



CNR

*Institute  
for Complex Systems*



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# Collective change of state and transport of information in biological groups

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# Collective behaviour in animal groups



movie by C. Carere - Starflag

## *Flocks*

Global order  
Scale free correlations - Collective turns

*Pnas 105 (2008), Pnas 107 (2010), Nature Phys 10 (2014), Jstat 2015*



movie by S. Melillo, SWARM

## *Swarms*

No global order  
Correlations – quasi critical behavior

*Plos Comp. Biol 10 (2014) . , Phys Rev Lett 113 (2014)*

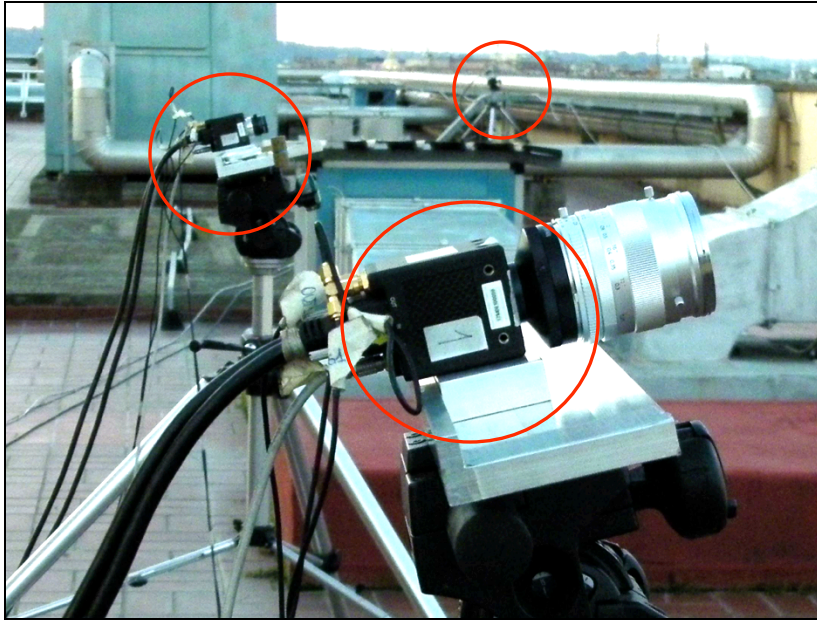
*Collective Response*

## collective turns in flocks of birds



quick collective change of state  
induced or spontaneous  
fast mechanism for information propagation

