Homogeneous isotropic turbulence: Particles, color is ω_z *Duane Rosenberg, Everest wall, OakRidge, 2015*

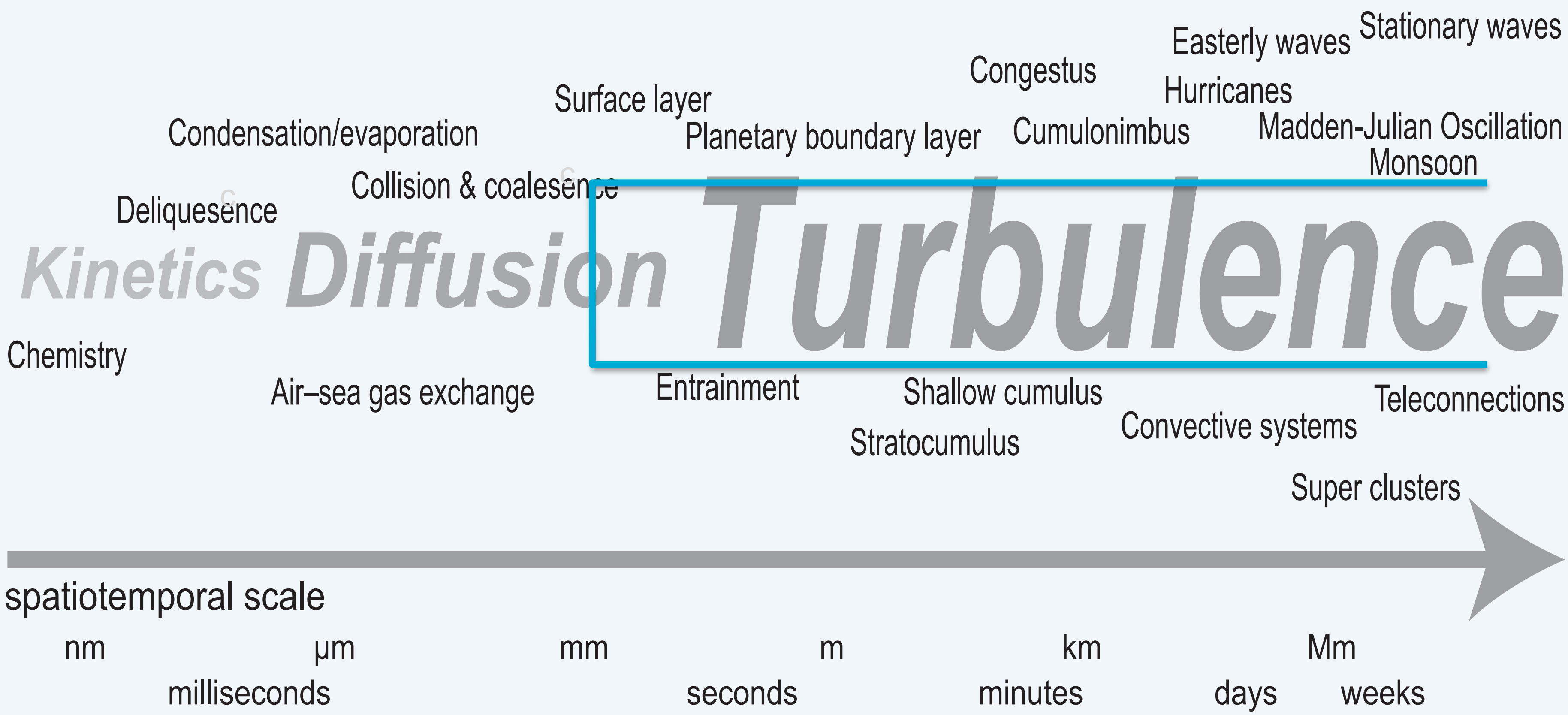
Stably stratified rotating turbulence What's different?

Annick Pouquet[1,2]

Corentin Herbert[3], Raffaele Marino[4*], Pablo Mininni[5], Cecilia Rorai[6*] & Duane Rosenberg[7]

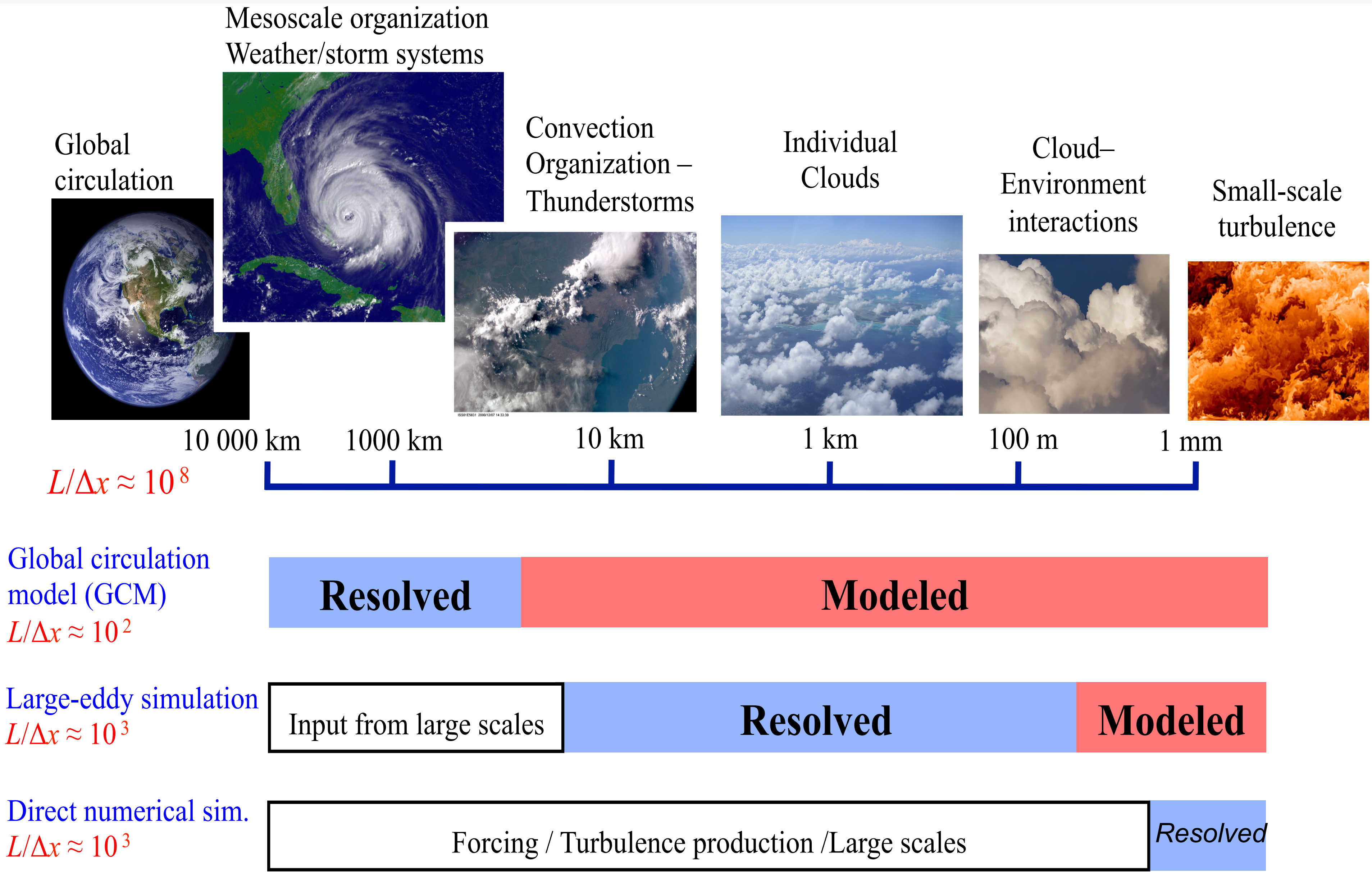
1: LASP; 2: NCAR; 3: Weizmann; 4: Berkeley; 5: U. Buenos Aires; 6: Nordita; 7: OakRidge

NSF/XSEDE - ASC090050 & TG-PHY100029 and Yellowstone (ASD/NCAR); INCITE/DOE - DE-AC05-00OR22725 * *NSF/CMG 1025183*

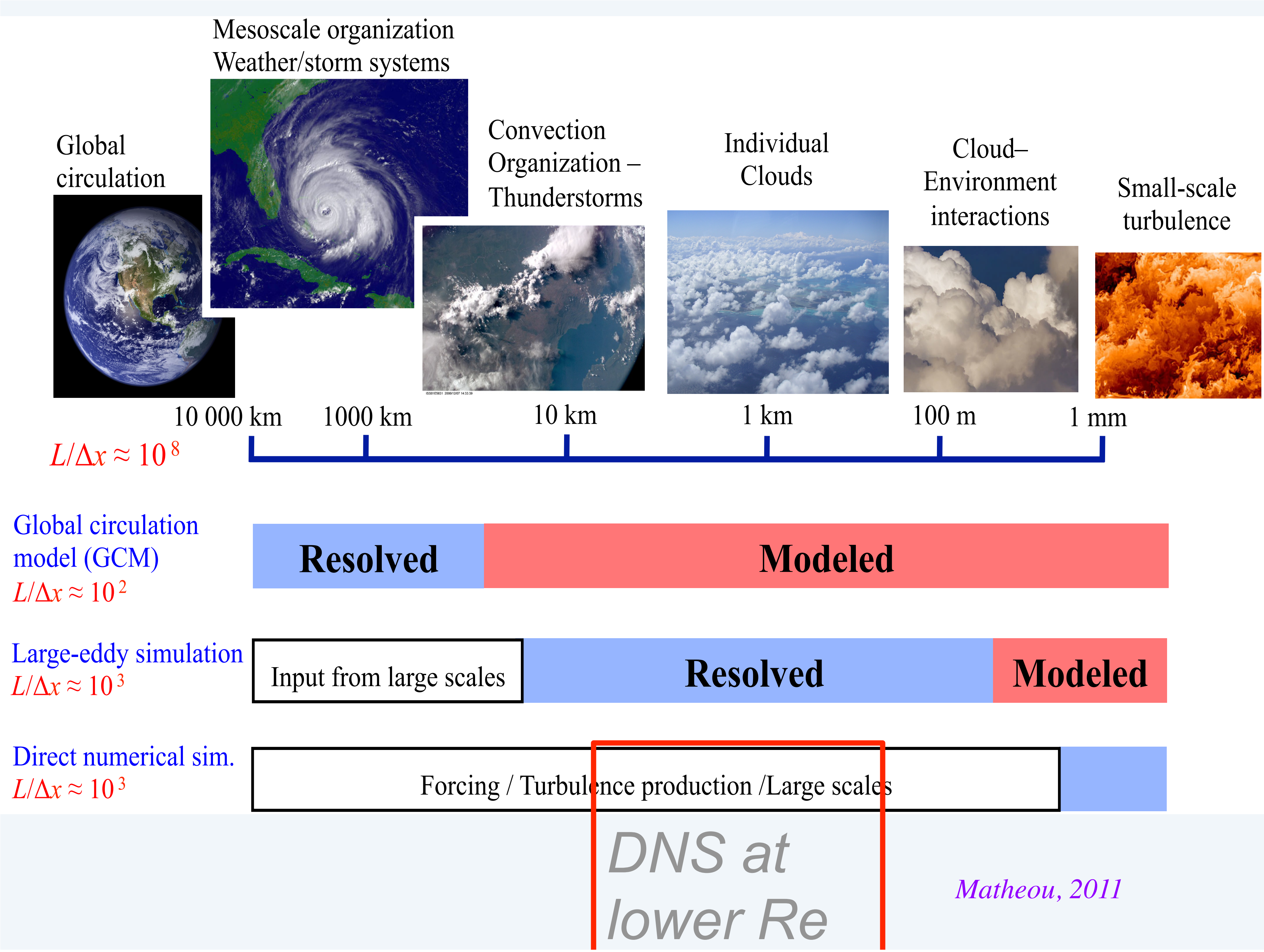


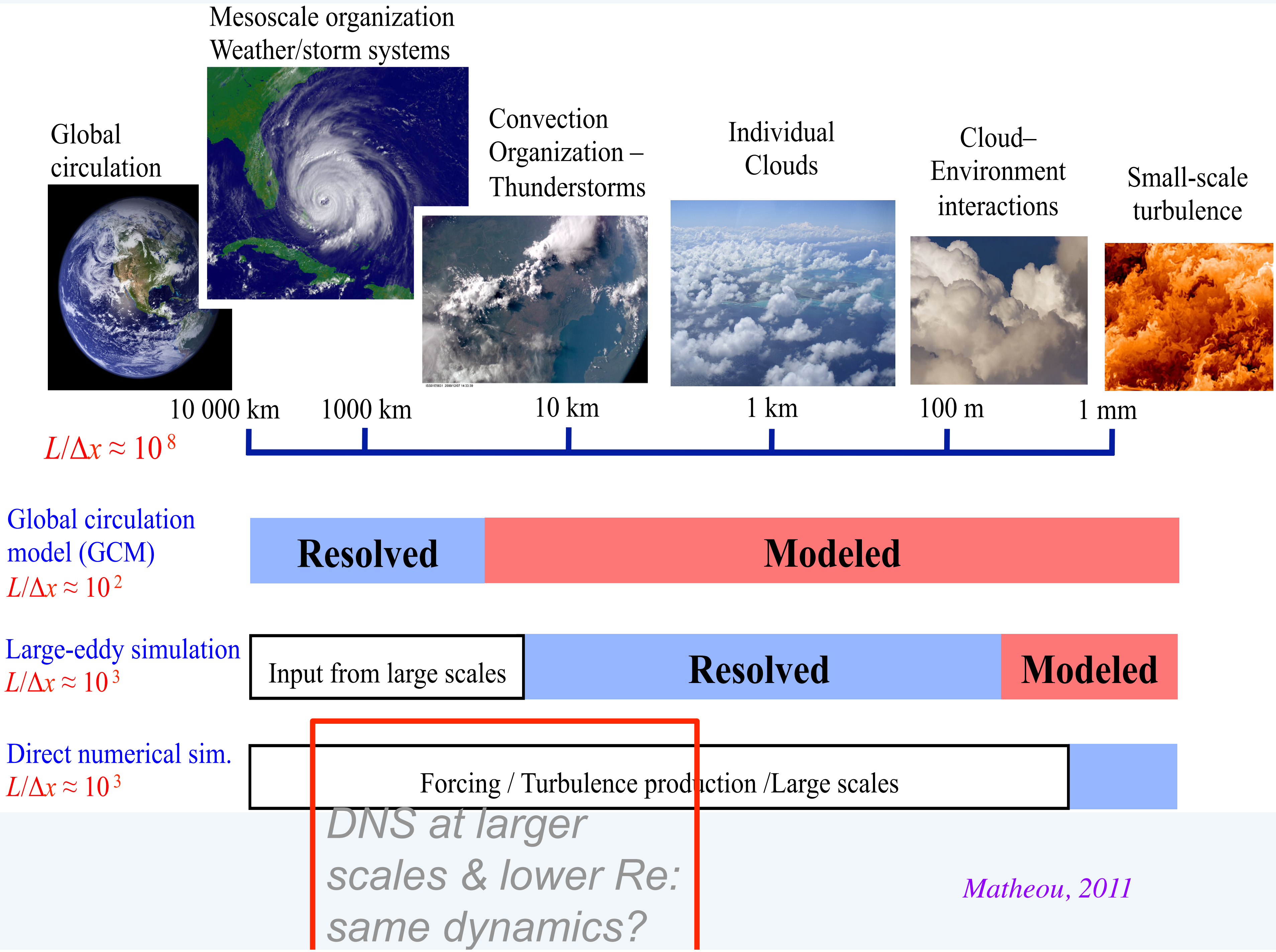
``Cloud controlling factors''

Bringuier et al., 2009



Matheou, 2011


Mesoscale organization
Weather/storm systems
Global circulation
Convection Organization – Thunderstorms
Individual Clouds
Cloud–Environment interactions
Small-scale turbulence
10 000 km
1000 km
10 km
1 km
100 m
1 mm
$L/\Delta x \approx 10^8$
Global circulation model (GCM)
$L/\Delta x \approx 10^2$
Resolved
Modeled
Large-eddy simulation
$L/\Delta x \approx 10^3$
Input from large scales
Resolved
Modeled
Direct numerical sim.
$L/\Delta x \approx 10^3$
Forcing / Turbulence production /Large scales
DNS at lower Re
Matheou, 2011


Mesoscale organization
Weather/storm systems
Global
circulation
Convection
Organization –
Thunderstorms
Individual
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Input from large scales
Resolved
Modeled
Direct numerical sim.
$L/\Delta x \approx 10^3$
Forcing / Turbulence production /Large scales
DNS at larger
scales & lower Re:
same dynamics?
Matheou, 2011

Boussinesq equations

$$\frac{D}{Dt}\boldsymbol{u} + f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \nabla p + N\theta\hat{\boldsymbol{z}} = \nu\nabla^2\boldsymbol{u} + \mathcal{F}$$

$$\frac{D}{Dt}\theta - Nw = \kappa\nabla^2\theta$$

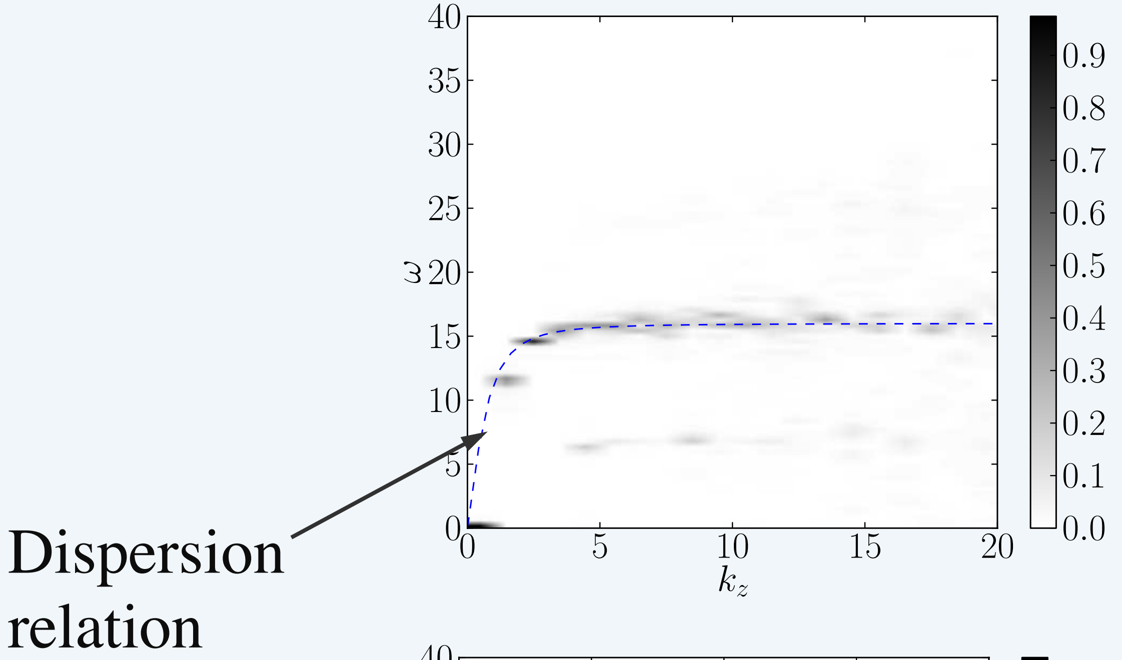
$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla$$

Frequency of (gravity-inertial) waves:

$\omega^2(k) = k^{-2} [N^2 k^2_{perp} + f^2 k^2_{//}]$

Wave dispersion broadening in rotating flows



Ro=0.015, N=0, Re~ 5000

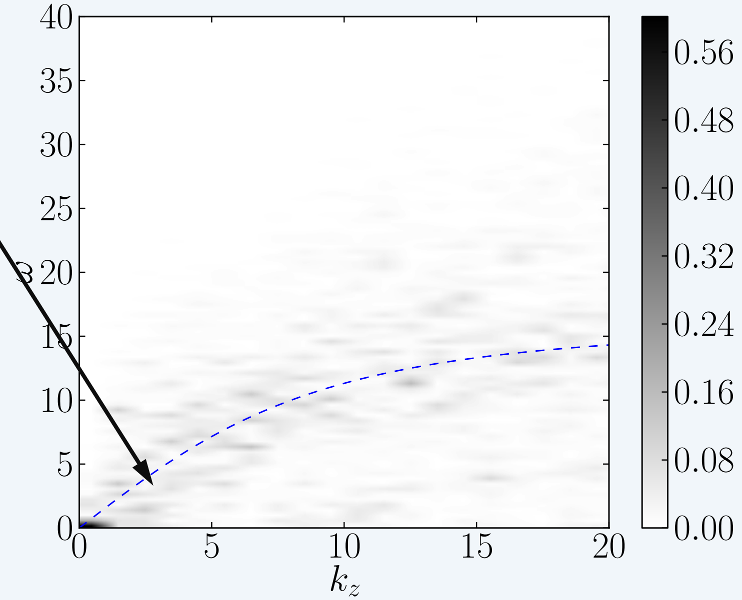
$\omega(k) = f\, k^{-1} k_{//}$

$k_x = 0$

$k_y = 1$

&

$k_y = 10$



Clark di Leoni et al. 2014

Boussinesq equations

$$\frac{D}{Dt}\boldsymbol{u} + f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \nabla p + N\theta\hat{\boldsymbol{z}} = \nu\nabla^2\boldsymbol{u} + \mathcal{F}$$

$$\frac{D}{Dt}\theta - Nw = \kappa\nabla^2\theta$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla$$

Four dimensionless parameters:

Re= UL/ν >> 1 , Pr= ν/ϰ =1,

Ro= U/[Lf] << 1 , Fr= U/[LN]<< 1

R_B= Re Fr^2

Frequency of (gravity-inertial) waves:

$\omega_k = [1/k]\sqrt{N^2 k_{perp}^2 + f^2 k_{//}^2}$

Do **waves** alter the overall dynamics?

