Plastic flow of foams and emulsions in a channel: experiments, theory and simulations

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FLOMAT2015, "Flowing Matter across the scales"
Istituto Nazionale di Studi Romani, Rome, Italy 26th March 2015

Motivations

Soft-glassy materials (foams, emulsions, gels…)

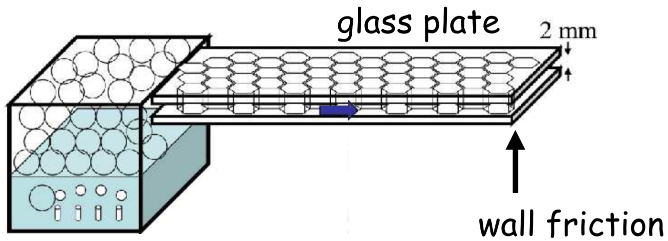
✓Yield Stress (solid-like below)

✓Non-newtonian (above yield)

✓Heterogeneities

✓Effect of confinement

Shear localisation in **foams**



(B. Dollet, J. Rheol. **54**, 741 (2010))

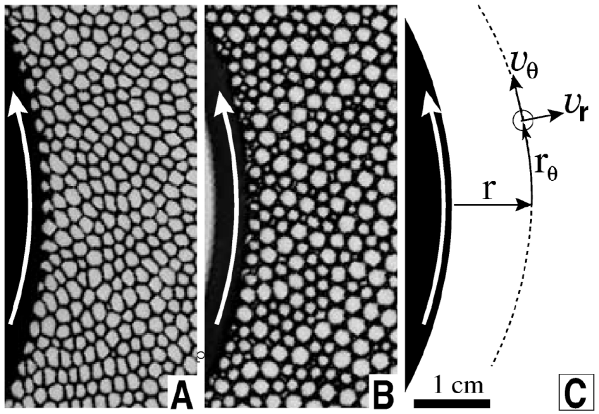
foams ≠ emulsions

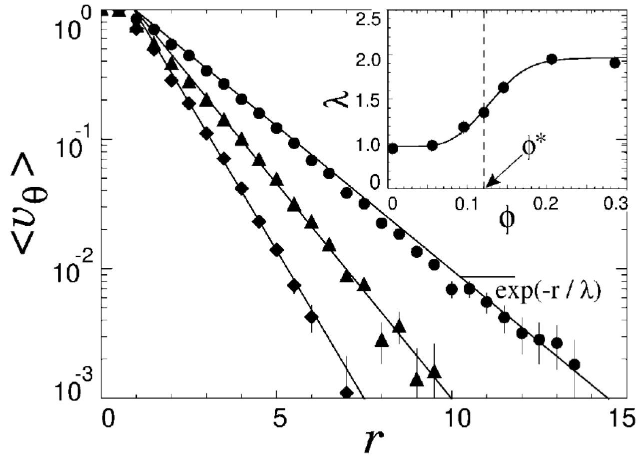
foams are inherently light scattering
(no refractive index matching possible)

→ **confinement (friction)**

+

complex rheology (non-locality)





(G. Debrégeas et al, Phys. Rev. Lett. **87**, 178306 (2001))

Wall friction: (local) continuum model with viscous drag

Drag force of a bubble sliding over a solid wall

$$\mathbf{F}_D \propto Ca^\alpha$$

(F.P. Bretherton, JFM **10**, 166 (1961))
(N.D. Denkov et al, PRL **100**, 138301 (2008))
(G. Katgert et al, PRE **79**, 066318 (2009))

