Non-Universal and Non-Asymptotic Turbulence in Magnetic Fields

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Incompressible Magnetohydrodynamics Fluid Model

$$\partial_t \boldsymbol{v} = -\boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla p + \mathbf{j} \times \mathbf{B} + \mathrm{Re}^{-1} \Delta \boldsymbol{v}$$

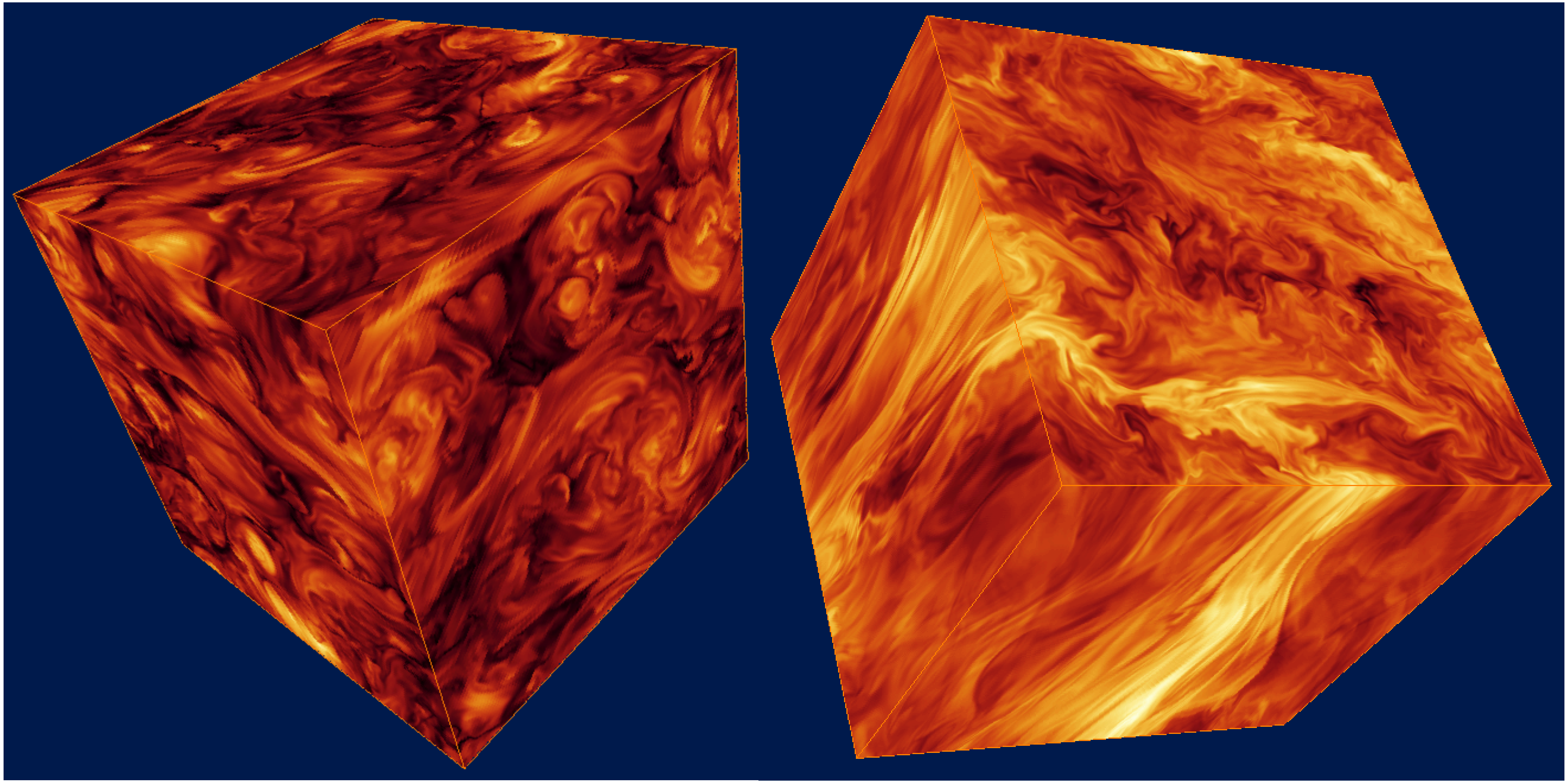
$$\partial_t \mathbf{B} = \nabla \times (\boldsymbol{v} \times \mathbf{B}) + \mathrm{Rm}^{-1} \Delta \mathbf{B}$$

$$\nabla \cdot \boldsymbol{v} = \nabla \cdot \mathbf{B} = 0\,, \ \mathbf{j} = \nabla \times \mathbf{B}$$

Reynolds numbers: $\mathrm{Re} = \frac{\ell_0 v_0}{\mu}$ $\quad \mathrm{Rm} = \frac{\ell_0 v_0}{\eta}$

magnetic Prandtl number: $\mathrm{Pr_m} = \frac{\mathrm{Rm}}{\mathrm{Re}}$

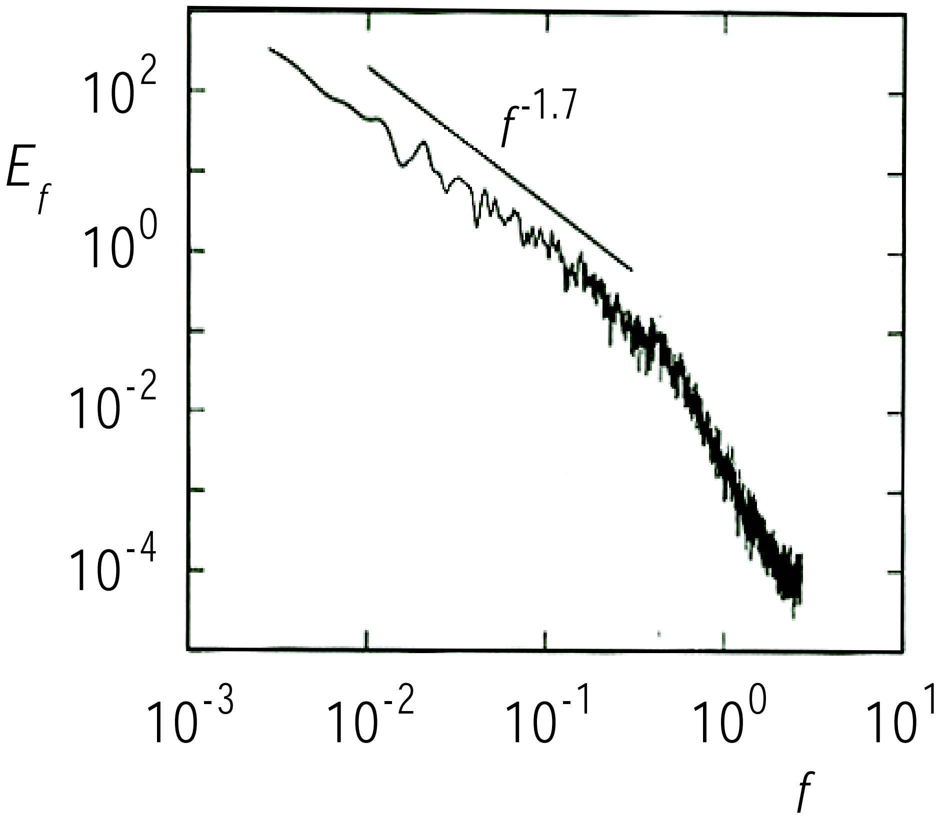
Numerically Generated MHD Turbulence

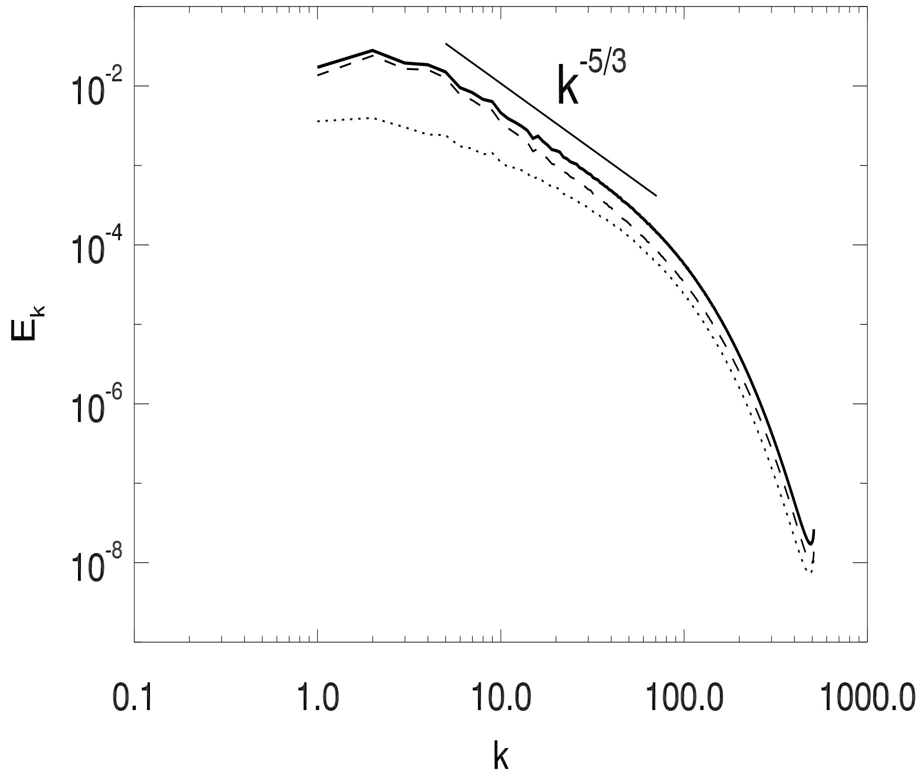


$\langle \mathbf{B} \rangle = 0$ $\langle \mathbf{B} \rangle = 5 b_{\mathrm{rms}} \hat{e}_z$

$\mathrm{Re} \sim \mathcal{O}(10^3)$, $\mathrm{Pr}_m = 1$

Solar Wind Energy Spectrum





WIND probe (solar distance 1 a.u.)

Leamon et al., J. Geophys. Res., 1998

1024^3 DNS, decaying MHD turbulence

Müller & Grappin, PRL, 2005

Müller & Biskamp, PRL, 2000

Elsässer Picture of MHD

Elsässer variables $\boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \mathbf{b}$, $\langle \mathbf{B} \rangle = \mathbf{B}_0$

$$\nabla \cdot \boldsymbol{z}^{\pm} = 0$$

$$\partial_t \boldsymbol{z}^{\pm} = \pm B_0 \partial_{\|} \boldsymbol{z}^{\pm} - \boldsymbol{z}^{\mp} \cdot \boldsymbol{\nabla} \boldsymbol{z}^{\pm} - \nabla p^* + \frac{\mathrm{Re}^{-1} + \mathrm{Rm}^{-1}}{2} \Delta \boldsymbol{z}^{\pm}$$

Elsässer 1950

►exact nonlinear solutions: $\boldsymbol{z}^{\pm} = 0$, $\boldsymbol{z}^{\mp} = \boldsymbol{z}^{\mp}(x \mp B_0 t)$

►only counter-propagating Alfvén pulses interact nonlinearly

►Alfvén time: $\tau_{\mathrm{A}} = \ell_{\|} / B_0$

►$\boldsymbol{\tau_{\mathrm{NL}} \gg \tau_{\mathrm{A}}}$

inherent anisotropy

►nonlinear energy flux $\perp B_0$

Shebalin et al. 1983, Grappin 1986

Scale Non-Locality of MHD Turbulence

- Navier-Stokes turbulence (velocity field) **Galilei-invariant**
- mean velocity dynamically irrelevant
  $\Rightarrow$ scale locality

- MHD turbulence (magnetic field) **not Galilei-invariant**
- large-scale magnetic field dynamically relevant (all scales of motion !)
  $\Rightarrow$ scale non-locality

non-universality of MHD turbulence

Turbulence in Mean Magnetic Fields

incompressible magnetohydrodynamics, homogeneous mean field $\mathbf{B}_0$
turbulence dynamics description (high Reynolds numbers)