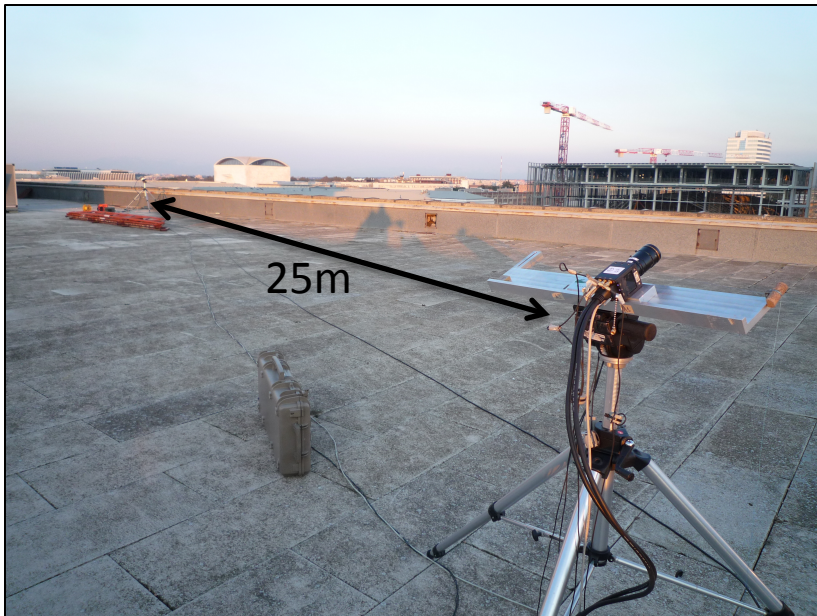
# stereo experiments



trifocal system

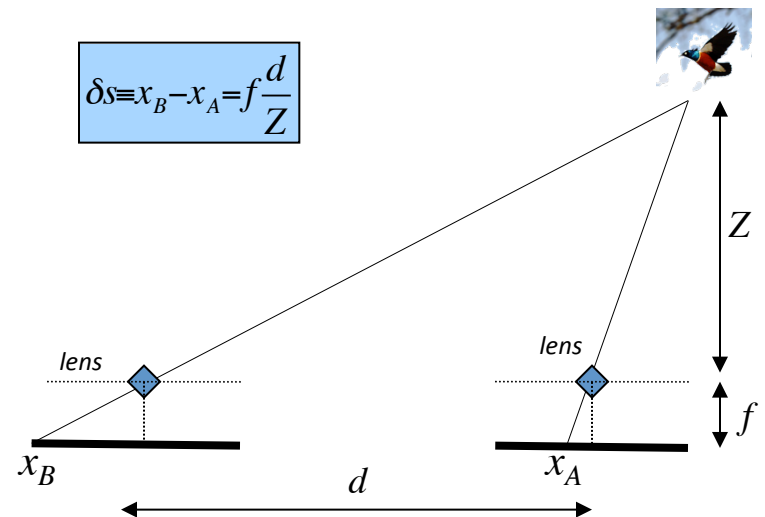


- IDT-Red Lake M5
- 4 Megapixel
- monochromatic
- 170 fps
- Schneider lenses



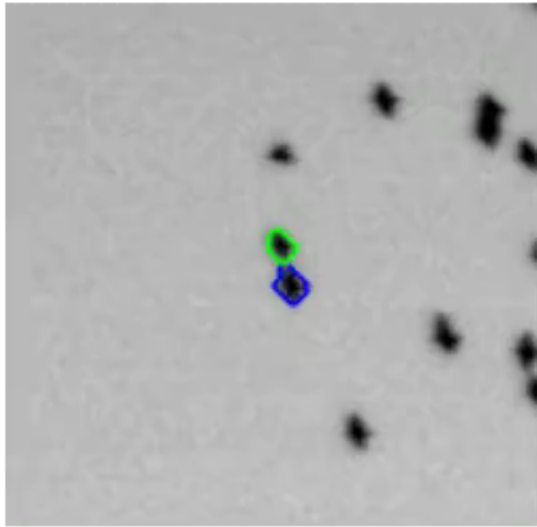
## Stereometry

$$\delta s \equiv x_B - x_A = f \frac{d}{Z}$$

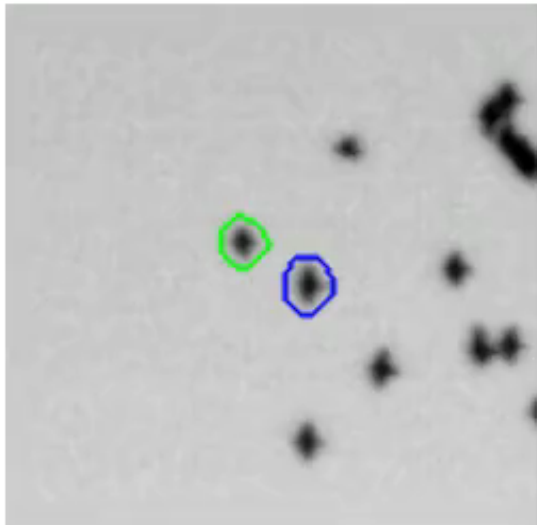
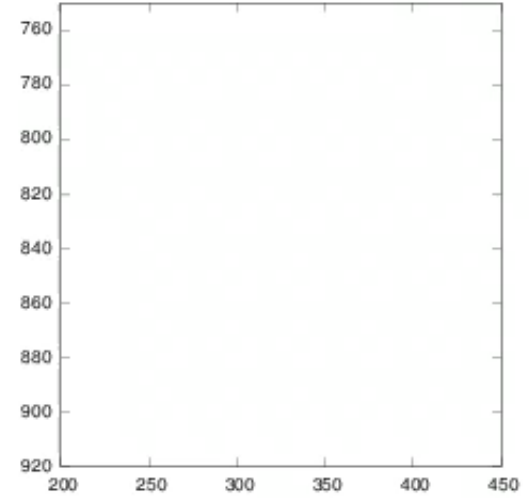