Stable Boussinesq stratification

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} = -\nabla P - Nbe_z \qquad + \mathrm{F}$$
$$\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b = Nw\ ,$$
$$\nabla\cdot\mathbf{u} = 0\ .$$

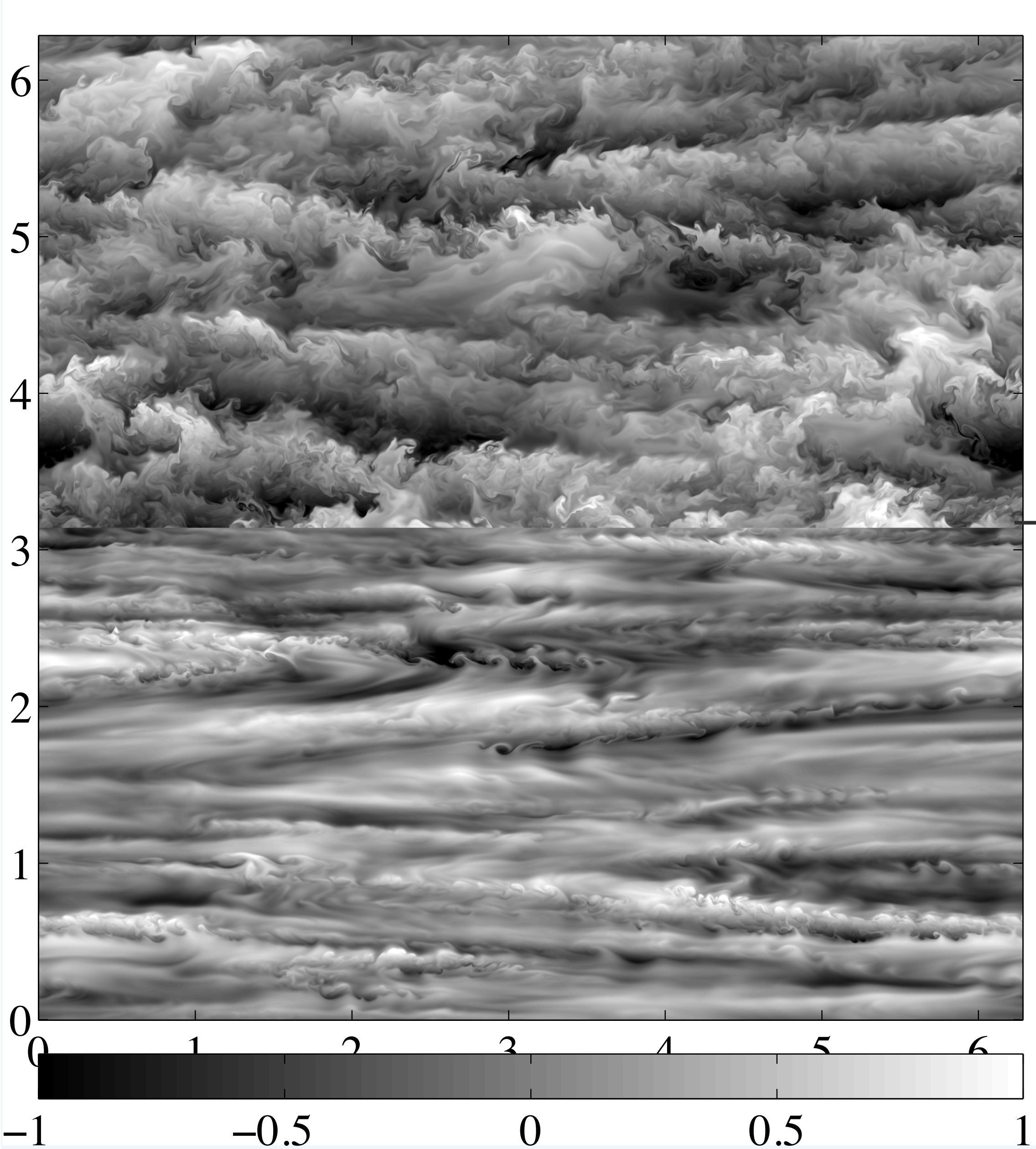
* $F_z=1$ → $L_z=u_{perp}/N$: buoyancy scale L_B *(Billant Chomaz 2001)*

* *Together with using div u=0* → $L_z/L_{perp} = u_z/u_{perp} = Fr << 1$

* Measure of vertical nonlinearity: $R_Z = u_z L_z/\nu = Fr^2\ u_{perp} L_{perp}/\nu$

→ $R_Z = Fr^2\ Re = \varepsilon/[\nu N^2] = R_B$: *Buoyancy Reynolds number*

Stratification, no rotation: Temperature fluctuations, xz slice, Re ~ 24000, 2048^3 grids, K_F ~2-3

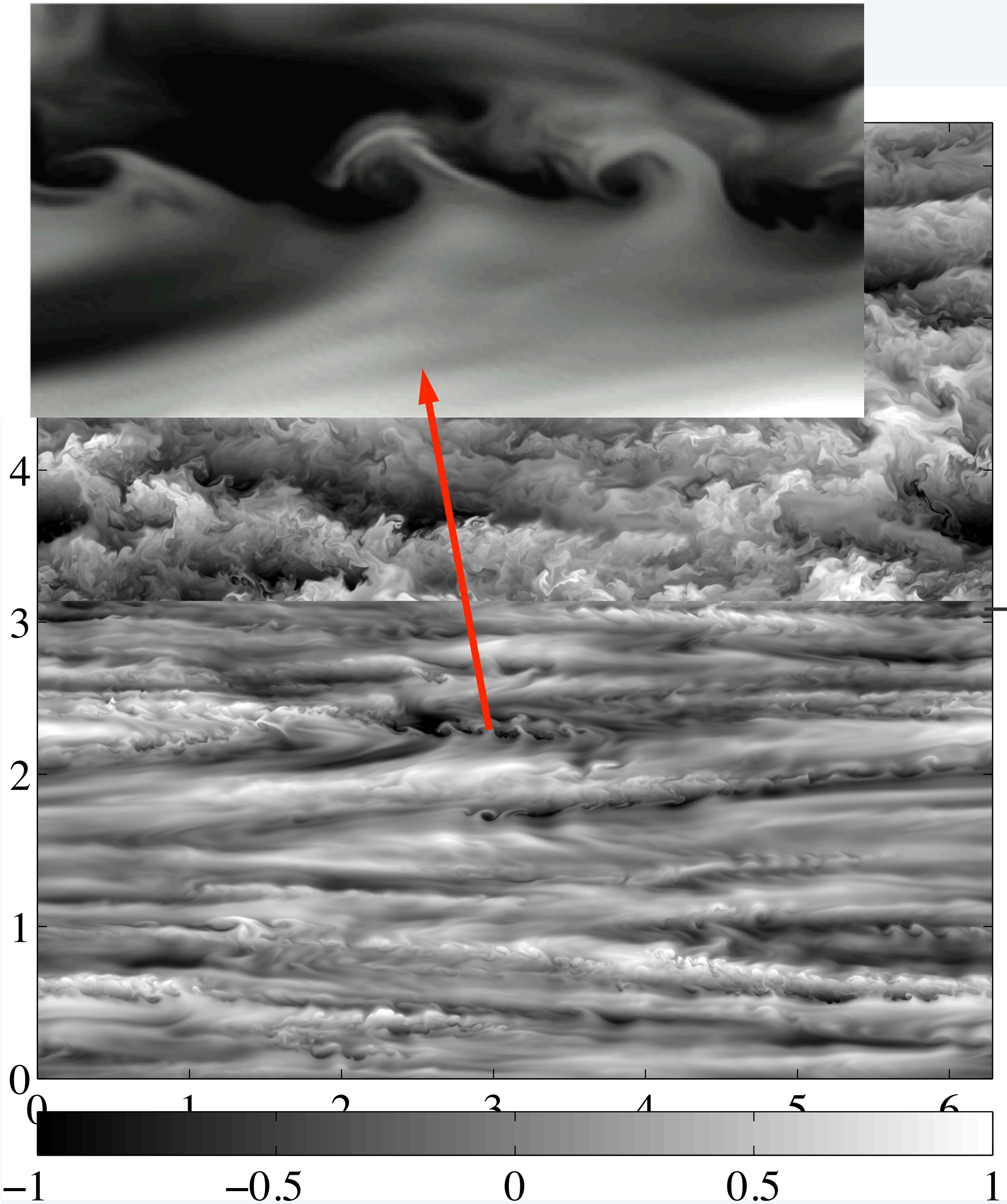


$R_B = ReFr^2$: buoyancy

Fr ~ 0.11 *(N=4)*
R_B ~ 300

Fr ~ 0.03 *(N=12)*
R_B ~ 22

Rorai et al., 2014



Pure stratification

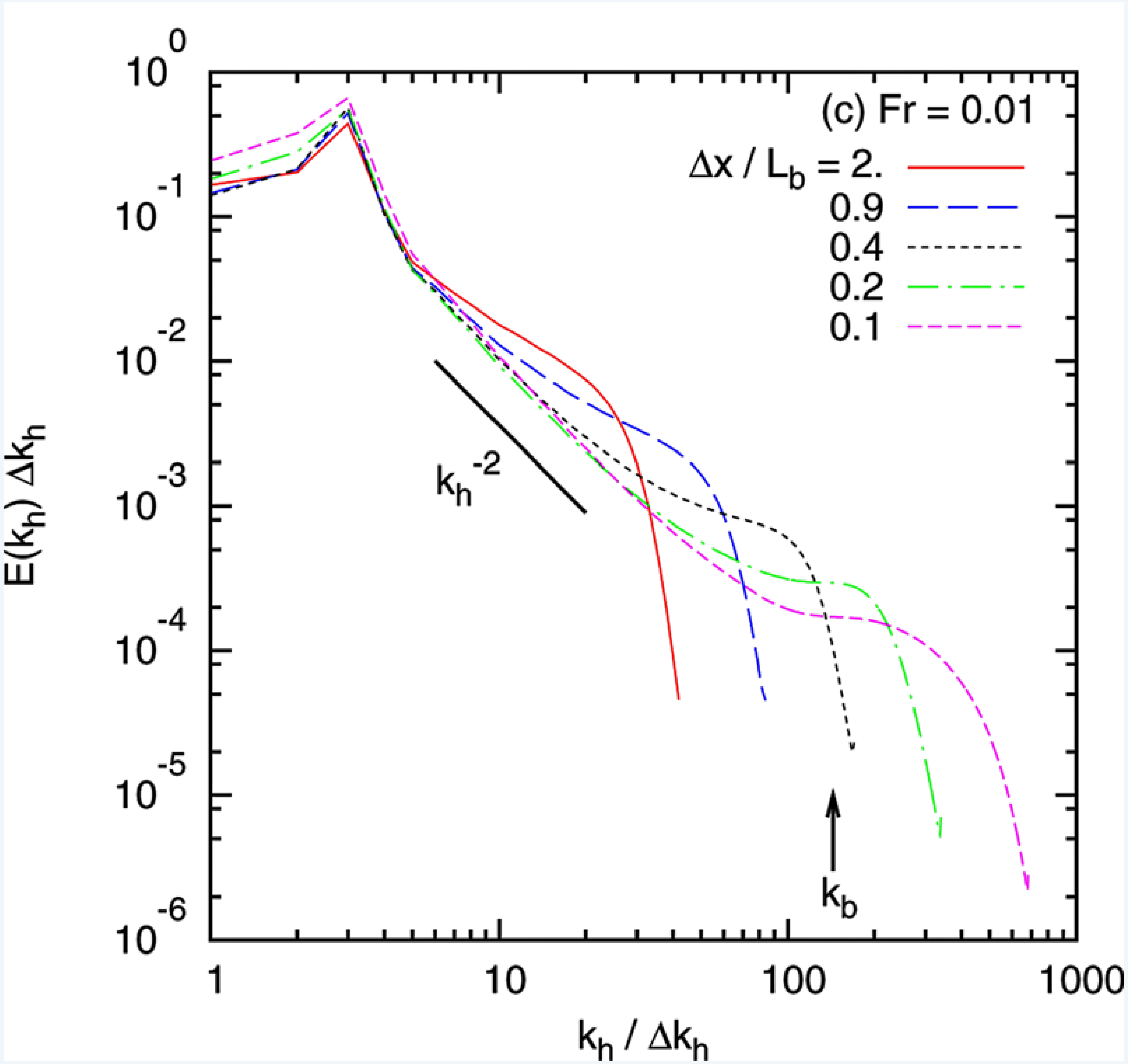
$Fr \sim 0.11, R_B \sim 300$

$Fr \sim 0.03, R_B \sim 22$

Rorai et al., 2014

Need for resolving characteristic scales:

Stratified turbulence with increasing horizontal resolution



Buoyancy (vertical) scale:

$L_B \sim N^{-1}$: $Fr = U/[L_B N] = 1$

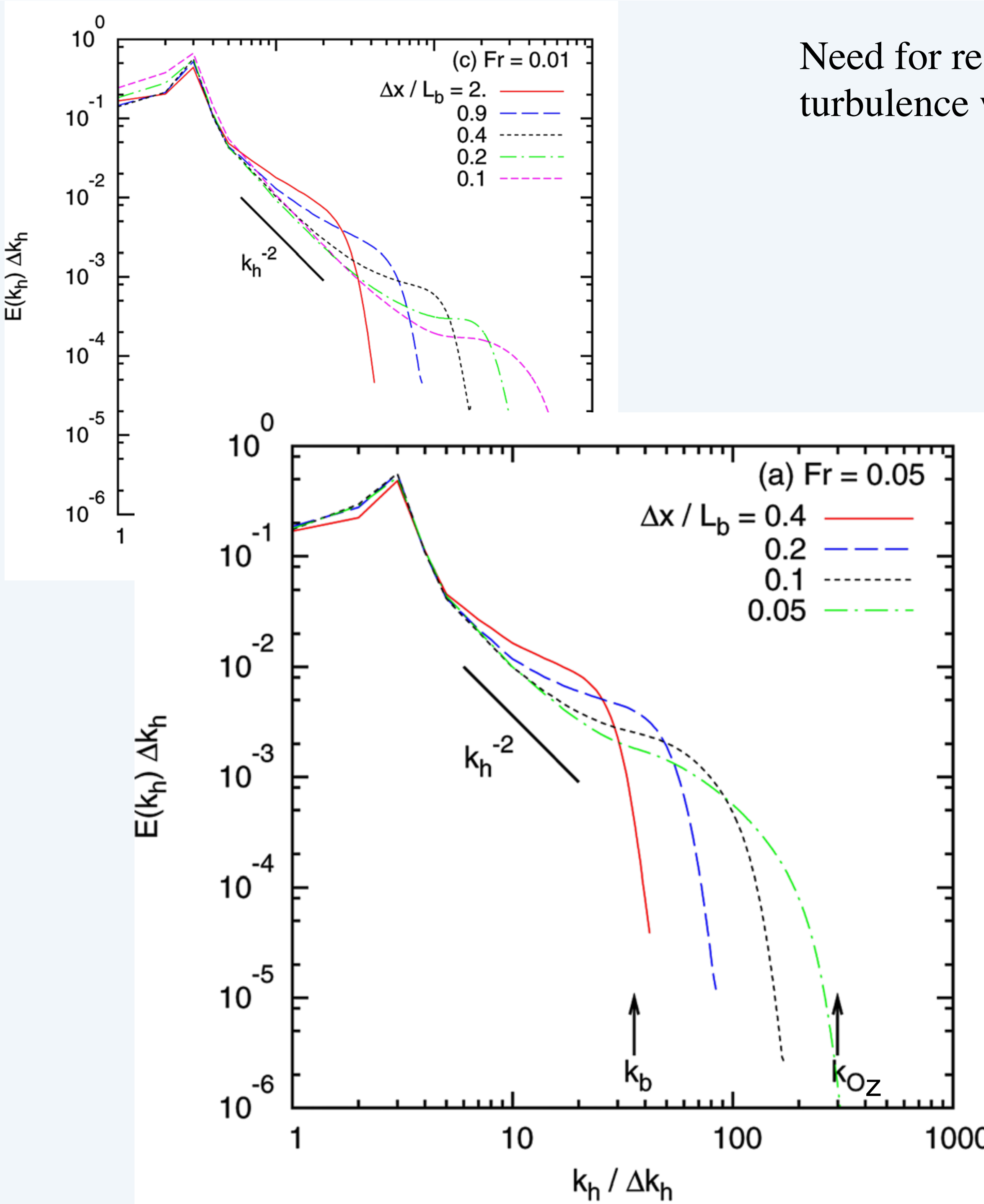
Ozmidov scale:

$L_{oz} \sim N^{-3/2}$: $T_{NL} = T_w$

(unresolved here)

Waite, PoF 2011

Need for resolving characteristic scales: Stratified turbulence with increasing horizontal resolution



Buoyancy (vertical) scale:

$L_B \sim N^{-1}$: $Fr = U/[L_B N] = 1$

Ozmidov scale:

$L_{oz} \sim N^{-3/2}$: $T_{NL} = T_w$

(

← *Now resolved here*

Waite, PoF 2011

Horizontal vorticity

$\Delta x/L_b = 0.9$

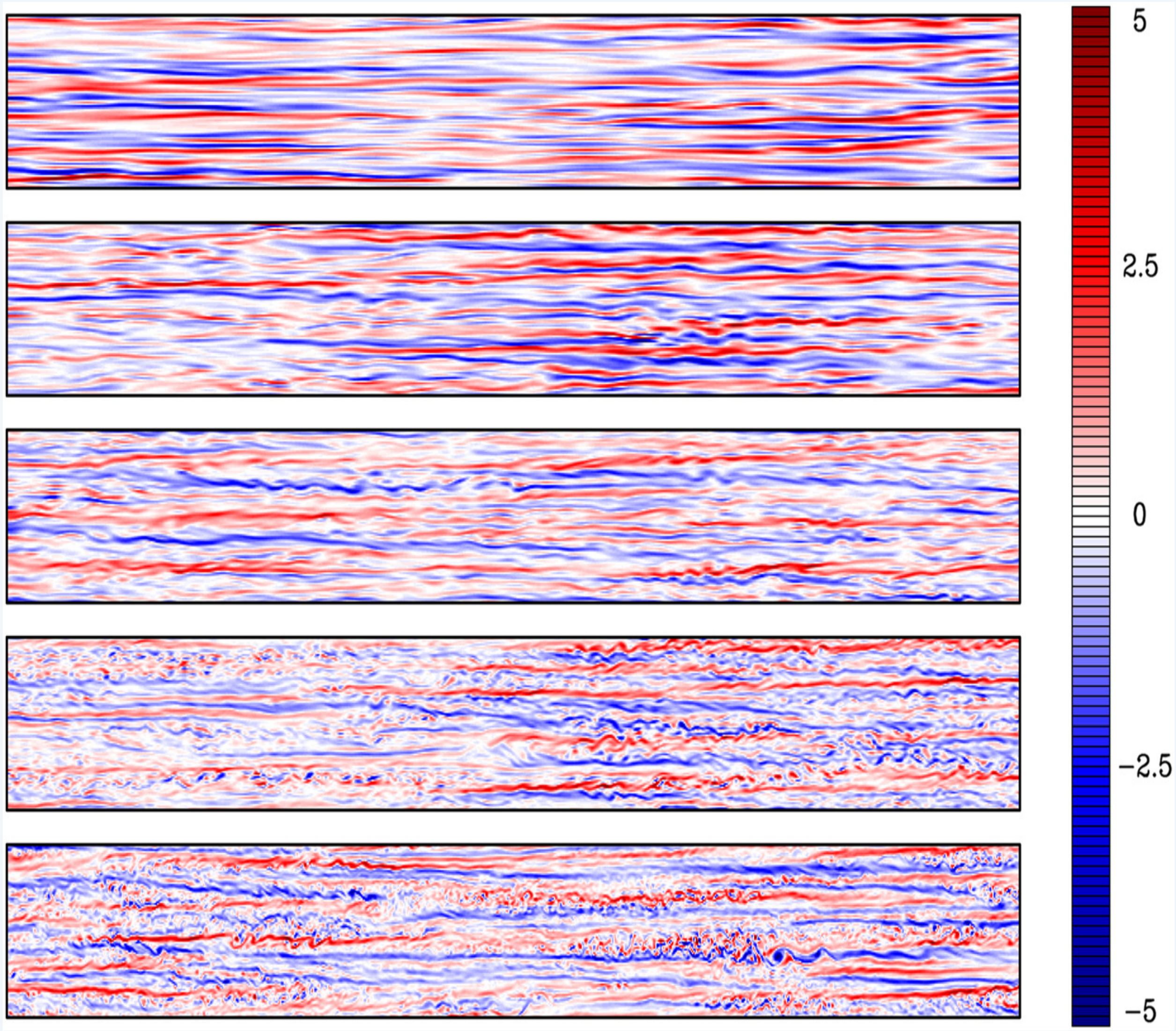


FIG. 2. (Color online) Vertical slices through the $y=0$ plane of ω_y/N for $Fr=0.02$ and (from top to bottom) $\Delta x/L_b = 0.9, 0.4, 0.2, 0.1$, and 0.05 (runs B1-B5 in Table I). For clarity, only half the domain $0 \leq x \leq L/2$ is shown. All fields are plotted at the end of the simulation.