- **timescales** $\tau_\perp^{ac}$, $\tau_\|^{ac}$
  $\tau_{NL} = (k_\perp^3 v_{k_\perp})^{-1/2}$, $\quad k \sim \ell^{-1}$
  $\tau_A = (k_\| B_0)^{-1}$
  $\tau_A^{rms} = (k_\perp b_{rms})^{-1}$
- **cascade model** $\varepsilon \sim F(\tau_\perp^{ac}, \tau_\|^{ac}, \mathbf{B}_0, \ldots)$
- **coherence relation** $G(\tau_\perp^{ac}, \tau_\|^{ac}) = 0$

Nonlinear Paradigms and Spectral Signatures

Alfvén-wave interaction phenomenologies

- **weak** ($\tau_\perp^{ac} \gg \tau_\parallel^{ac}$): $E(k_\perp) \sim k_\perp^{-2}$

  Ng & Bhattacharjee,1996, Galtier et al., 2000

- **strong** ($\tau_{NL} \sim \tau_A$): $E(k_\perp) \sim k_\perp^{-5/3}$ or $E(k_\perp) \sim k_\perp^{-3/2}$

  Goldreich-Sridhar,1995, Boldyrev,2005

- **3D IK** ($\tau_A^{rms} \sim \tau_A$): $E(k) \sim k^{-3/2}$, isotropic scaling

  Iroshnikov 1964, Kraichnan 1965

Numerical Experiment

configuration

- MHD turbulence + strong mean magnetic field $B_0 = 5b_{\text{rms}}$
- large-scale forcing ("isotropic", frozen structure of large-scale fluctuations)

observation

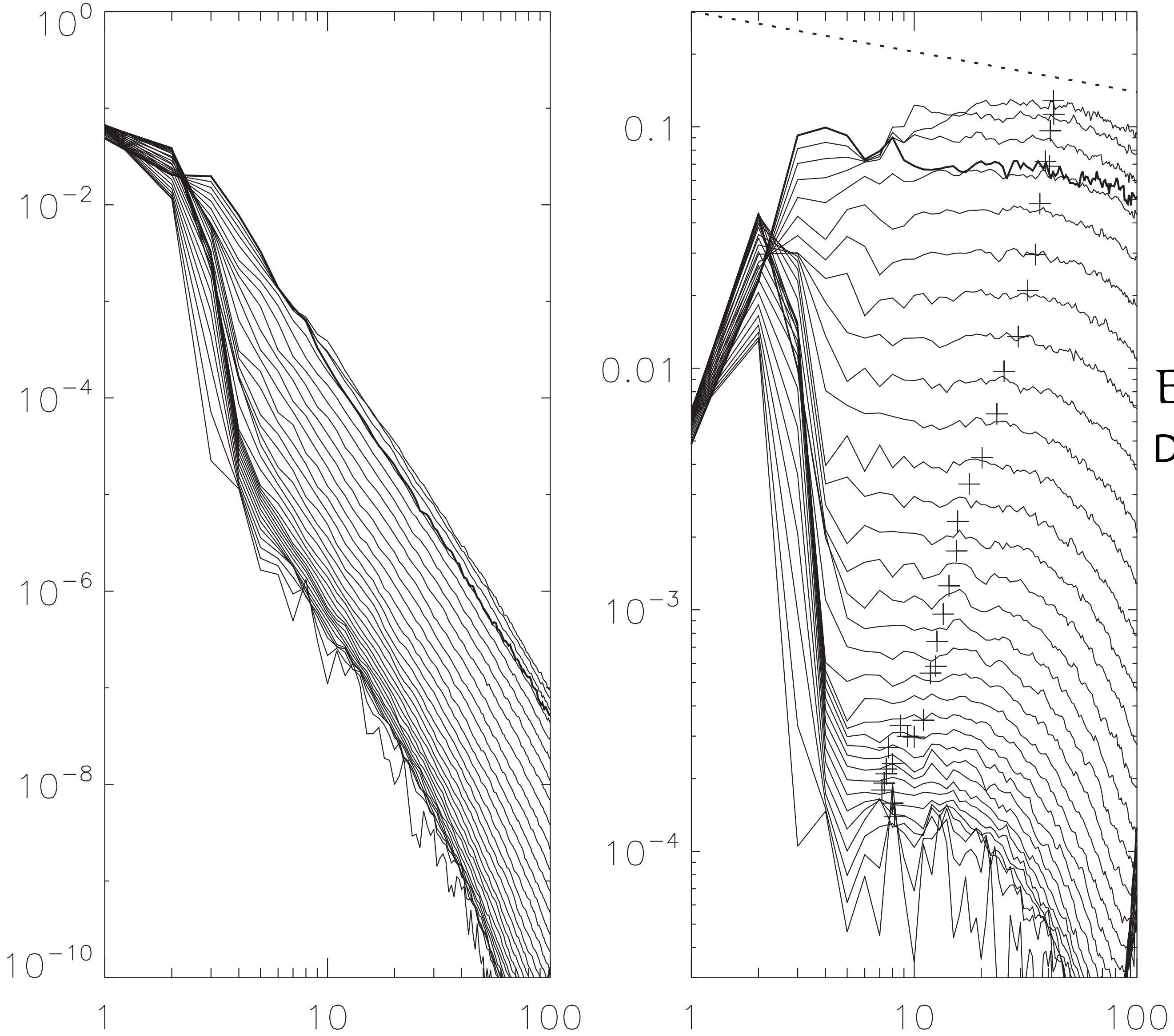
- moderate **amplitude anisotropy** of spectrum, **isotropic scaling** $\sim k^{-3/2}$
- **scale-independent anisotropy** $\sim b_{\text{rms}}/B_0$
- significant **field-parallel** energy flux

large-scale conditions important $\rightarrow$**non-universality**

Lee et al.,2010, Krstulovic et al. 2014

New Regime of Turbulence

$E^{3D}(k, \theta)$



$E^{3D}(k, \theta)k^{3/2+2}$

Dotted: $\sim k^{-5/3-2}$

ray spectra taken along $\mathbf{R}$ at different angles $\theta = \measuredangle(\mathbf{B}_0, \mathbf{R})$

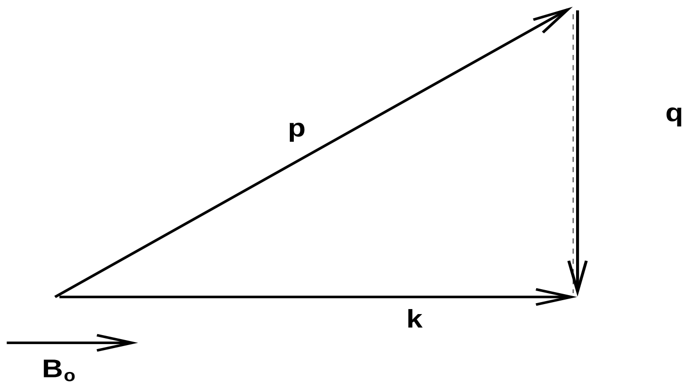
Müller & Grappin, 2010; Grappin et al., arxiv

A Different Regime of MHD Turbulence

- scaling isotropy
- extended spectral energy distribution beyond critical balance cone
- necessary precondition: energetic equipartition at largest scales

- three-dimensional Iroshnikov-Kraichan turbulence ?

Resonant Nonlinear Triad Interaction



Convolution constraint on three-mode interactions: $\mathbf{k} = \mathbf{p} + \mathbf{q}$

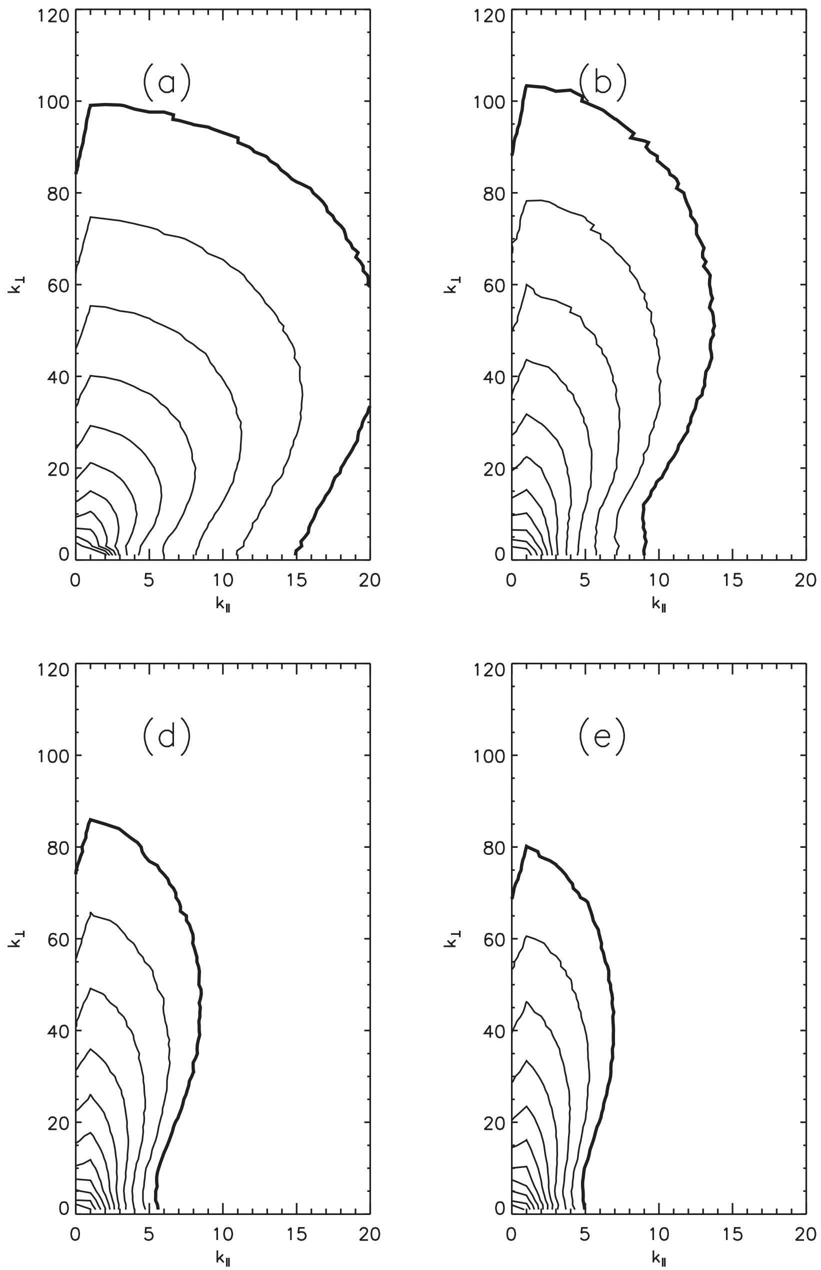
Weak turbulence

Resonance condition: $\omega(\mathbf{k}) = \omega(\mathbf{p}) + \omega(\mathbf{q})$

