On the role of the helicity in the energy transfer in three-dimensional turbulence

Ganapati Sahoo and Luca Biferale

University of Rome Tor Vergata, Italy

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Introduction

- Energy and enstrophy are conserved in 2D Navier-Stokes equations.
- Forward cascade of energy is blocked, since enstrophy is positive and definite. (Boffetta, Ann. Rev. Fluid Mech 2012)
- Energy and Helicity are invariants of 3D Navier-Stokes equations.
- Both cascade forward, from large scales to small scales. (Chen, Phys. Fluids 2003)
- Helicity could be positive or negative.
- Each Fourier mode of velocity could be decomposed into positive and negative helical modes.

What happens when we change the relative weight of the positive and the negative helicity modes?

Helical decomposition

- Following Waleffe, Phys. Fluids (1992)

$$\begin{aligned}\mathbf{u}(\mathbf{k},t) &= \mathbf{u}^{+}(\mathbf{k},t)+\mathbf{u}^{-}(\mathbf{k},t),\\ \mathbf{u}^{\pm}(\mathbf{k},t) &= u^{\pm}(\mathbf{k},t)\mathbf{h}^{\pm}(\mathbf{k})\end{aligned}$$

where $\mathbf{h}^{\pm}(\mathbf{k})$ are the eigenvectors of the curl operator $i\mathbf{k}\times\mathbf{h}^{\pm}(\mathbf{k})=\pm k\mathbf{h}^{\pm}(\mathbf{k})$, $u^{\pm}(\mathbf{k},t)$ are the time-dependent scalar co-efficients.

- Projection operator:

$$\mathcal{P}^{\pm}(\mathbf{k})\equiv\frac{\mathbf{h}^{\pm}(\mathbf{k})\otimes\mathbf{h}^{\pm}(\mathbf{k})^{*}}{\mathbf{h}^{\pm}(\mathbf{k})^{*}\cdot\mathbf{h}^{\pm}(\mathbf{k})}$$

$$\mathbf{u}^{\pm}(\mathbf{k},t)=\mathcal{P}^{\pm}(\mathbf{k})\mathbf{u}(\mathbf{k},t)$$

- Decimated Navier-Stokes equations in Fourier space:

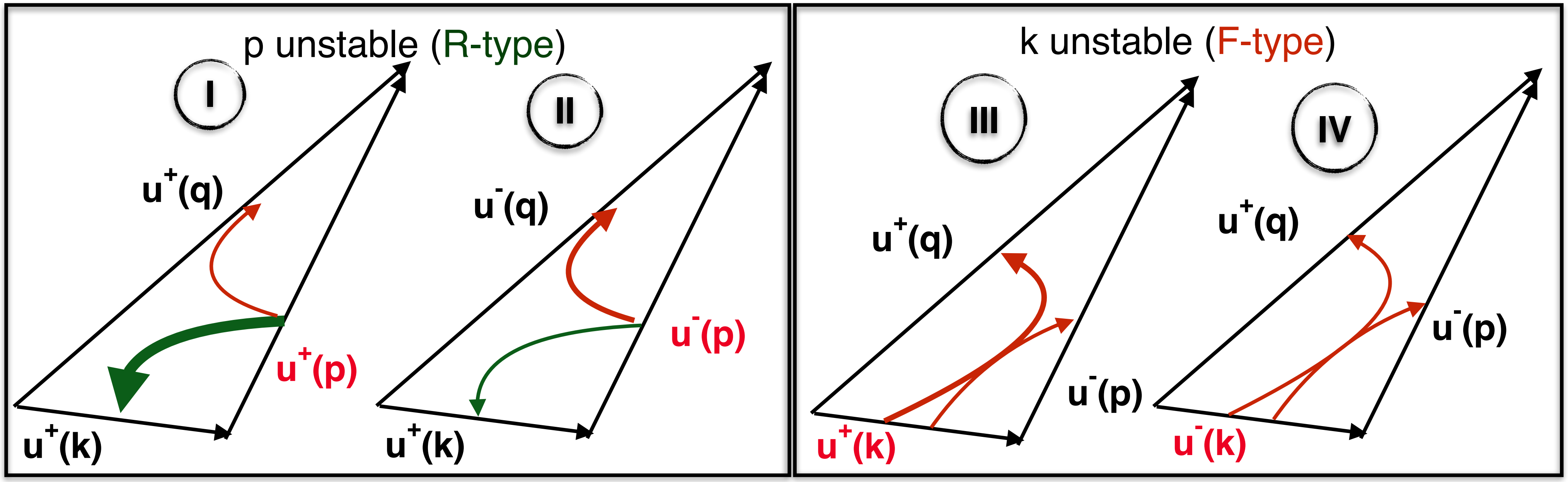
$$\partial_t\mathbf{u}^{\pm}(\mathbf{k},t)=\mathcal{P}^{\pm}(\mathbf{k})\mathbf{N}_{u^{\pm}}(\mathbf{k},t)+\nu k^2\mathbf{u}^{\pm}(\mathbf{k},t)+\mathbf{f}^{\pm}(\mathbf{k},t)$$