# the real enemy: blobs

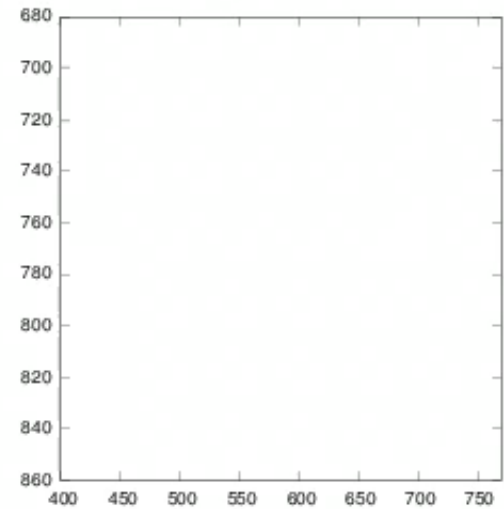
frame 1



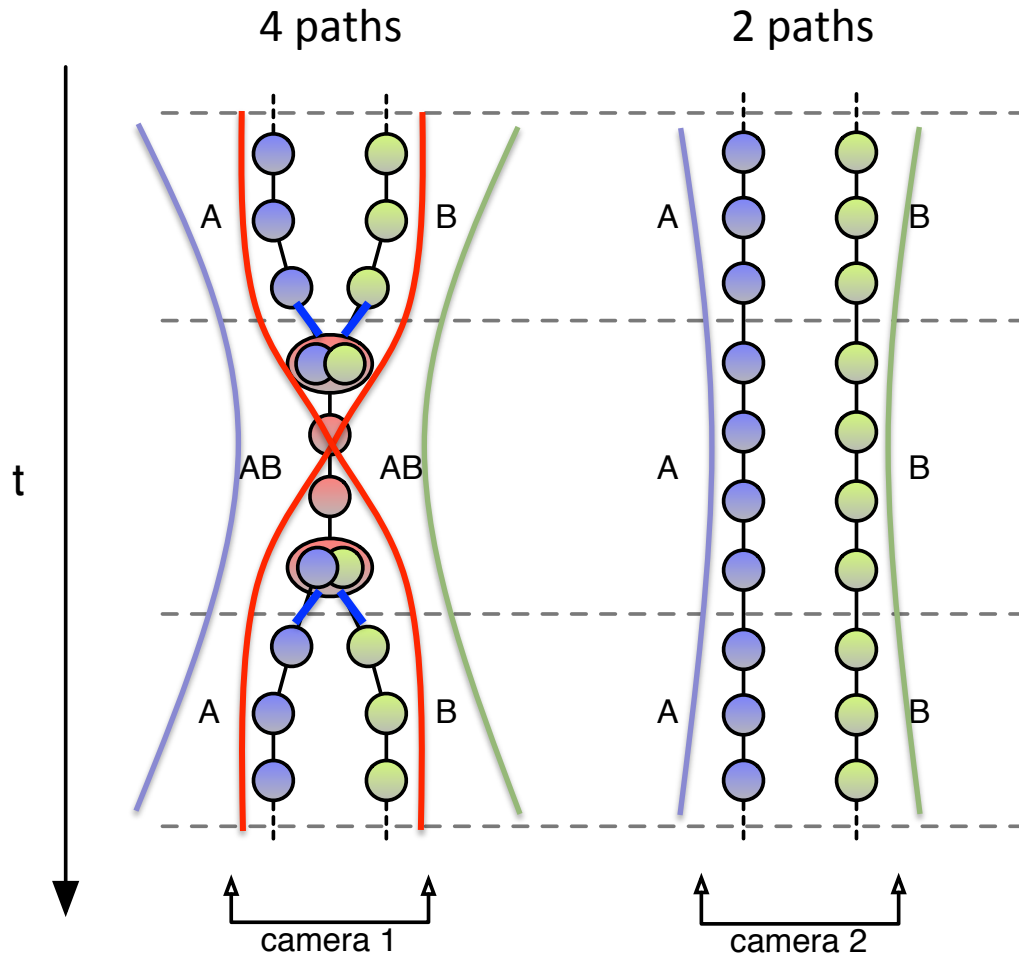
right camera



left camera



