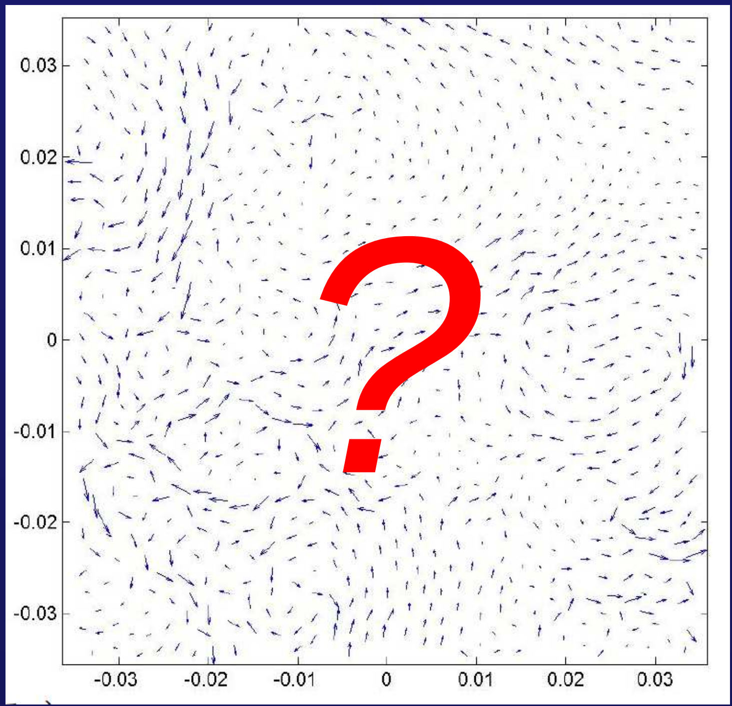
Shear-induced First-order transition

…towards lower density



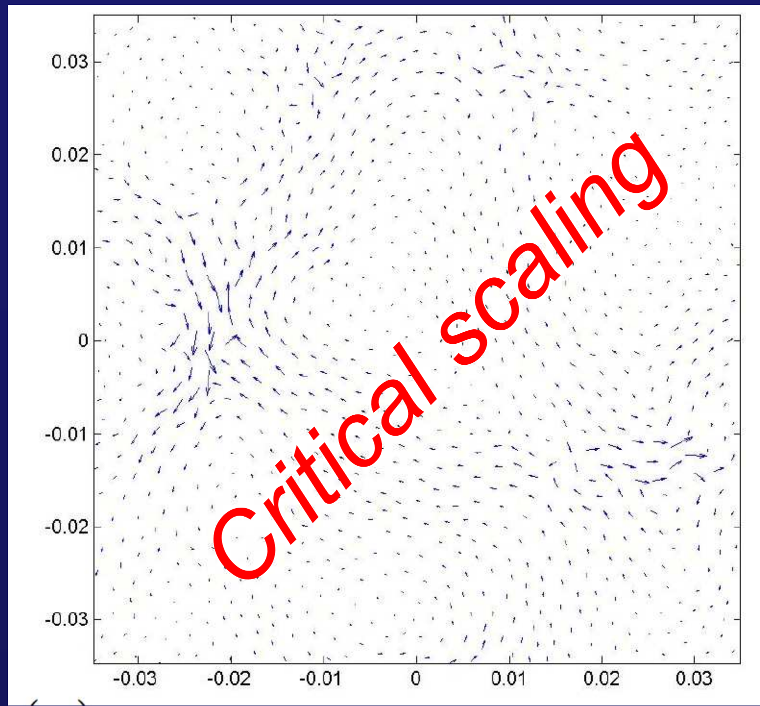
Foam Simulations

unjammed