linear drag →

$$\mathbf{F}_D = -\beta \mathbf{v}$$

force balance for a plane **Couette flow**

$$\frac{d\sigma}{dz} = \beta v(z)$$

for a Newtonian liquid $\sigma = \eta \dot{\gamma}$

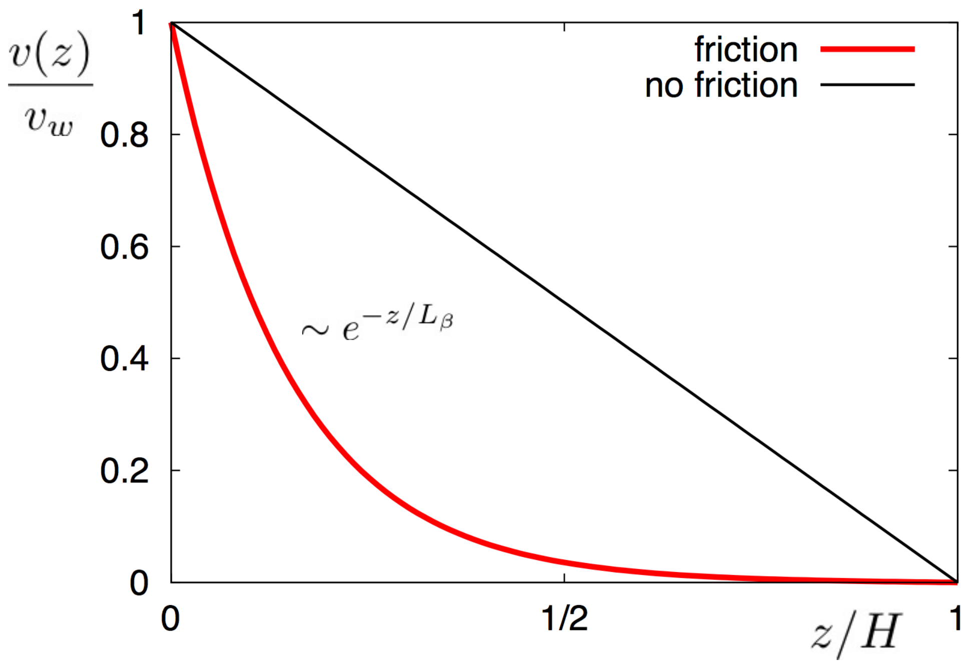
→

$$v(z) = v_w \frac{\sinh(z/L_\beta)}{\sinh(H/(2L_\beta))}$$

$$L_\beta = \sqrt{\frac{\eta}{\beta}}$$

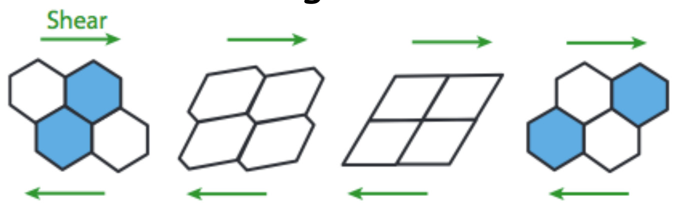
friction length

(E. Janiaud et al, Phys. Rev. Lett. **97**, 038302 (2006))