# create all paths



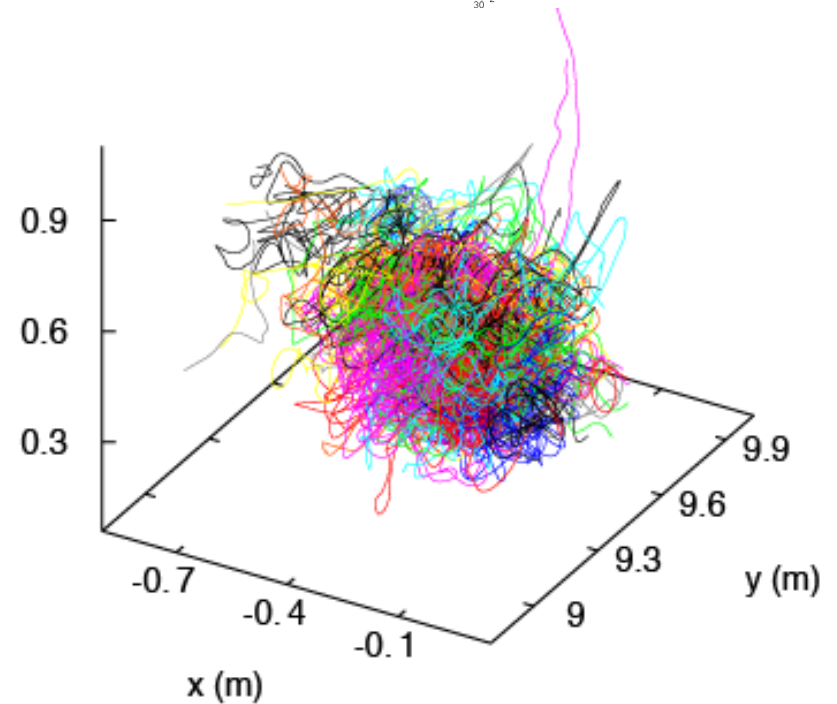
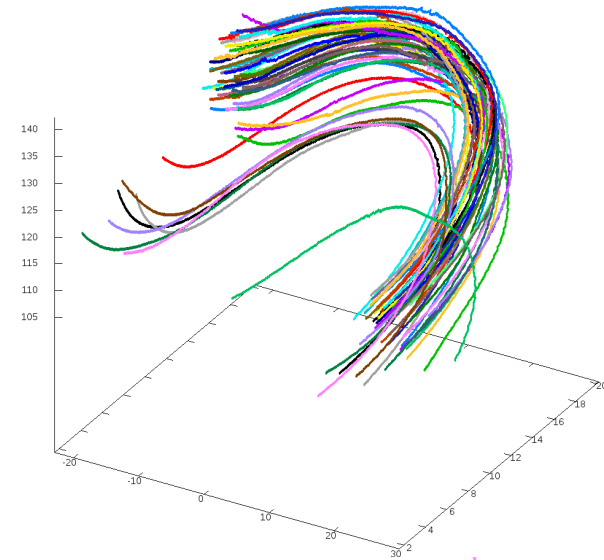
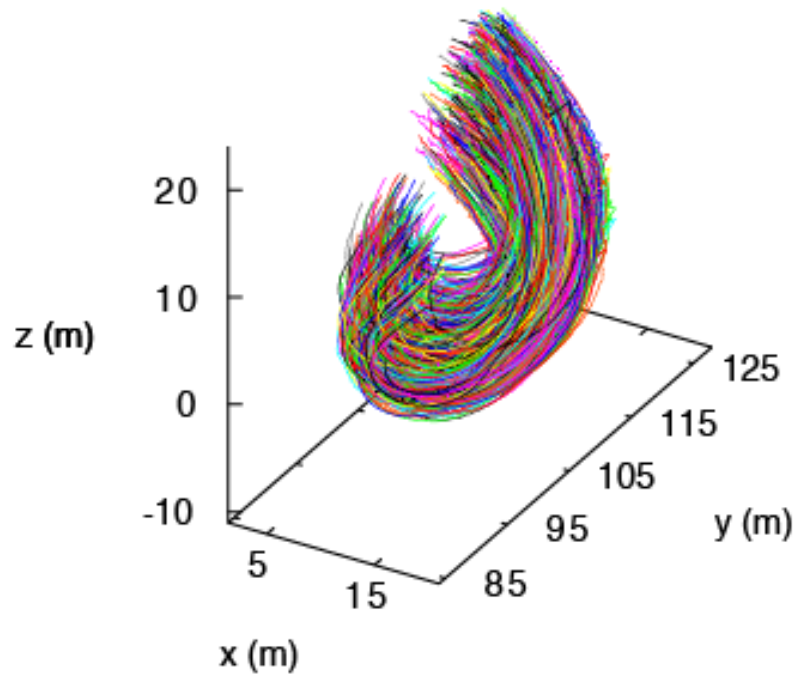
score matrix