Alfvén waves: $\omega(\mathbf{k}) = \mathbf{k} \cdot \mathbf{B}_0 = k_{\|} B_0$

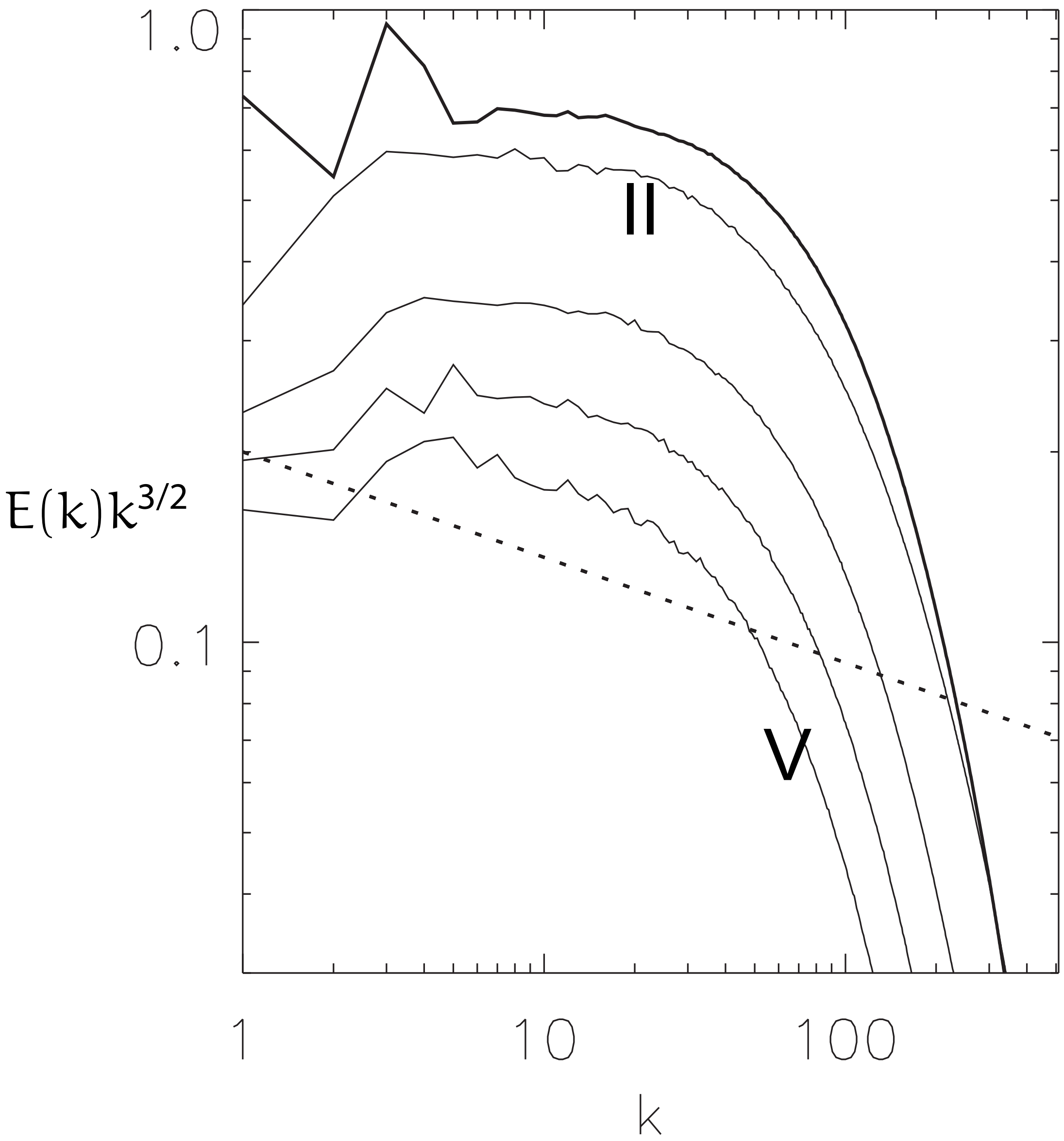
Resonance condition implies $q_{\|} = 0$, i.e. no field-parallel cascade