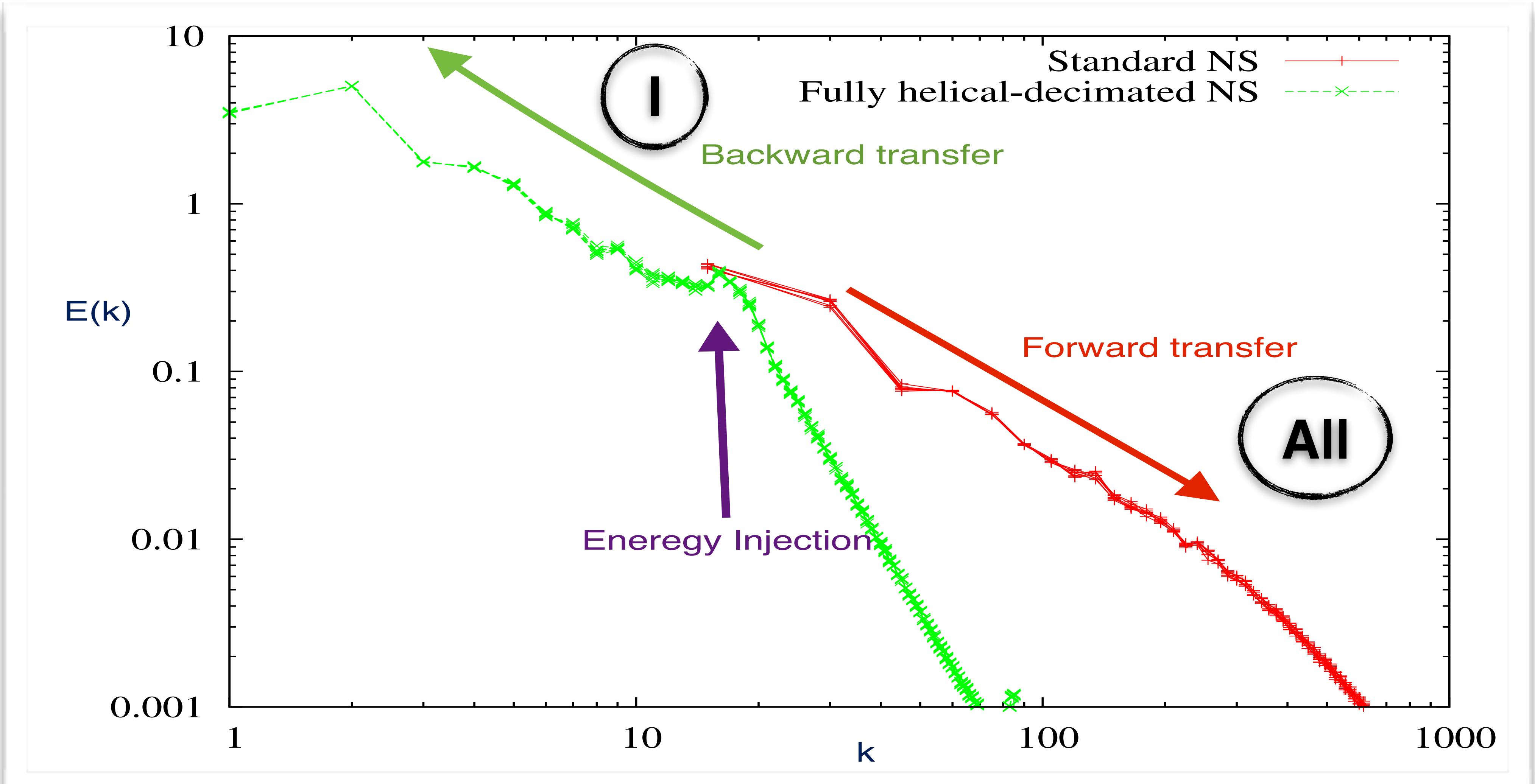
where ν is kinematic viscosity and $\mathbf{f}$ is external forcing.

- The non-linear term $\mathbf{N}_{u^{\pm}}(\mathbf{k},t)=\mathcal{FT}(\mathbf{u}^{\pm}\cdot\boldsymbol{\nabla}\mathbf{u}^{\pm}-\boldsymbol{\nabla}p)$, contains 8 possible triadic interactions $\mathbf{q}=\mathbf{k}+\mathbf{p}$ which fall into four classes.

Classes of triadic interactions in NS equations

$$\mathbf{N}_{\mathbf{u}^{\pm}}(\mathbf{q}) = \mathcal{F}\mathcal{T}\left[\mathbf{u}^{\pm}(\mathbf{k})\cdot\nabla\mathbf{u}^{\pm}(\mathbf{p})\right];\ \mathbf{q}=\mathbf{k}+\mathbf{p};\ k\le p\le q$$





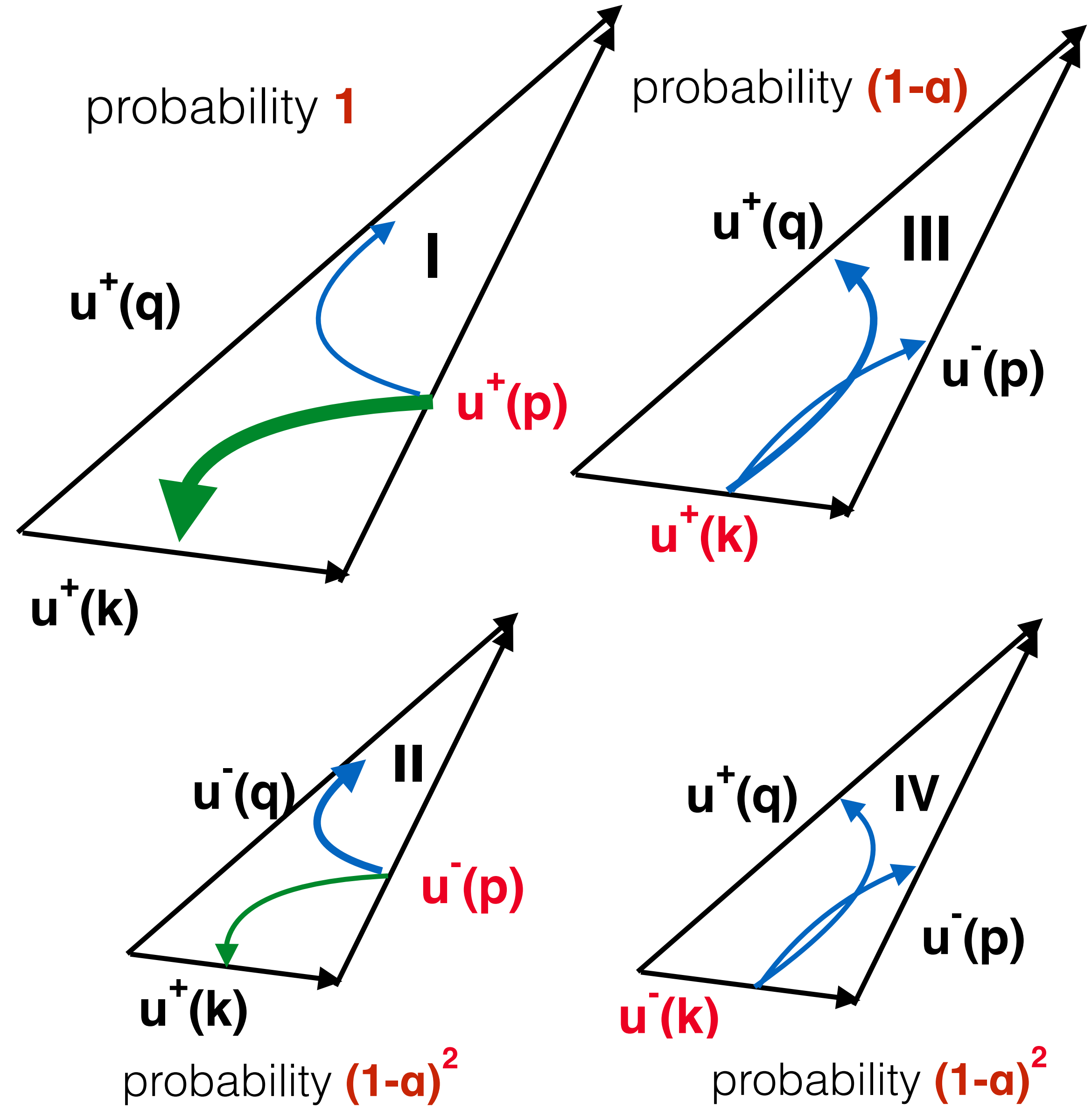
Partial Helical-decimation

What happens in between??
when we give different weights to different class of triads…