$\phi \sim 0.84$

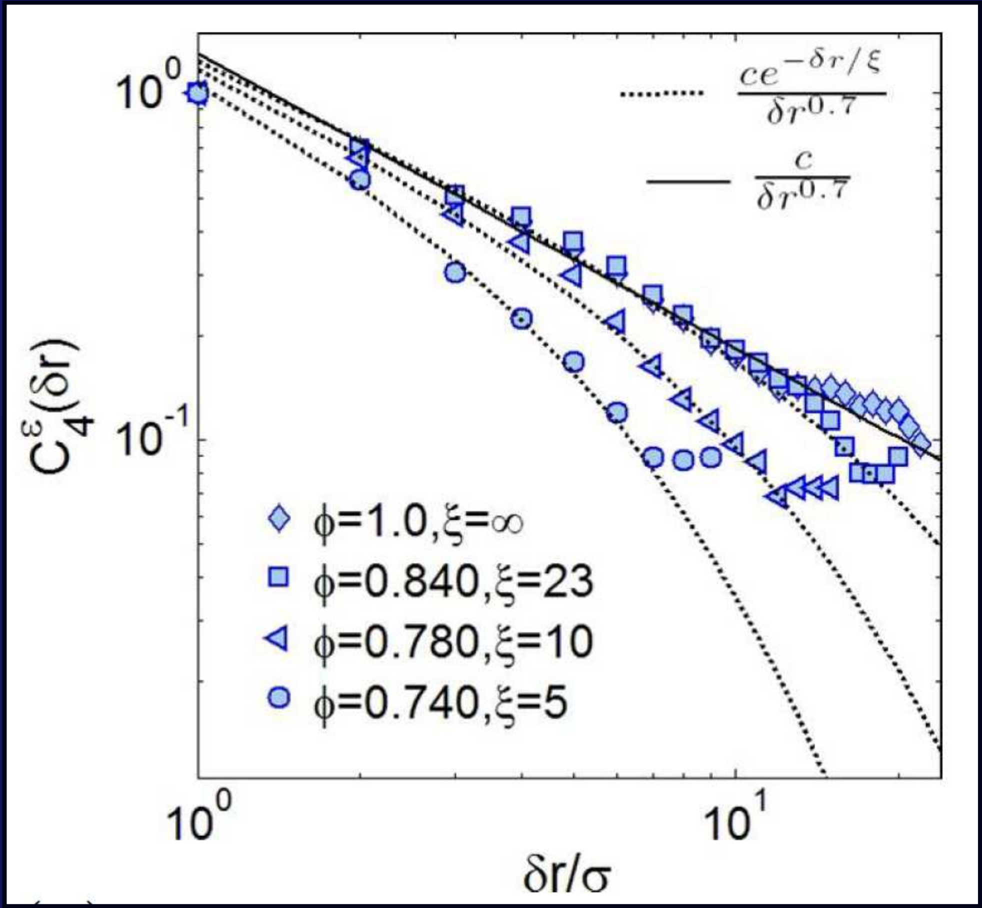
jammed

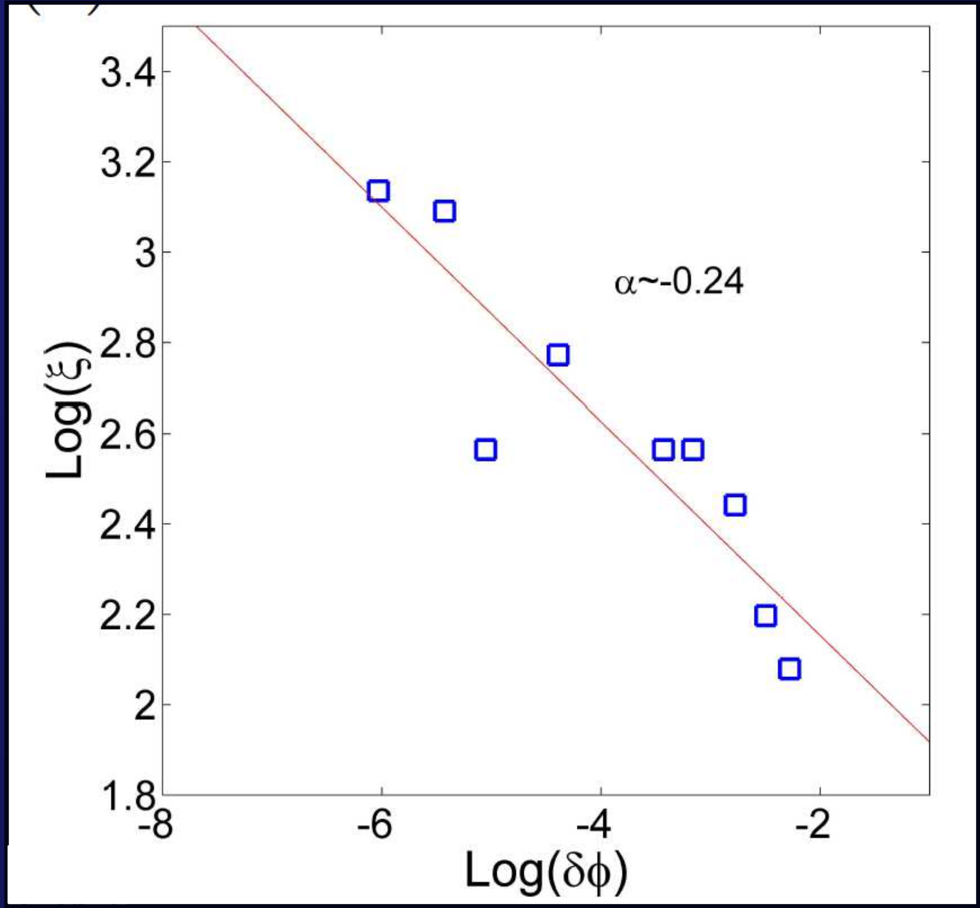


$\phi \sim 1$

Collaboration with M. van Hecke (Leiden)