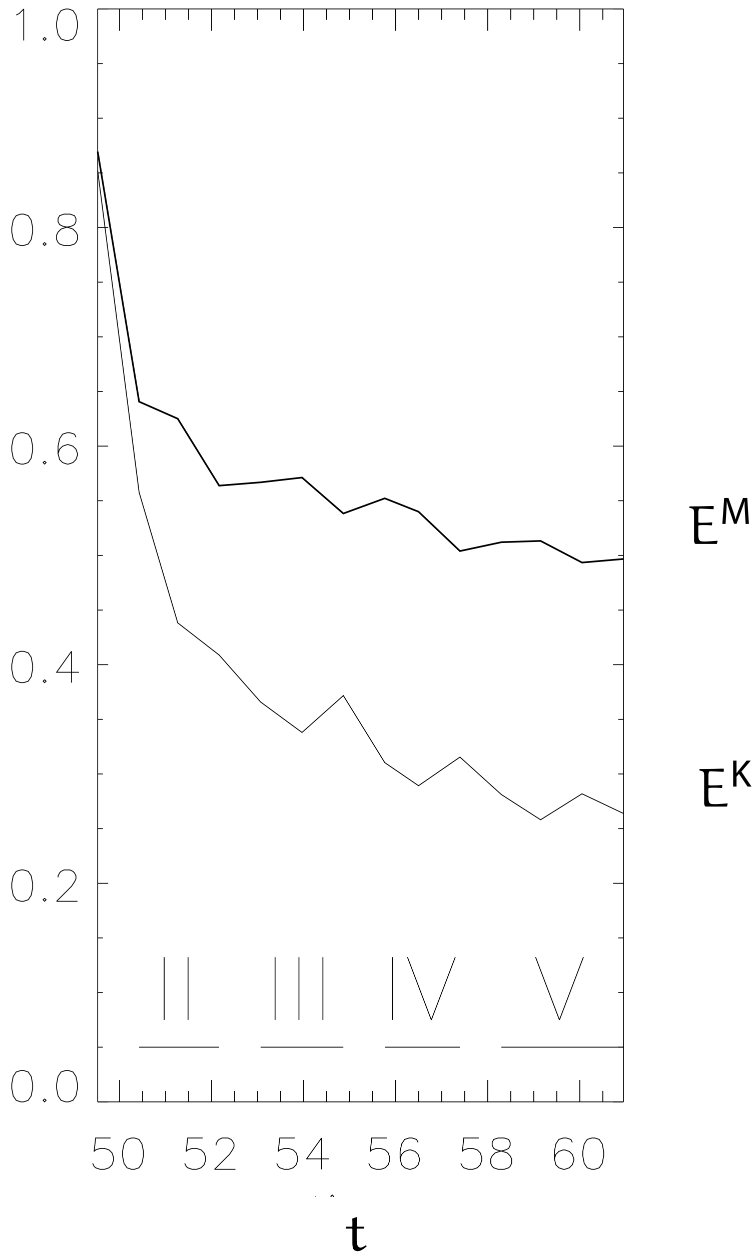
Relaxing Large-Scale Equipartition



Fourier energy isocontours
plane containing $\mathbf{B}_0$

Decaying Turbulence



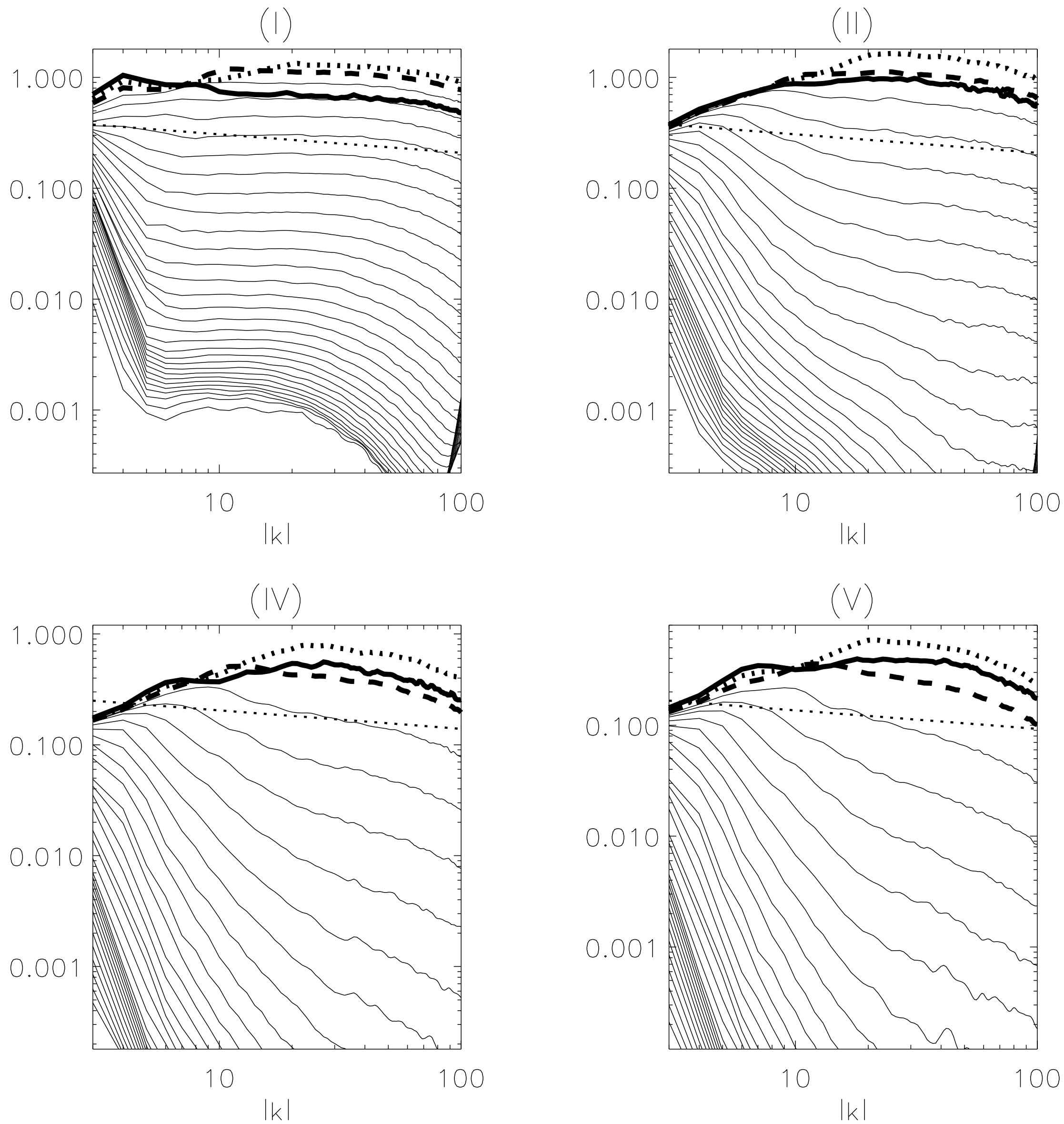


Summary

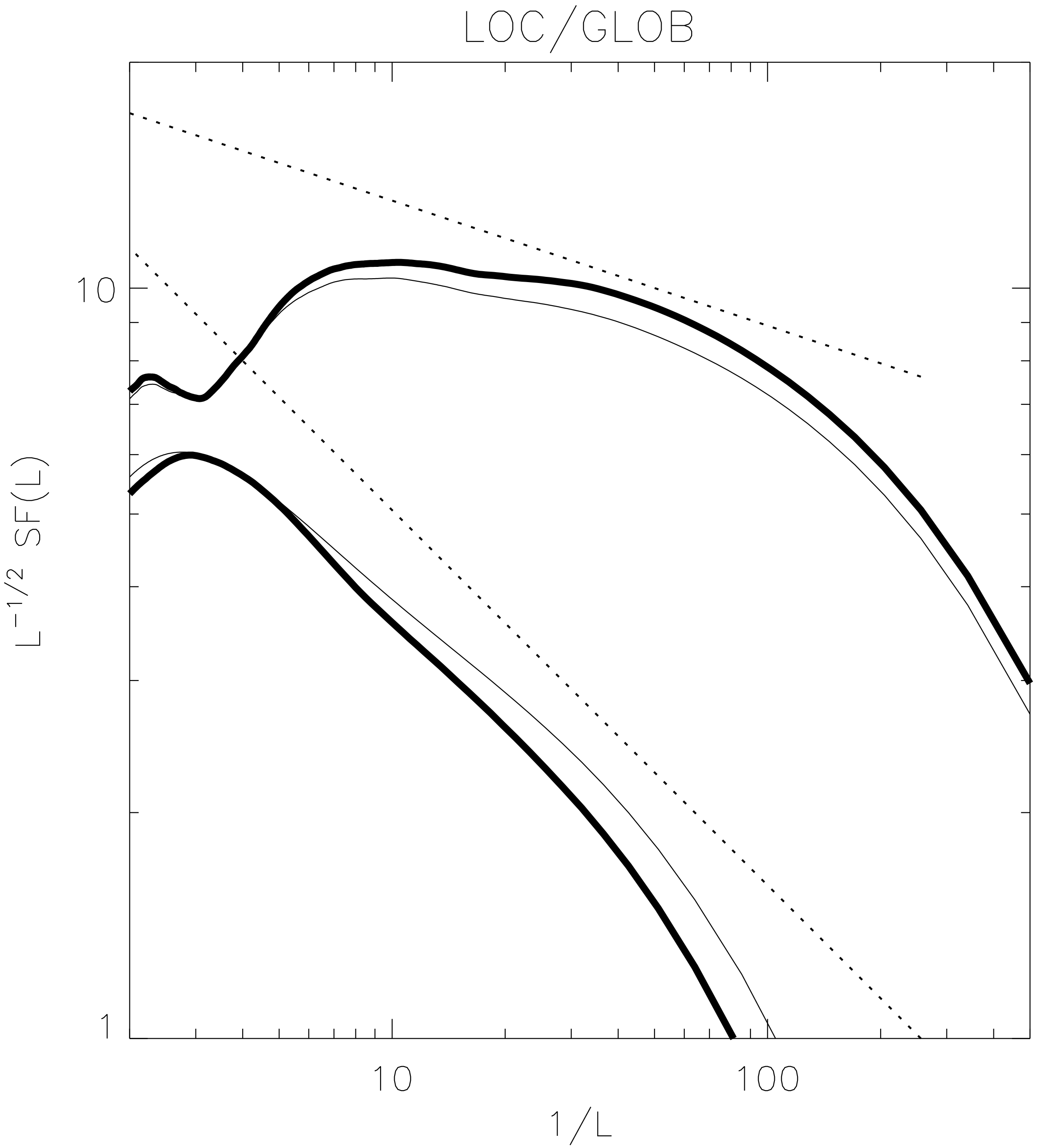
- DNS of MHD turbulence with strong mean magnetic field, large-scale isotropic driving incompatible with standard theory
- new weak regime of MHD turbulence based on IK cascade
- includes quasi-resonant energy flux $\parallel \mathbf{B}_0$
- existence depends crucially on kinetic/magnetic energy ratio (large scales)

`arxiv:1312.3459`

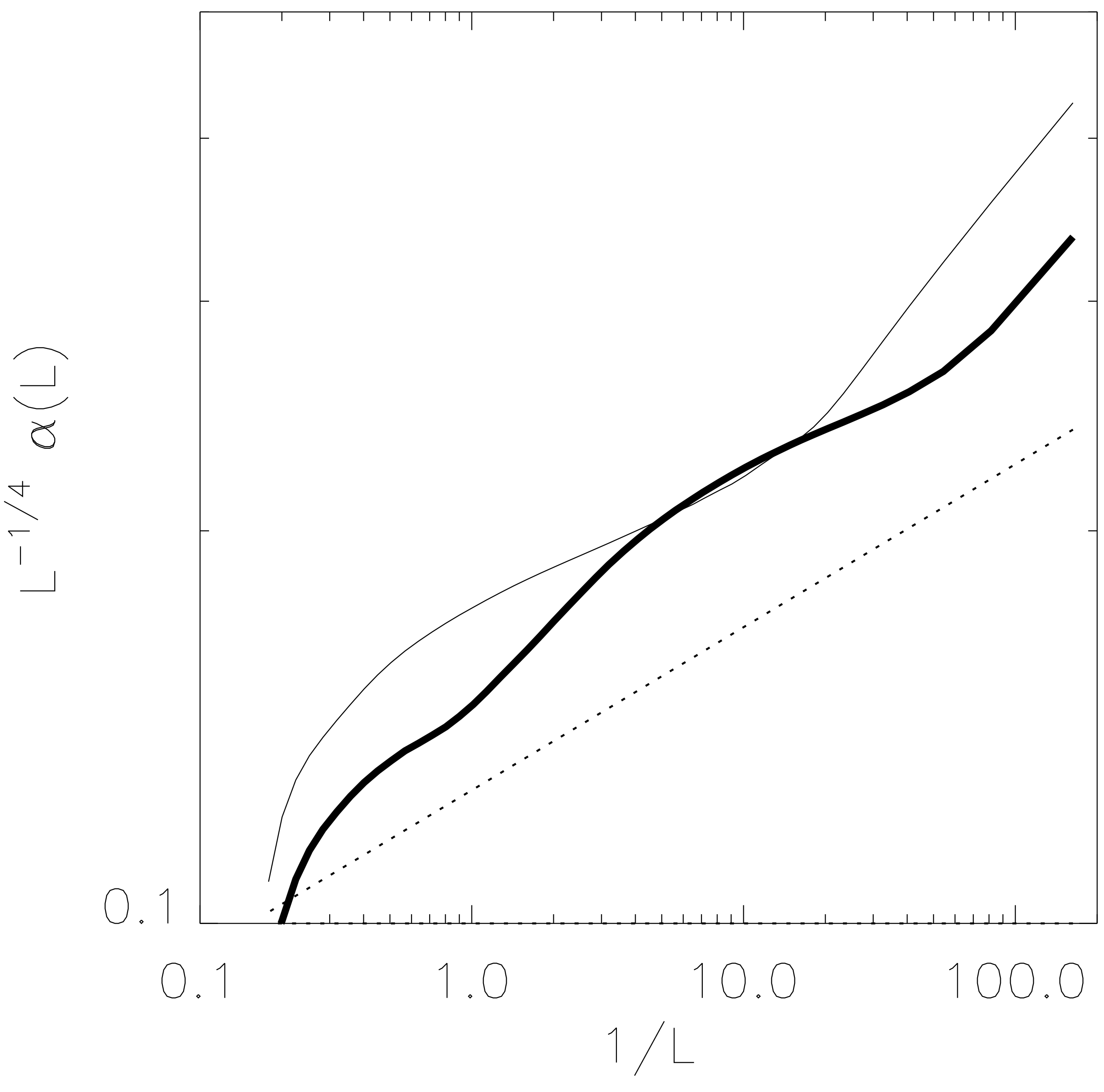
Spectral Energy Distribution



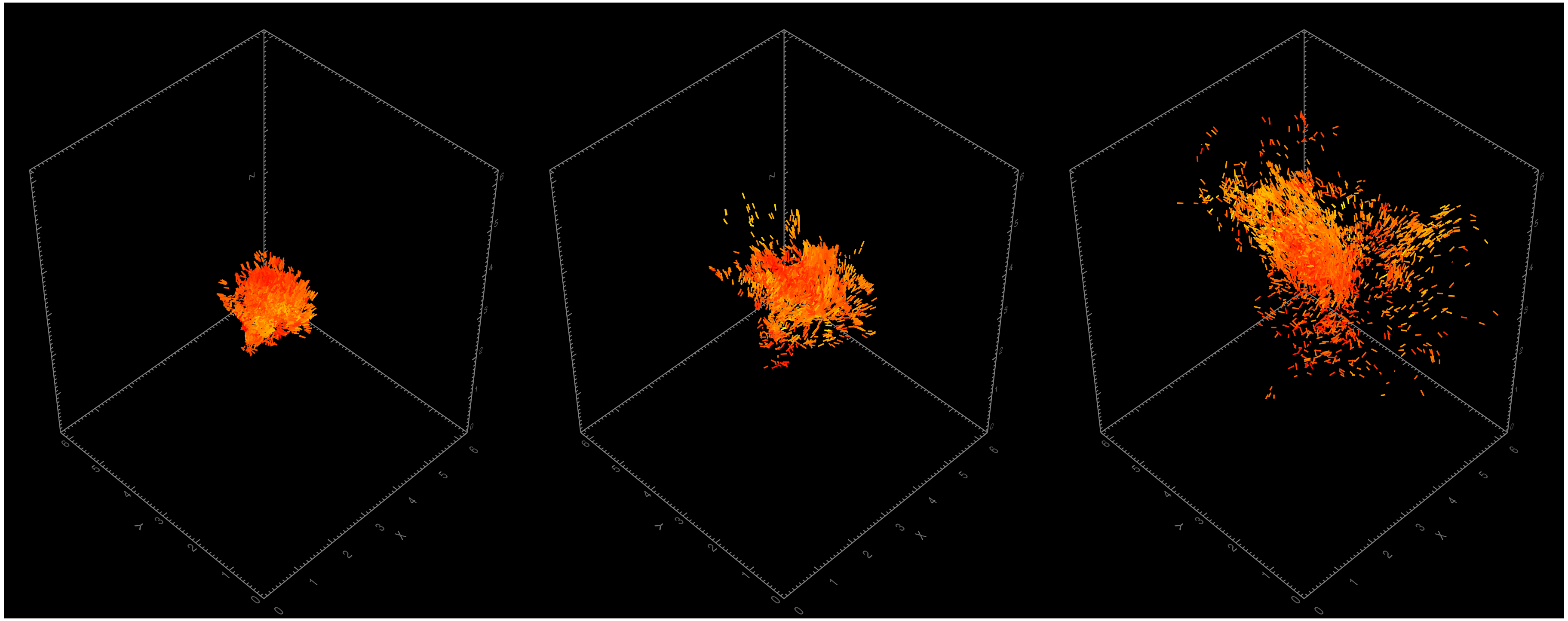
Local vs. Global Frame of Reference



Scale-dependent Alignment ?



Lagrangian Perspective

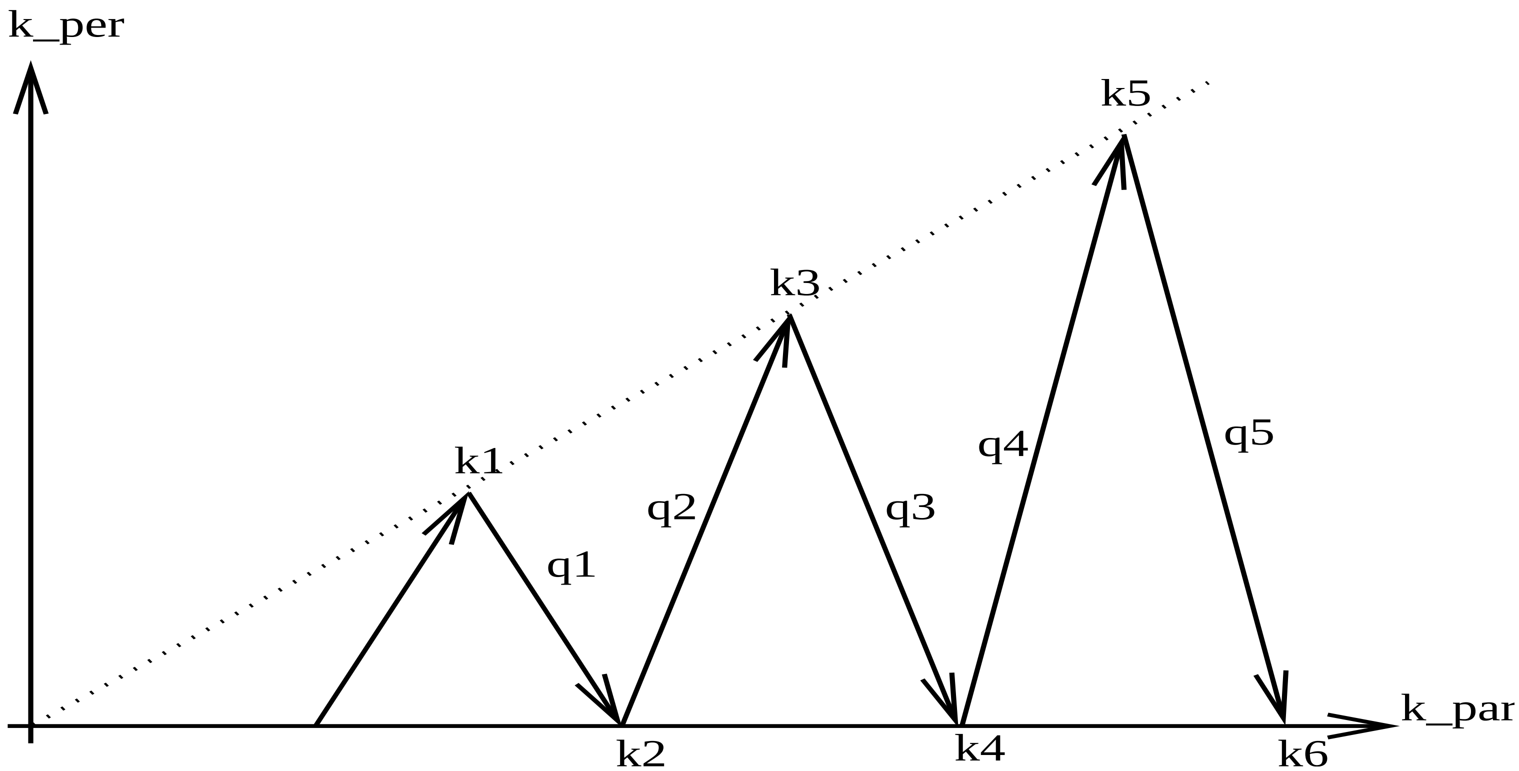


Eulerian frame of reference, $f(\mathbf{r}, t)$
Lagrangian comoving frame of ref., $f(\mathbf{r}(t), t)$ mit $\dot{\mathbf{r}}(t) = \boldsymbol{v}(\mathbf{r}, t)$