- Modified projection operator:

  $$\mathcal{P}^{+}_{\alpha}(\mathbf{k})\mathbf{u}(\mathbf{k},t) = \mathbf{u}^{+}(\mathbf{k},t) + \theta_{\alpha}(\mathbf{k})\mathbf{u}^{-}(\mathbf{k},t)$$

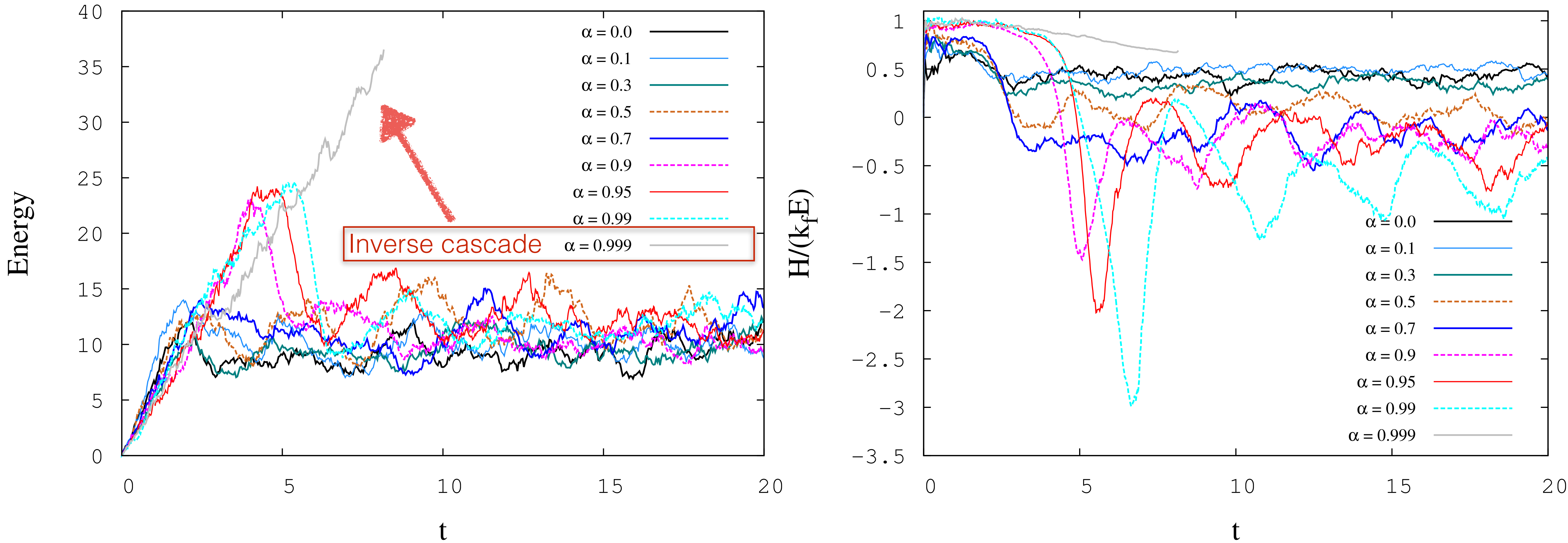
  where $\theta_\alpha(\mathbf{k})$ is 0 with probability α and is 1 with probability $1-\alpha$.
- We consider triads of Class-I with probability 1, Class-III with probability $1-\alpha$ and Class-II and Class-IV with probability $(1-\alpha)^2$.
- $\alpha = 0 \rightarrow$ Standard Navier-Stokes.
  $\alpha = 1 \rightarrow$ Fully helical-decimated NS.
- Critical value of α at which forward cascade of energy stops?
  alternatively, inverse cascade of energy stops if forced at small scales.



$$\mathbf{N}_{\mathbf{u}^{\pm}}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^{\pm}(\mathbf{k})\cdot\nabla\mathbf{u}^{\pm}(\mathbf{p})\right];\, \mathbf{q}=\mathbf{k}+\mathbf{p};\, k\le p\le q$$

Evolution of Energy and helicity

- Pseudo-spectral DNS on a triply periodic cubic domain of size $L = 2\pi$ with resolutions up to 512^3 collocation points.



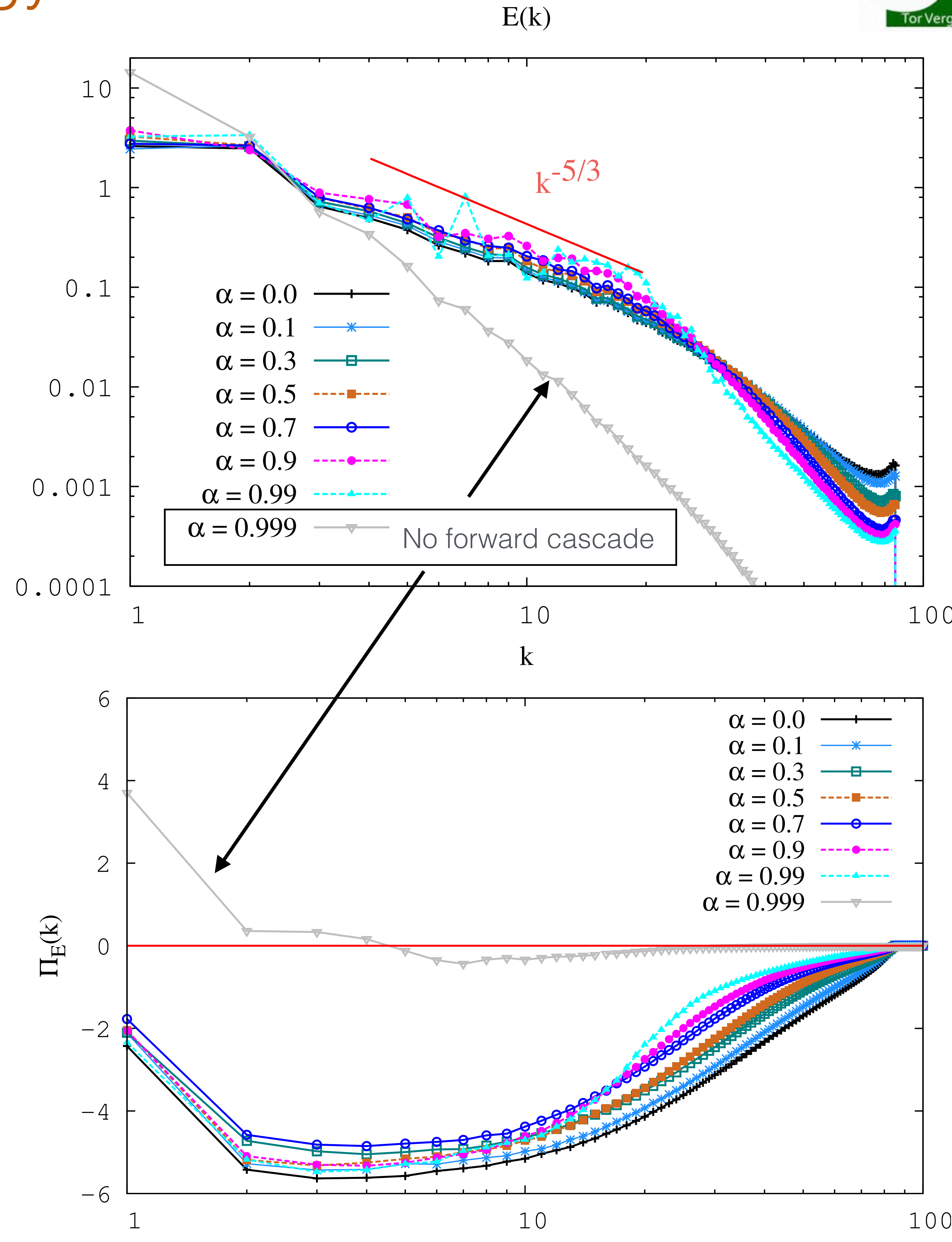
- The peaks suggest the building up of the energy at forced large scales before being able to transfer to the small scales.