2. Transition to low density



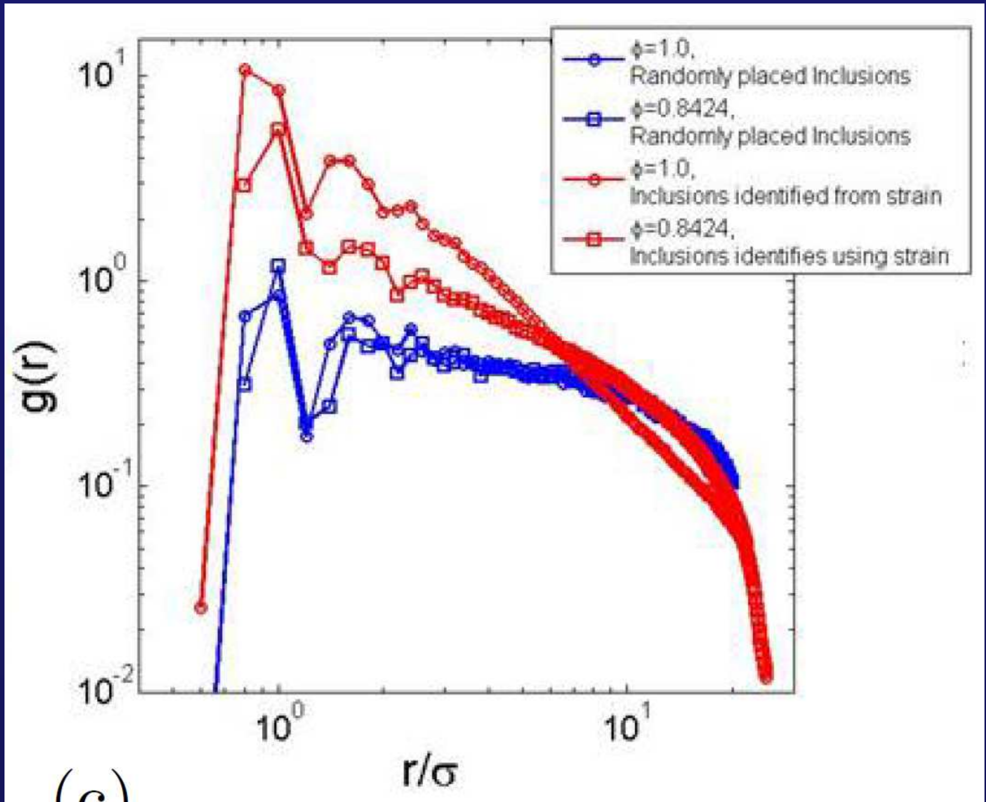


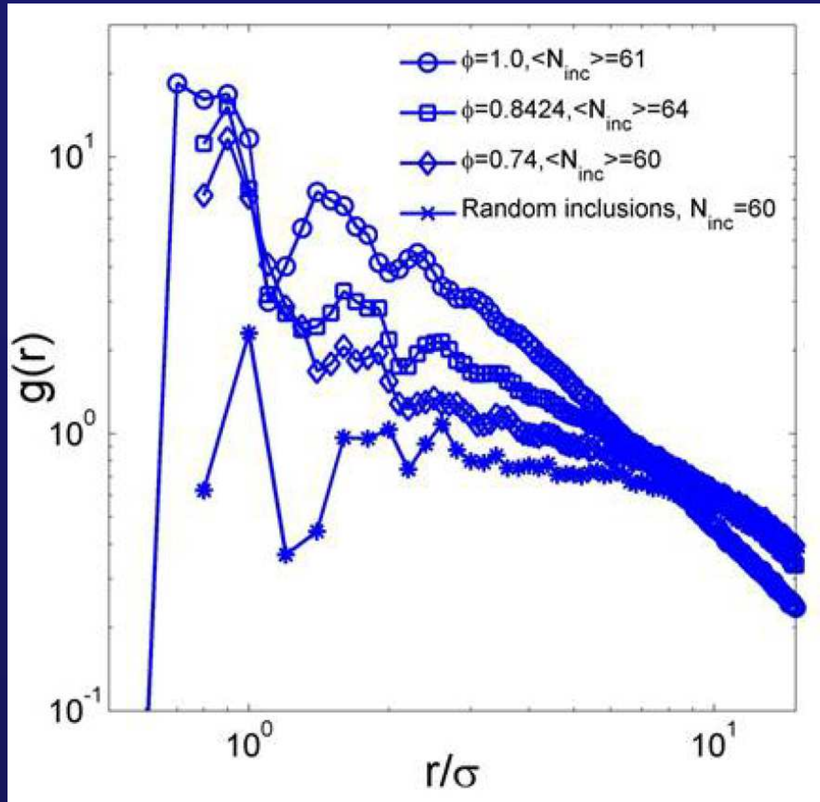
$$C_4^{\epsilon}(\delta r) = \int_0^{2\pi} C_{\epsilon}(\delta r, \theta) cos(4\theta) d\theta$$