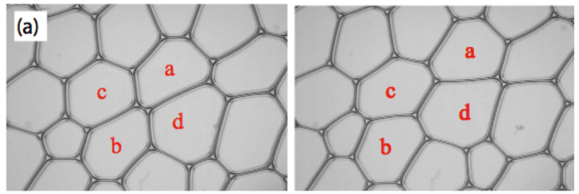


Non-local rheology

Plastic rearrangements: T1 event

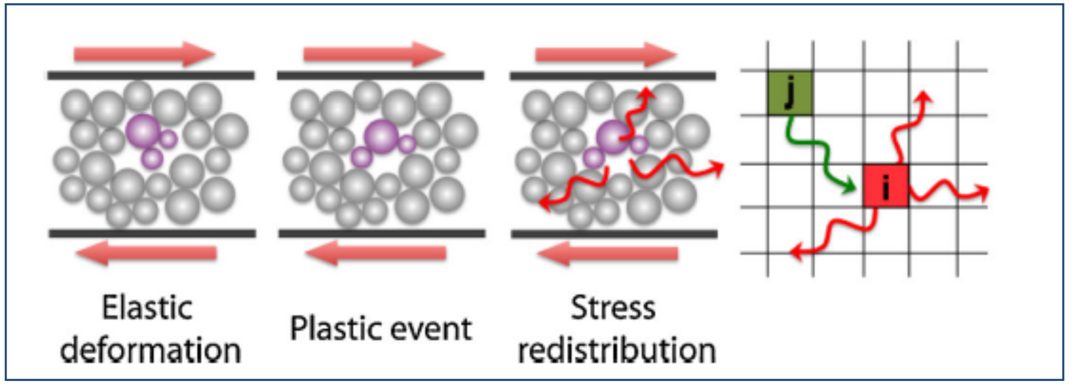


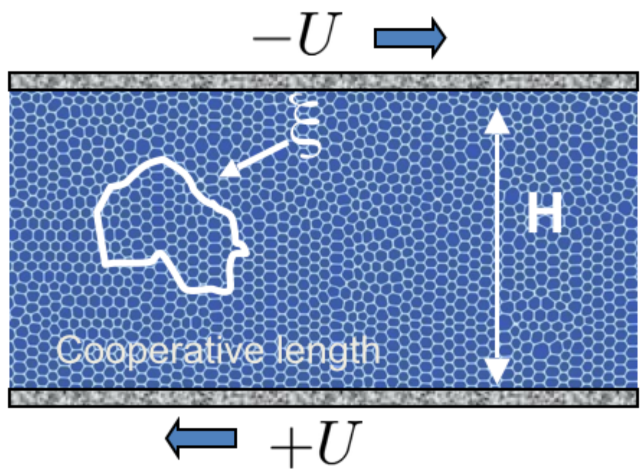
(S. Cohen-Addad et al ARFM **45**, 241 (2013))



(A. Kabla et al JFM **587**, 45 (2007))

Non-local effects: **Kinetic Elasto-Plastic (KEP)** model





kinetic equation for stress probability distribution

$$\partial_t P_i + \dot{\gamma_i}^{(0)} \partial_{\sigma_i} P_i = \mathcal{L}(P, P)$$

continuum limit

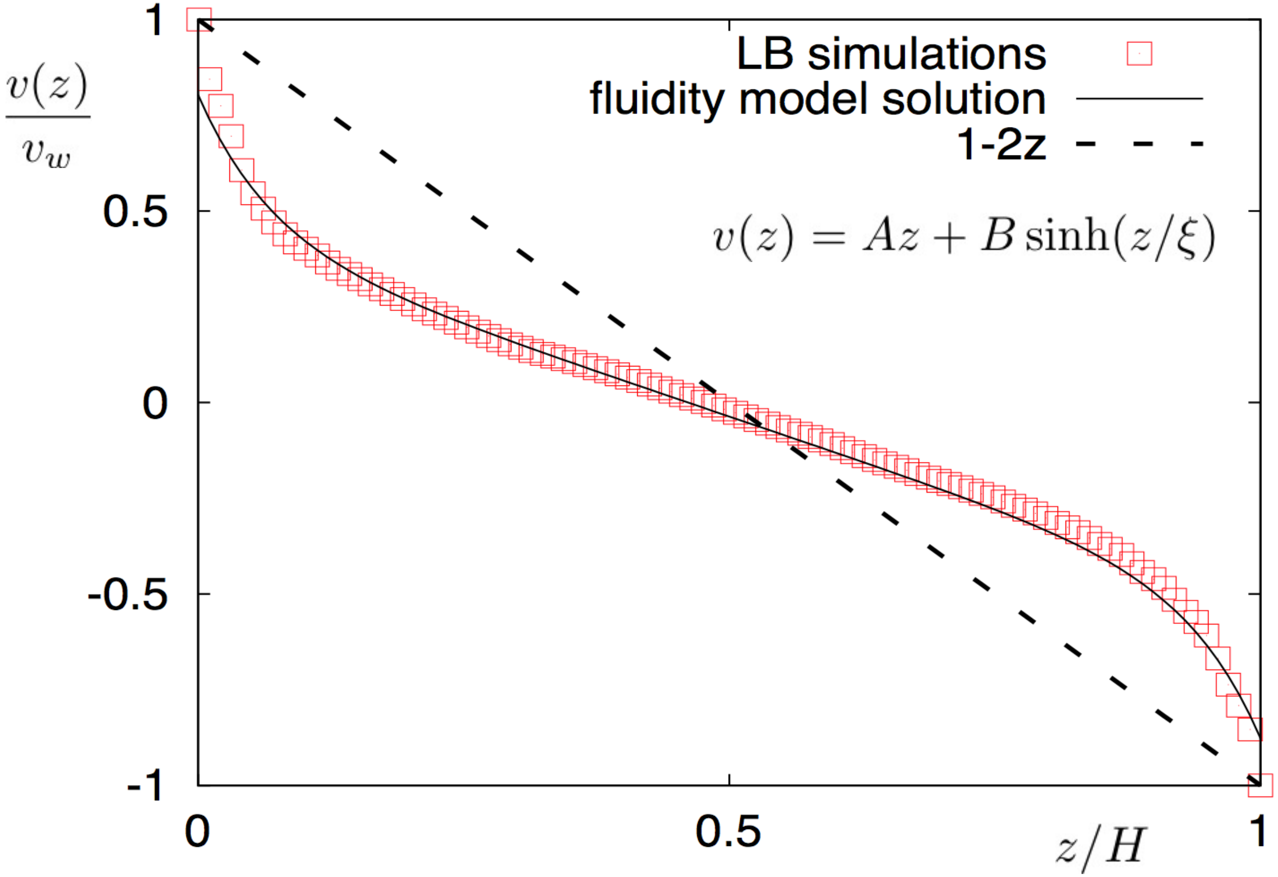
diffusion equation for the **FLUIDITY** $f = \frac{\dot{\gamma}}{\sigma}$