- The cascade of energy starts only when helicity becomes active, i.e., modes with negative helicity becomes energetic.

- With increase in α the peak grows, a signature of inverse cascade.

Robustness of energy cascade

- Spectra for all values of α showing $k^{-5/3}$ suggest the forward cascade of to be strongly robust.
- Unless we kill almost all the modes of one helicity-type energy always finds a way to reach small scales.
- The energy flux also remains unaffected by the decimation until α is very close to 1.
- **Critical value of α is ~ 1 !**

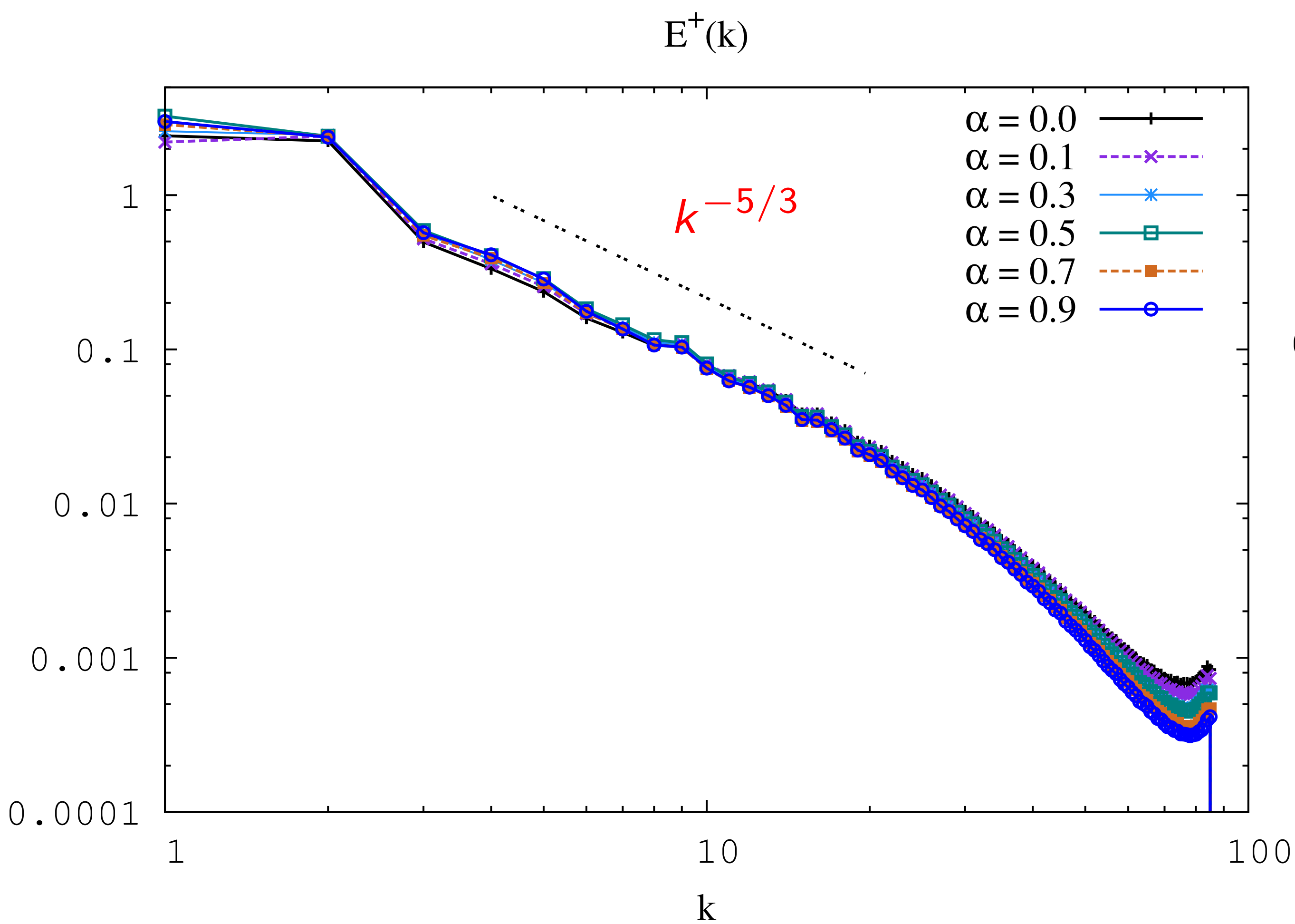


Reaction of negative modes

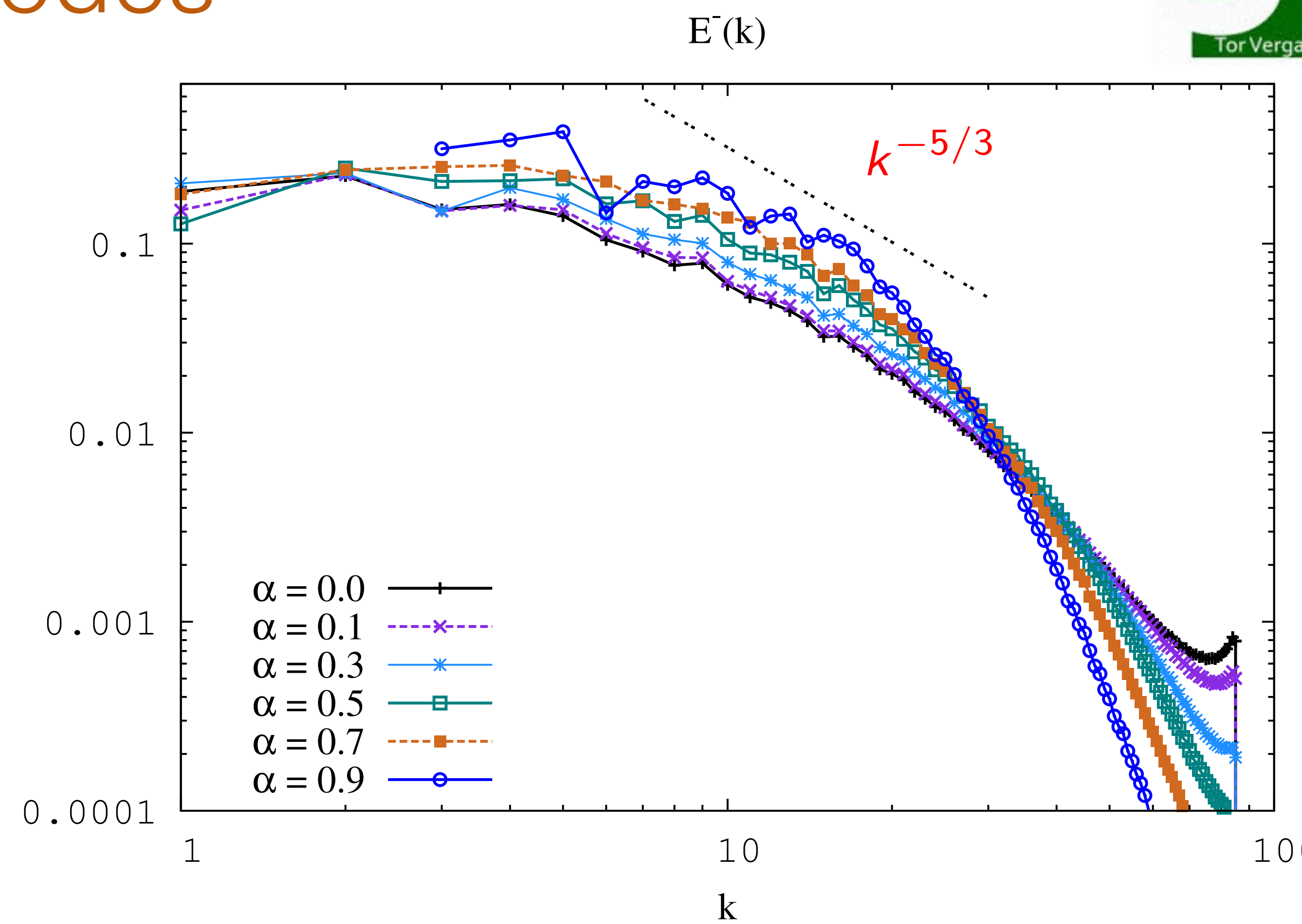
Chen, Phys. Fluids 2003

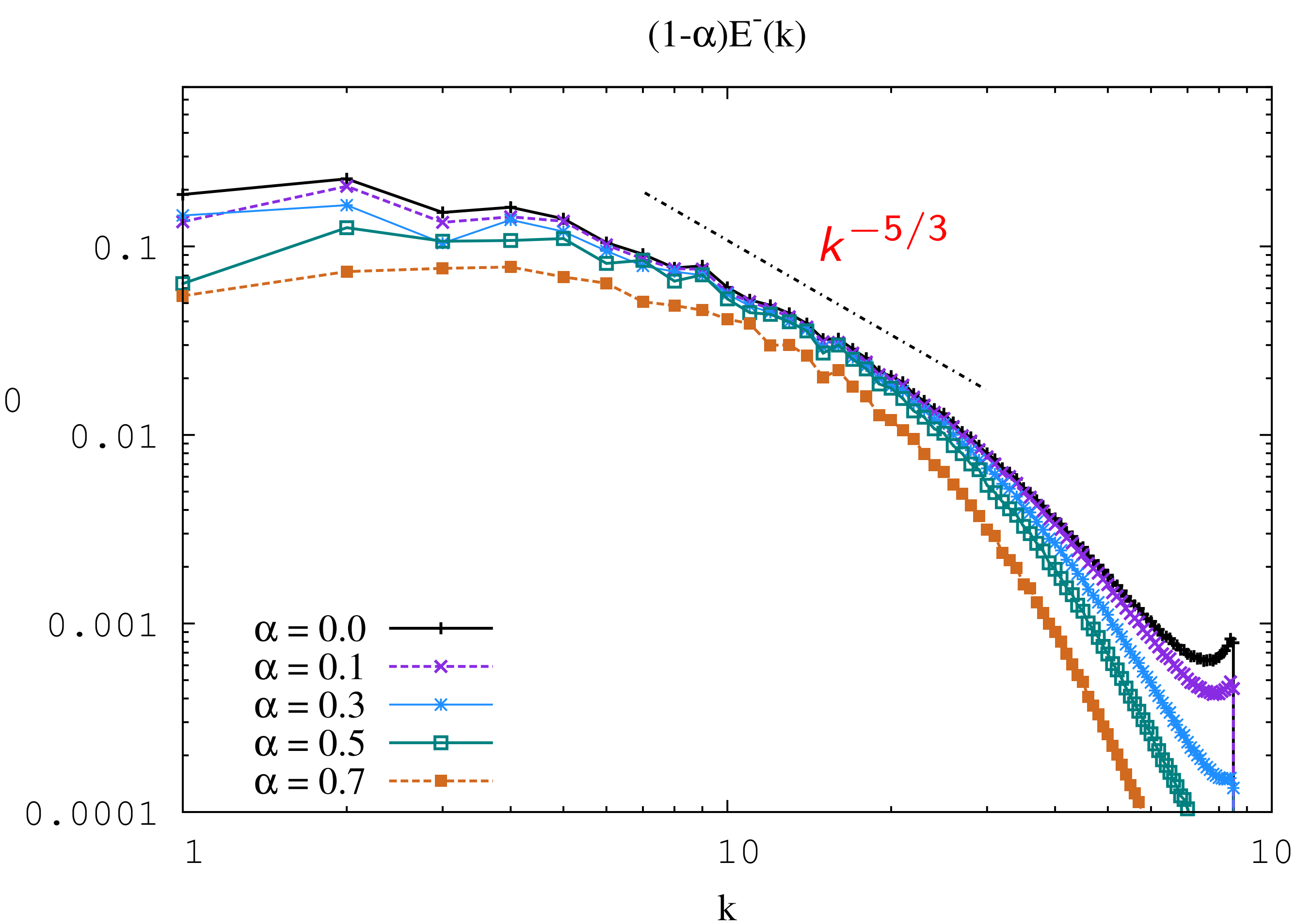
$$E^{\pm}(k) \sim C_1 \epsilon_E^{2/3} k^{-5/3}\left[1 \pm C_2\left(\frac{\epsilon_H}{\epsilon_E}\right)k^{-1}\right],$$

