Hydrodynamics of 1D dissipative models with conserved momentum

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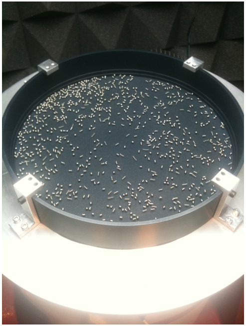
Outline

Lasanta, Manacorda, Puglisi & Prados, "Submitted"

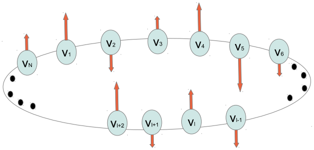
Why are dissipative system relevants?

- Many non equilibrium systems are characterized by an irreversible dissipation of energy (e.g. granular media) $\longrightarrow$ Ubiquitous



- Intrinsically out-of-equilibrium
- Injection of energy $\longrightarrow$ steady state
- Gradients controlled by dissipation (not by boundary conditions)

The Model



Dynamics (Markov chain)

- 1-D lattice of Maxwell molecules of granular matter with N sites
- At a given time p each site l possesses a velocity $v_{l,p}$
- In an elementary step a pair of next neigbours is chosen at random $(l, l+1)$ collides

$$v_{l,p+1} = v_{l,p} - \frac{1+\alpha}{2}(v_{l,p} - v_{l+1,p})$$

$$v_{l+1,p+1} = v_{l+1,p} + \frac{1+\alpha}{2}(v_{l,p} - v_{l+1,p})$$

- With $0 < \alpha \leq 1$

Fluctuating hydrodynamics description

- Momentum conservation $\rightarrow v_{l,p} + v_{l+1,p} = v_{l,p+1} + v_{l+1,p+1}$
- Energy is not conserved ($\alpha \neq 1$)
  $v_{l,p+1}^2 - v_{l,p}^2 + v_{l+1,p+1}^2 - v_{l+1,p}^2 = \frac{1}{2}(\alpha^2 - 1)(v_{l,p} - v_{l+1,p})^2 < 0$

Microscopic balance equations

- Momentum
  - Momentum Current $\rightarrow j_{l,p} = \delta_{y_p,l} \frac{1+\alpha}{2}(v_{l,p} - v_{l+1,p})$
  - Balance equation

$$v_{l,p+1} - v_{l,p} = -j_{l,p} + j_{l-1,p}$$

- Energy
  - Energy current $\rightarrow J_{l,p}^E = (v_{l,p} + v_{l+1,p}) j_{l,p}$
  - Dissipation $\rightarrow d_{l,p} = \frac{\alpha^2-1}{4}[\delta_{y_p,l}(v_{l,p} - v_{l+1,p})^2 + \delta_{y_p,l-1}(v_{l-1,p} - v_{l,p})^2]$
  - Balance equation

$$v_{l,p+1}^2 - v_{l,p}^2 = -J_{l,p}^E + J_{l-1,p}^E + d_{l,p}$$

Mesoscopic description

- Quasielastic limit $\alpha \sim 1$ *Local equilibrium approximation*
- Large system size $N \gg 1$
- Momentum Continuity equation

$$\partial_t v(x,t) = -\partial_x j(x,t),$$
$$j(x,t) = -\partial_x v(x,t) + \xi^j(x,t)$$

- Energy Continuity equation

$$\partial_t E(x,t) = -\nu(E(x,t) + v^2(x,t)) - \partial_x J^E(x,t),$$
$$J^E = -\partial_x E(x,t) + \xi^{J^E}(x,t).$$

- Dissipation noise term subdominant $O(L^{-3})$
- Noises are Gaussian and White
- Macroscopic dissipation coefficient

$$\nu = (1-\alpha^2)L^2$$

Homogeneus cooling state (HCS)

- Well studied case $\rightarrow u(x,t) = 0$ and $T_{HCS}(x,t) = T(t=0)e^{-\nu t}$

Stability of HCS

- Rescaled fields

$$U(x,t) = u(x,t)/\sqrt{T_{HCS}(t)},$$
$$\tilde{T} = T(x,t)/T_{HCS}(t),$$

- Stabilty analysis

$$\frac{\partial \delta U(k,t)}{\partial t} = \left(\frac{\nu}{2} - k^2\right)\delta U(k,t)$$

  - Critical value of the effective dissipation coefficient

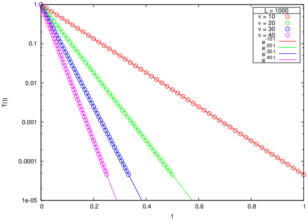
$$\nu < 8\pi^2$$

  - Equivalent to the critical length in the IHS model for granular matter

Comparison theory-simulation: averages

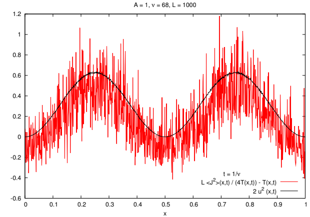
Temperature decay(Haff law)

$$T(t) = e^{-\nu t}$$



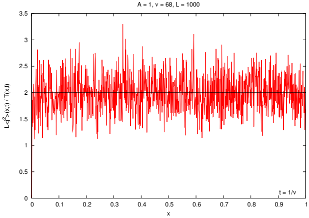
Energy current noise term amplitude

$$\langle \eta^2 \rangle \propto \frac{1}{L} 4T(T + u^2)$$



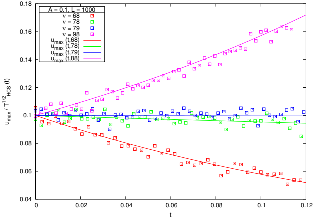
Moment current noise amplitude

$$\langle \xi^2 \rangle \propto \frac{1}{L} 2T$$



Instability

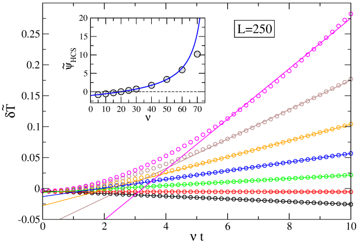
$$U(x, t) \sim e^{\frac{\nu - \nu_C}{2} t}$$



Comparison theory-simulation: Surprise!!!

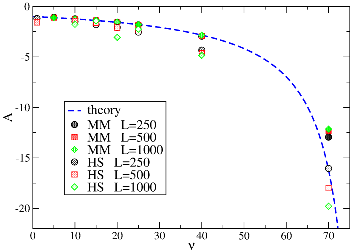
Correlations alter HCS decay

$$T(t) = T_{HCS}(t)\left[1 + \frac{1}{L}\tilde{\psi}_{HCS}\nu t + \mathcal{O}(L^{-2})\right]$$



Correlations

$$\tilde{D}(x) = -A\cos\left[\pi\sqrt{\frac{\nu}{\nu_C}}(1-2x)\right], \quad A = \frac{\pi\sqrt{\frac{\nu}{\nu_C}}}{\sin\left(\pi\sqrt{\frac{\nu}{\nu_C}}\right)}$$



Summary

Main conclusions

- Well-defined scheme based on
  - balance equations
  - a few transport coefficients
- The instability of HCS is recovered
- The spatial correlations are derived analytically
- The spatial correlation alter the temperature decay

Outlook

- Possibility to be a general result in dissipative systems?
- MFT and Large Deviations
- Other physical situations (driven systems)
  - Stochastic thermostat
  - Boundary thermostat
  - Uniform shear flow