$$\xi^2 \Delta f + (f_b(\sigma) - f) = 0$$

$f \propto \Gamma$ rate of plastic events

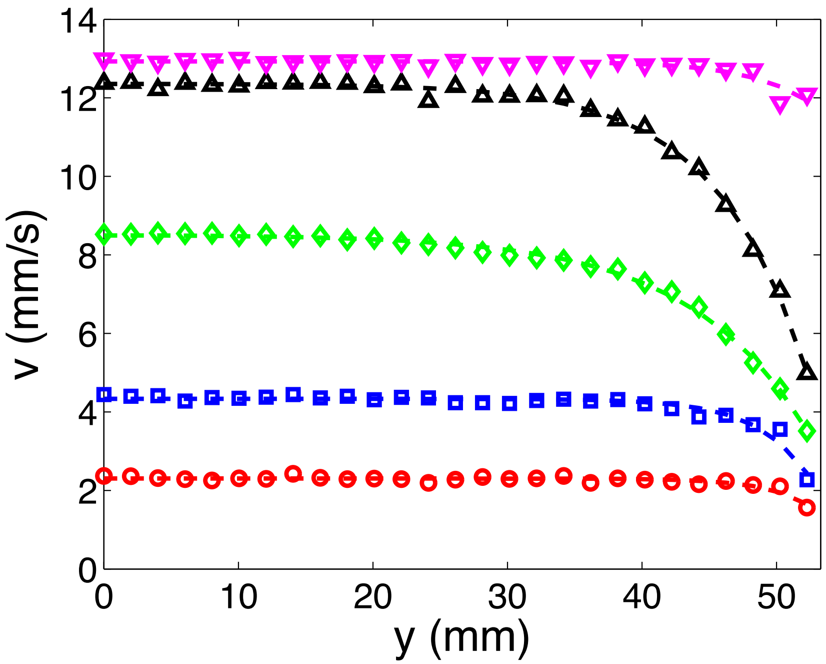
(L. Bocquet et al, Phys. Rev. Lett **103**, 036001 (2009))