	A	B
AA	10	4
AB	7	7
BA	7	7
BB	4	10



2 real birds

# global multi-path recursive algorithm



## basic questions about collective turns

- what is the origin of the turn?
  - spatially localized or extended?
  - endogenous or exogenous?
- how does the information spread across the flock?
  - what kind of propagation (dispersion) law?
  - damped or undamped propagation?

effective decision-making crucially depends on these last two issues



100

-5

0

5

10

15

0

5

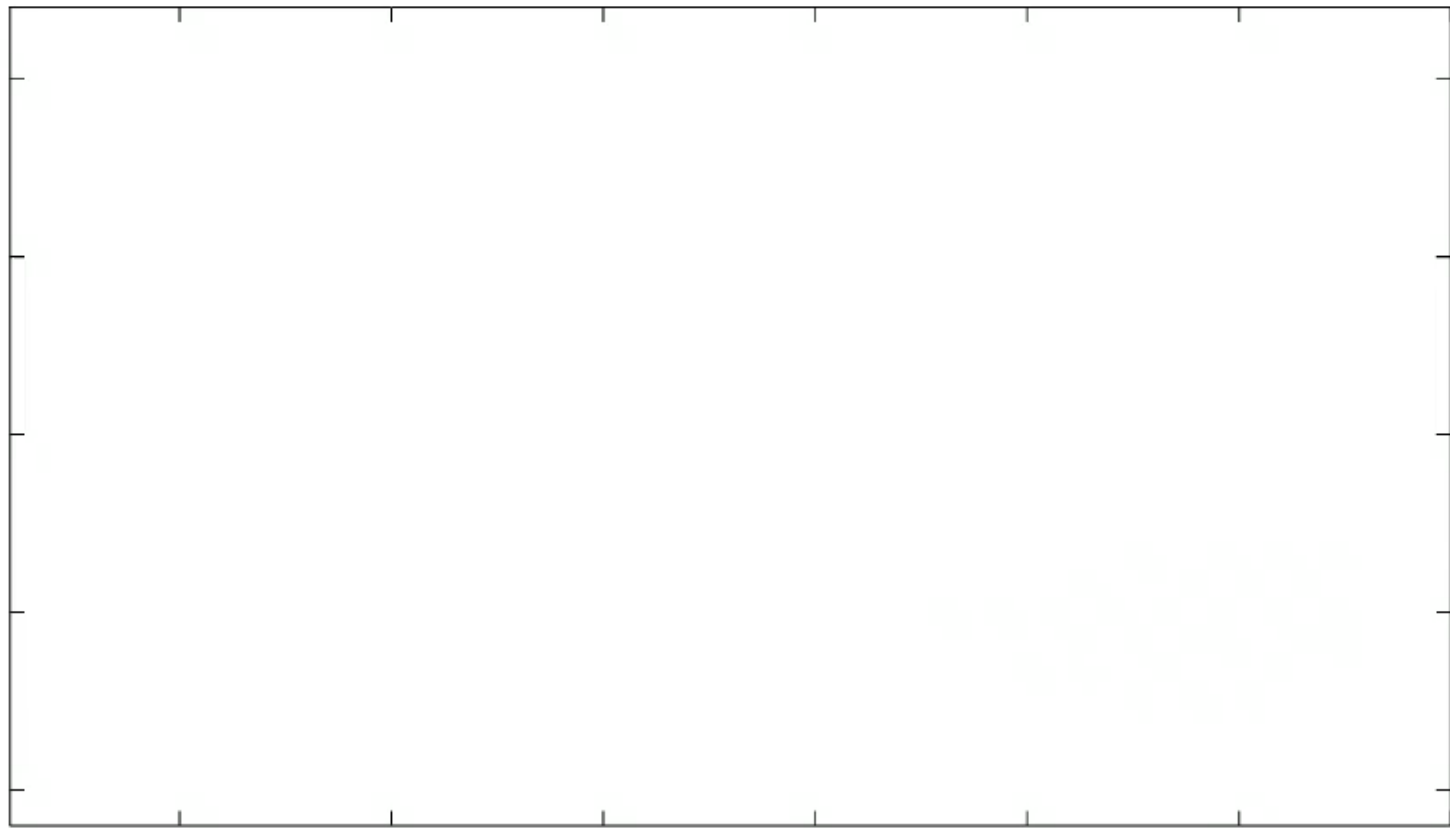
10

15

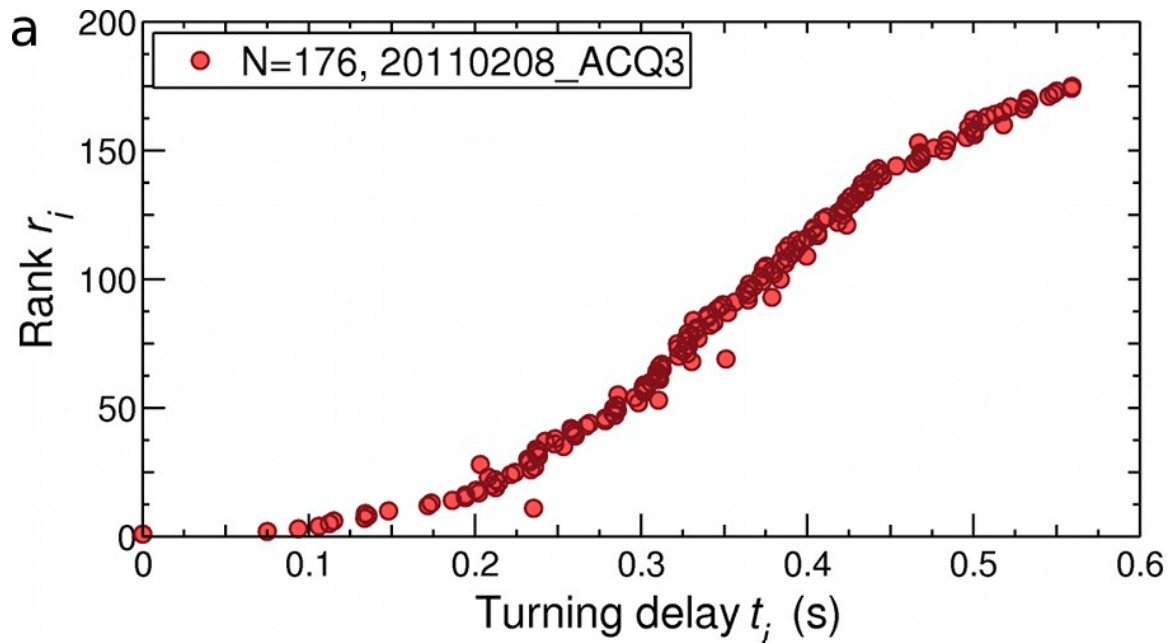
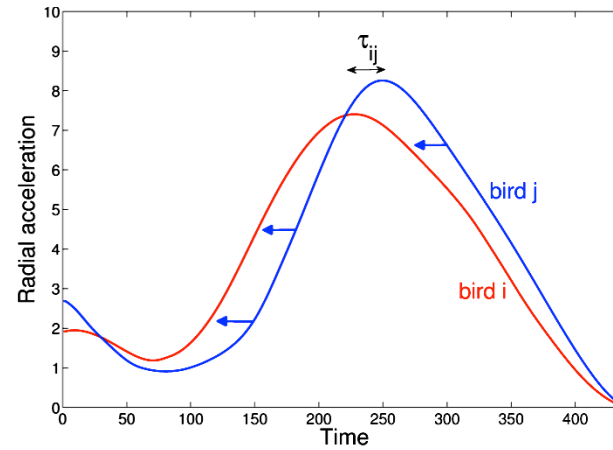
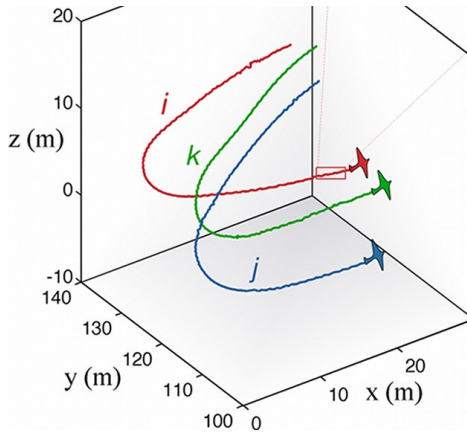
20