- ►tracing massless test particles in the flow
- ►measuring turbulent fluctuations along tracer trajectory
- ►tracer positions $\longrightarrow$ diffusion, dispersion, two-point statistics

Ricochet Process

Realizes energy flow along directions oblique w.r.t. $\mathbf{B}_0$



Process based on two basic triads to transfer prolongations along two directions in Fourier space

Dependent on dominant perpendicular cascade process populating excitations within the CB region.

Start near Fourier origin requires externally excited fluctuations (e.g. isotropic large-scale forcing)

Lagrangian Frequency Spectrum

- Autocorrelation along n tracer trajectories: $R(s) = \langle v(t)v(t+s)\rangle_n / \langle v^2\rangle_n$
- Frequency spectrum: $E^L(\omega) = 2\int_0^\infty \mathrm{d}sR(s)\cos(\omega s)$, $\omega \geq 0$

With $\omega \sim (\tau_{ac})^{-1}$

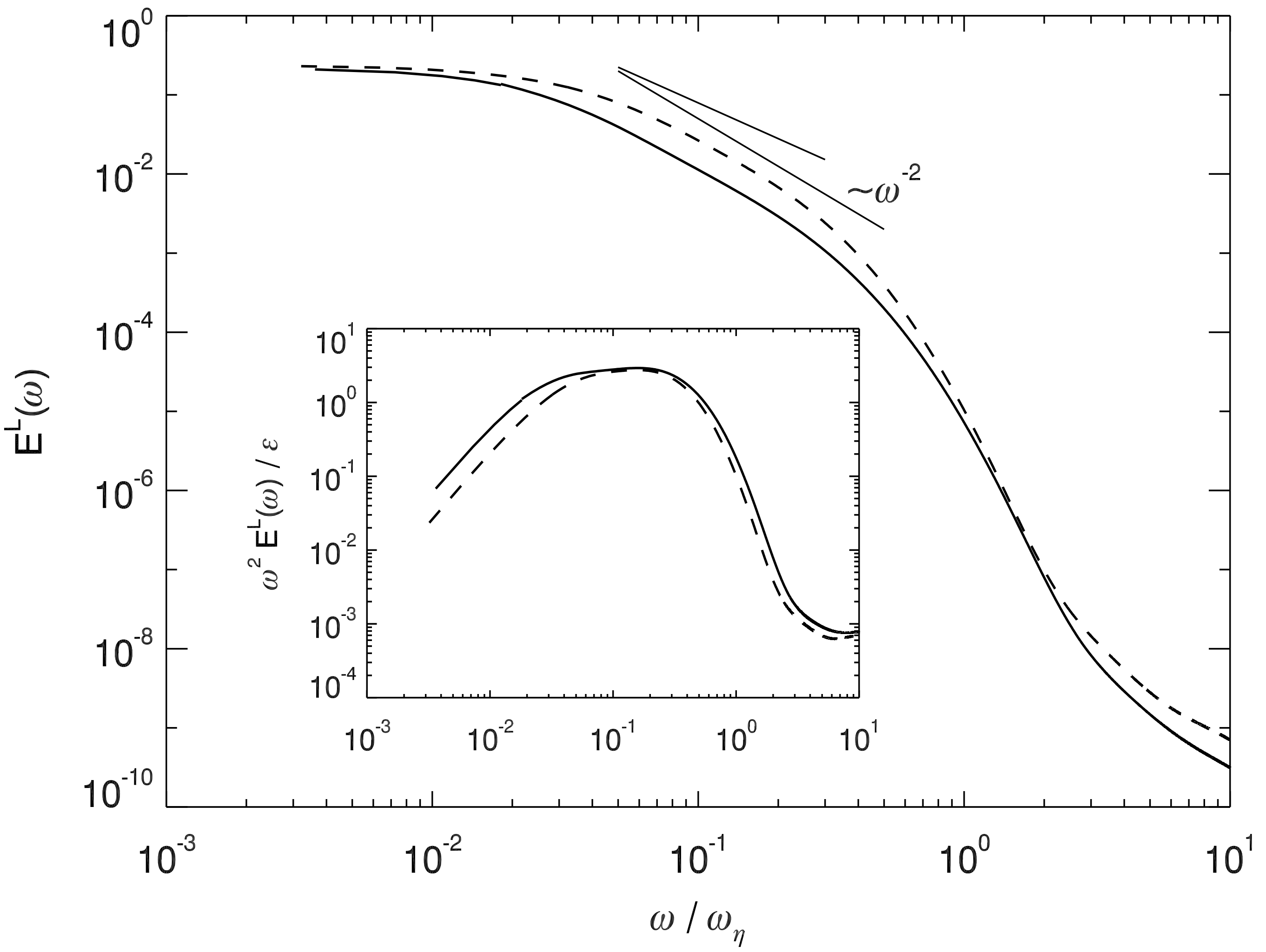
$$\omega E^L(\omega) \sim kE(k) \sim z_\ell^2$$

$$\Rightarrow z_\ell^2/\tau_{cas} \sim \varepsilon \Leftrightarrow E^L(\omega) \sim \varepsilon\tau_{cas}\tau_{ac}$$

- **Strong turbulence:** $\tau_{cas} \sim \tau_{ac}$ $\qquad \Rightarrow E^L(\omega) \sim \varepsilon\omega^{-2}$
- **Weak turbulence:** $\tau_{cas} \gg \tau_{ac}$ $\qquad \Rightarrow E^L(\omega) \sim \varepsilon^{1/2}b_0\omega^{-3/2}$