where ϵ_E is the mean energy dissipation rate and ϵ_H is the mean helicity dissipation rate.



- The $E^+(k)$ does not change with decimation.
- Invariance of parity is restored through scaling of $E^-(k)$ by the factor (1-α).





Summary

- As we increase decimation of the modes with negative helicity (α), the contribution of triads leading to inverse energy cascade grows.

- The forward cascade of energy is very robust in 3D turbulence. It requires only a few negative modes to act as catalyst to transfer energy forward.

- Only when α is very close to 1, i.e., we decimate almost all modes of one helical sign, inverse energy cascade takes over the forward cascade.

- We observe a strong tendency to recover parity invariance even in the presence of an explicit parity-invariance symmetry breaking ($\alpha > 0$).

- What about abrupt symmetry breaking at some k_c?
  - can we stop the cascade by killing all negatives modes from $k > k_c$?
  - or can we start it at our wish (killing all modes up to k_c)?
- What about intermittency in the forward cascade regime at changing α?

Classes of triadic interactions in NS equations

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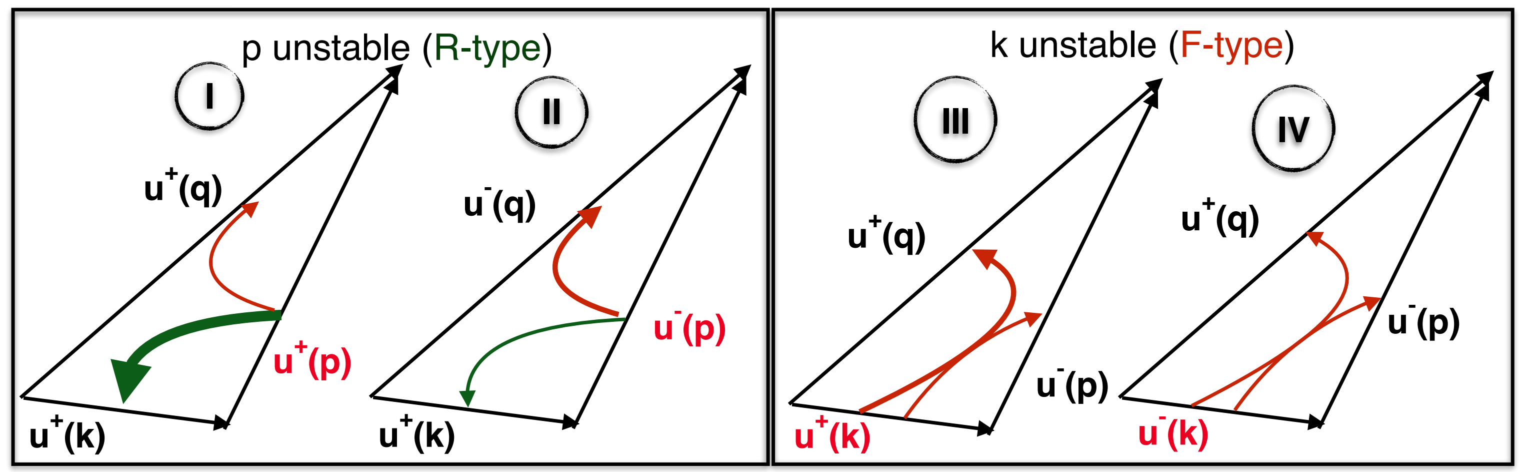
R-type: When large wavenumbers have same sign, middle one is unstable and could transfer energy to both small and large wavenumbers;

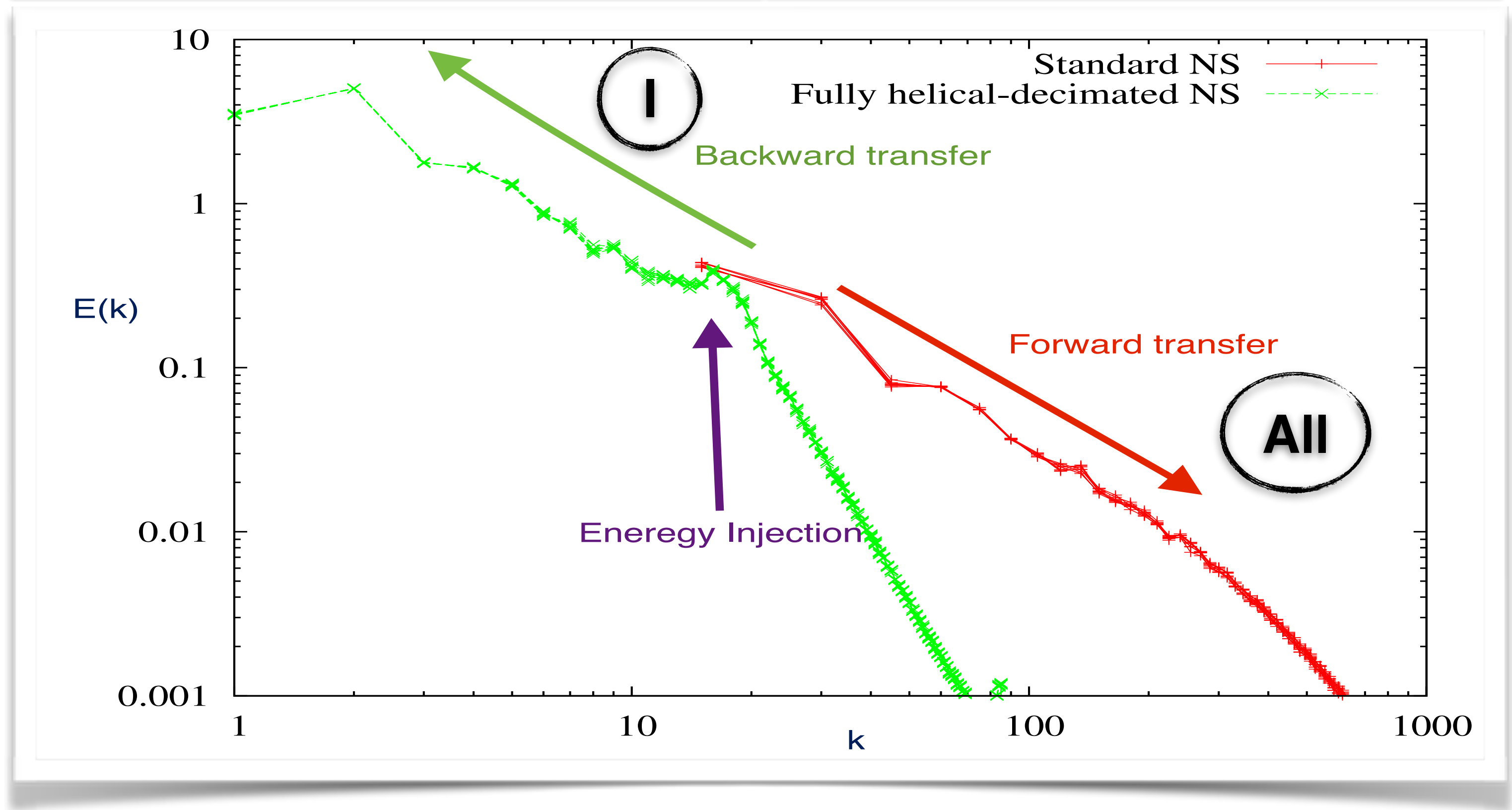
- predominantly to the smallest wavenumber if it has the same sign [Class-I (+, +, +)].
- mixed transfer if smallest wavenumber has the opposite sign [Class-II (+, -, -)].