25

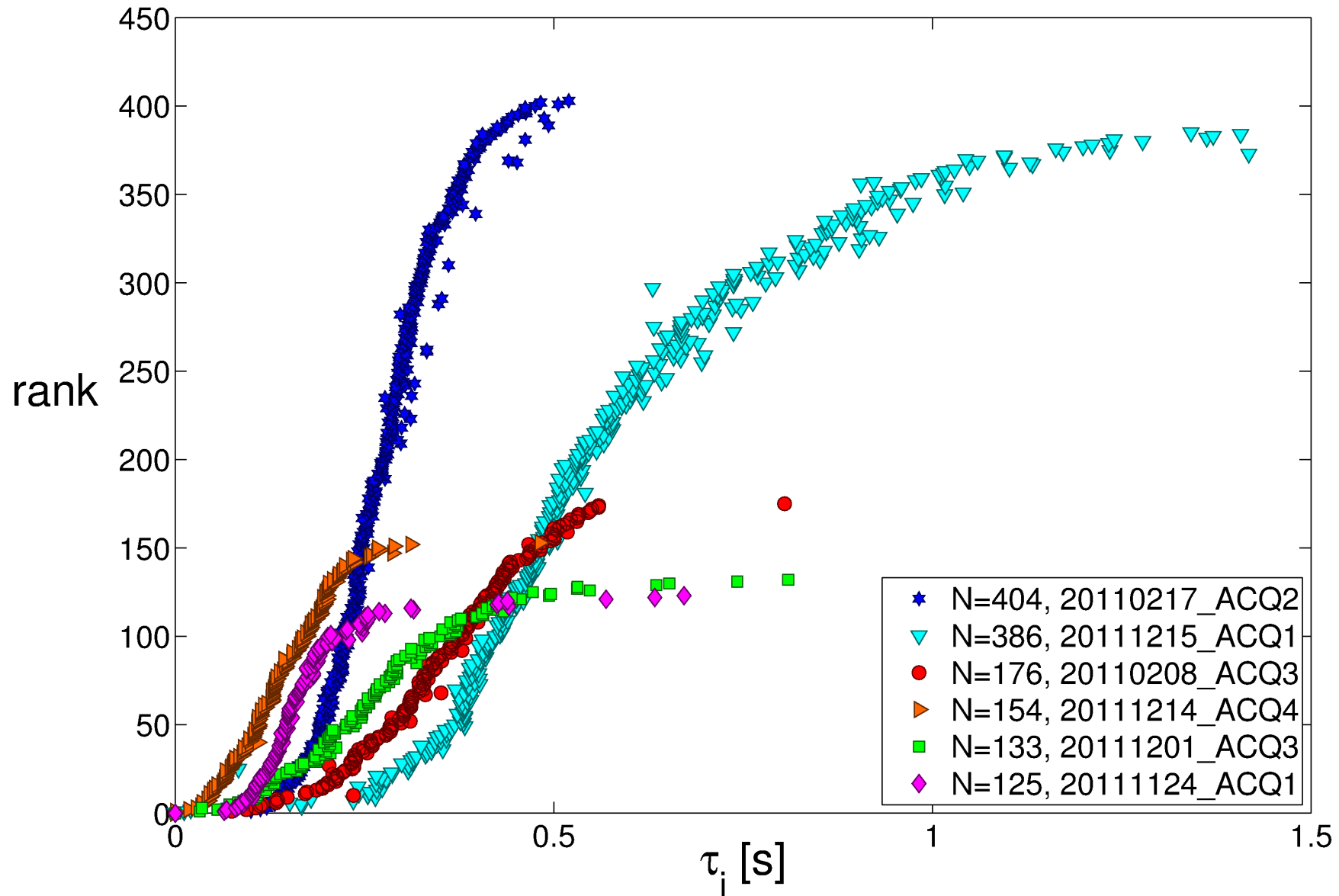
30



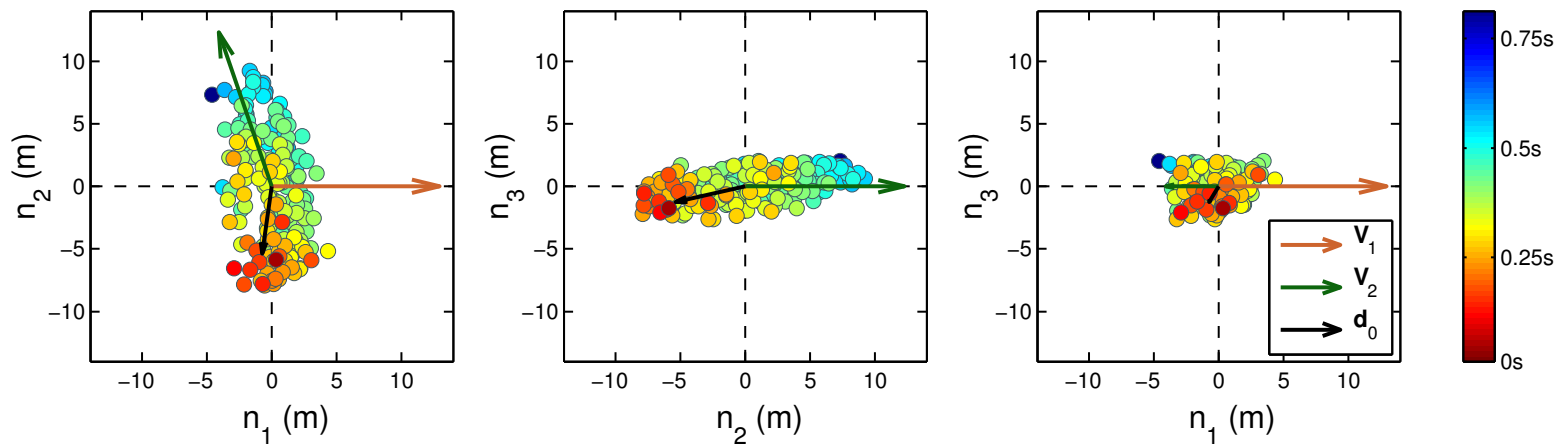
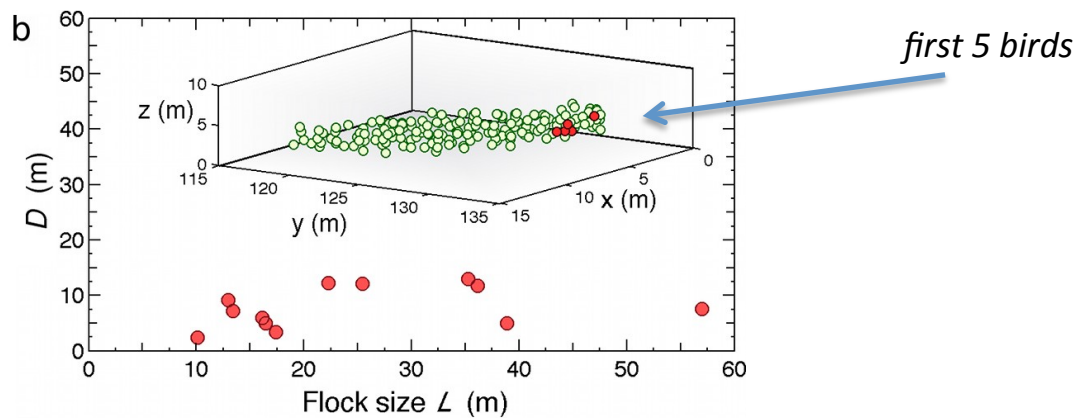
# mutual delay $\tau_{ij}$ and ranking



# ranking curve



# localized start of the turn

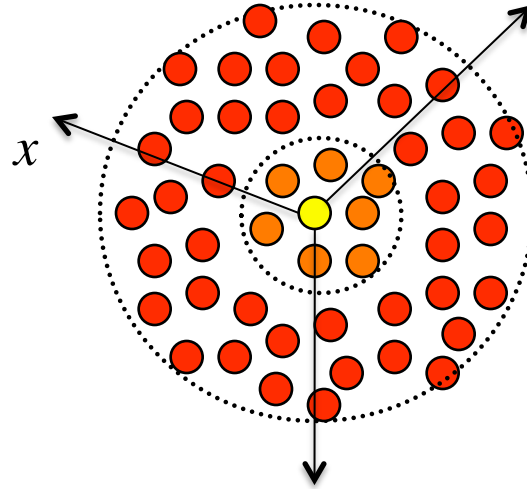


the turn starts **localized at the edges** and then it **propagates** across the flock