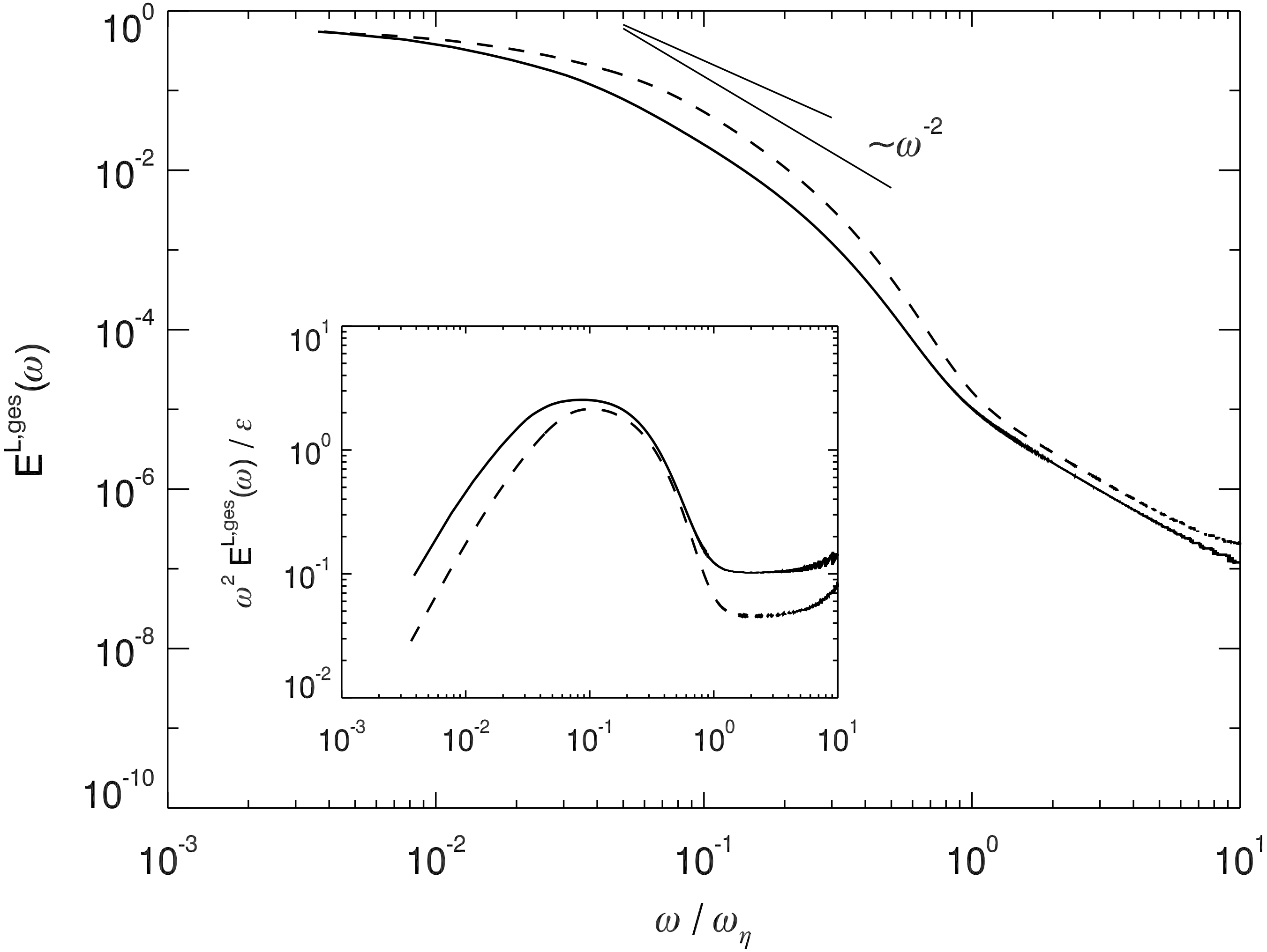
Busse, Müller, Gogoberidze, PRL, 2010

Navier-Stokes



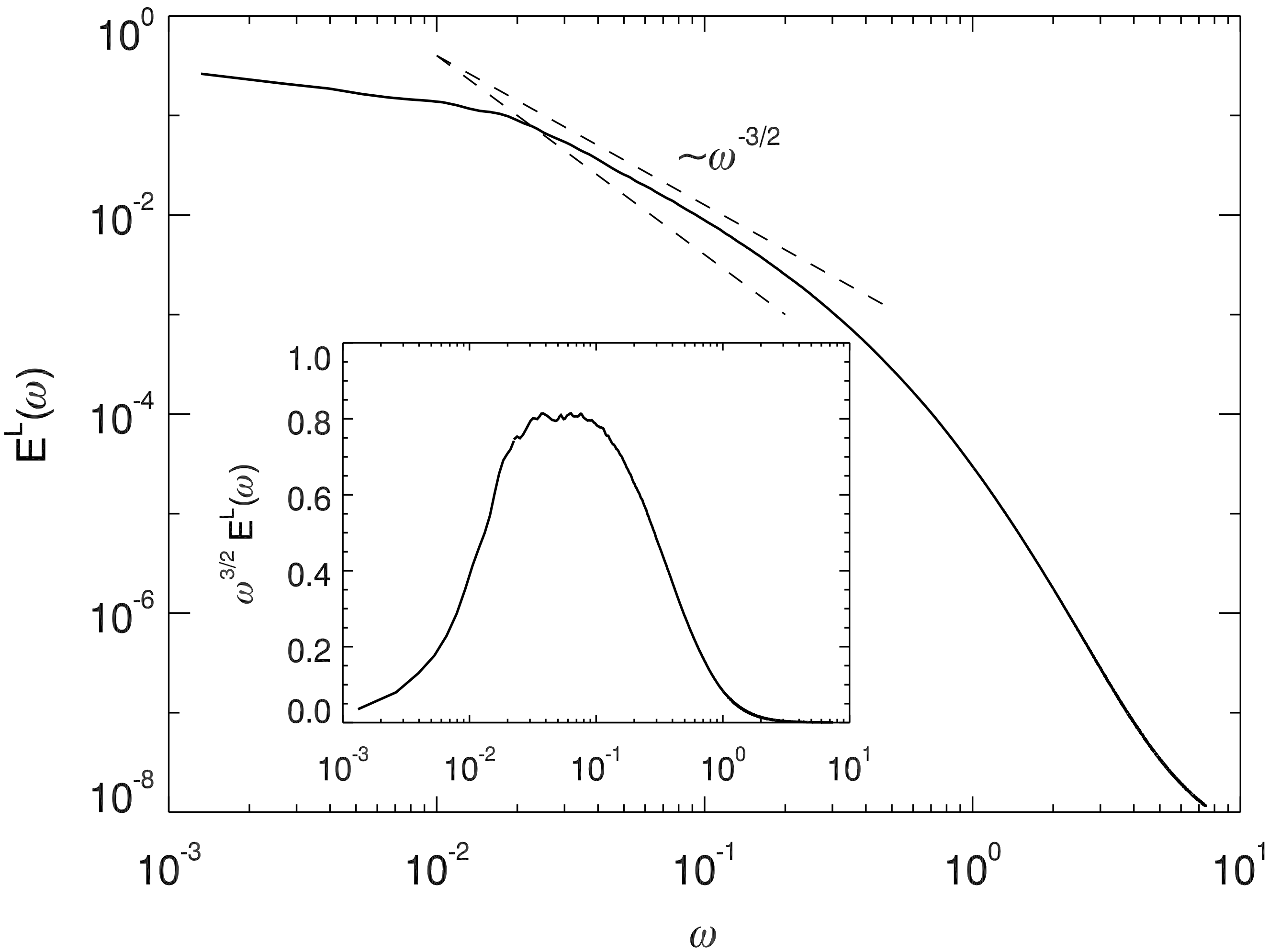
Frequency spectrum, 1024^3**,** 512^3 **(dashed)**

MHD, $B_0 = 0$



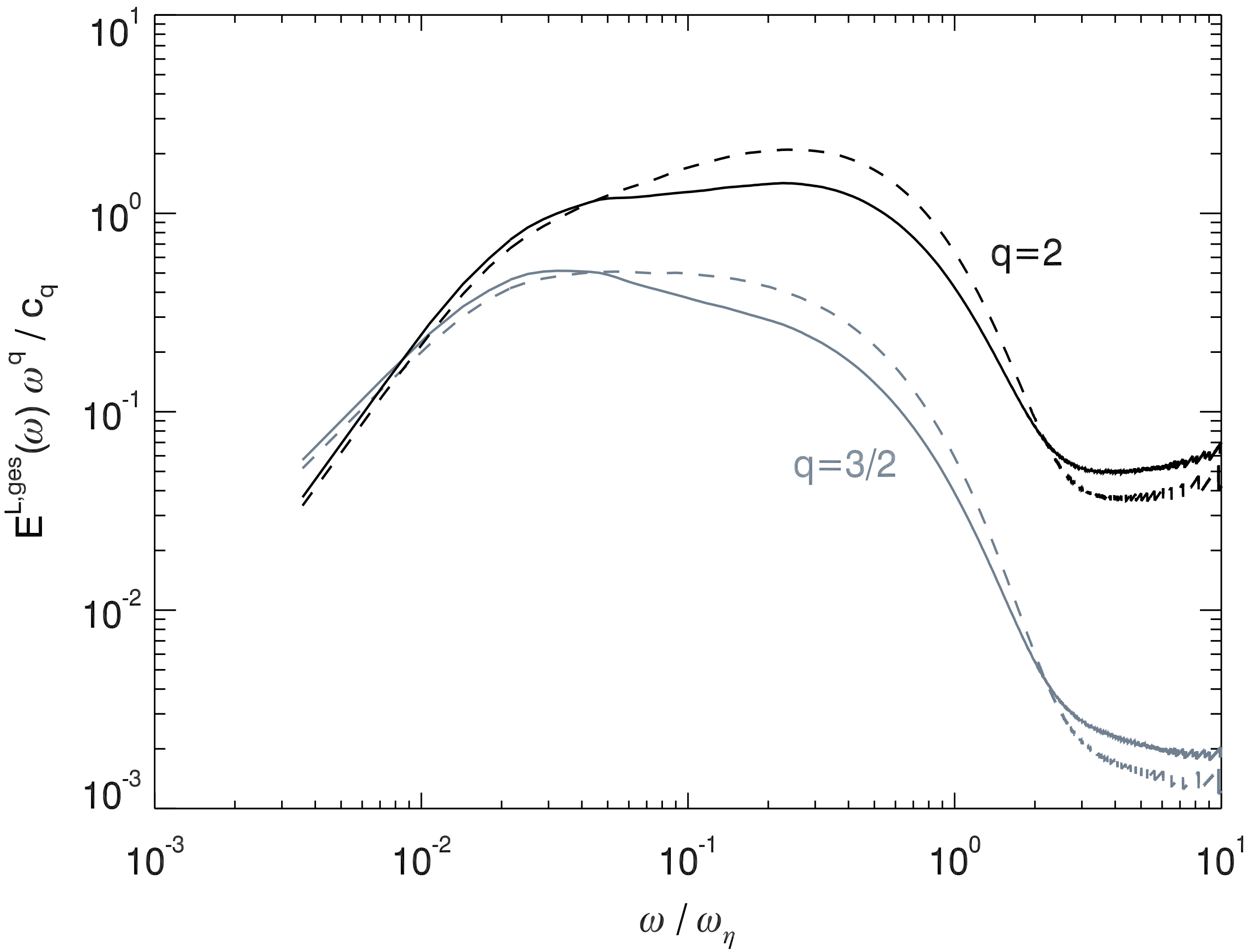
Frequency spectrum, MHD, $B_0 = 0$, 1024^3, 512^3 (dashed)

2D MHD



Frequency spectrum, 2D MHD, 4096^2

MHD, $B_0 = 5b_{\mathbf{rms}}$



Frequency spectrum, MHD, $B_0 \approx 5b_{\mathbf{rms}}$, $\parallel$ (dashed), $\perp$

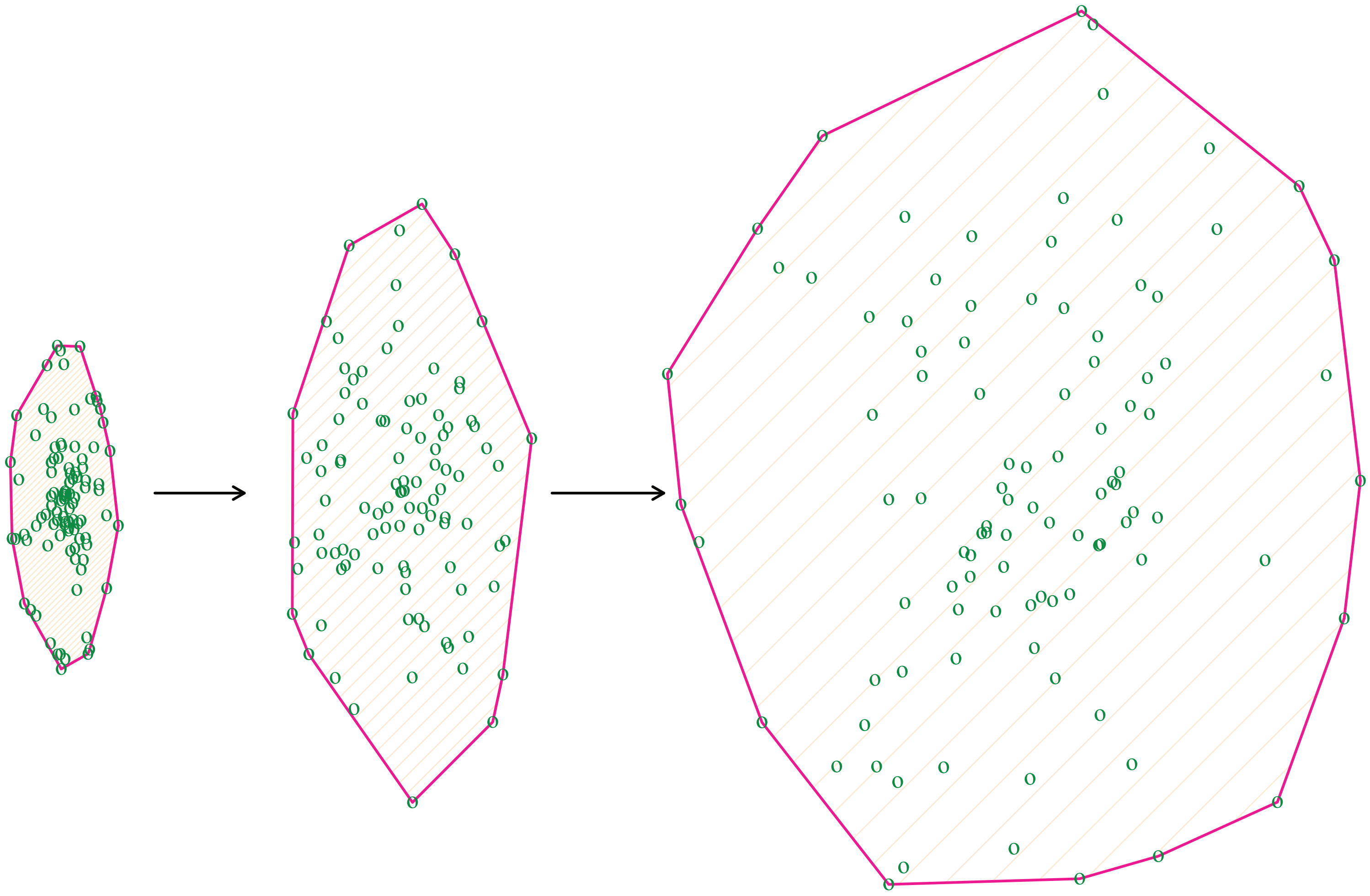
Lagrangian Statistics

- passive Lagrangian tracers (10^6–10^9)
- probing advecting environment + tracer trajectories
- complementary to Eulerian diagnostics