KEP: results for a Couette flow

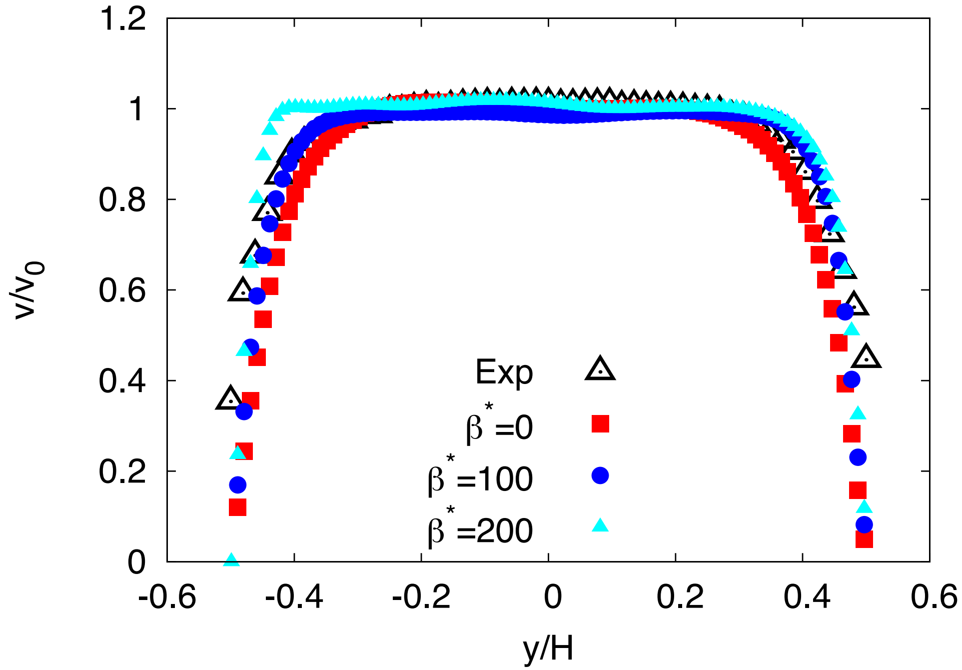


Flow in a channel: Velocity profiles

Experiments
Half-channel (symmetrised) profiles



Simulations + Experiment
Profiles normalised by the centerline velocity



$$v(y) = v_0 \frac{\cosh(H/2L_v) - v_s - (1 - v_s)\cosh(y/L_v)}{\cosh(H/2L_v) - 1}$$

fitting function → L_v velocity localisation length

FACTS:

1) Exp and sim AGREE: <u>Shear localisation</u> with a characteristic length depending on the flow-rate/friction-parameter

2) Exp and sim DISAGREE: Slip vs no-slip at the walls (different boundary conditions…)

3) Slip velocity much lower than predicted by local models (Janiaud et al, Phys. Rev. Lett. 97, 038302 (2006)) → ELASTICITY and **NON-LOCALITY**

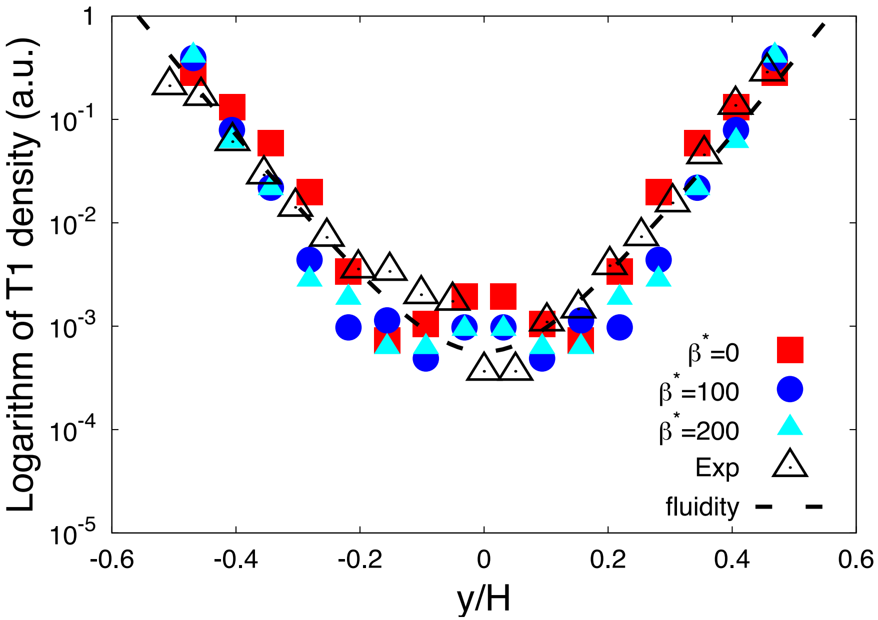
Connection with KEP: Rate of plastic events