# ranking and propagation

if the turn starts localized then:

$$\text{rank} = (\text{density } \rho) \times (\text{space traveled by the turn } x)^3$$

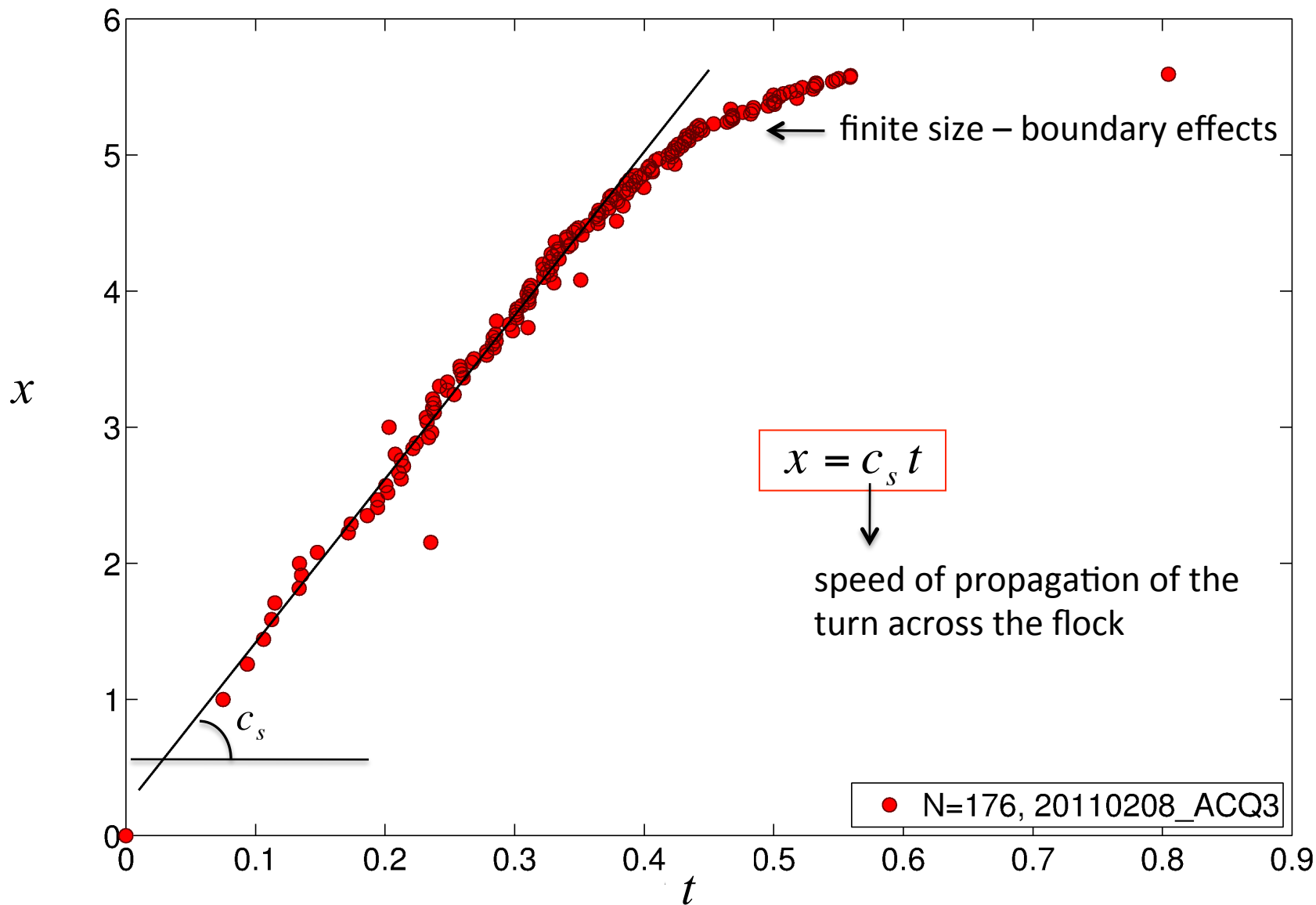


- rank: 1
- rank: 2-8
- rank: 9-38