V. Cikkadi, E. Woldhuis, M. van Hecke, P. Schall (*preprint* 2015)

2. Transition to low density

Pair correlation of active spots

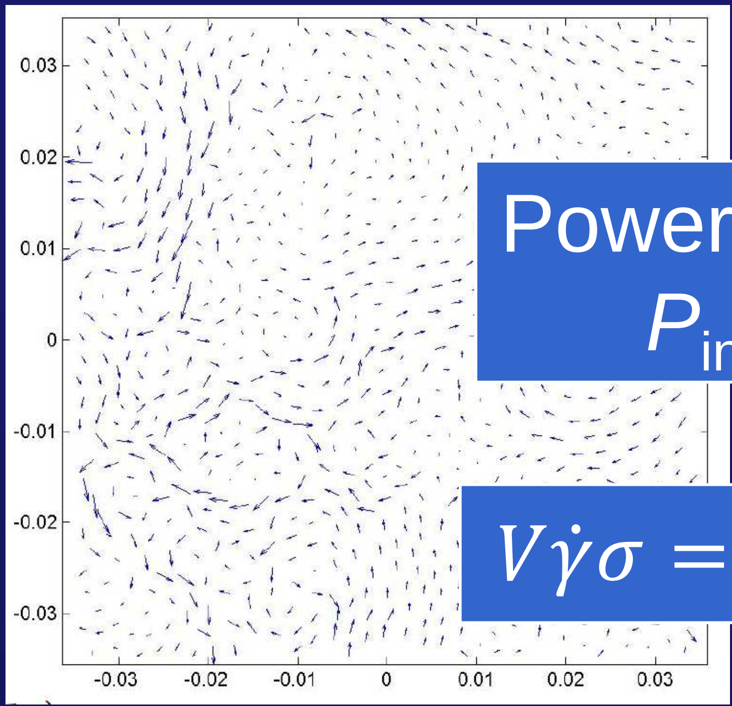


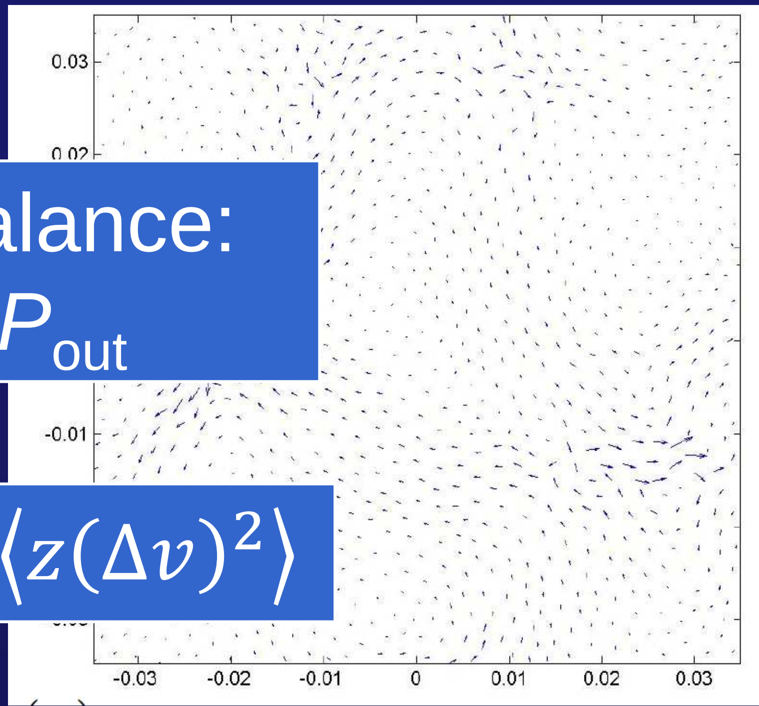


V. Cikkadi, E. Woldhuis, M. van Hecke, P. Schall (*preprint* 2015)

Multiscaling relations

Scaling of velocity fluctuations?





Power Balance:

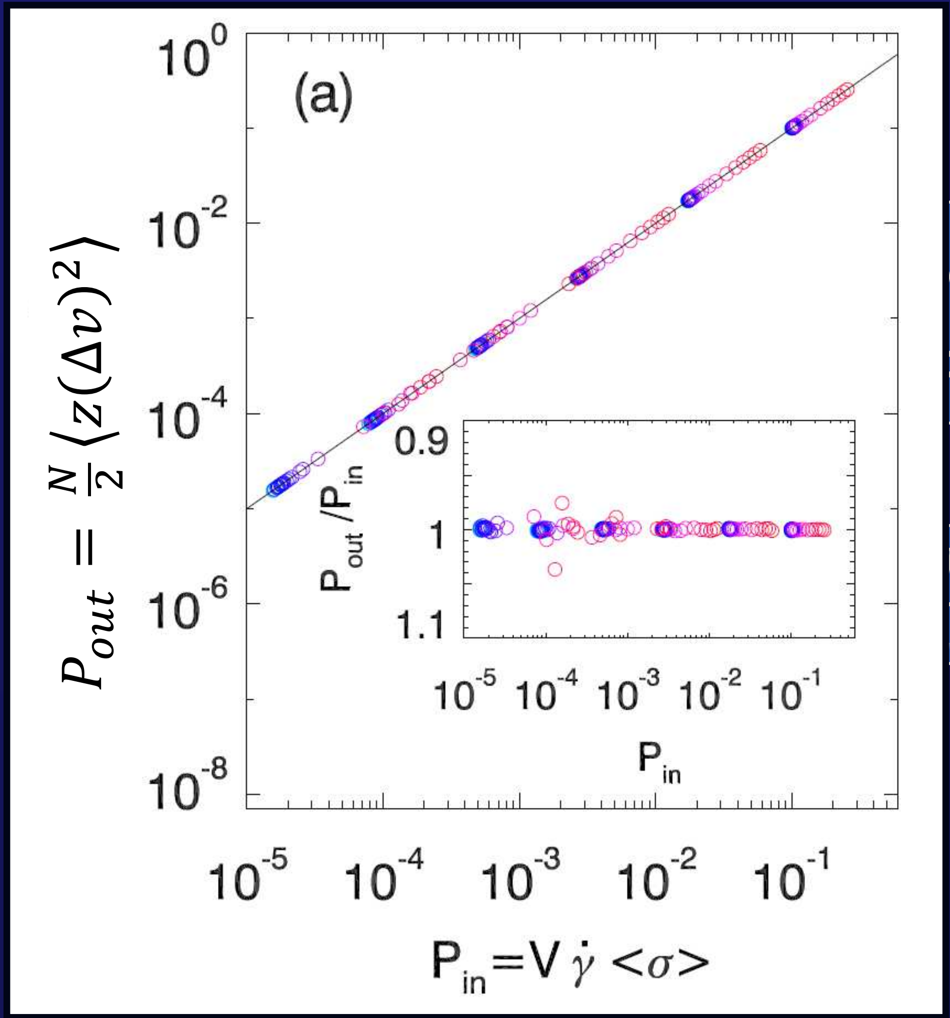
$$P_{\text{in}} = P_{\text{out}}$$

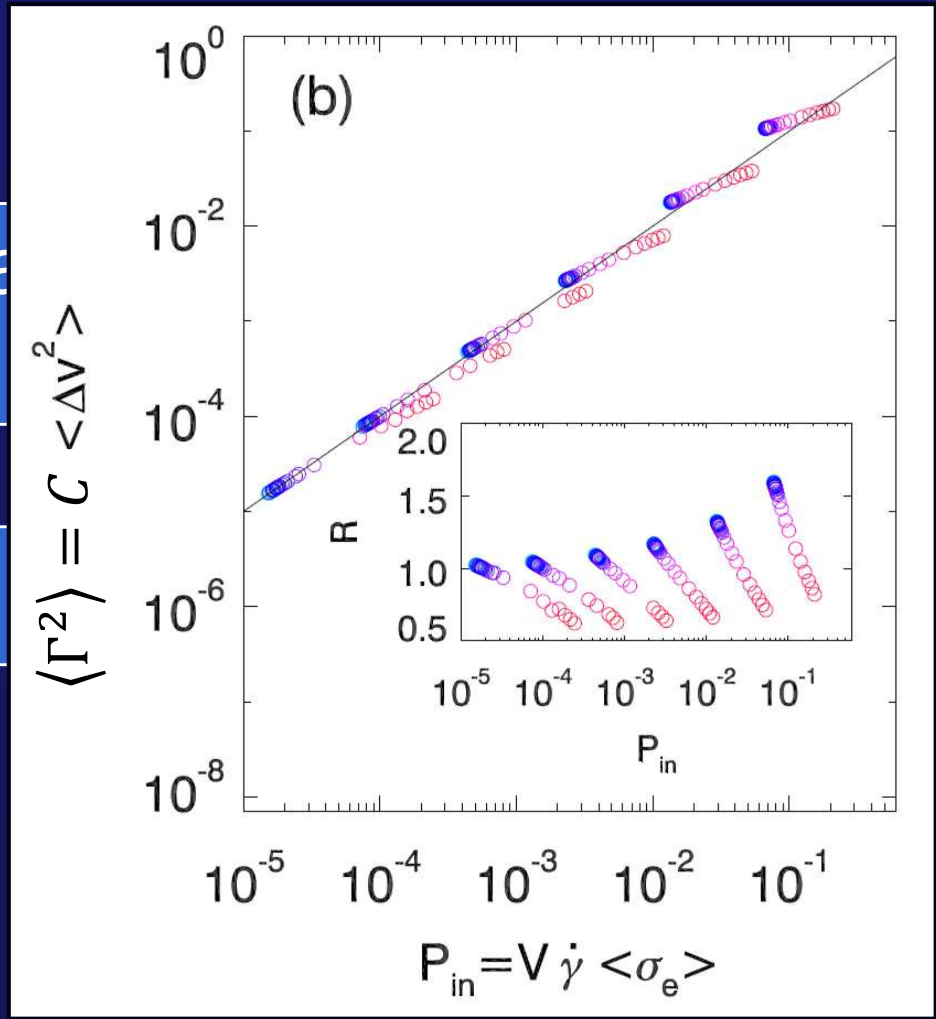
$$V\dot{\gamma}\sigma = \frac{N}{2}\left\langle z(\Delta v)^2 \right\rangle$$

E. Woldhuis, V. Chikkadi, P. Schall, M. van Hecke (*preprint* 2015)

Multiscaling relations

Scaling of velocity fluctuations?