(Non-local) fluidity equation

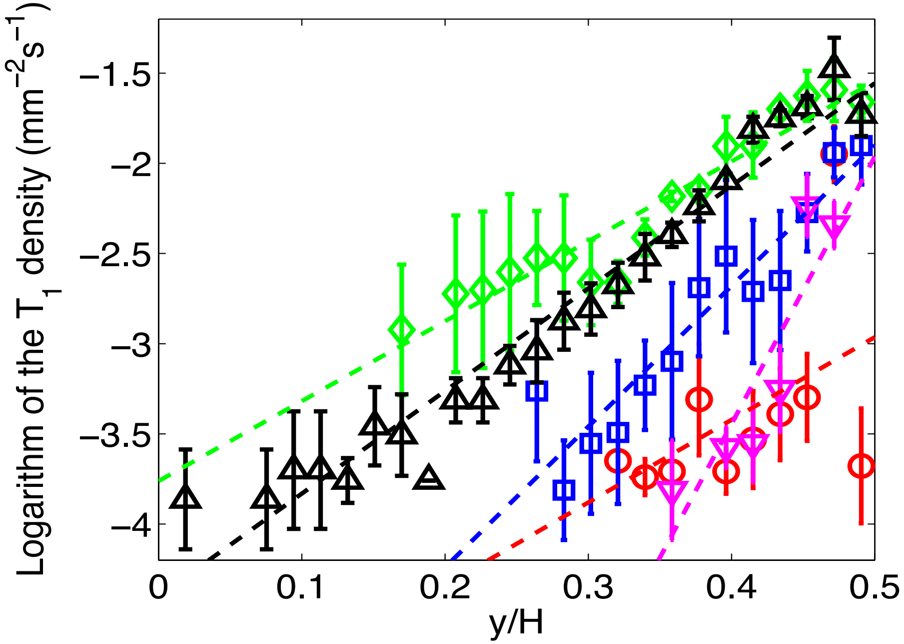
$$\xi^2 \Delta f(\mathbf{r}) = f(\mathbf{r}) - f_b[\sigma(\mathbf{r})] \qquad f = \frac{\dot{\gamma}}{\sigma}$$