additional benefits

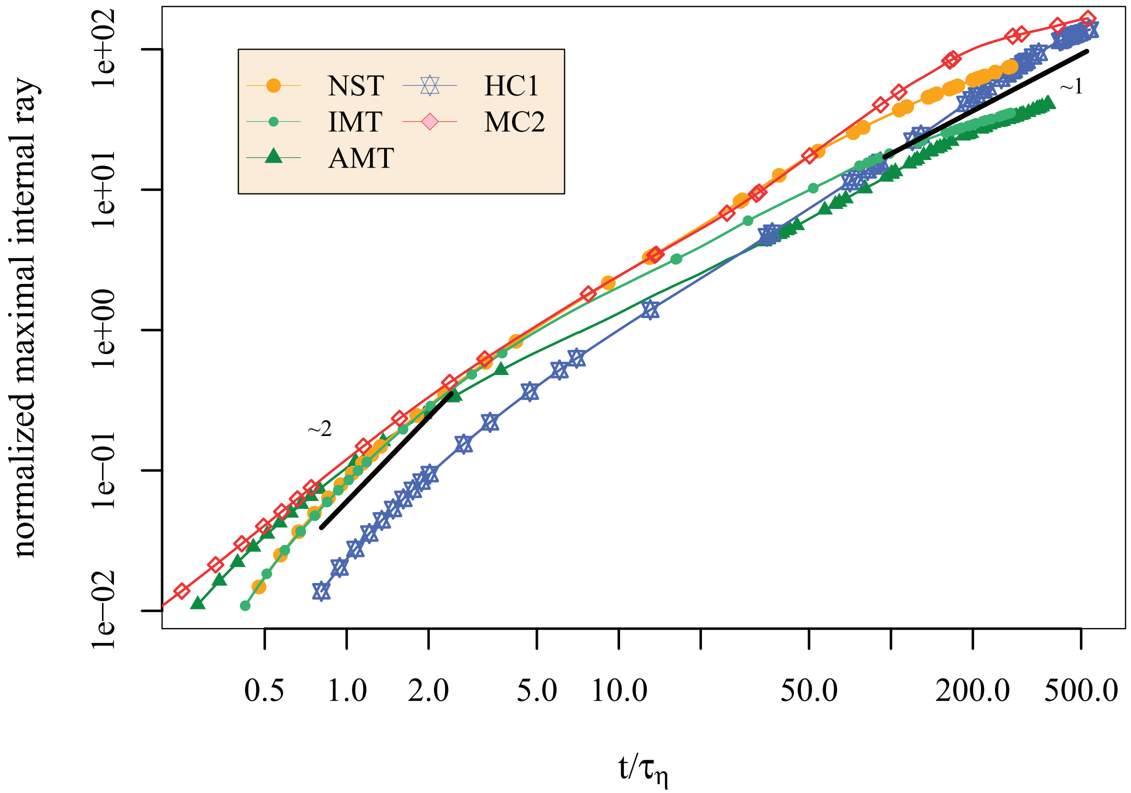
- diffusion, mixing and pair-dispersion
- frequency spectrum
- convex hull diagnostic
- Lagrangian coherent structures (LCS), Lyapunov-exponents

Convex Hull



►Lagrangian coherent structures (ridges of finite-time Lyapunov exponent field)

Pair Dispersion



$$\langle \ell^2(t)\rangle = 2\langle v^2\rangle \int_0^t d\tau (t-\tau)\langle v(t)v(t-\tau)\rangle/\langle v^2\rangle = \begin{cases} \langle v^2\rangle t^2, & t \ll T_L \\ 2\langle v^2\rangle T_L t, & t \gg T_L \end{cases}$$

$\tau_\eta = (\mu/\varepsilon_K)^{1/2}$ (Kolmogorov time scale)

Numerical Experiment

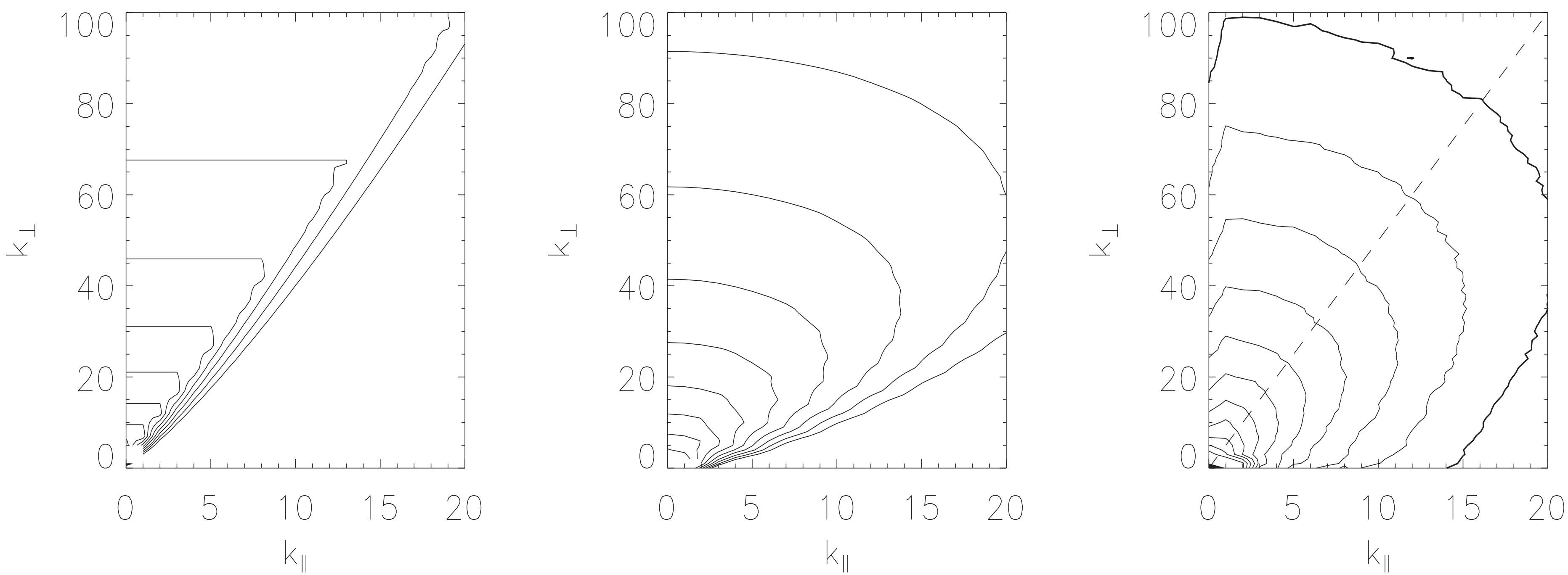
pseudospectral simulations

- spatial resolution $512^3 \Rightarrow \mathrm{Re} \in [1.4, 6.1] \cdot 10^3$
- $\mathrm{Pr_m} = \mu/\eta \in 0.5 - 2$
- $\mathrm{Pr} = \mu/\kappa \in 0.5 - 2$

configuration

- common Rayleigh-Bénard configuration too constraining
- periodicity along $g \Rightarrow$ **elevator instability**
- quasi-periodic boundary conditions, suppressing modes with $k_z = 0$

Fourier Energy Distribution ($k_{\|}$-$k_{\perp}$ plane)

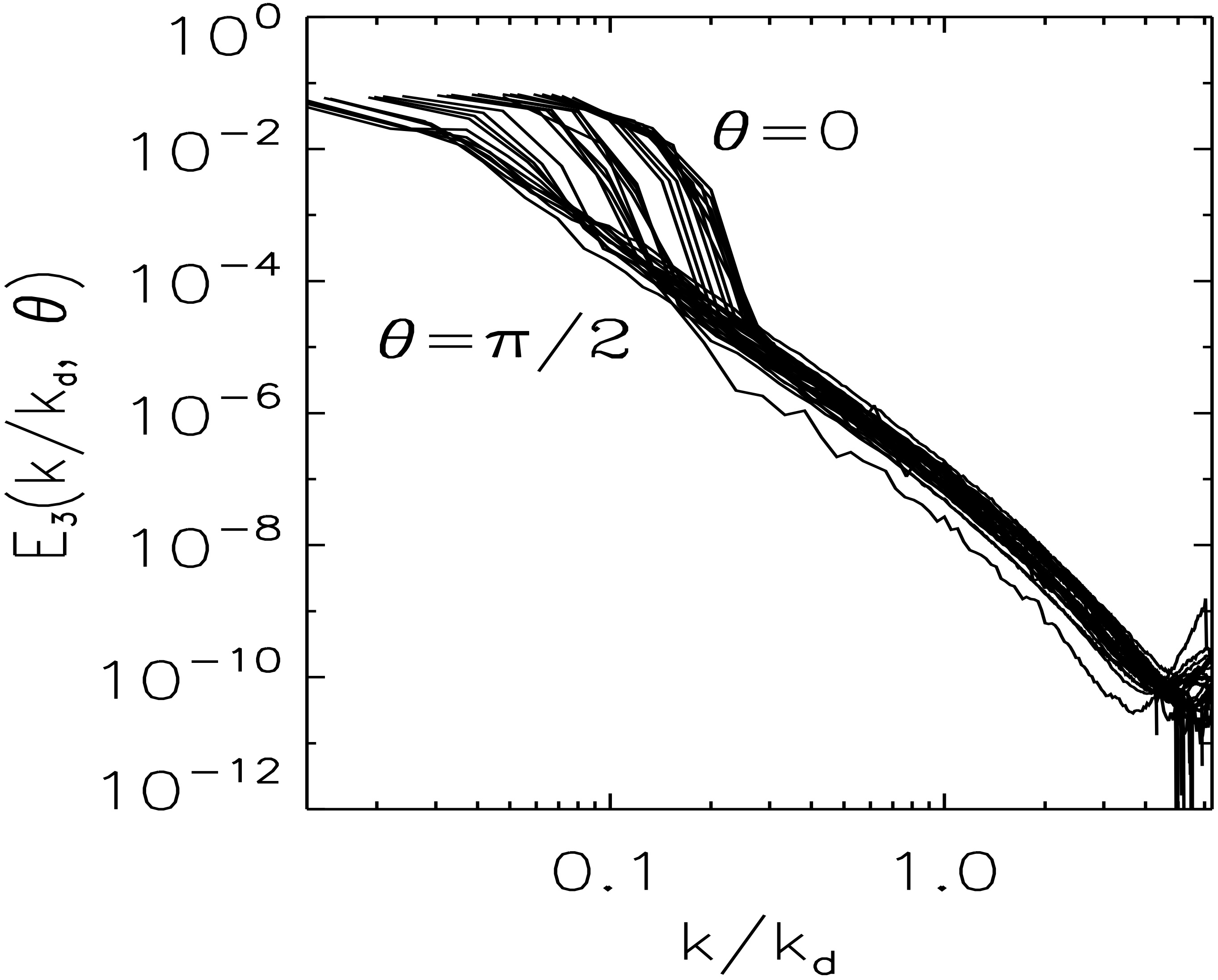


Left: Critical balance cone (local frame): $k_{\|} \sim k_{\perp}^{2/3}$

Middle: CB cone subject to fluctuations around mean direction $\sim \frac{b_{\perp}}{B_0} \simeq \frac{1}{5}$

Right: DNS with isotropic large-scale driving

Universality



$$E_3(k,\theta) = A(\theta)k^{-m-2} = A_0(k/k_d)^{-m-2}, \qquad A(\theta) \simeq k_d(\theta)^{m+2}$$

Causality

Generalization of GS-critical balance: $\tau_\perp^{ac} \sim \tau_\|^{ac} \sim \tau_A$

Incompressible MHD ($B_0 \lesssim 2-3$): $\tau_{NL_\perp} \sim \tau_A$

If transfer in planes perpendicular to B_0 governed by IK cascade:

- $\tau_\perp^{ac} \sim \tau_{A_\perp} = (k_\perp b_{rms_\perp})^{-1}$
- $\tau_A < \tau_{A_\perp} < \tau_{NL}$

Relaxation of weak turbulence constraint ($\tau_A \ll \tau_{NL}$)
$\rightarrow$ possibility of **quasi-resonant cascade**, allows small $q_\| \sim q_\perp \frac{b_{rms_\perp}}{B_0}$

Nonlinear Energy Flux

Isotropic K41 flux:

$$F_{K41} \sim k v_k^3 \qquad (k^{-5/3})$$

Iroshnikov-Kraichnan flux:

$$F_{IK} \sim k b_k^2 b_q^2 / B_0 \qquad (k^{-3/2})$$

F_{IK} approximately reduced by factor $\frac{b_q}{B_0}$

comparison with quasi-resonant flux (triad counting)

Ensemble of triads reduced through quasi-resonance constraint by factor $\frac{b_{rms}}{B_0}$