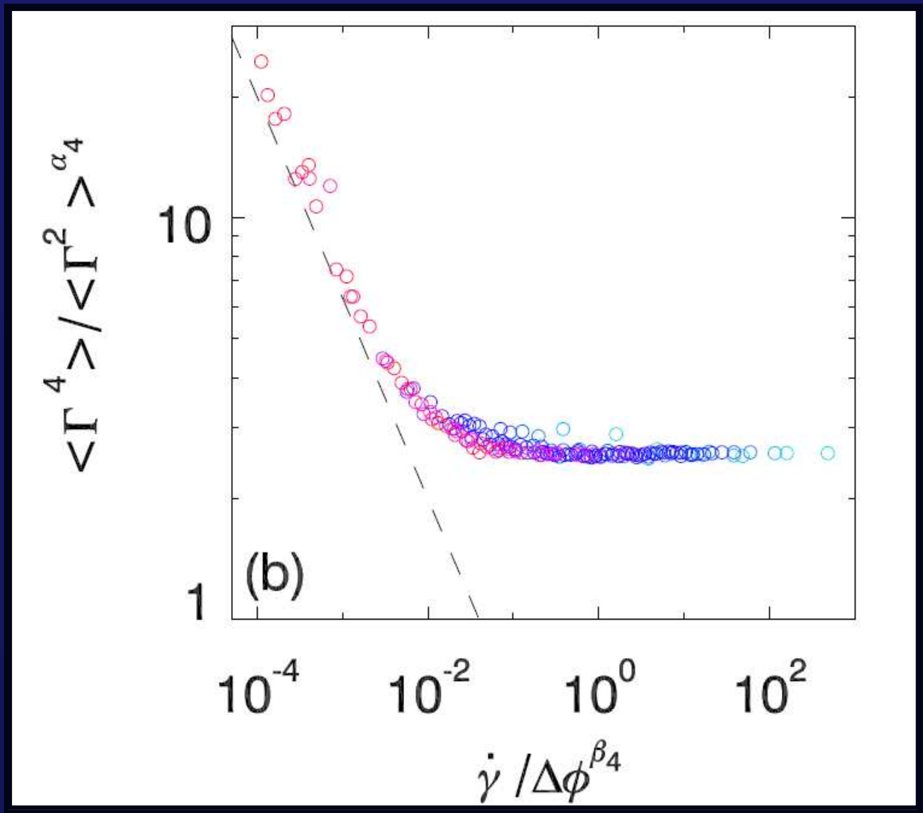




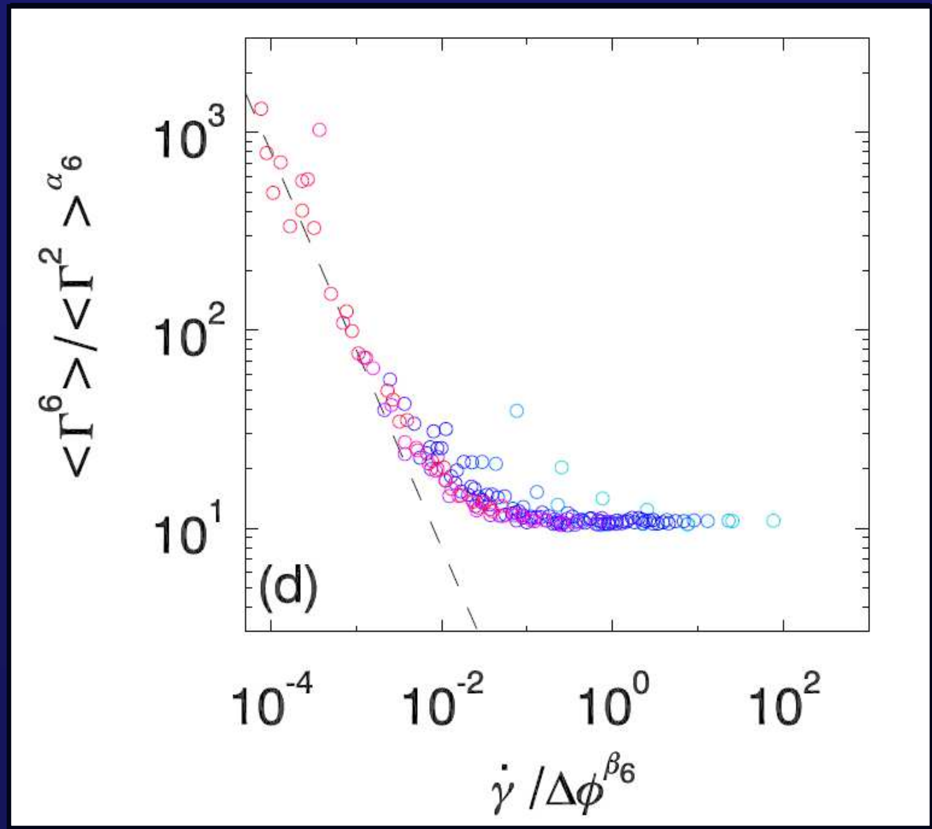
E. Woldhuis, V. Chikkadi, P. Schall, M. van Hecke (*preprint* 2015)

Multiscaling relations

Scaling of higher moments



4th moment



6th moment

Multiscaling relations

Scaling of higher moments

Temporal heterogeneity $$E_d = \langle \Delta v^2 \rangle_x / \langle \Delta v^2 \rangle_{xt}$$