$\Delta x/L_b = 0.05$

Stratified turbulence: resolving the buoyancy scale L_b

Waite, PoF 2011

Geostrophic Balance

$$\frac{D}{Dt}\boldsymbol{u} + \boxed{f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \nabla p + N\theta\hat{\boldsymbol{z}}} = \nu\nabla^2\boldsymbol{u} + \mathcal{F}$$

$$\frac{D}{Dt}\theta - Nw = \kappa\nabla^2\theta$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

→ Hydrostatic balance in the vertical

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla,$$

Boussinesq → Geostrophic Balance

$$\frac{D}{Dt}\boldsymbol{u} + \boxed{f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \nabla p + N\theta\hat{\boldsymbol{z}}} = \nu\nabla^2\boldsymbol{u} + \mathcal{F}$$

$$\frac{D}{Dt}\theta - Nw = \kappa\nabla^2\theta$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla,$$

Take the curl → "thermal winds"

$N\,\partial_y\theta = -\,f\,\partial_z u_x$

$N\,\partial_x\theta = f\,\partial_z u_y$

Boussinesq → Geostrophic Balance

$$\frac{D}{Dt}\boldsymbol{u} + f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \nabla p + N\theta\hat{\boldsymbol{z}} = \nu\nabla^2\boldsymbol{u} + \mathcal{F}$$

$$\frac{D}{Dt}\theta - Nw = \kappa\nabla^2\theta$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla,$$

Take the curl → "thermal winds"

$N \, \partial_y\theta = - f \, \partial_z u_x$

$N \, \partial_x\theta = f \, \partial_z u_y$

One more step (Hide '72): dot product with f ***zxu*** *& horizontal average*

→ Creation of helicity through rotation and stratification

$$\langle u_\perp \cdot \nabla \times u_\perp \rangle_\perp = \frac{N}{f}\langle \theta \frac{\partial w}{\partial z} \rangle_\perp$$

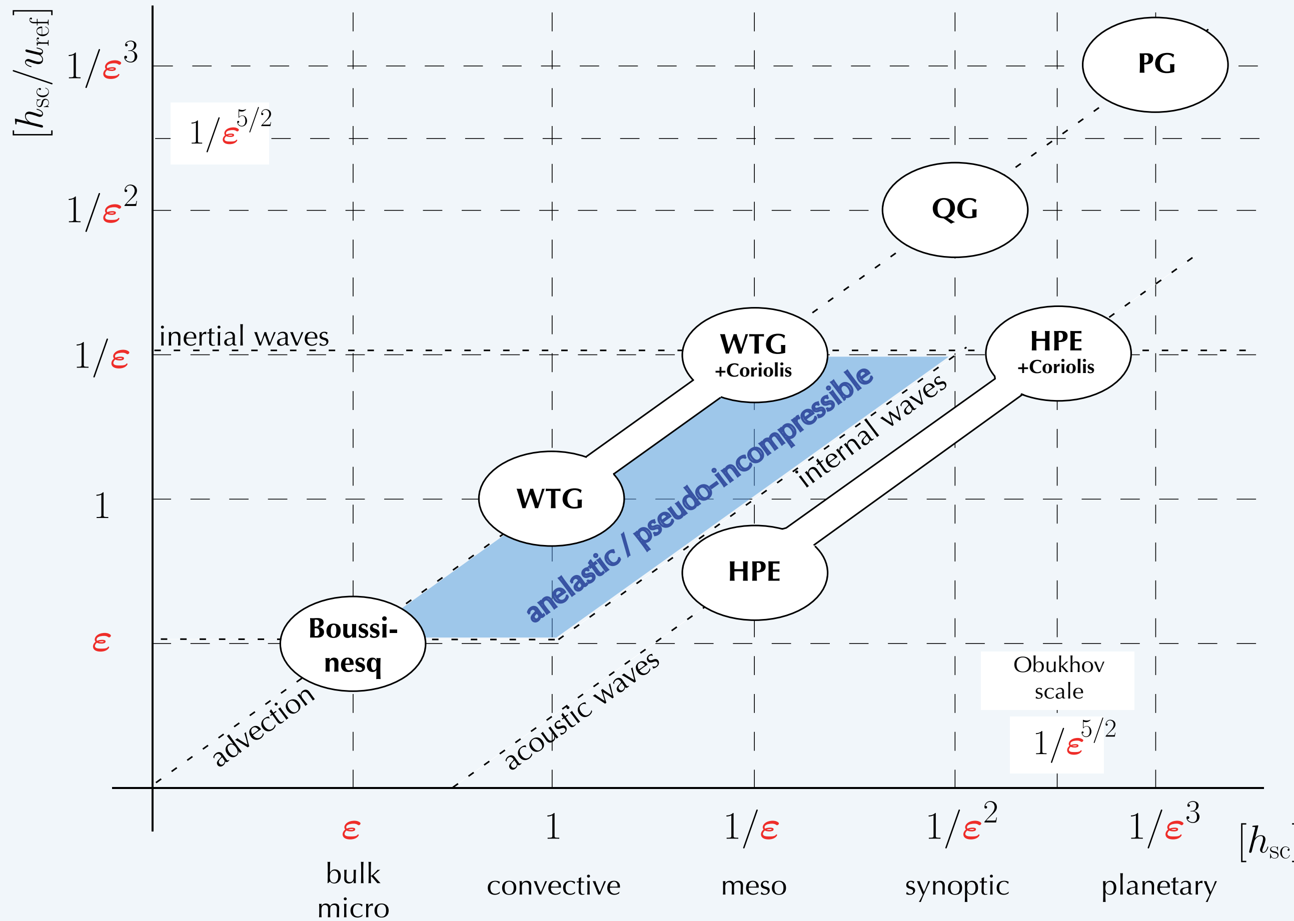
Recovered classical single-scale models:

$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\frac{t}{\varepsilon}, \boldsymbol{x}, \frac{z}{\varepsilon})$	Linear small scale internal gravity waves
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(t, \boldsymbol{x}, z)$	Anelastic & pseudo-incompressible models
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon t, \varepsilon^2 \boldsymbol{x}, z)$	Linear large scale internal gravity waves
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon^2 t, \varepsilon^2 \boldsymbol{x}, z)$	Mid-latitude Quasi-Geostrophic Flow
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon^2 t, \varepsilon^2 \boldsymbol{x}, z)$	Equatorial Weak Temperature Gradients
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon^2 t, \varepsilon^{-1} \xi(\varepsilon^2 \boldsymbol{x}), z)$	Semi-geostrophic flow
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\underline{\varepsilon^{3/2}} t, \underline{\varepsilon^{5/2}} x, \underline{\varepsilon^{5/2}} y, z)$	Kelvin, Yanai, Rossby, and gravity Waves

Klein, 2010

Atmospheric Flow Regimes

ε: *Froude number*
h_{sc}: *density scale height*



R.K., Ann. Rev. Fluic

Scaling regimes and model equations for atmospheric flows. The weak-temperature-gradient (WTG) and hydrostatic primitive equation (HPE) models cover a wide range of spatial scales assuming the associated advective and acoustic timescales, respectively. The anelastic and pseudoincompressible models for realistic flow regimes cover multiple spatiotemporal scales (Section 4.3). For similar graphs for near-equatorial flows, see Majda 2007b, Majda & Klein 2003. PG, planetary geostrophic; QG, quasi-geostrophic.

Klein, 2010

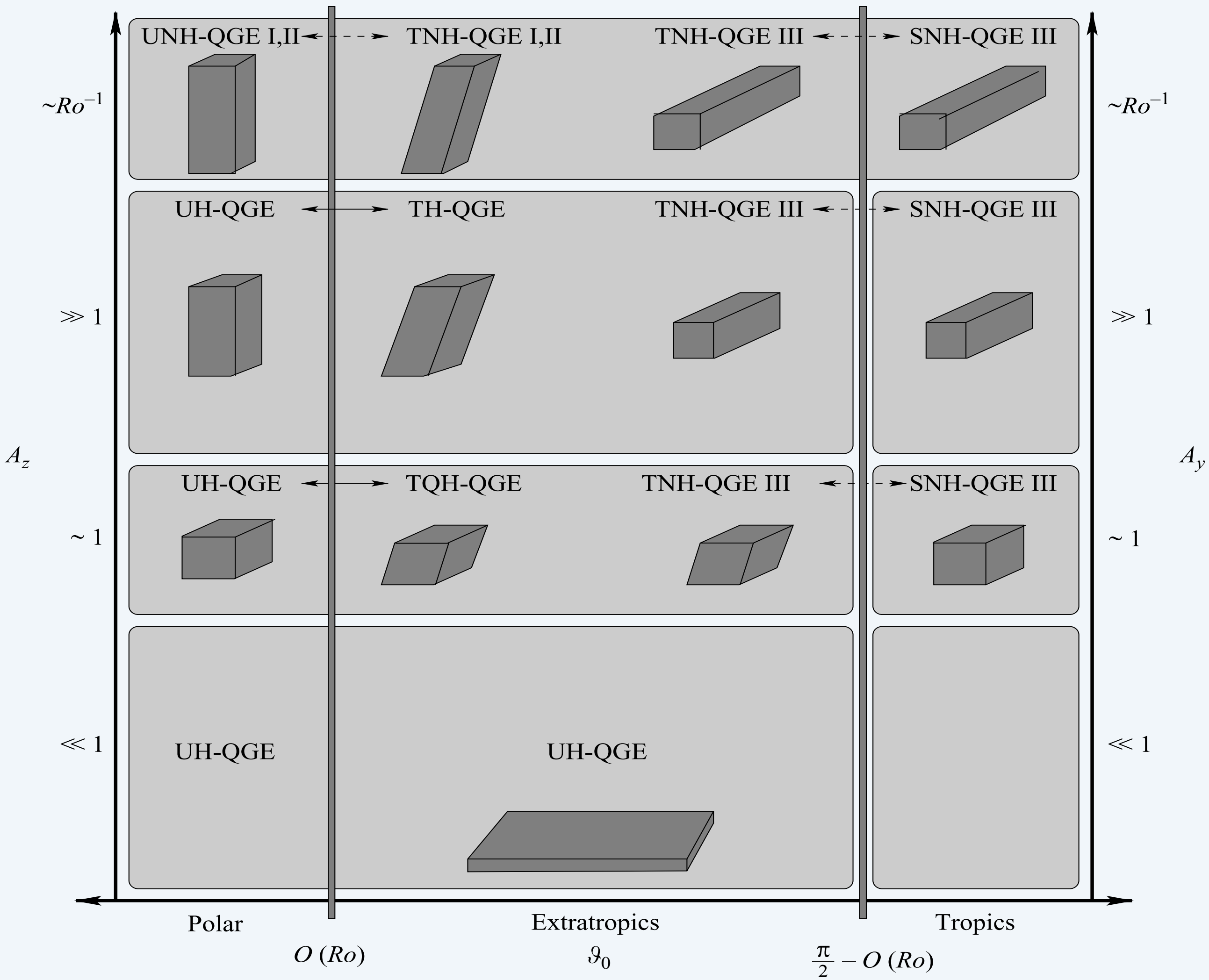
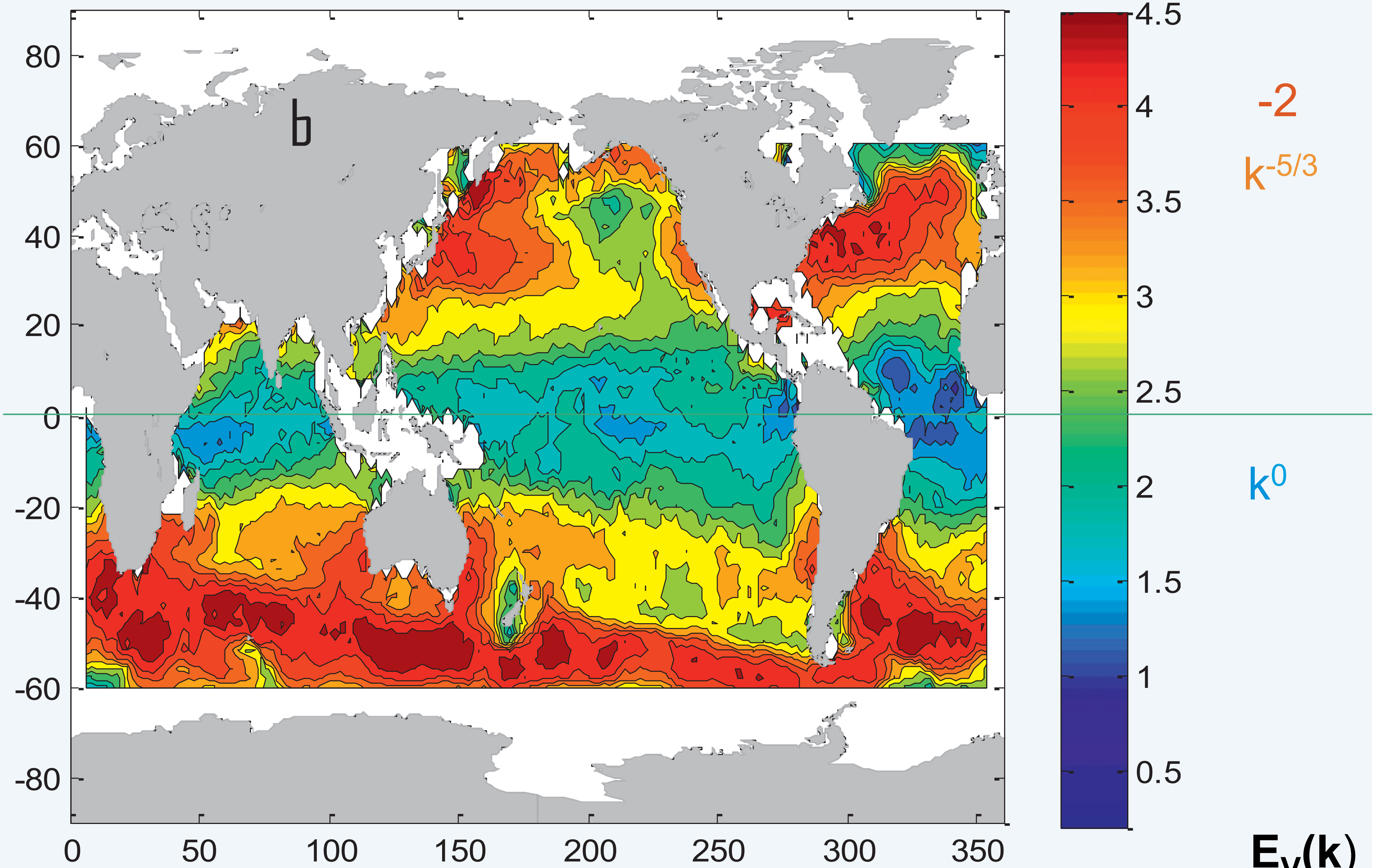


FIGURE 4. Classification of the reduced U–Upright, T–Tilted, S–Sideways QG models (see table 4) as a function of the colatitude ϑ_0, and the spatial aspect ratios A_z or A_y. H–hydrostatic, QH–quasi-hydrostatic, NH–non-hydrostatic. With the exception of TNH-QGE III A_z distinguishes between all models in the polar and extratropical regions where $A_y = O(1)$, while A_y distinguishes between the tropical QGE and TNH-QGE III for which $A_z = O(1)$. The symbol $\longleftrightarrow$ indicates a continuous transition between different models while ⇠⇢ indicates extension of a model to the polar or equatorial regions.

Jason de-noised altimeters: $S_{ea}S_{surface}H_{eight}$



Xu & Fu JPO 2012

Resolution of 7km, range of 50-500km, 1 s average, for 6 months.

Δ of 1 minute

What’s different in rotating &/or stratified turbulence (R/ST)?

- *Direct and inverse cascades in homogeneous isotropic turbulence*
- Bi-directional constant-flux energy cascades & oceanic mixing
- Bolgiano-Obukhov scaling and the role of potential energy
- Development of large vertical velocity in stratified flows
- Role of helicity (velocity-vorticity correlations)

Homogeneous, isotropic turbulence: Navier-Stokes eqs.

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} - \nu \Delta \mathbf{u} = -\nabla P + \mathrm{F}$$

$$\nabla \cdot \mathbf{u} = 0 .$$

One dimensionless parameter: Re= UL/ν >> 1

u: velocity , P: pressure

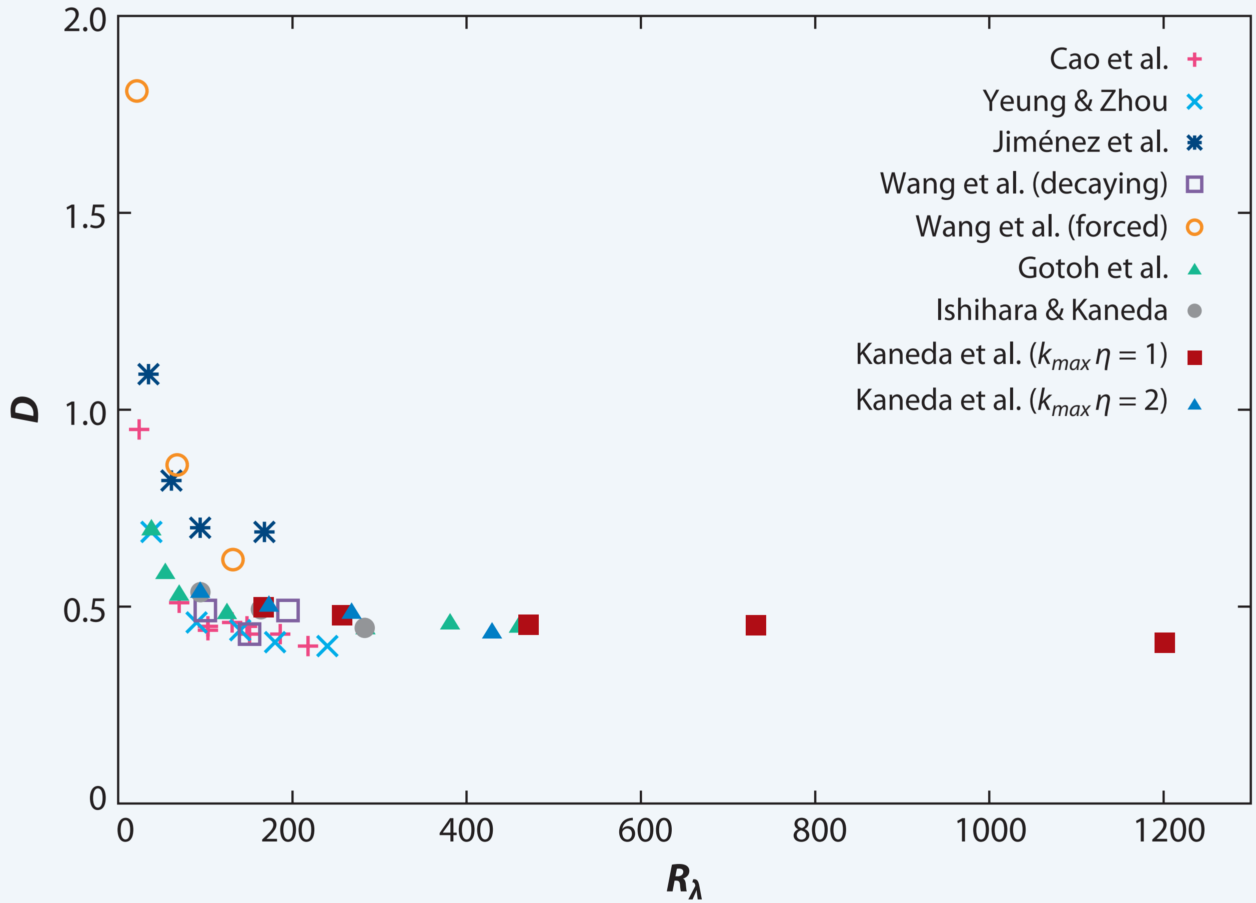
ν: viscosity , F: Force

Invariants (ν=0, F=0):

3D: Energy $<u^2/2>$ & *helicity* $<\boldsymbol{u}.\nabla\mathrm{x}\boldsymbol{u}>$

2D: Energy $<u^2/2>$ & *enstrophy* $<|\nabla\mathrm{x}\boldsymbol{u}|^2>$

- Direct numerical simulations: process study for a range of parameters: *Dissipation for homogeneous isotropic turbulence*



Kaneda et al., 2003; Ishihara & Kaneda, Ann. Rev. 2009

Vorticity $\omega = \nabla \times u$

Direct numerical simulation of homogeneous isotropic turbulence

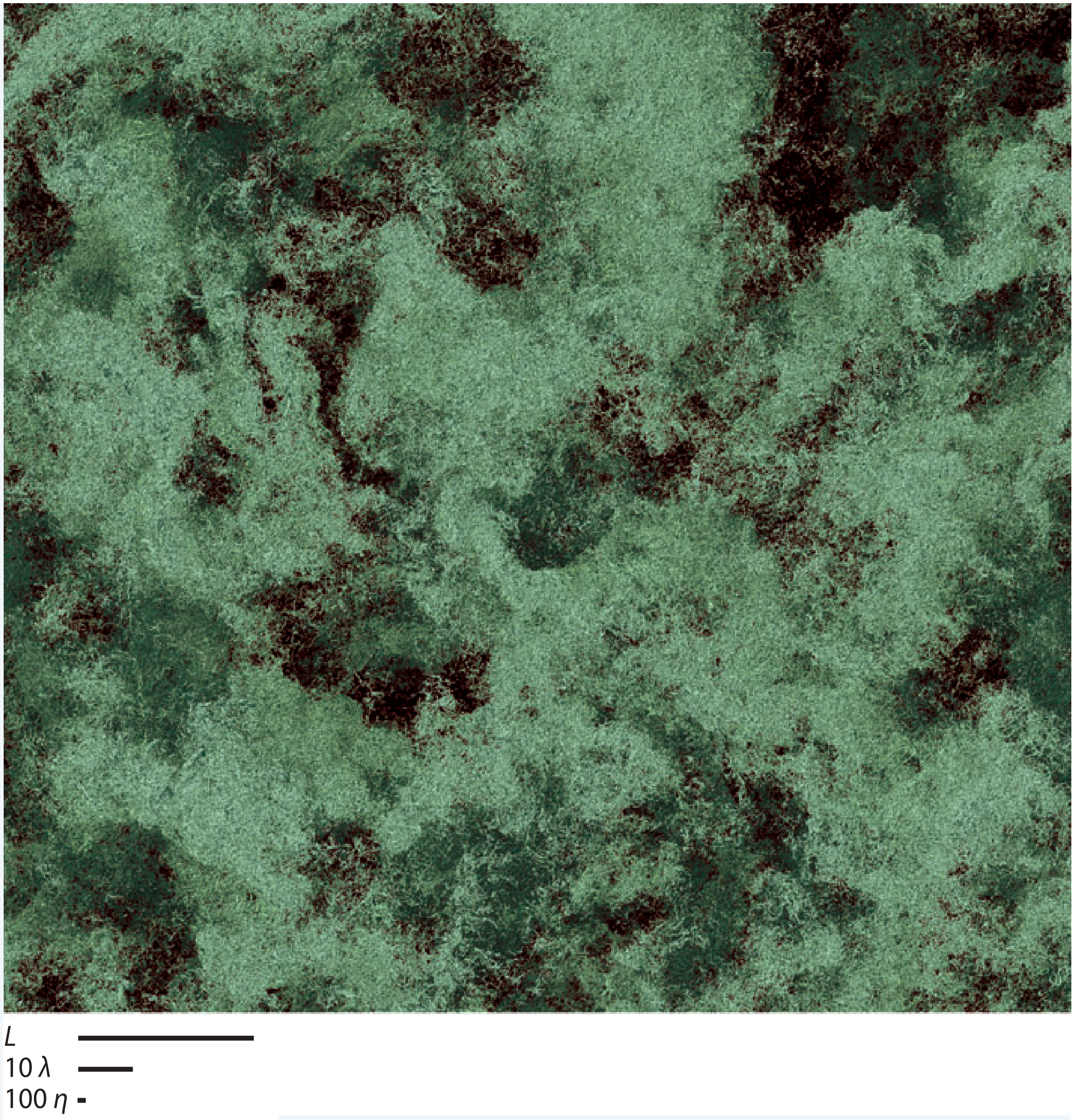
Incompressible, 3D Navier-Stokes

Periodic boundary conditions

64+ billion grid points

(4096^3)

Ishihara Kaneda '03, Earth sim.