shear-rate/shear-stress

Normalised distribution of T1's across the channel

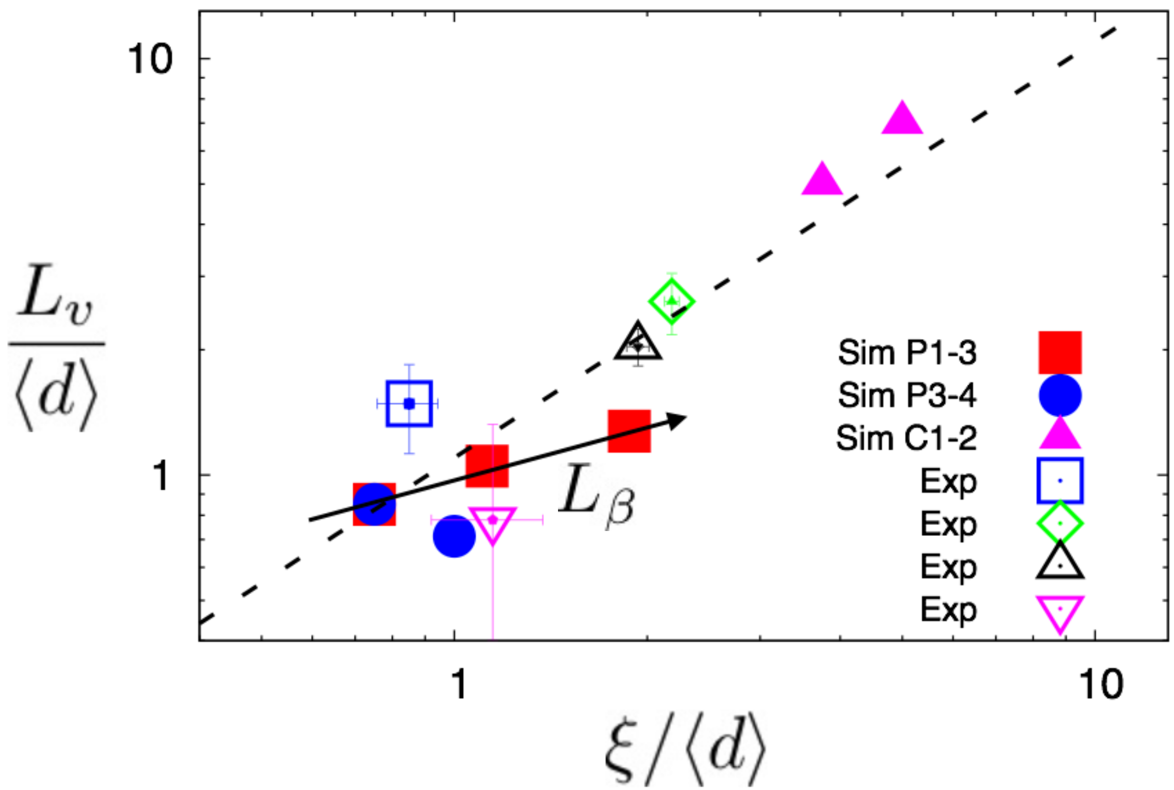


Experimental (symmetrised) distributions of T1's + exp fit



Shear localisation length vs Plastic localisation length

Velocity localisation length from cosh fit of profiles **vs** Plastic cooperativity length from exp fit of symmetrised rate of T1's



$L_v \sim \xi$ → **Plastic flow**

L$_v$ grows with ξ and L_β

(B. Dollet, AS, M. Sbragaglia, J. Fluid Mech. **766**, 556-589 (2015))

Statement of the mathematical problem (for a Couette flow)

HERSCHEL-BULKLEY RHEOLOGY

$$\sigma = \sigma_Y + K\dot{\gamma}^a$$

+

FLUIDITY EQUATION
(non-locality)

$$\xi^2 \frac{d^2 f(z)}{dz^2} = f(z) - f_b(\sigma)$$

$$f(z) \overset{def}{=} \frac{\dot{\gamma}}{\sigma} \equiv \frac{1}{\sigma}\frac{dv(z)}{dz} \qquad f_b(\sigma) = \frac{1}{\sigma}\left(\frac{\sigma - \sigma_Y}{K}\right)^{1/a}$$

+

FORCE BALANCE
Navier-Stokes
(non-constant stress)

$$\frac{d\sigma}{dz} = \beta v(z)$$

How to simplify the problem?

bulk fluidity

Assuming **a=1** in the Herschel-Bulkley equation → **Bingham plastic** → $f_b(\sigma) = \frac{\sigma - \sigma_Y}{K\sigma}$

Considering two **asymptotic** regimes:

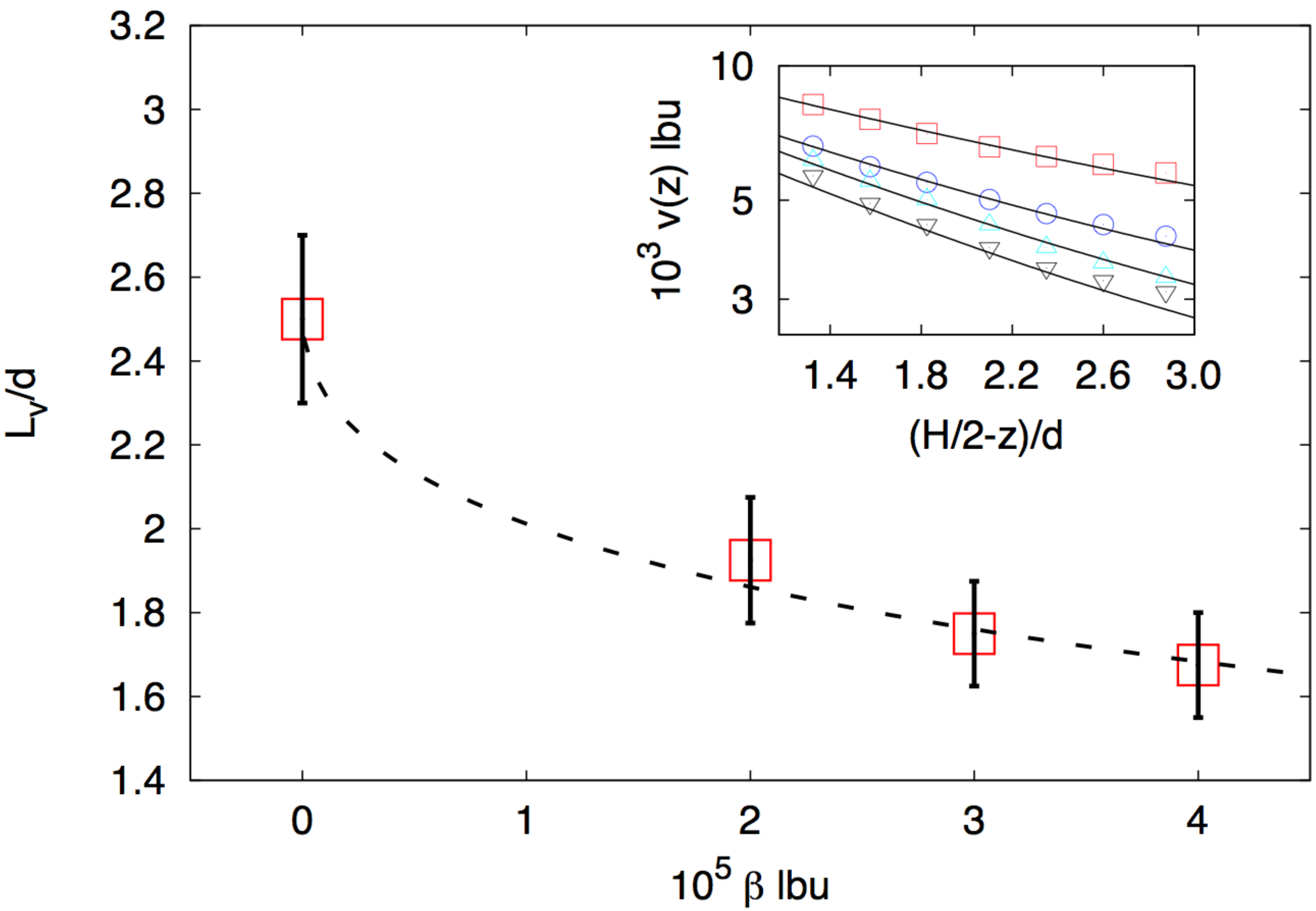
"Fluid" regime $\sigma \gg \sigma_Y$ → $f_b(\sigma) \approx \frac{1}{K} = const$

"Plastic" regime $\sigma \approx \sigma_Y$ → $\sigma = \sigma_Y + \tilde{\sigma}$ $\tilde{\sigma} \ll \sigma_Y$

$$f_b(\sigma) \approx \frac{\tilde{\sigma}}{K\sigma_Y}$$

Analytics vs LB numerical simulations for a Couette flow

$\sigma \gg \sigma_Y$



$$v(z) \sim e^{(z-H/2)/L_v}$$

$$L_v = \frac{\xi L_\beta}{\xi + L_\beta}$$

fitting value $\xi \approx 2.5\langle d \rangle$

(AS, B. Dollet, M. Sbragaglia, Colloids Surf. A (2015) DOI: 10.1016/j.colsurfa.2015.01.090)

Summarizing…

1) Combined **experimental/numerical/theoretical study** of foams/emulsions flowing in a channel

2) **First experimental measurement** of the rate of plastic events in a **Poiseuille flow of foams**

3) **Innovative numerical method** combining two capabilities:
   i) it provides realistic structures of foams/emulsions;
   ii) it naturally incorporates elastic and viscous contributions to stresses

4) **Shear localisation** length grows with the **characteristic length of rate of plastic events**

5) **Wall friction** acts adding up as an **extra-localisation** mechanism

6) **Agreement with analytical results** for a Couette flow

Thank you!!!