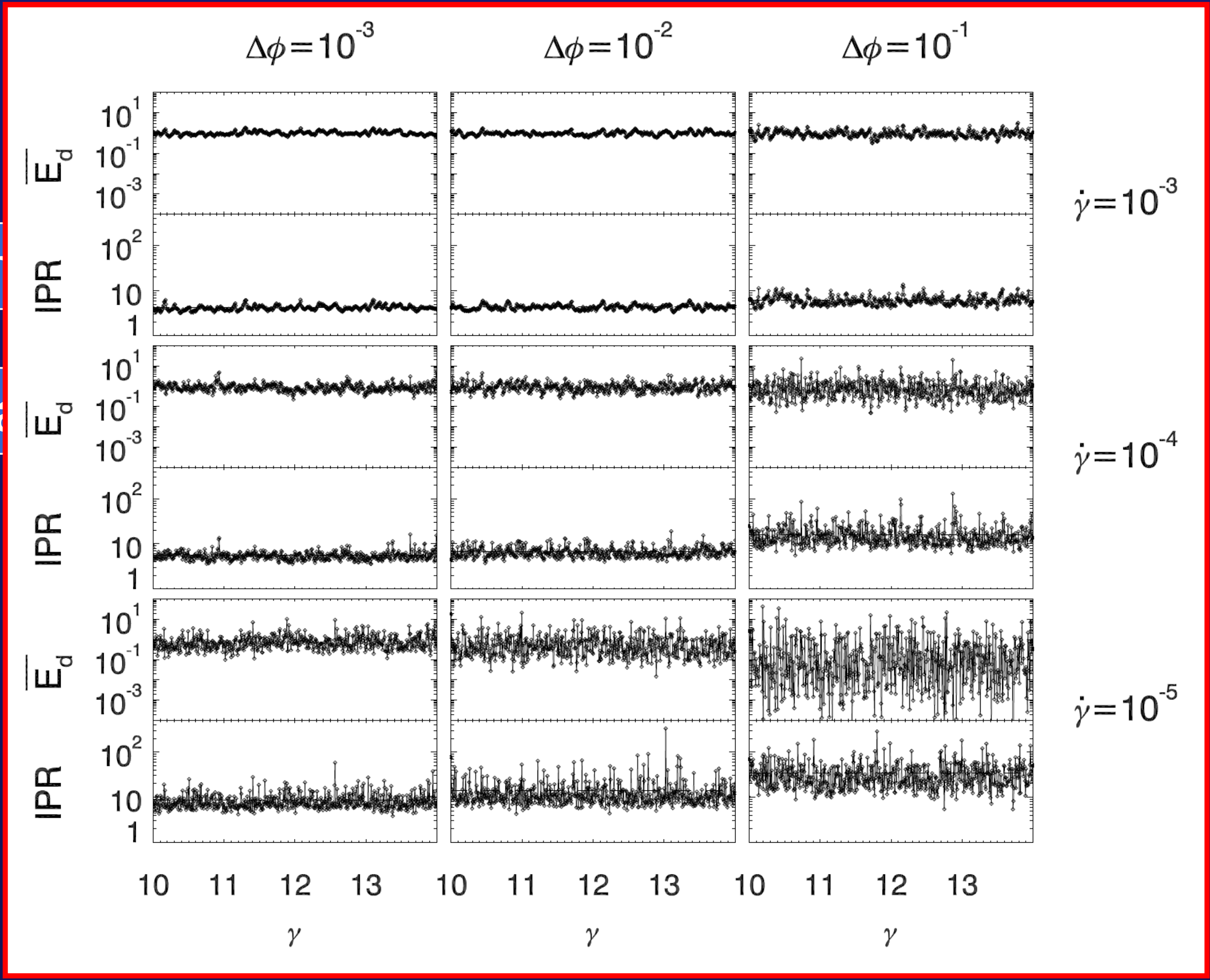
Spatial heterogeneity $$IPR = \langle \Delta v^4 \rangle / \langle \Delta v^2 \rangle^2$$

Multiscaling relations

Sc

Ter

Spa



Conclusions

- Elastic coupling
  → highly correlated, intermittent flow
- Material failure (yielding, banding)
  → Nonequilibrium phase transitions
- Evidence of Universality
- Multiscaling: intermittency towards slow, dense flow

Thanks to ....

Vijay Chikkadi (PhD, now Post doc)

Dmitry Denisov (Post Doc)