NEW! De Bruyn-Kops, 2015: 4096×8092^2 with stratification

NEW! Kaneda, 2015: 12288^3 homogeneous isotropic turbulence

Vorticity $\omega = \nabla \times u$

Direct versus inverse cascades

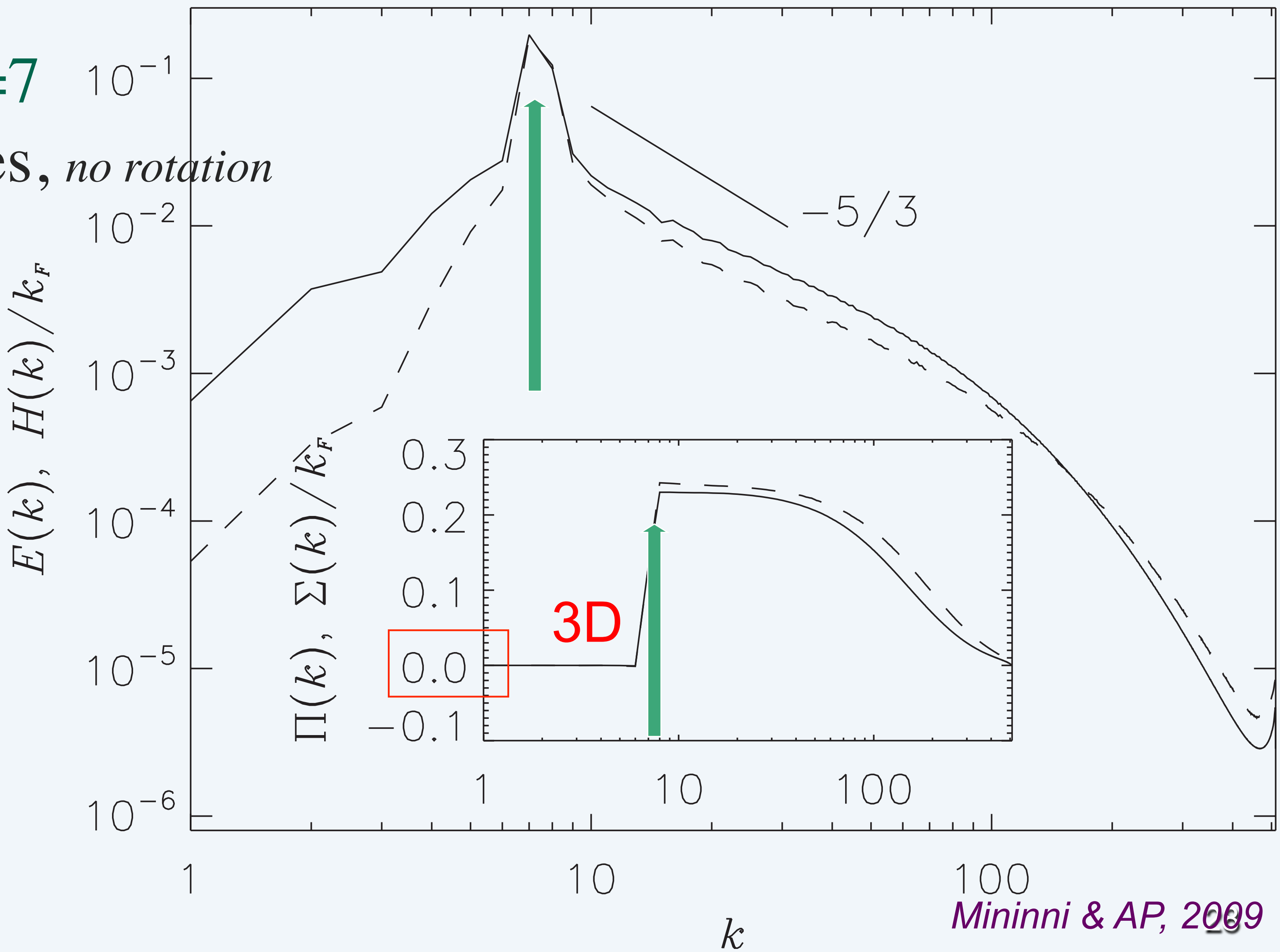
Spectra & fluxes in the isotropic case:
Energy ———
Helicity H= u.ω - - -

Forced @ k=7

Navier Stokes, *no rotation*

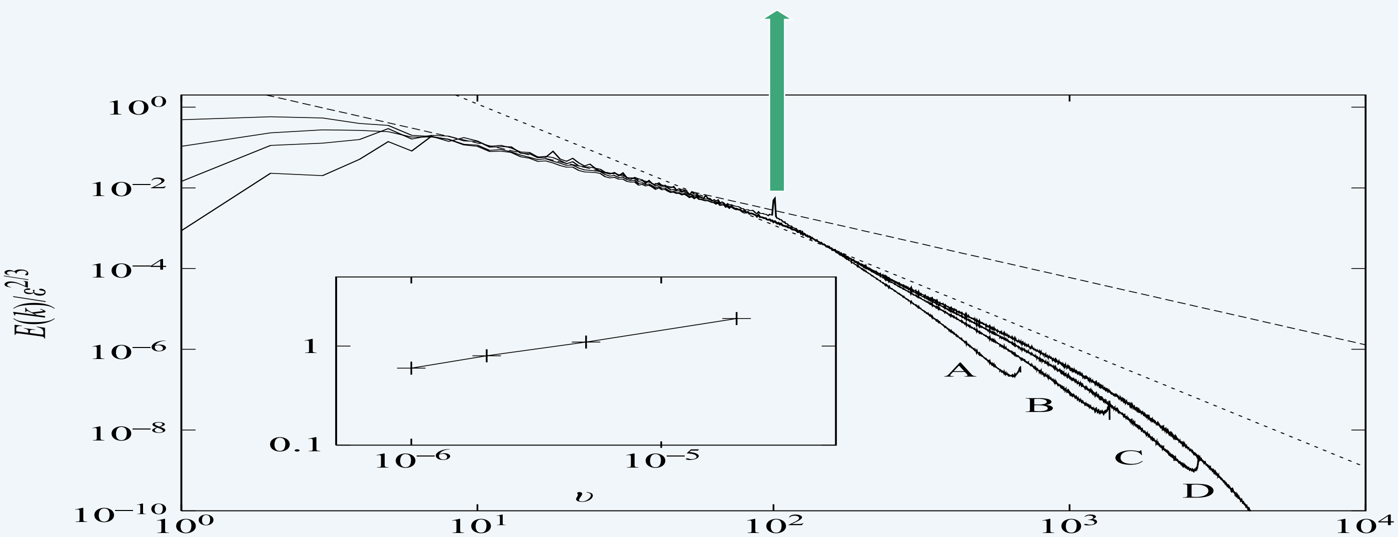
Re=1200

1536^3 grid



Mininni & AP, 2009

2D



Forced @ k=100

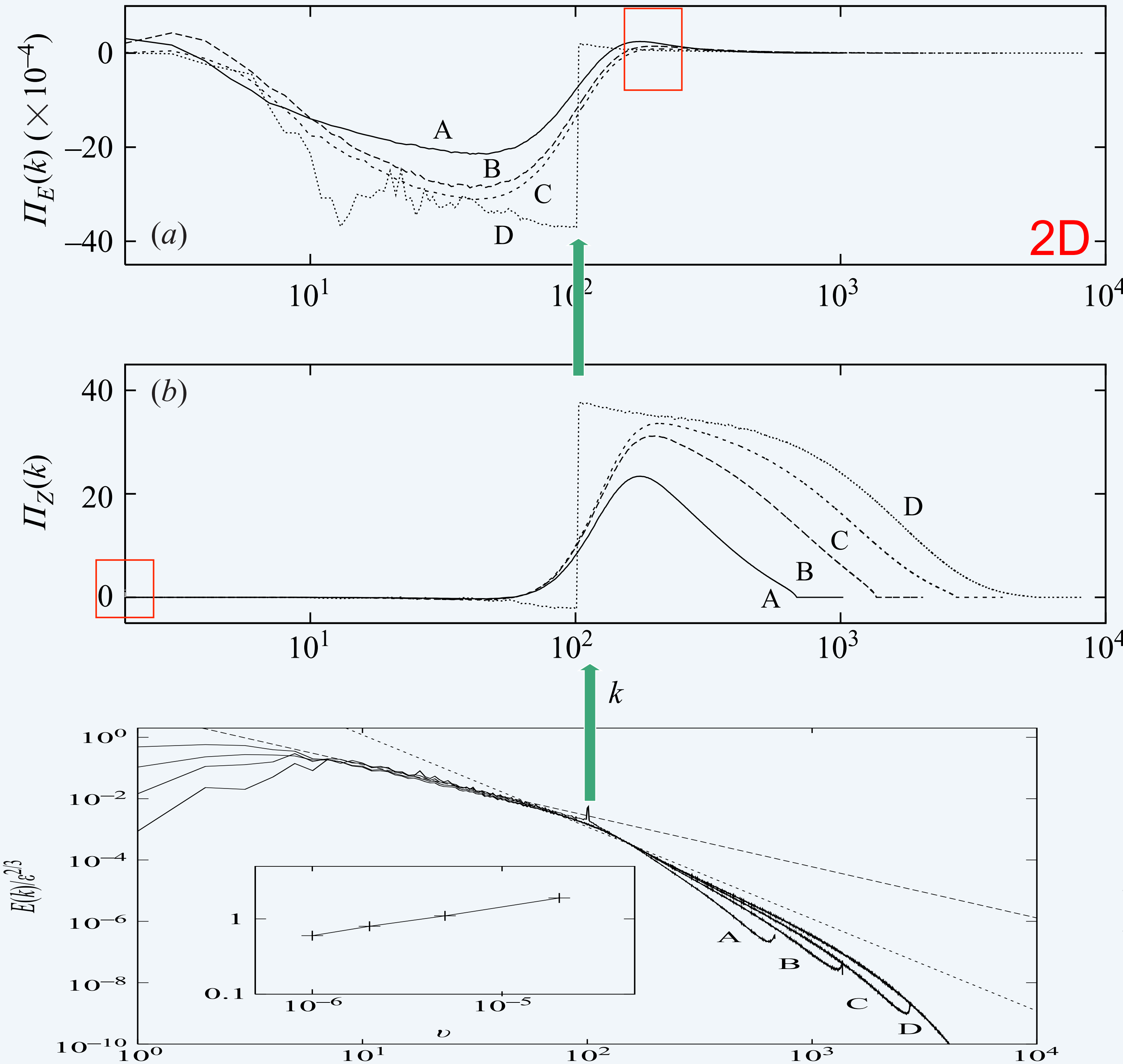
** Friction*

Grid up to 16384^2

$E(k) = C\,\varepsilon^{2/3} k^{-5/3}$, $C \sim 6$

$E(k) \sim k^{-(3+x)}$

Boffetta 2007



0

Flux of **Energy**

and of

Enstrophy

Forced @ k=100

** Friction*

Grid up to 16384^2

$E(k)= C\ \varepsilon^{2/3}\ k^{-5/3}$, C~6

$E(k) \sim k^{-(3+x)}$

Boffetta 2007

Paradigm with 2 invariants like energy & enstrophy:

2D: Dual but mutually exclusive system with an
inverse cascade of energy & a direct enstrophy cascade

3D: Direct cascade of energy, and direct helicity cascade

BUT … role of:

1) Aspect ratio / Anisotropy
2) Imposed magnetic field
3) Imposed rotation / stratification
4) Helicity decimation
5) And more

3D, T-HI, 2D2C force, $A=L_z/L_x=1/64$ with $S = L_f/L_z$

Turbulent viscosity, Navier-Stokes, no rotation, 128³ grid

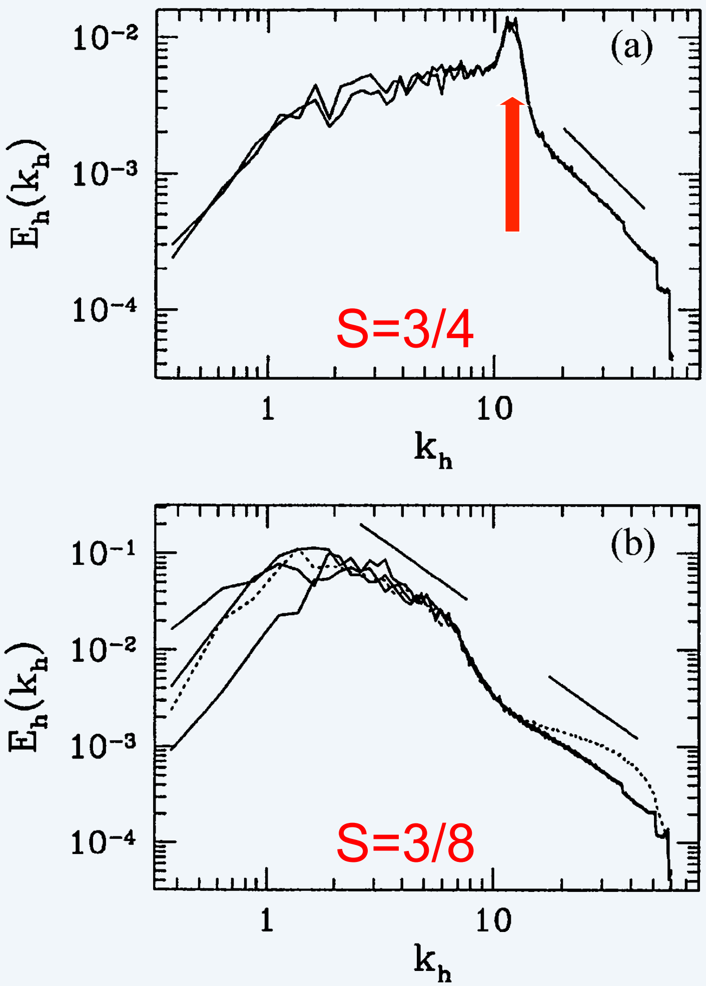


FIG. 2. (upper) $A = 1/64$, Ro $= \infty$, $S = 0.75$ (statistically steady); (lower) $A = 1/64$, Ro $= \infty$, $S = 0.375$: eddy viscosity (solid line) with time increasing upwards; hyperviscosity (dotted line). The lines are $E_h \propto k_h^{-5/3}$.

Smith et al. PRL 1996

3D, T-HI, 2D2C force, $A=L_z/L_x=1/64$ with $S = L_f/L_z$

Turbulent viscosity, Navier-Stokes, no rotation, 128^3 grid

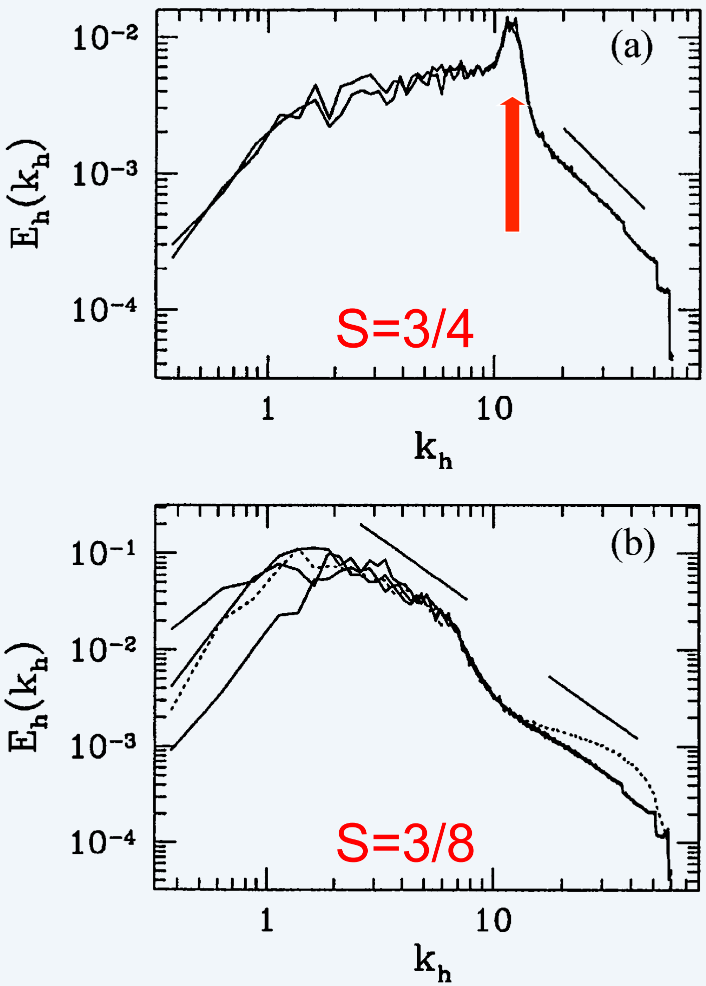


FIG. 2. (upper) $A = 1/64$, Ro $= \infty$, $S = 0.75$ (statistically steady); (lower) $A = 1/64$, Ro $= \infty$, $S = 0.375$: eddy viscosity (solid line) with time increasing upwards; hyperviscosity (dotted line). The lines are $E_h \propto k_h^{-5/3}$.