Dissipative Regions

Estimating end of inertial range:

$$\tau_{\text{diss}} \sim \tau_{\text{flux}}$$

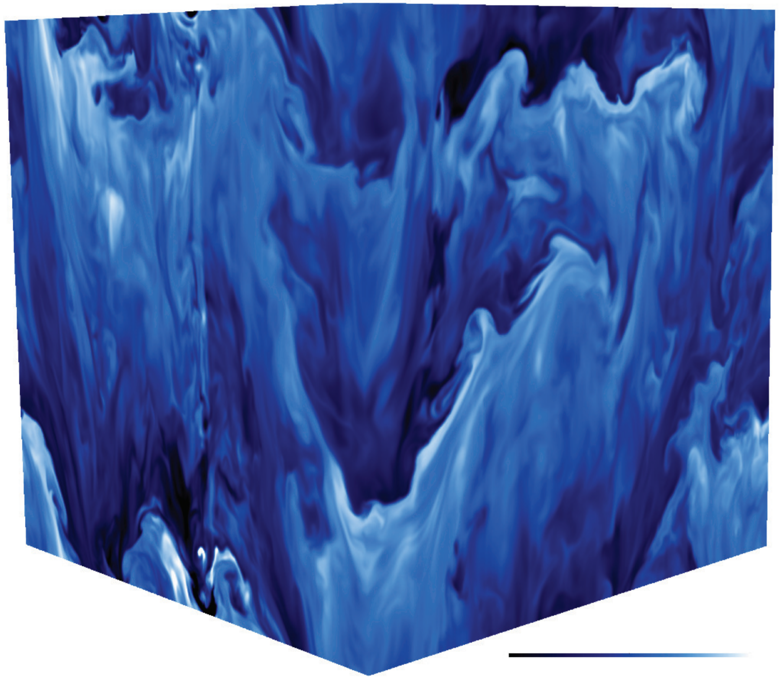
$$\tau_{\text{diss}}^{-1} \sim \nu k^2,\ \tau_{\text{flux}_{\|}} \sim \frac{k u_k^2}{b_{\text{rms}}},\ \tau_{\text{flux}_{\perp}} \sim \frac{k u_k^2}{B_0}\ \text{(IK)}$$

$$\frac{k_{d\|}}{k_{d\perp}} \sim \frac{b_{\text{rms}}}{B_0}$$

Found in numerical simulations (Grappin & Müller 2010)

Summary

Turbulent Dynamo



convectively driven MHD turbulence

- Boussinesq approximation
- incompressible fluid with buoyancy
- temperature $T = T_0(z) + \Theta$
- $\langle \Theta \rangle = 0$, $\langle \partial_z T_0 \rangle =$ const.

$$\partial_t \omega = \nabla \times (\mathbf{v} \times \omega + \mathbf{j} \times \mathbf{b}) + \mu \Delta \omega - \nabla \Theta \times \mathbf{g}$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \eta \Delta \mathbf{b}$$

$$\partial_t \Theta = -\mathbf{v} \cdot \nabla (T_0 + \Theta) + \kappa \Delta \Theta$$

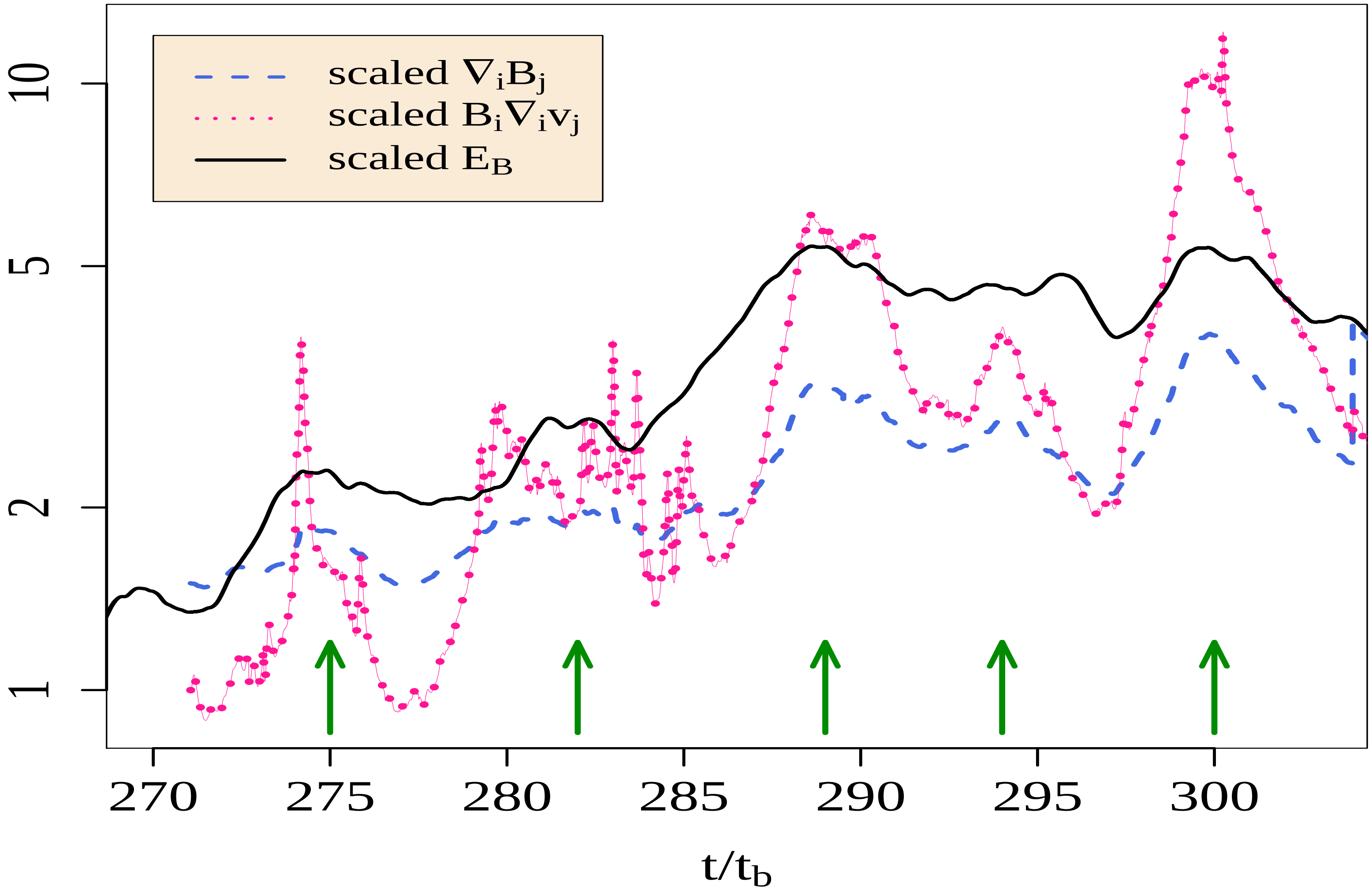
$$\omega = \nabla \times \mathbf{v} \qquad \mathbf{j} = \nabla \times \mathbf{b}$$

Characteristic timescale $\tau_b = (\alpha g |\nabla T_0|)^{-1/2}$

Shear Bursts

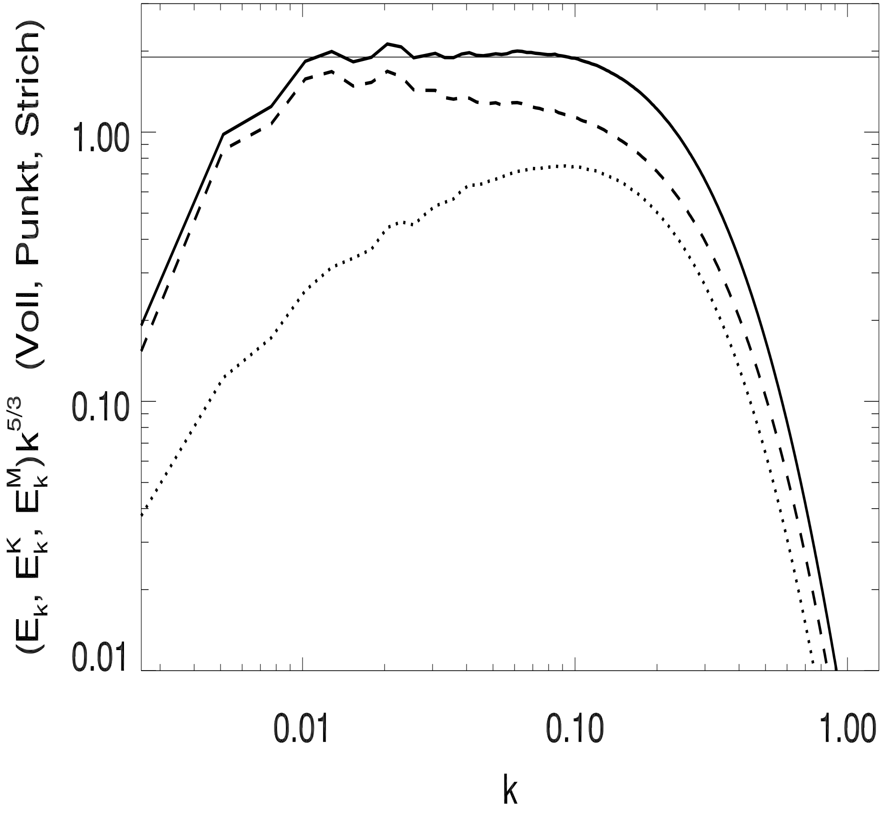
- intermittent and spontaneous emergence of coherent flows
- flows generate high local velocity gradients ($\longrightarrow$ **shear burst**)
- bursts show no preferred orientation, dissolve in turbulent environment
- no dependence on Prandtl number (like elevator mode)
- raise magnetic energy on time scales significantly longer than their lifetime

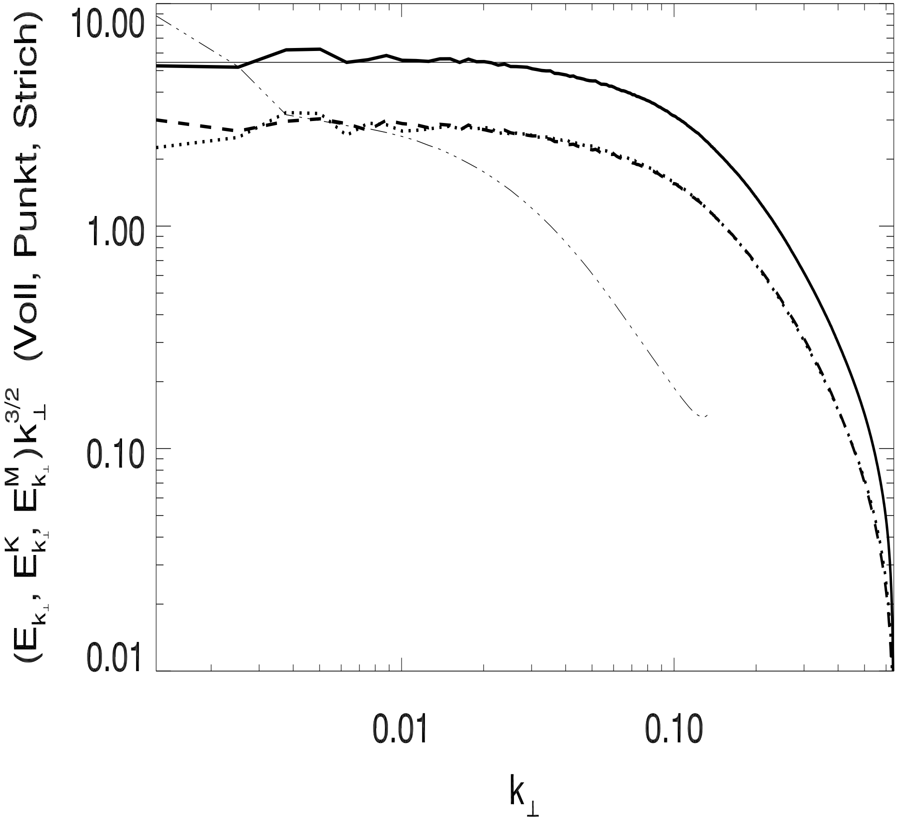
SB „Dynamo"



Pratt, Müller, A&A, 2013

Simulation Energy Spectra (MHD)





globally isotropic

anisotropic (strong mean magnetic field)