F-type: When large wavenumbers have opposite sign, smallest one is unstable and could transfer energy only to large wavenumbers, for both Class-III (+, -, +) and Class-IV (-, -, +).

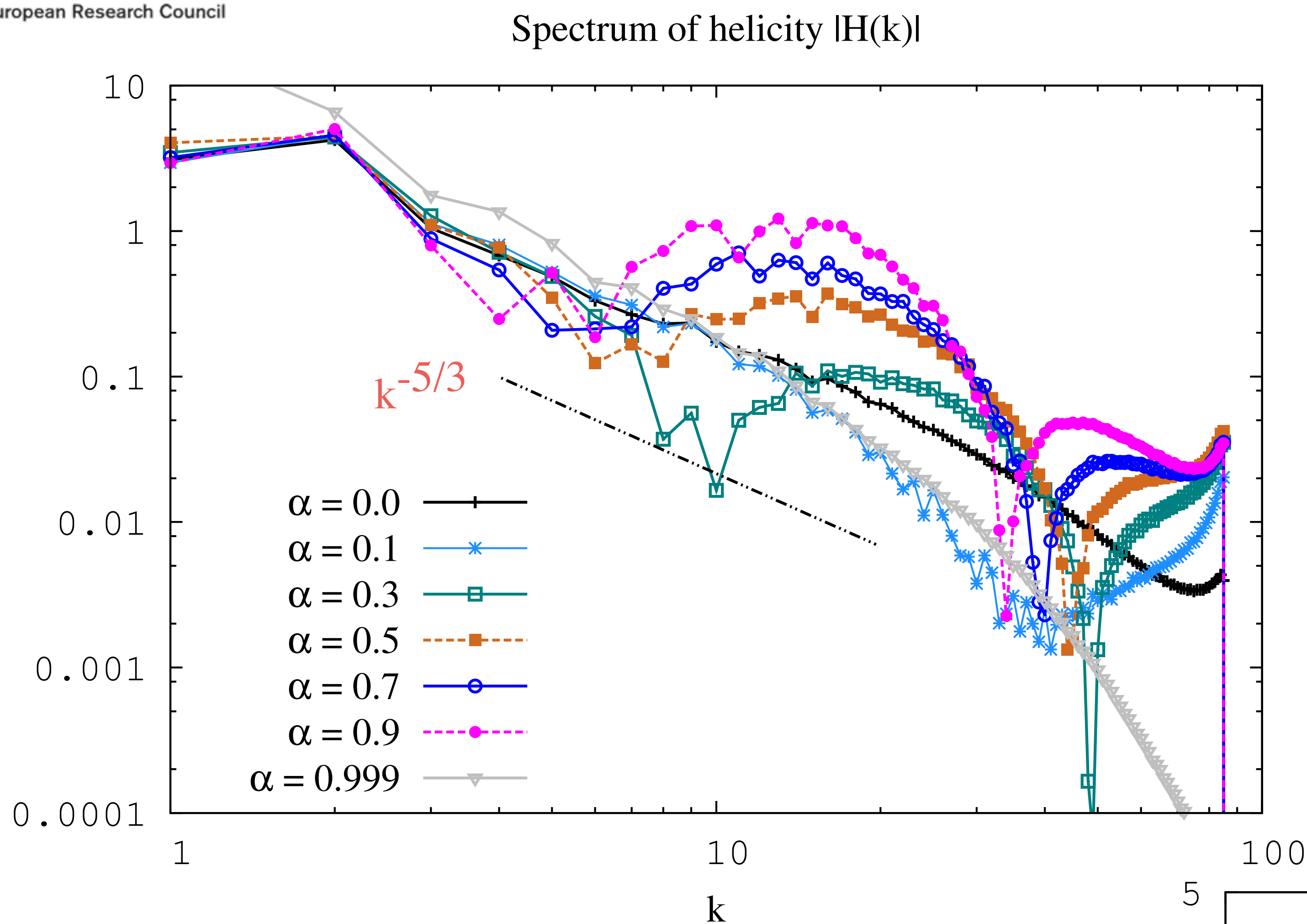
$$\mathbf{N}_{\mathbf{u}^{\pm}}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^{\pm}(\mathbf{k}) \cdot \nabla \mathbf{u}^{\pm}(\mathbf{p})\right]; \mathbf{q} = \mathbf{k} + \mathbf{p}; \, k \le p \le q$$