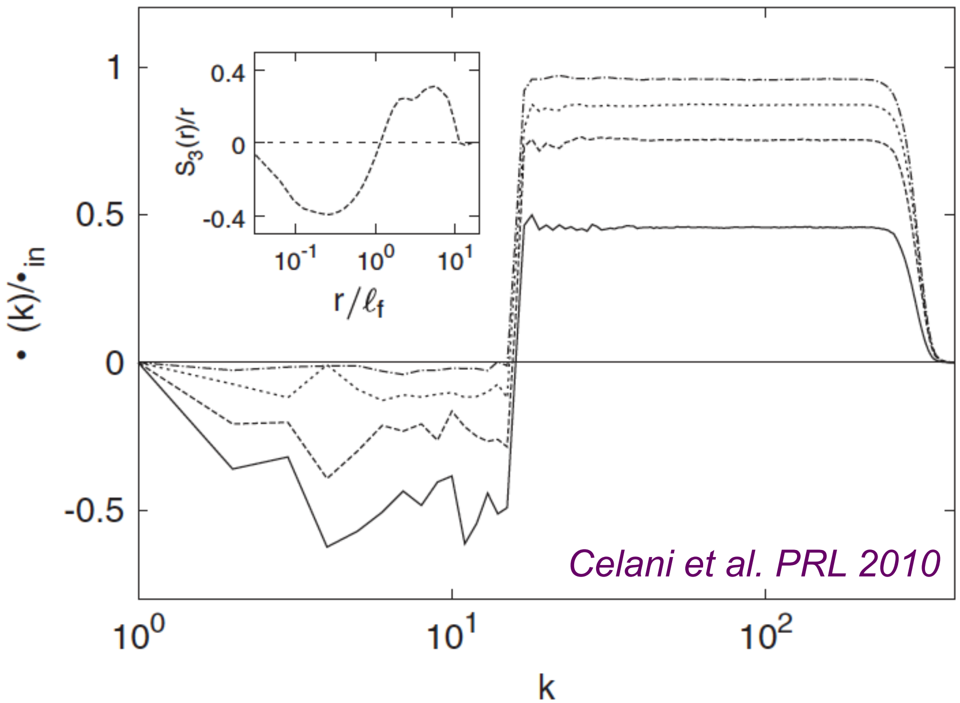
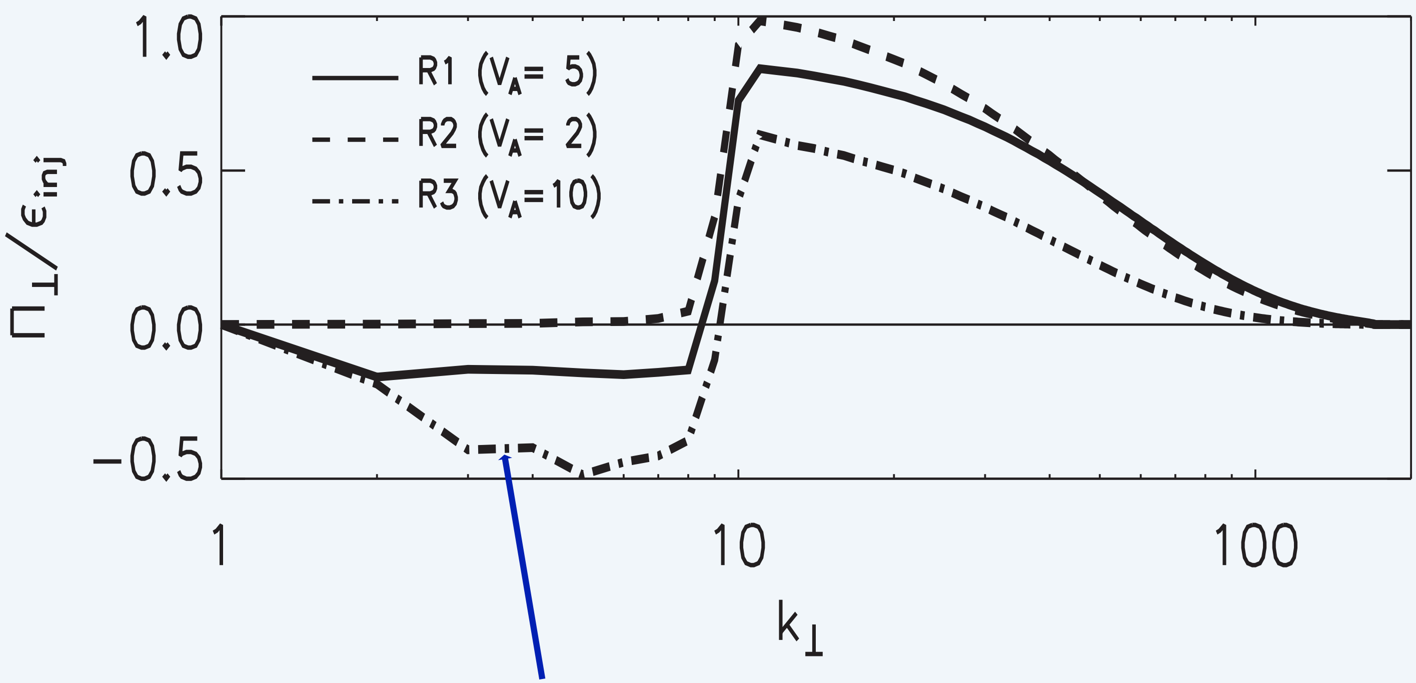


FIG. 2. Spectral flux of kinetic energy for various aspect rati $L_z/\ell_f = 1/8, 1/4, 3/8, 1/2$ (from bottom to top). Simulatio parameters as in Fig. 1. The inset reports the third order structur function of the velocity, $S_3(r)$, for $L_z/\ell_f = 1/4$.

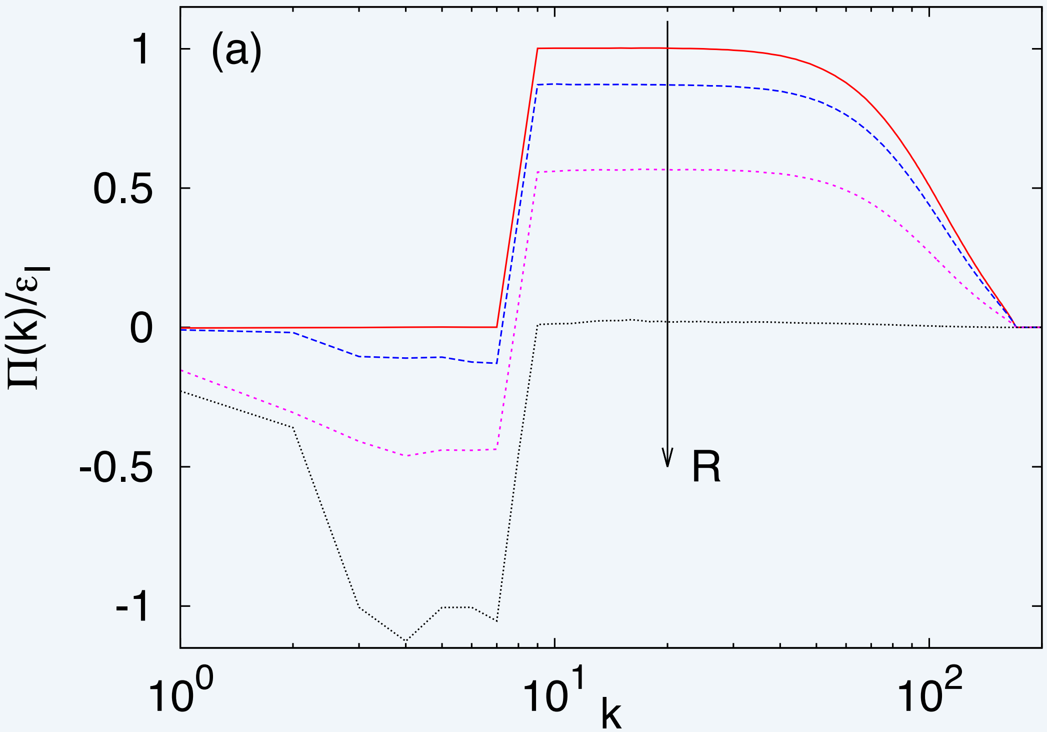
Kinetic energy flux in 3D MHD for various V_A



Stronger V_A

Alexakis, 2011

Energy flux in rotating flows with varying aspect ratios



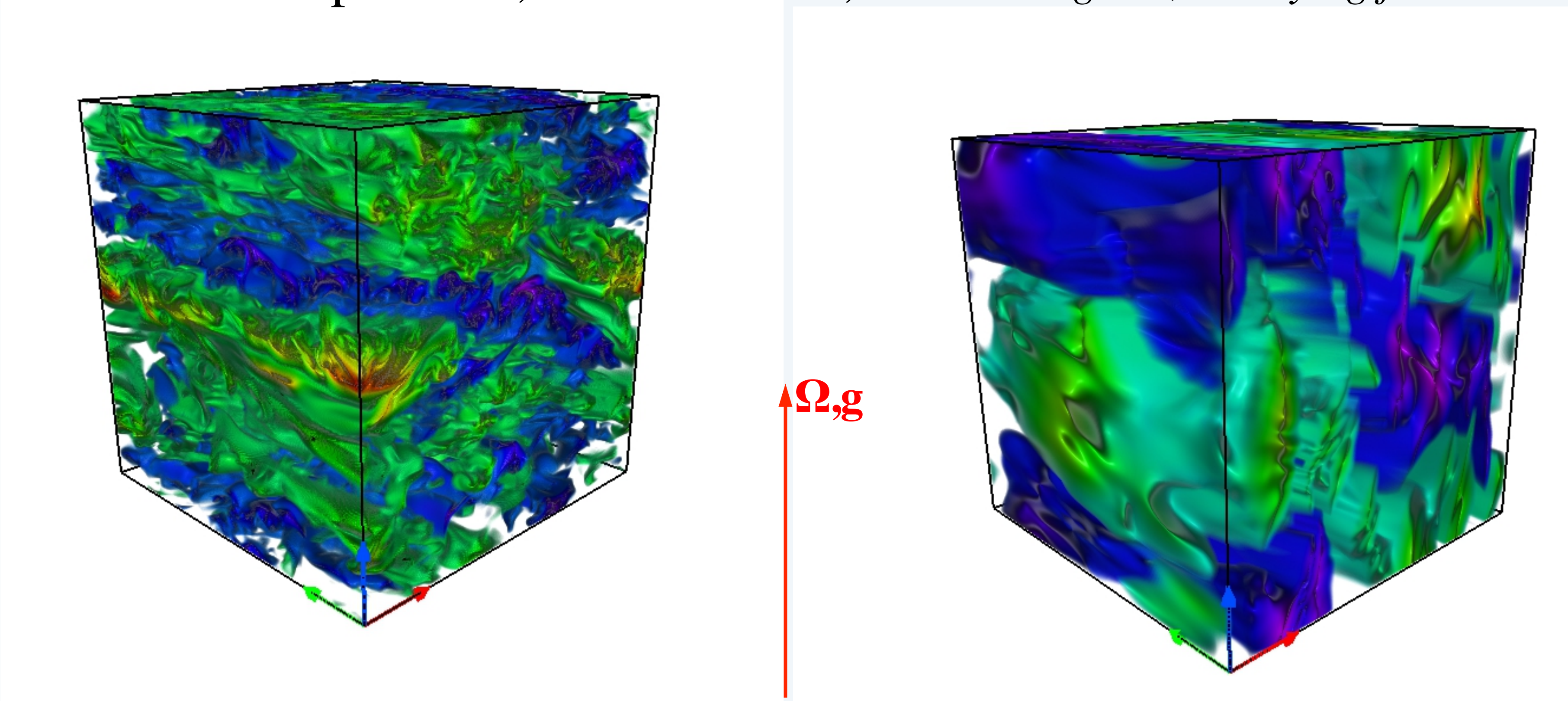
R~ f: rotation
$S = L_z / L_f$

R= 0, 1, 1.5, 5
with S=2

Deusebio et al. 2014

- What happens
  with rotation
  and
  stratification
  in an idealized setting?

Temperature, Re ~ 8000, *512^3 grids, decaying flows*

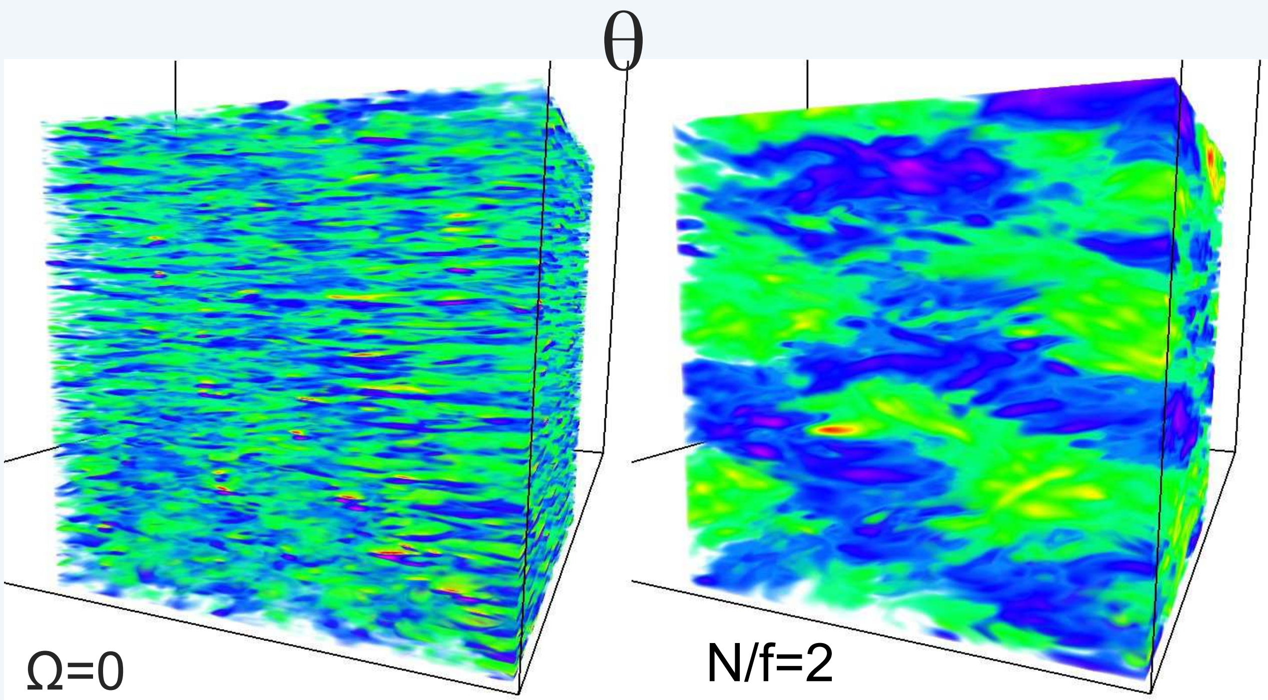


$Fr \sim 0.11$, $Ro \sim 0.4$,
$R_B = ReFr^2 \sim 100$, $N/f \sim 3.6$

$Fr \sim 0.025$, $Ro \sim 0.05$,
$R_B \sim 5$, $N/f = 2$

Rendering using Vapor

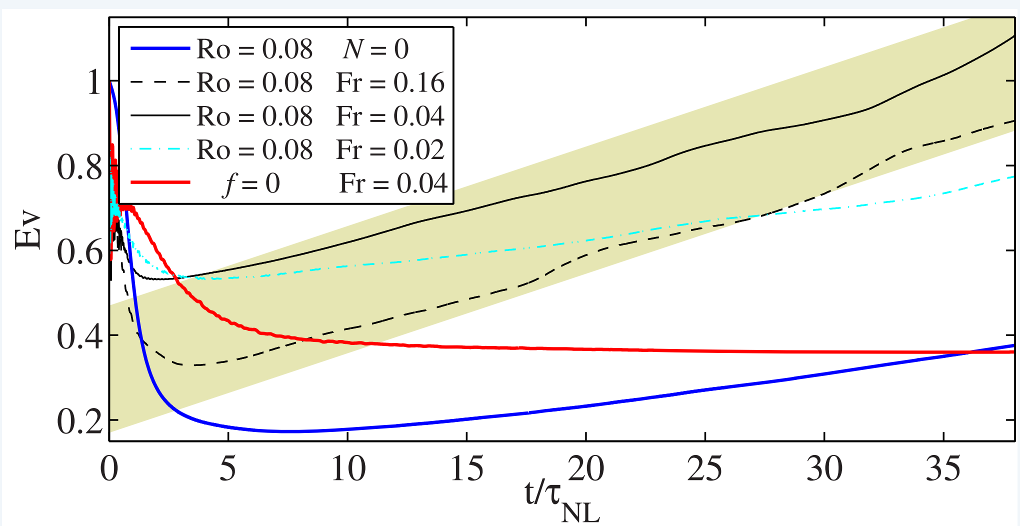
Marino et al., 2013



Rot + strat

Forcing at small scales

Re=1000



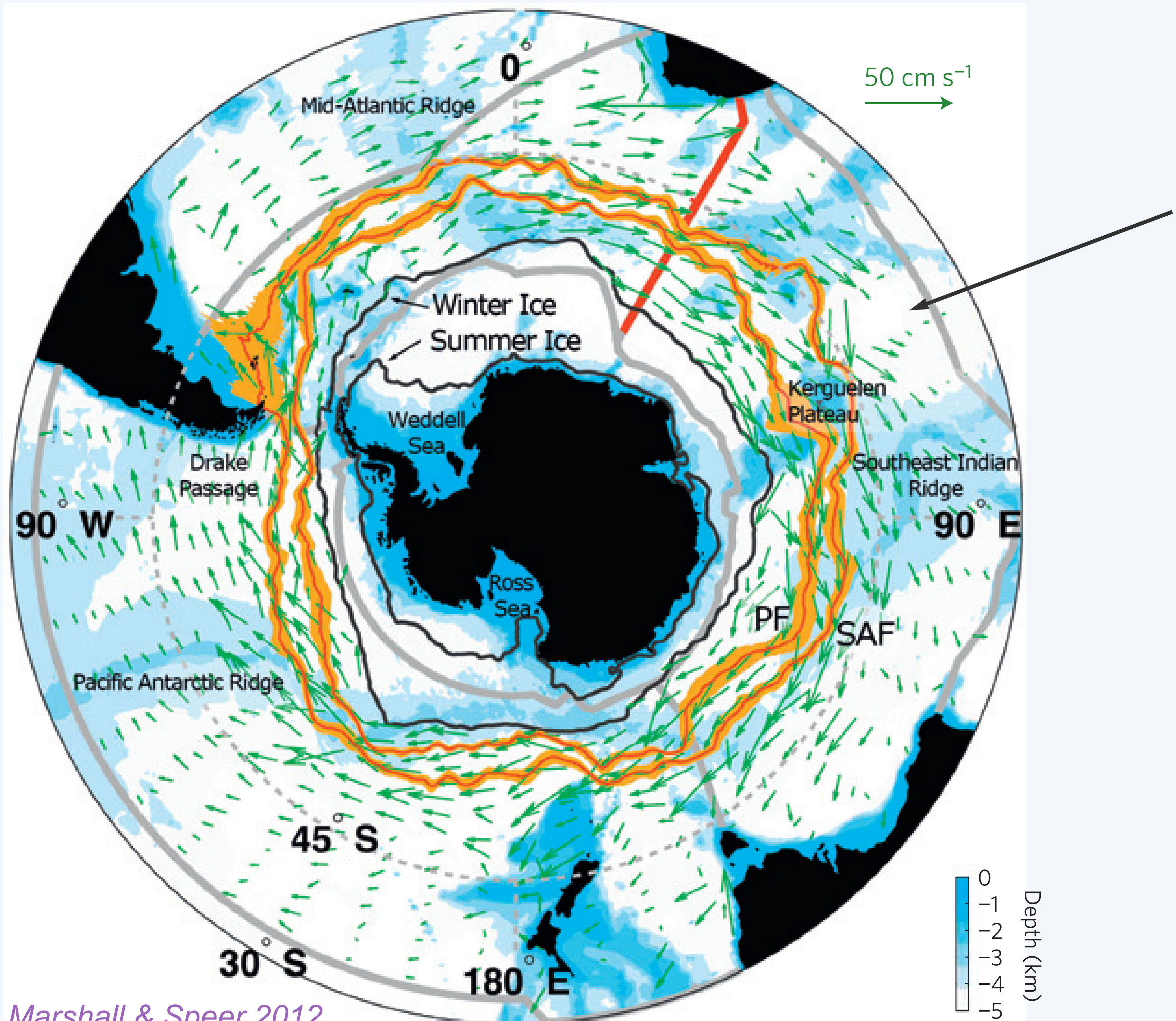
No rotation

E(t)

512^3 or 1024^3 grids

k_F=22 or 40

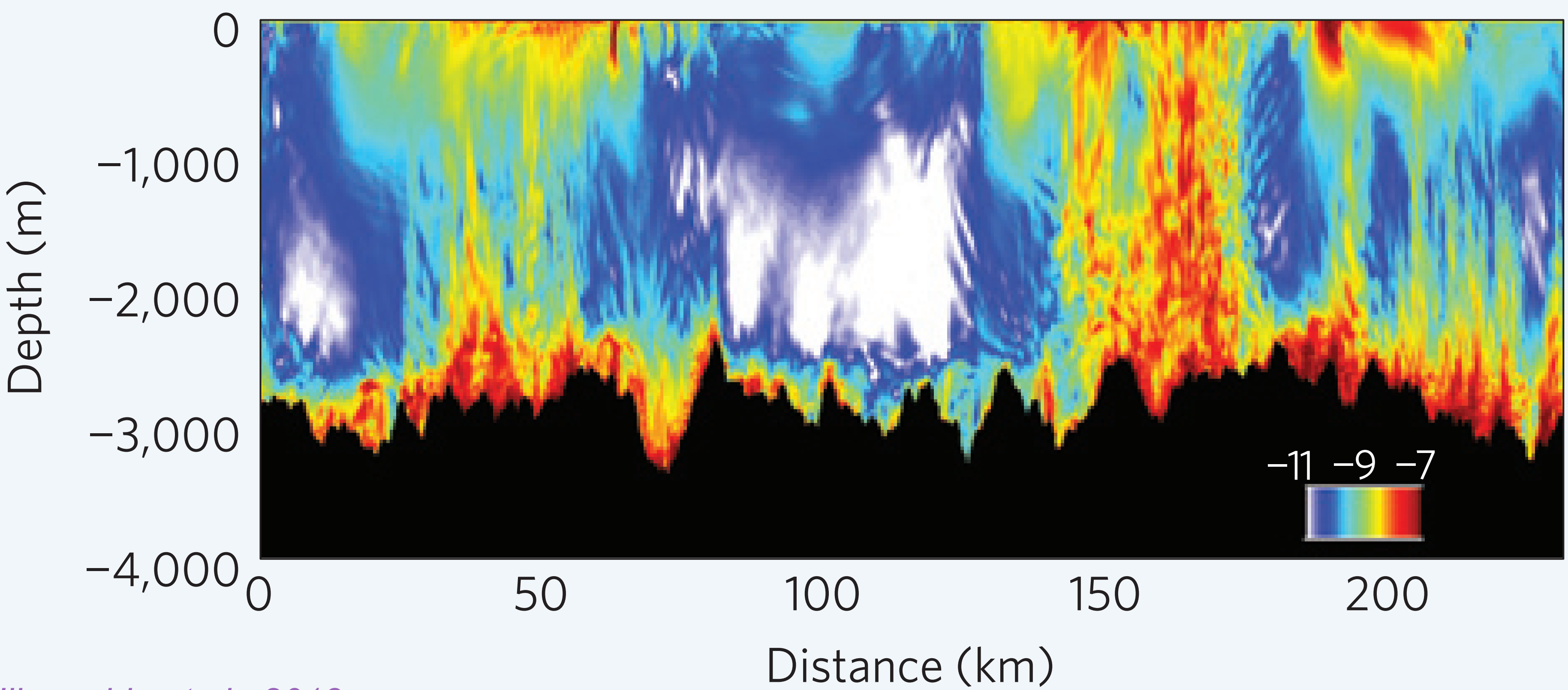
Marino et al., EPL 2013



Marshall & Speer 2012

Run: MIT-GCM,
N/f~ 4.7, Grid ~ 1200^2 X 200 points , 230x230 km^2 x 4km
U ~ 10cm/s, N=$7x10^{-4}$/s, high Prandtl number
R_{perp} ~ $7x10^7$, R_Z ~ $7x10^3$

Energy dissipation 10^{-10}
→ 10^{-8} W/kg



Nikurashin et al., 2013

Measurements in the Southern Ocean

← of flow speed

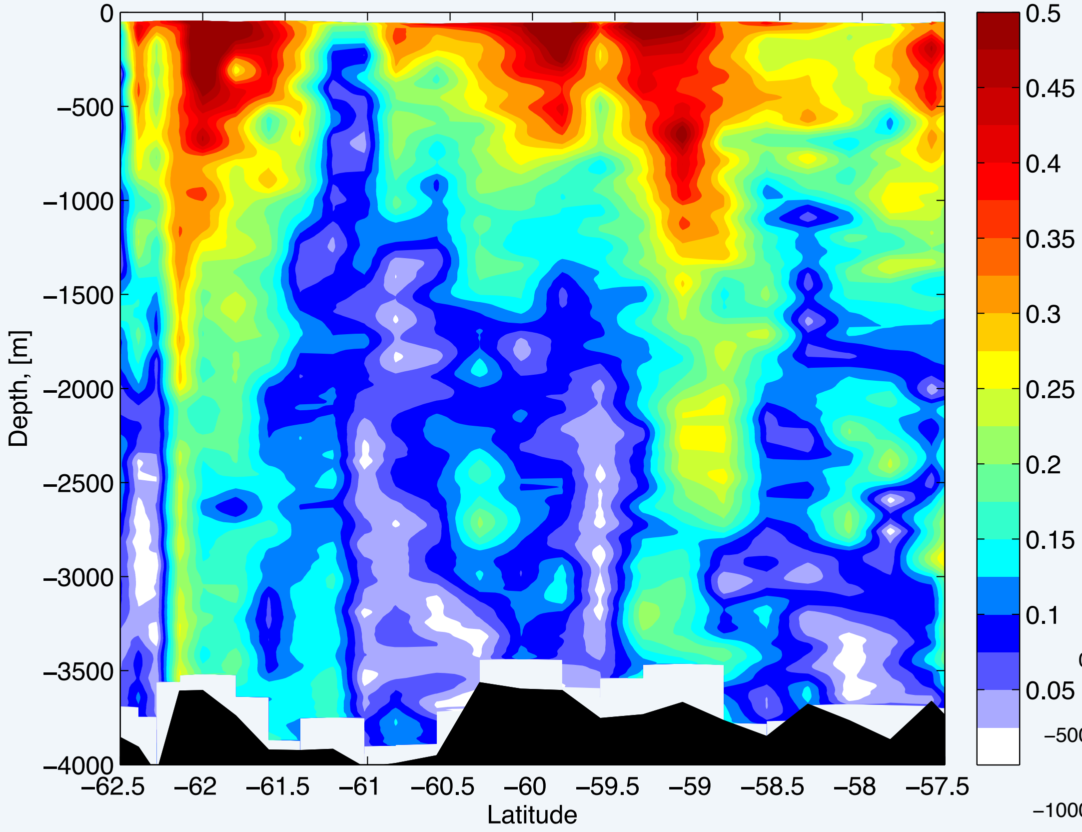


Figure 3-2: Flow speed (m s^{-1}) from the ALBATROSS section, Drake Passage.

and of buoyancy frequency

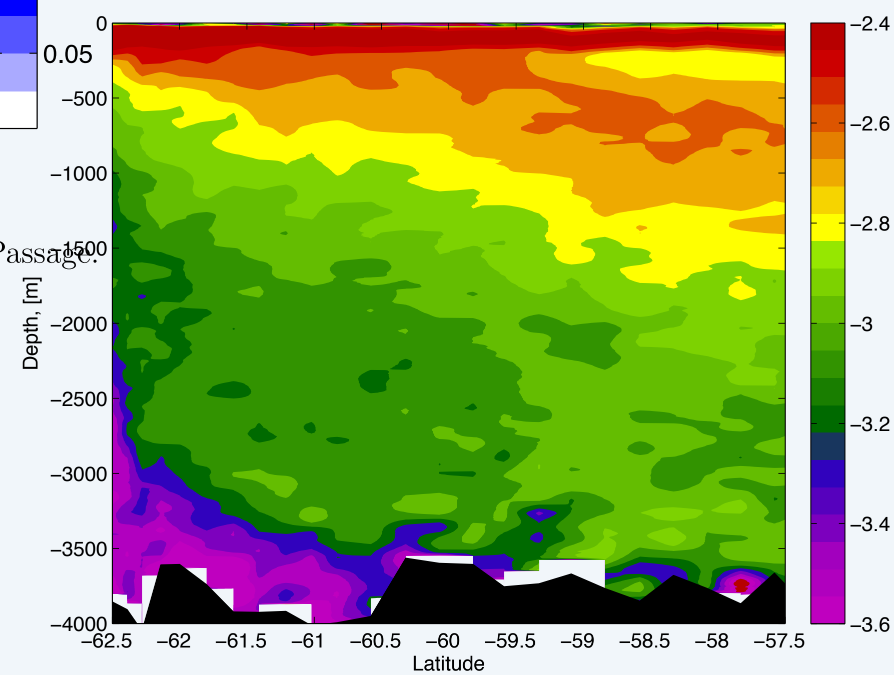
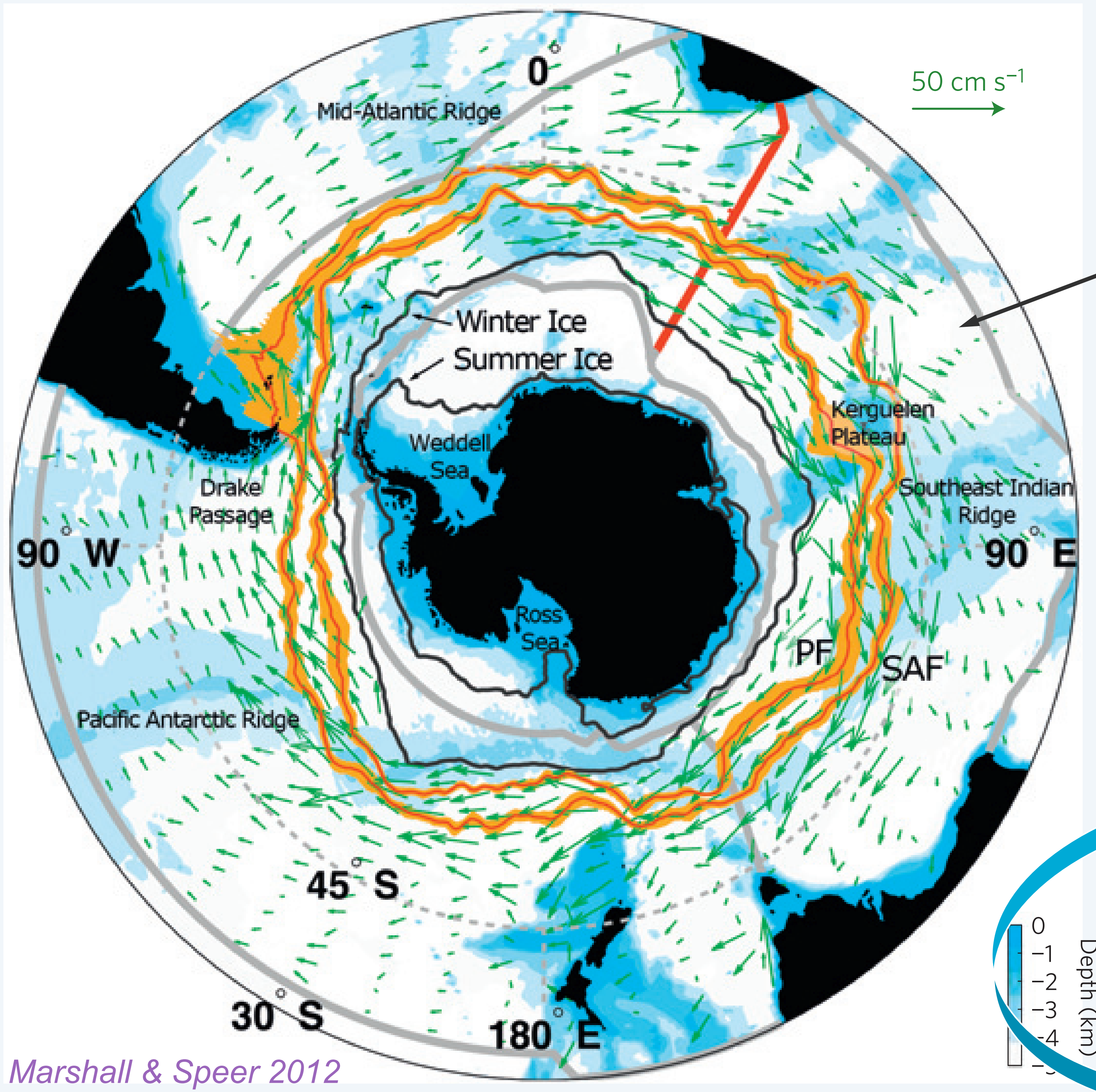


Figure 3-1: Buoyancy frequency (s^{-1}) in logarithmic scale from the ALBATROSS section, Drake Passage.

Nikurashin, 2009



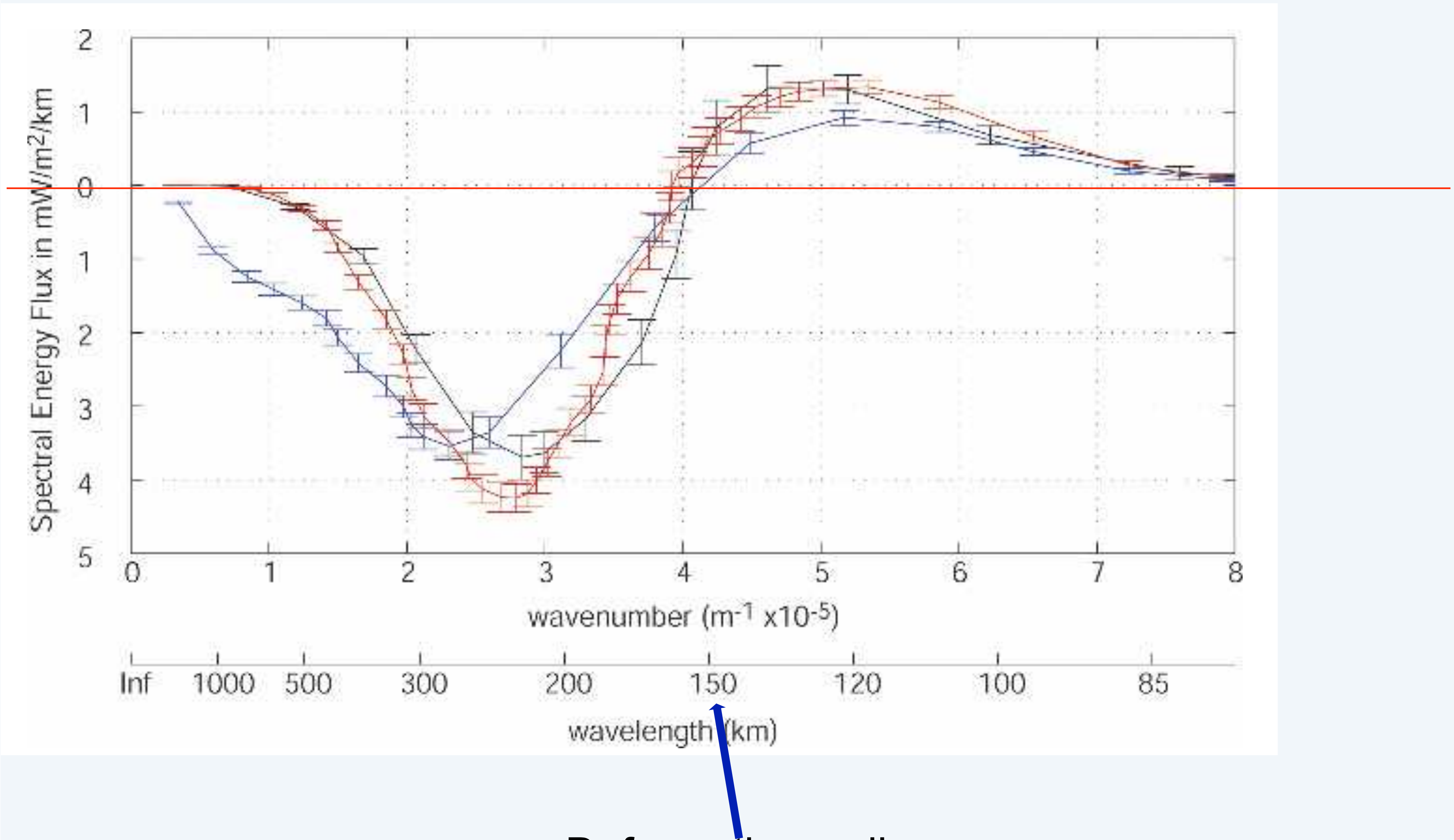
$5km^3$
next to the Kerguelen Plateau

$L_F \sim 500m$
$U \sim 0.04m/s$
$N = 0.001\ /s$
$f = N / 10$

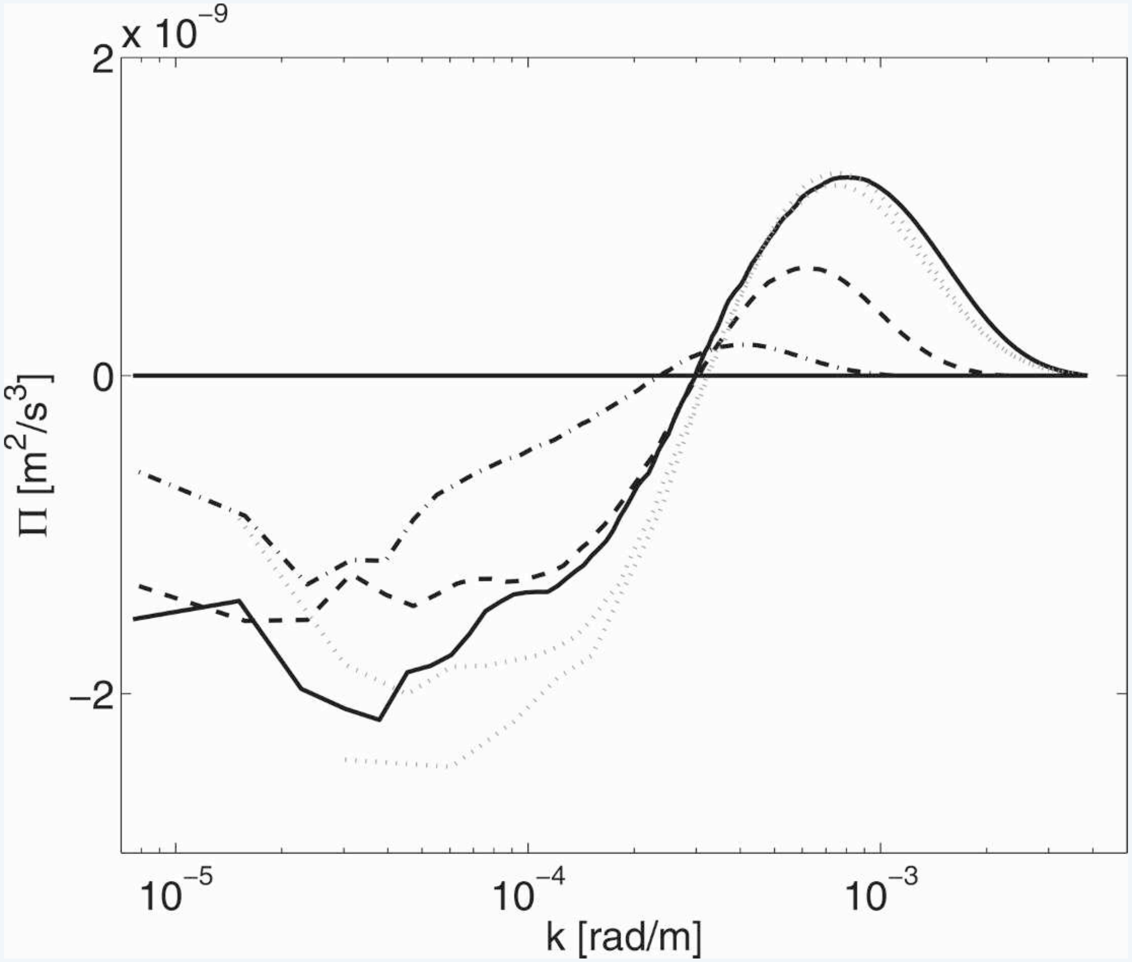
Fr= 0.08
Ro = 0.8
$\nu = 10^{-6}\ m^2/s$
$Re = 2.\ 10^7$
$R_B \sim 10^5$

DNS run
Boussinesq
2048^3 grid
$\nu = 8x\ 10^{-4}\ m^2/s$
$Re = 24000$
$R_B \sim 150$
$Pr = 1$

Kinetic energy flux in the ACC, 10+yrs data every 10 days ~ T_{NL}



Scott 2005



← Energy flux
and spectrum

ROMS

Forcing in momentum, fresh water & heat with restoring force, KPP & sponge layer

Down to 0.75km res.

(solid line)

*Larger range
for the inverse cascade
than for the direct one*

Capet et al., 2008

A paradox?

- *Capet et al. (2008), ROMS+KPP:*

… we hesitate to draw any strong conclusions about the efficacy of a mesoscale inverse KE *{Kinetic Energy}* cascade in our solutions, although our results indicate it does occur to some degree …

** Scott et al. (2011), oceanic data analysis:*

despite great effort in studying the ocean's energy budget in the last two decades, the bulk of the dissipation of the most energetic oceanic motions remains unaccounted for.