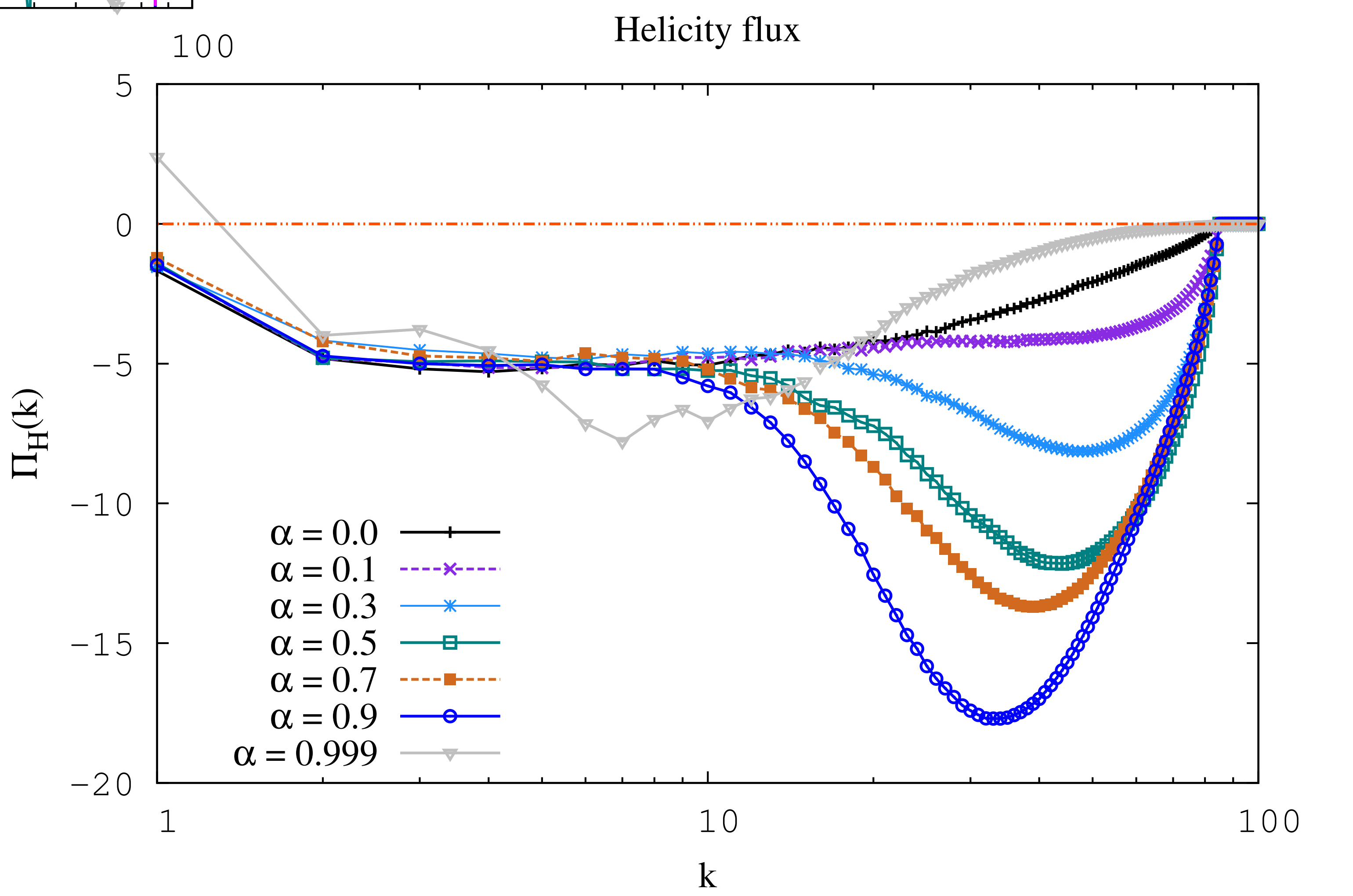


- Energy and helicity are conserved for each individual triad.
- Triads with only u⁺, i.e. Class-I, lead to reversal of energy cascade.
- Energy spectra in the inverse cascade regime shows a $k^{-5/3}$ slope.

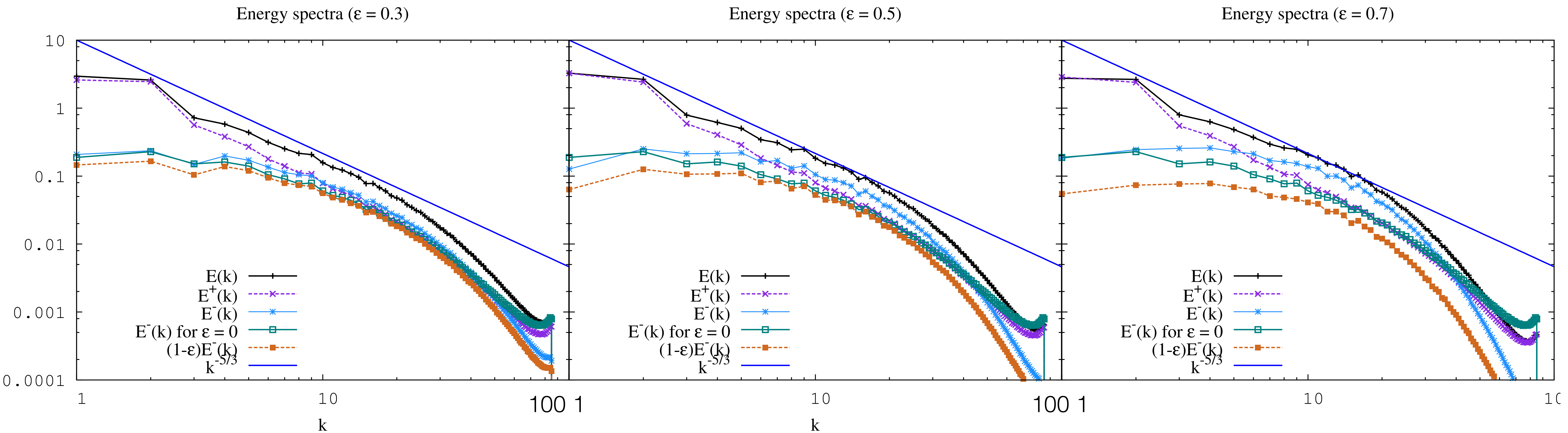


Helicity


Spectrum of helicity |H(k)|
10
1
0.1
0.01
0.001
0.0001
1
10
100
k
$k^{-5/3}$
α = 0.0
α = 0.1
α = 0.3
α = 0.5
α = 0.7
α = 0.9
α = 0.999


Helicity flux
5
0
−5
−10
−15
−20
1
10
100
k
$\Pi_H(k)$
α = 0.0
α = 0.1
α = 0.3
α = 0.5
α = 0.7
α = 0.9
α = 0.999

Recovery of parity invariance



- The $E^{-}(k)$ becomes higher than $E^{+}(k)$ in the inertial range with increasing α.
- Negative modes transfer energy more efficiently.
- Invariance of parity is restored through scaling of $E^{-}(k)$ by the factor (1-α)