Geophysical High Order Suite for Turbulence *(Gomez & Mininni)*

- Pseudo-spectral DNS, periodic BC cubic *(also 2D), single/double precision*; Runge-Kutta for incompressible Navier-Stokes, SQG & Boussinesq. Includes rotation, passive scalar(s), MHD + Hall term
- GHOST, from laptop to high-performance, parallelizes linearly up to 100,000 processors, using hybrid MPI/Open-MP (Mininni et al. 2011, Parallel Comp. 37)
- 3D Visualization: VAPOR (NCAR); and development @ OakRidge (D. Rosenberg)
- LES: alpha model & variants (Clark, Leray) for fluids & MHD
- Helical spectral (EDQNM) model for eddy viscosity & eddy noise
- *NEW!* Lagrangian particles *(w. A. Pumir, ENS)*
- *NEW!* Gross-Pitaevskii & Ginzburg-Landau *(with M. Brachet, ENS)*
- ***Data**, forced: 2048^3 Navier-Stokes and 1536^3 & 3072^3 with rotation, both w. or w/o helicity. Rotating stratified turbulence w. 2048^3 grids.*
- *Spin-down MHD: 1536^3 random + 6144^3 ideal & 2048^3 w. T-Green symmetry.*
- *Decaying rotating stratified flow, N/f~5, Re=5.5 10^4, 2048^3, 3072^3 & 4096^3 grids*

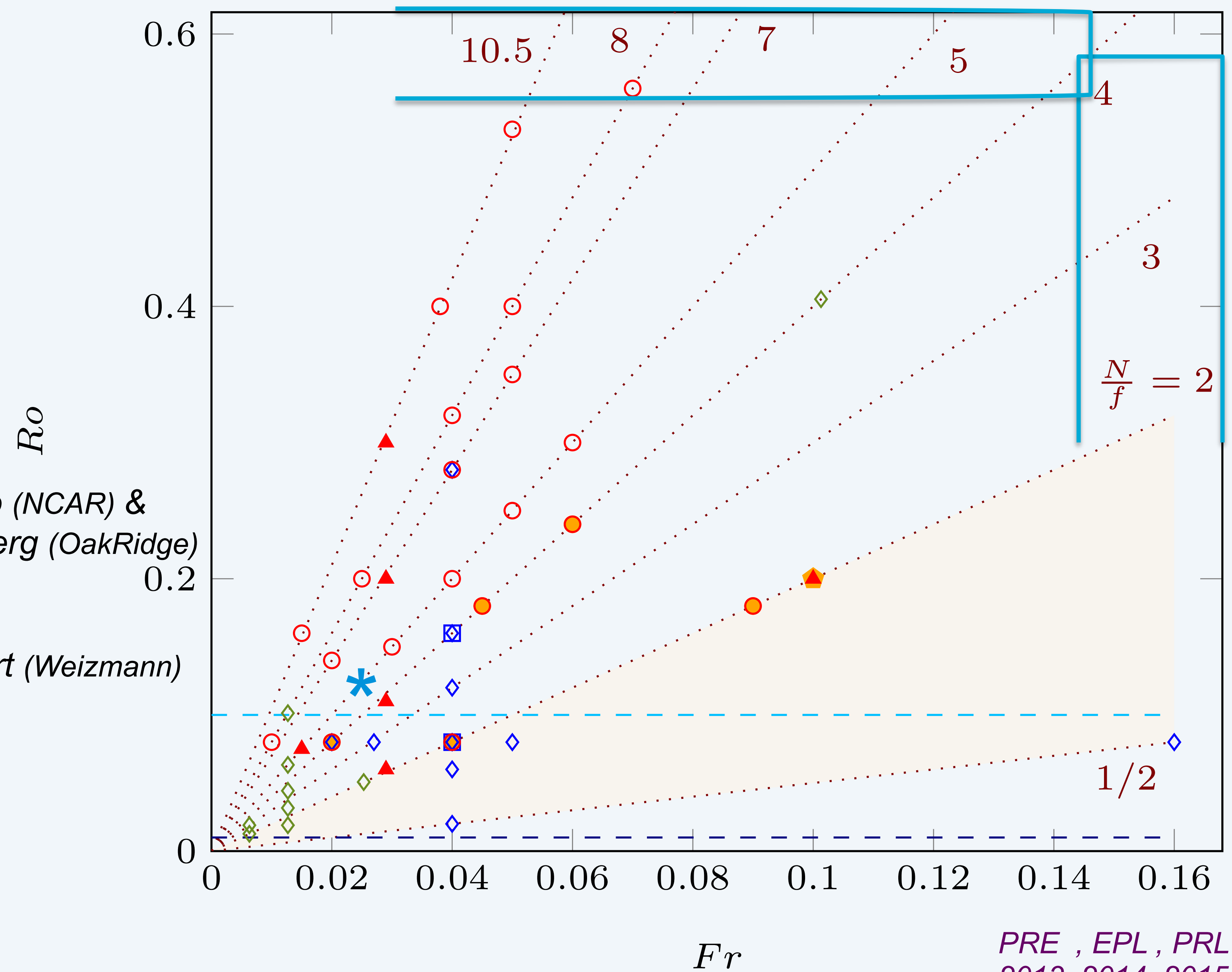
mininni@df.uba.ar duaner62@gmail.com

Rotating-stratified data

Runs:

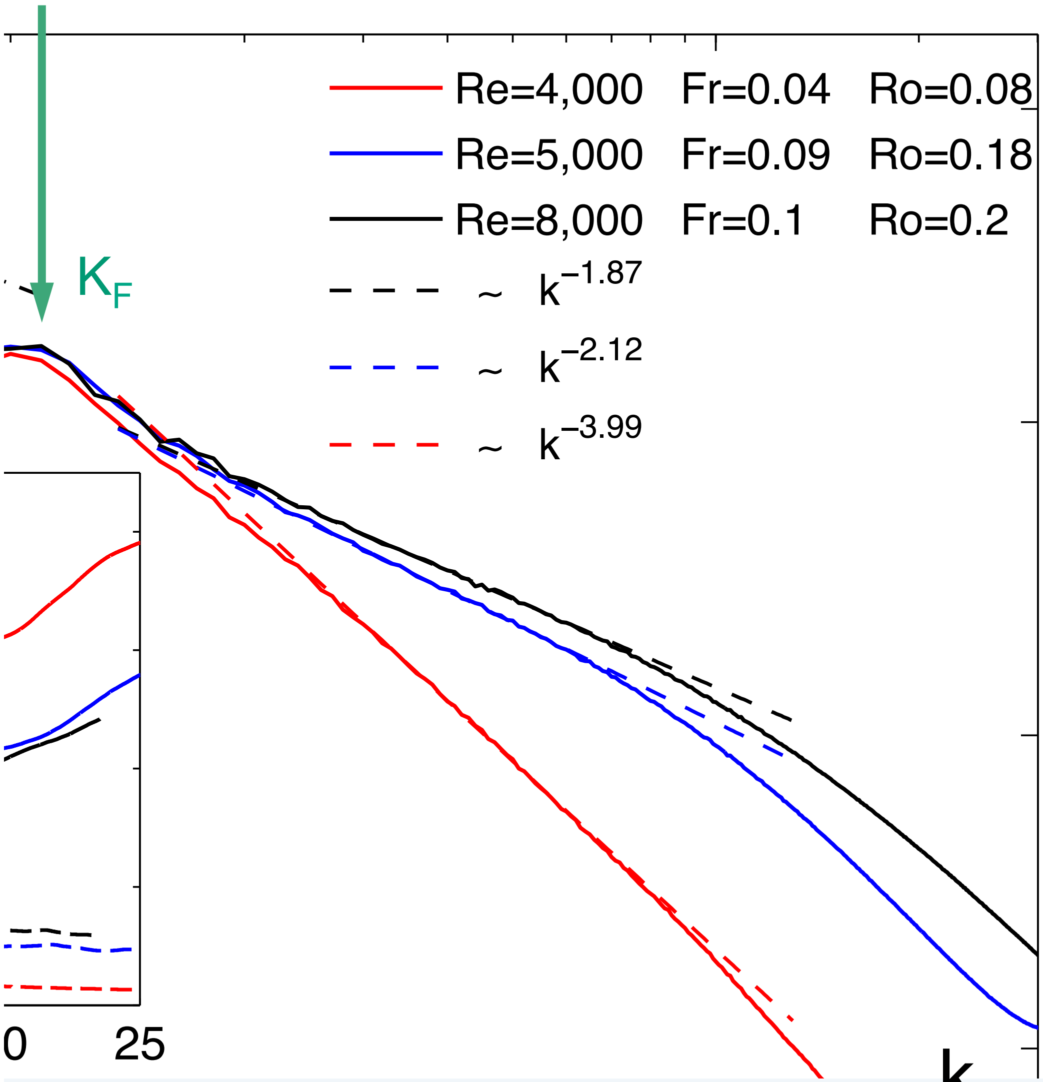
Raffaele Marino (NCAR) & Duane Rosenberg (OakRidge)

Diagram: Corentin Herbert (Weizmann)



PRE , EPL , PRL 2013, 2014, 2015

CPU: NSF (XSEDE & Yellowstone/NCAR); * : DOE/INCITE, 4096^3 grid

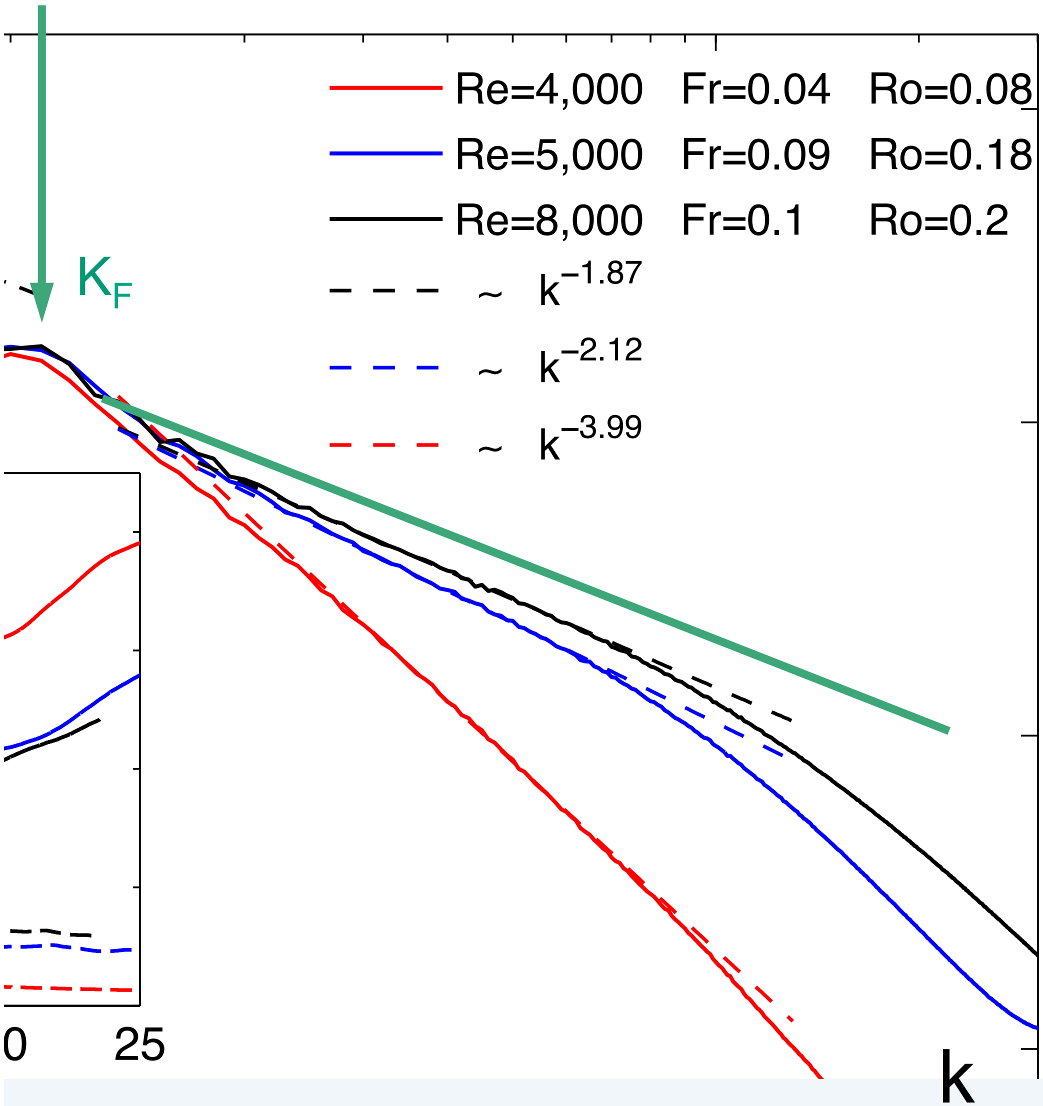


Forcing at K_F ~ 10

Small-scale spectra N/f =2 and for different parameters

R_B= Re Fr^2

R_B= 6 , 40 & 80



Forcing at K_F ~ 10

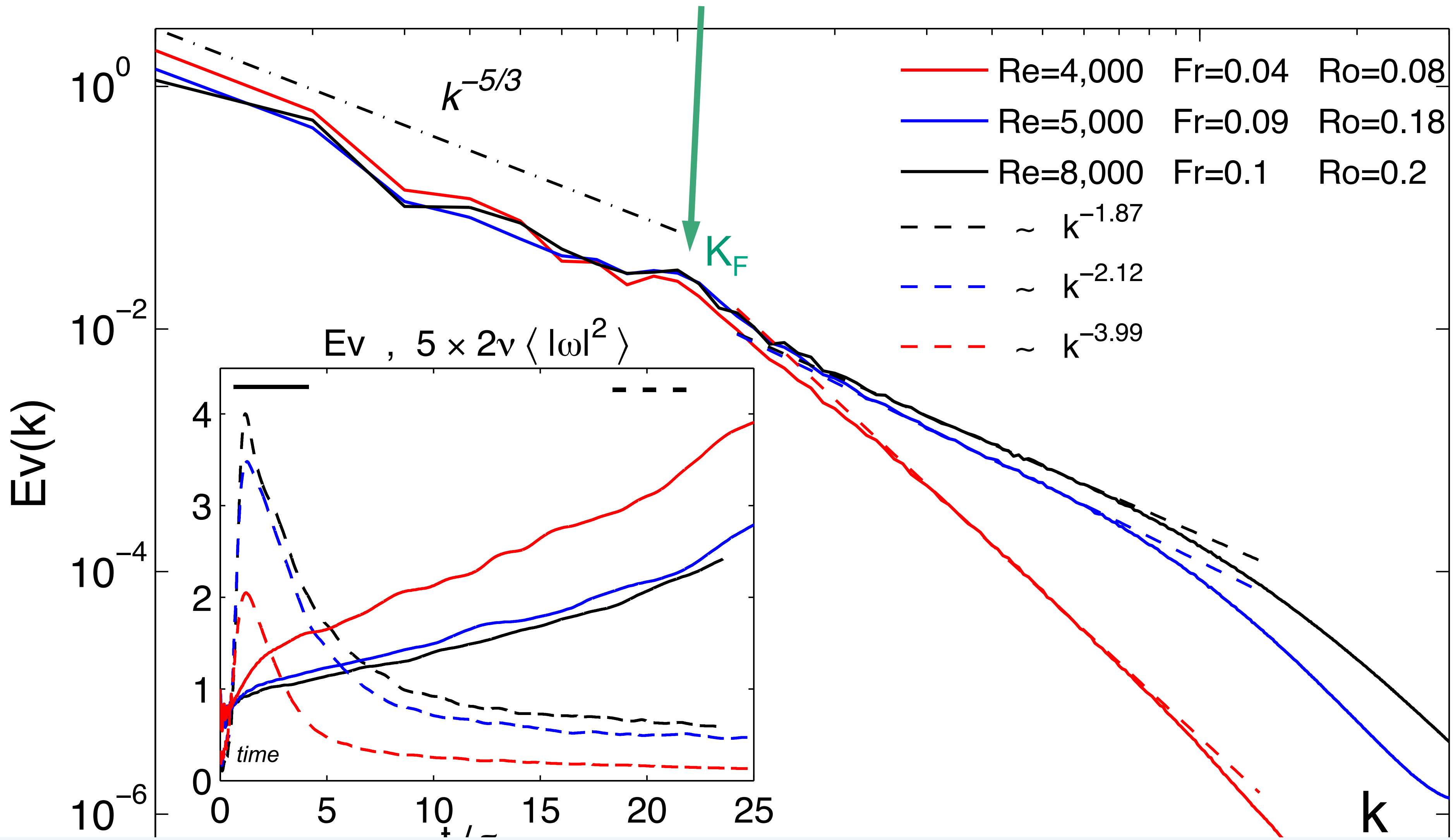
Small-scale spectra N/f =2 and for different parameters

R_B= Re Fr^2

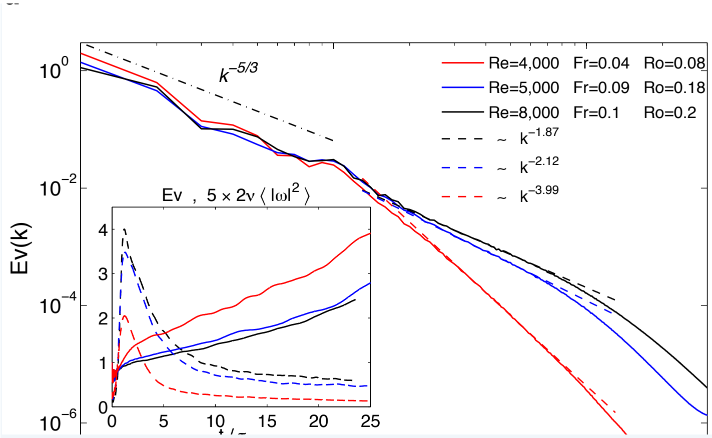
R_B= 6 , 40 & 80

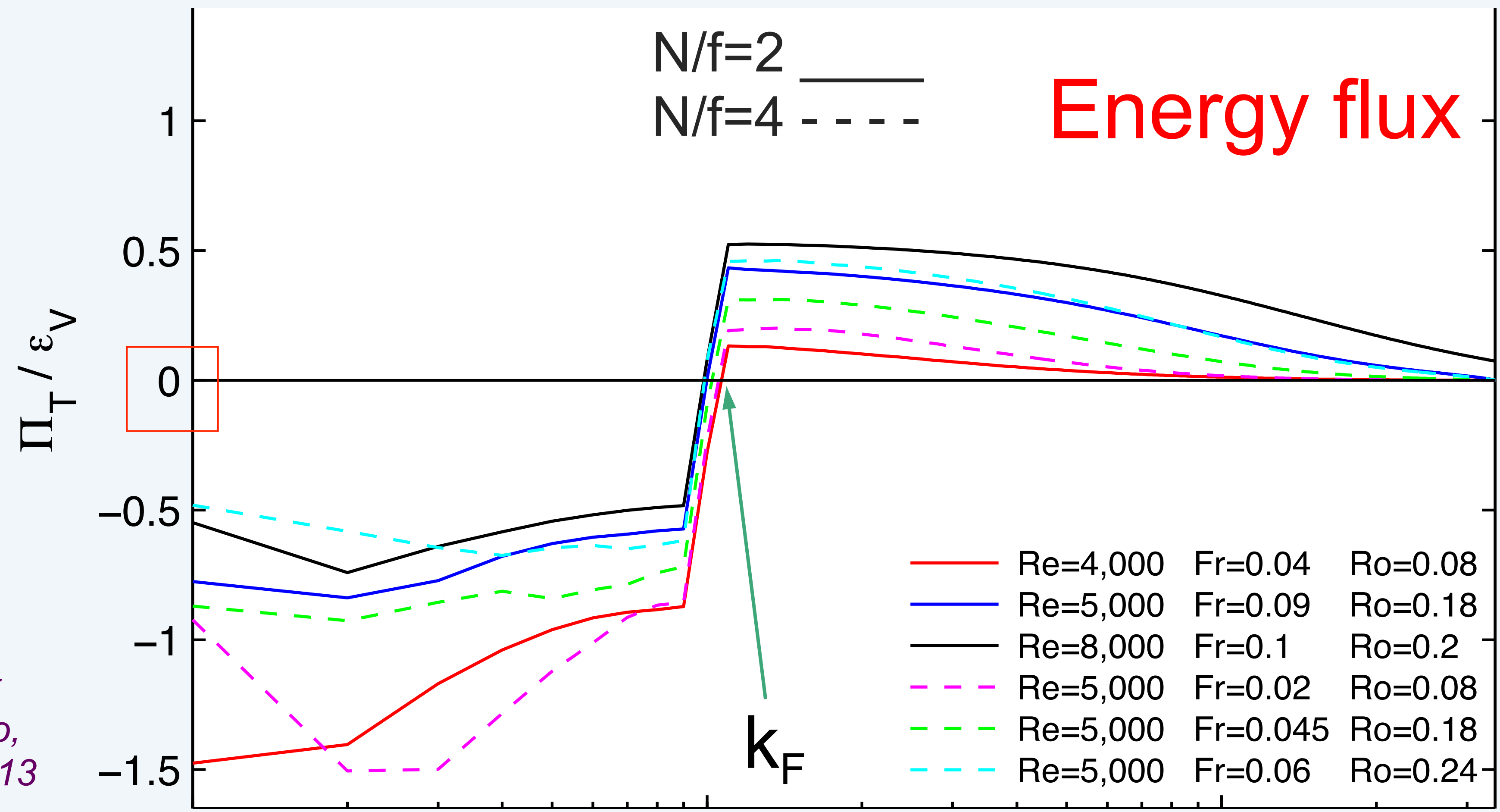
& 120,
$E(k) \sim k^{-1.77}$

Large-scale spectra, N/f=2

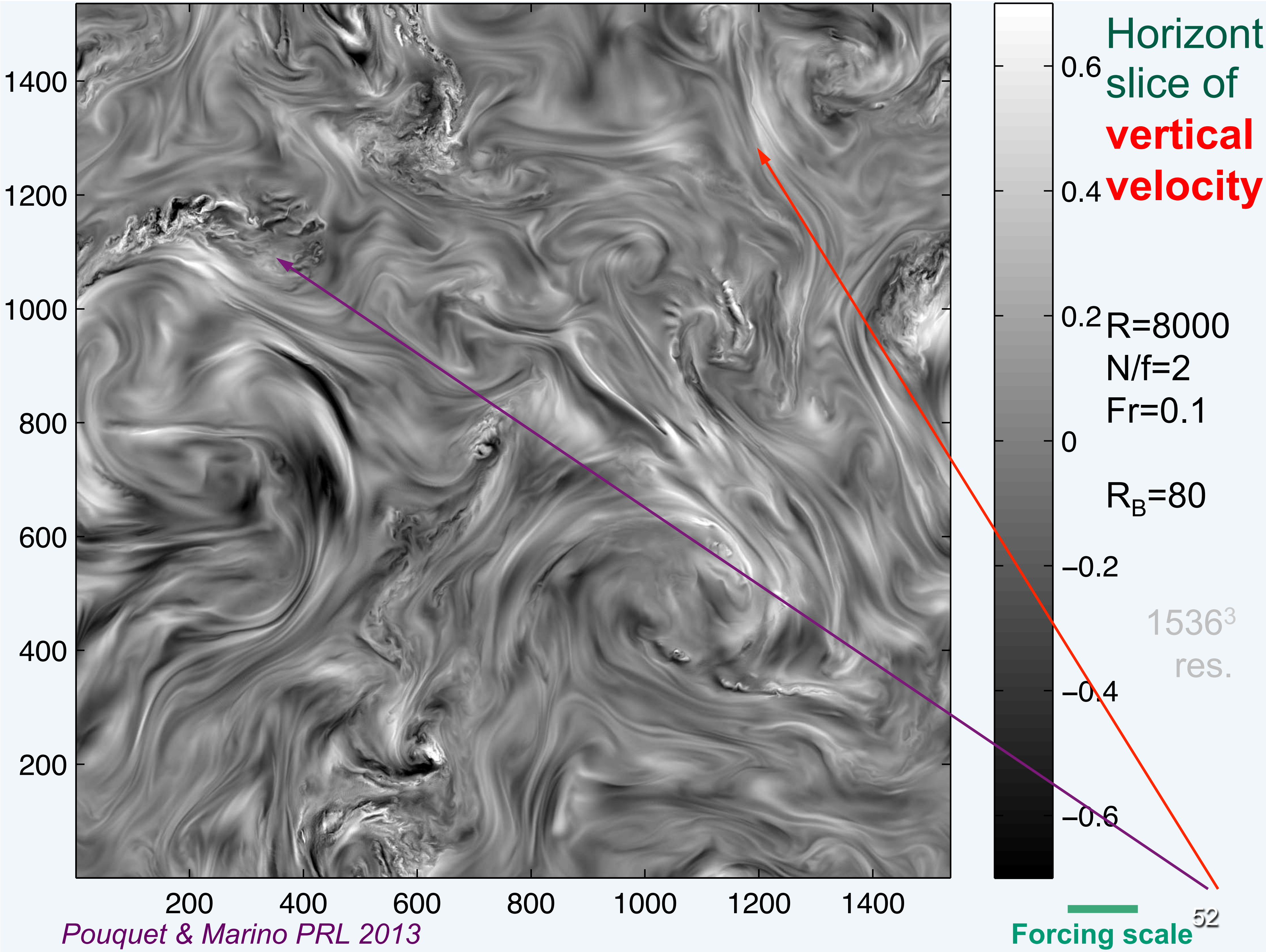


Temporal growth of energy (____) & stabilisation of energy dissipation (- - - -)

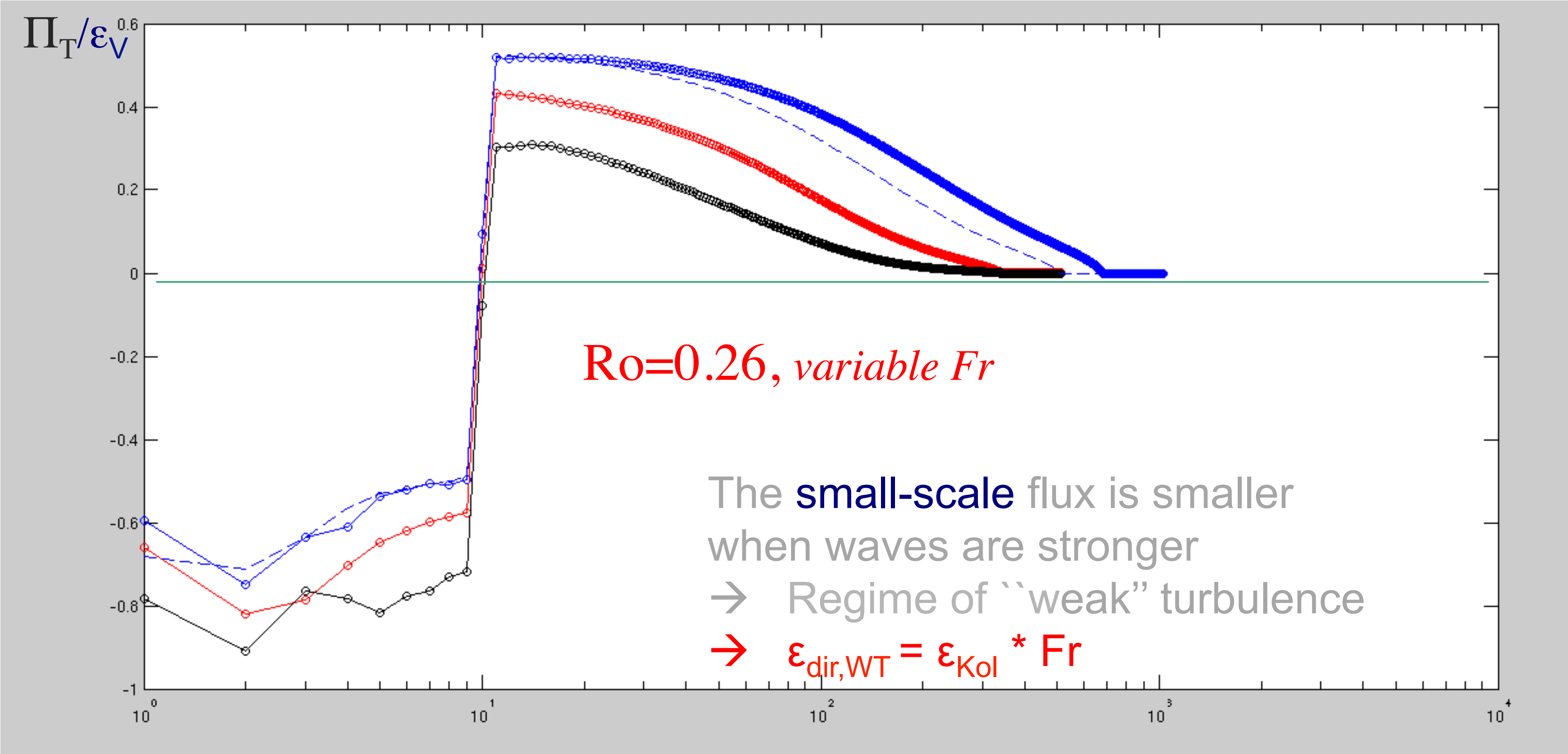




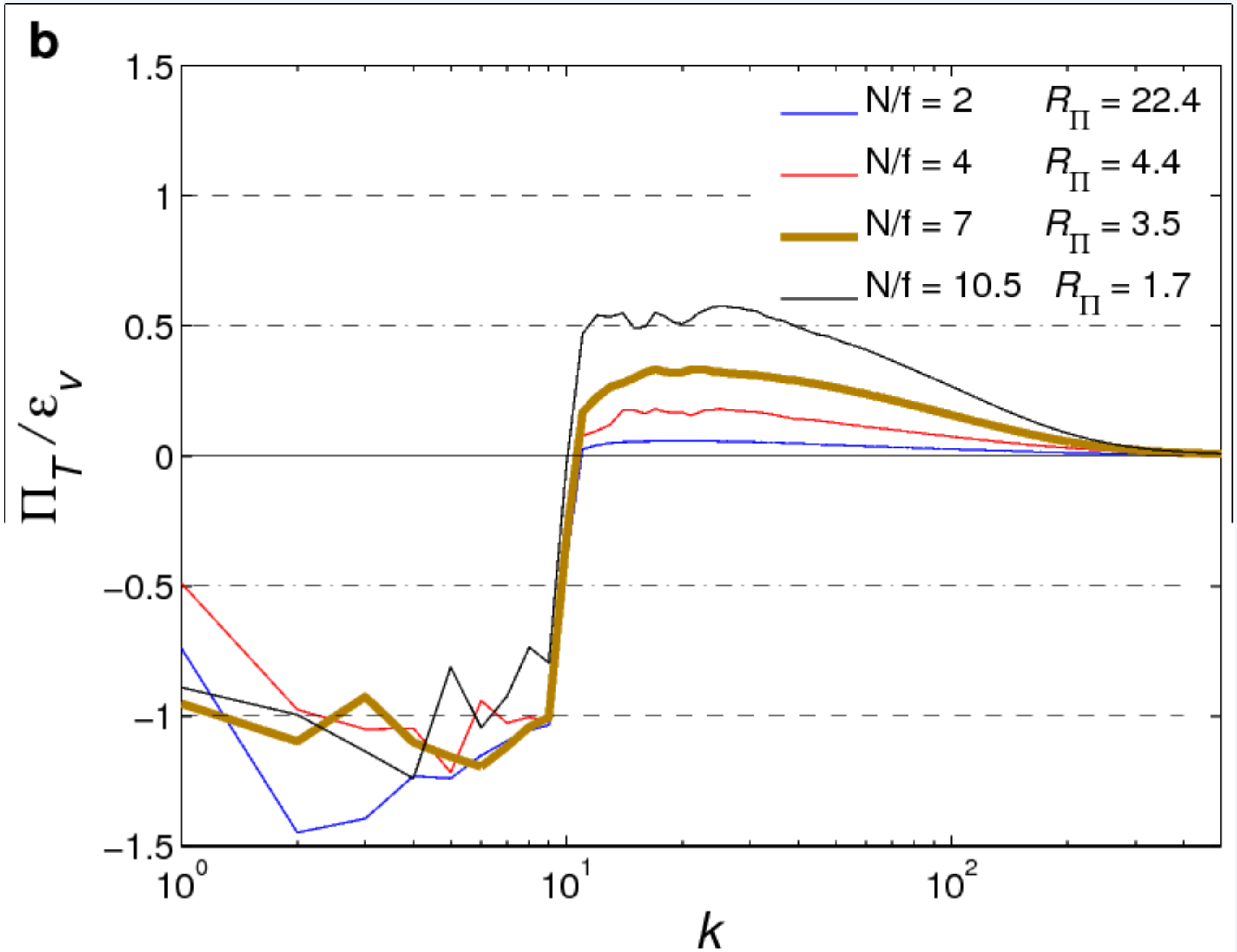
Pouquet & Marino, PRL 2013



Pouquet & Marino PRL 2013



Π_T/ε_V
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
10^0
10^1
10^2
10^3
10^4
Ro=0.26, *variable Fr*
The **small-scale** flux is smaller
when waves are stronger
→ Regime of ``weak'' turbulence
→ $\varepsilon_{dir,WT} = \varepsilon_{Kol} * Fr$



Fr=0.04, *variable Ro*

N/f=Ro/Fr, Ro=U/[Lf]

Ro~ 0.08

~ 0.16

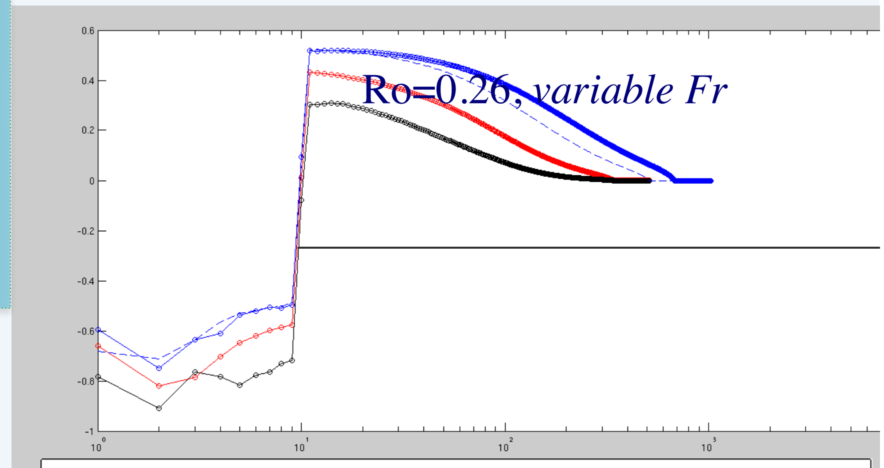
~ 0.28

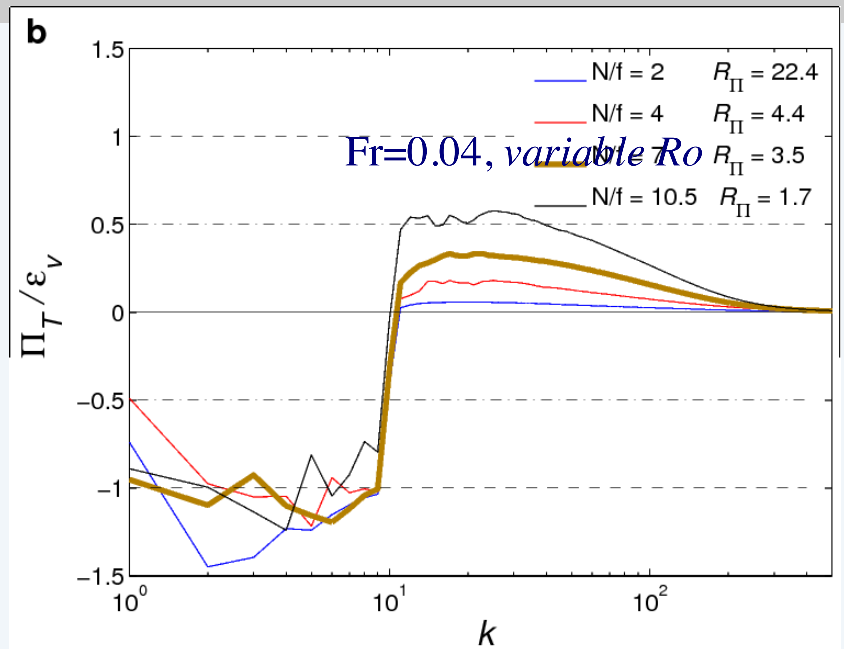
~ 0.45

The stronger the rotation, the larger is R_Π, i.e. the larger is the cascade to large scales ***relative*** *to that to small scales*

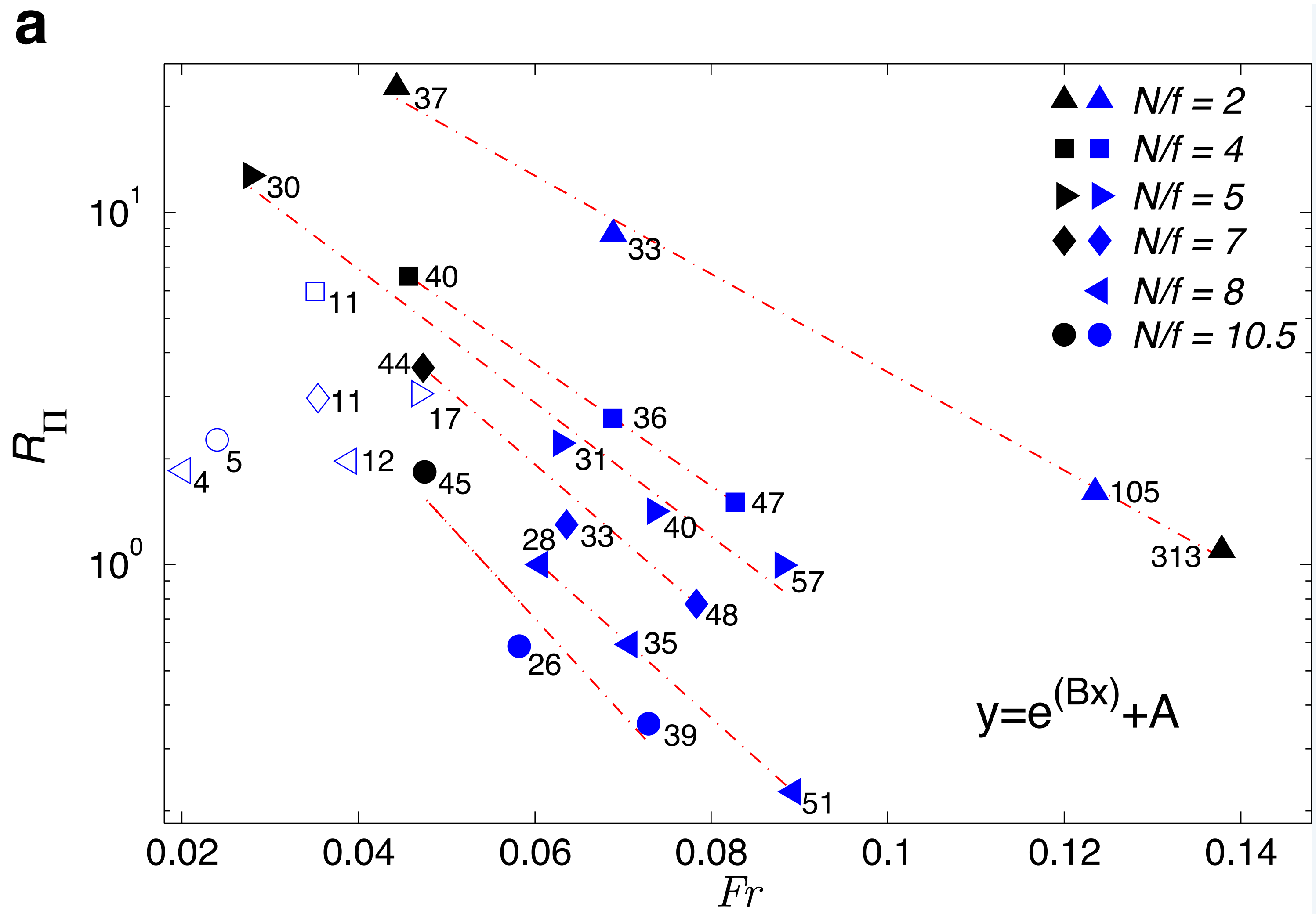
$\rightarrow \varepsilon_{LS} / \varepsilon_{ss} \sim [Fr * Ro]^{-1}$

$\sim \omega_{rms} [Nf]^{-1/2} Re^{-1}$



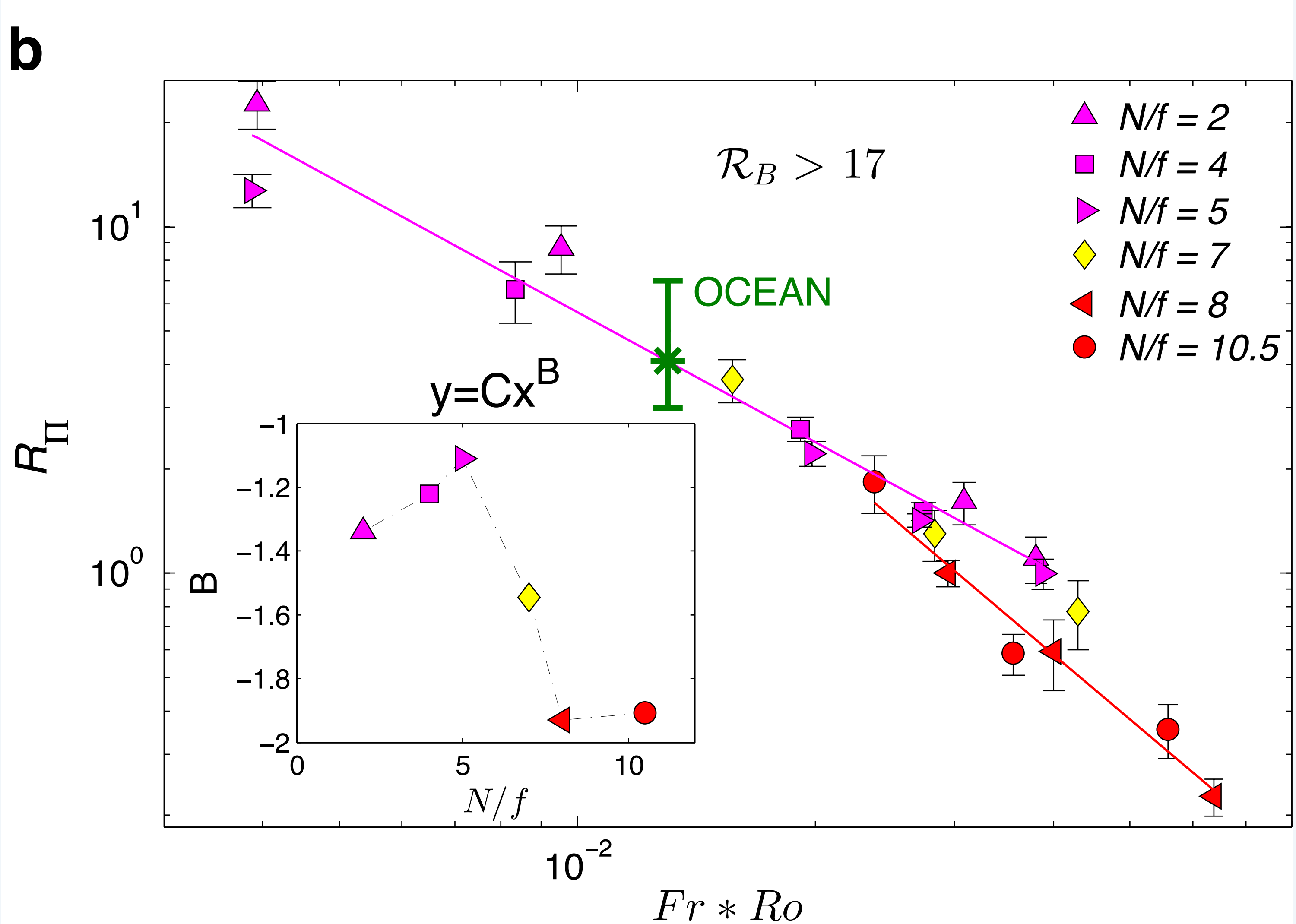


$R_\Pi = \varepsilon_{inv}/\varepsilon_{dir}$



* Point labeled with values of $R_B = Re\ Fr^2$

$R_\Pi = \varepsilon_{inv}/\varepsilon_{dir}$



R_B = Re Fr^2

Conclusion

Dual bi-directional constant flux cascades
are the norm, allowing for
long-time large-scale coherent structures
as well as small-scale mixing and dissipation