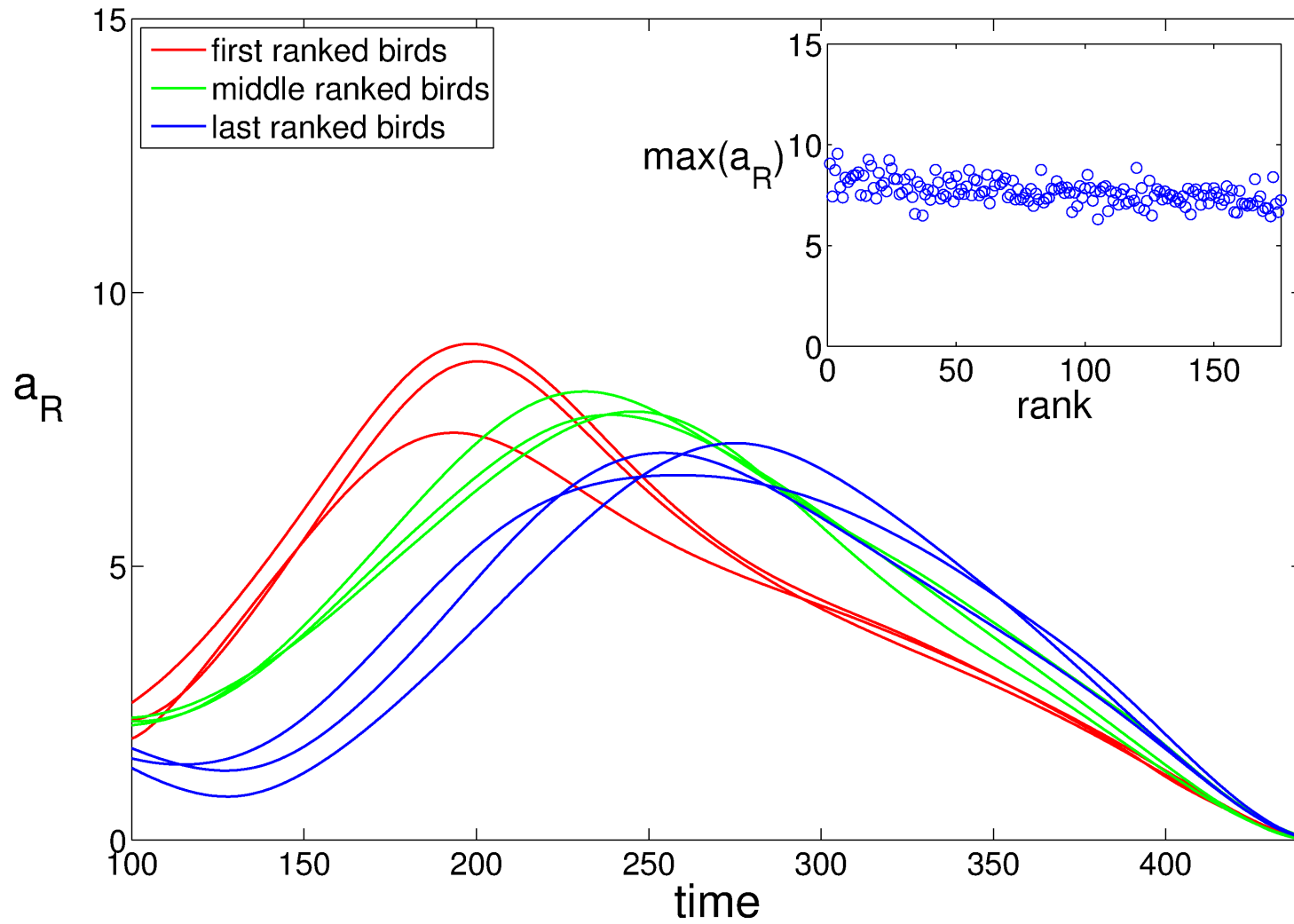
$$x(t) = \left[ \frac{r(t)}{\rho} \right]^{1/3}$$

# linear dispersion law

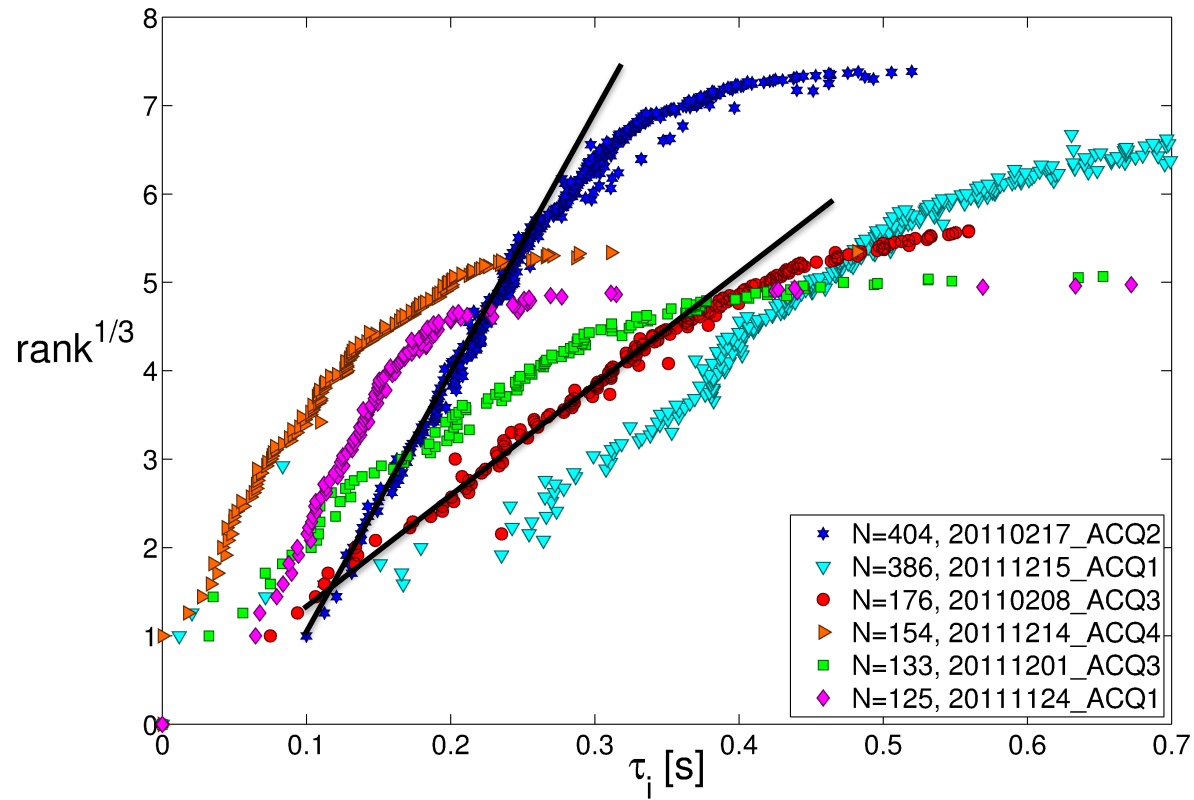
$$x \sim \text{rank}^{1/3}$$



# very weak attenuation



## flock-to-flock variability of $c_s$



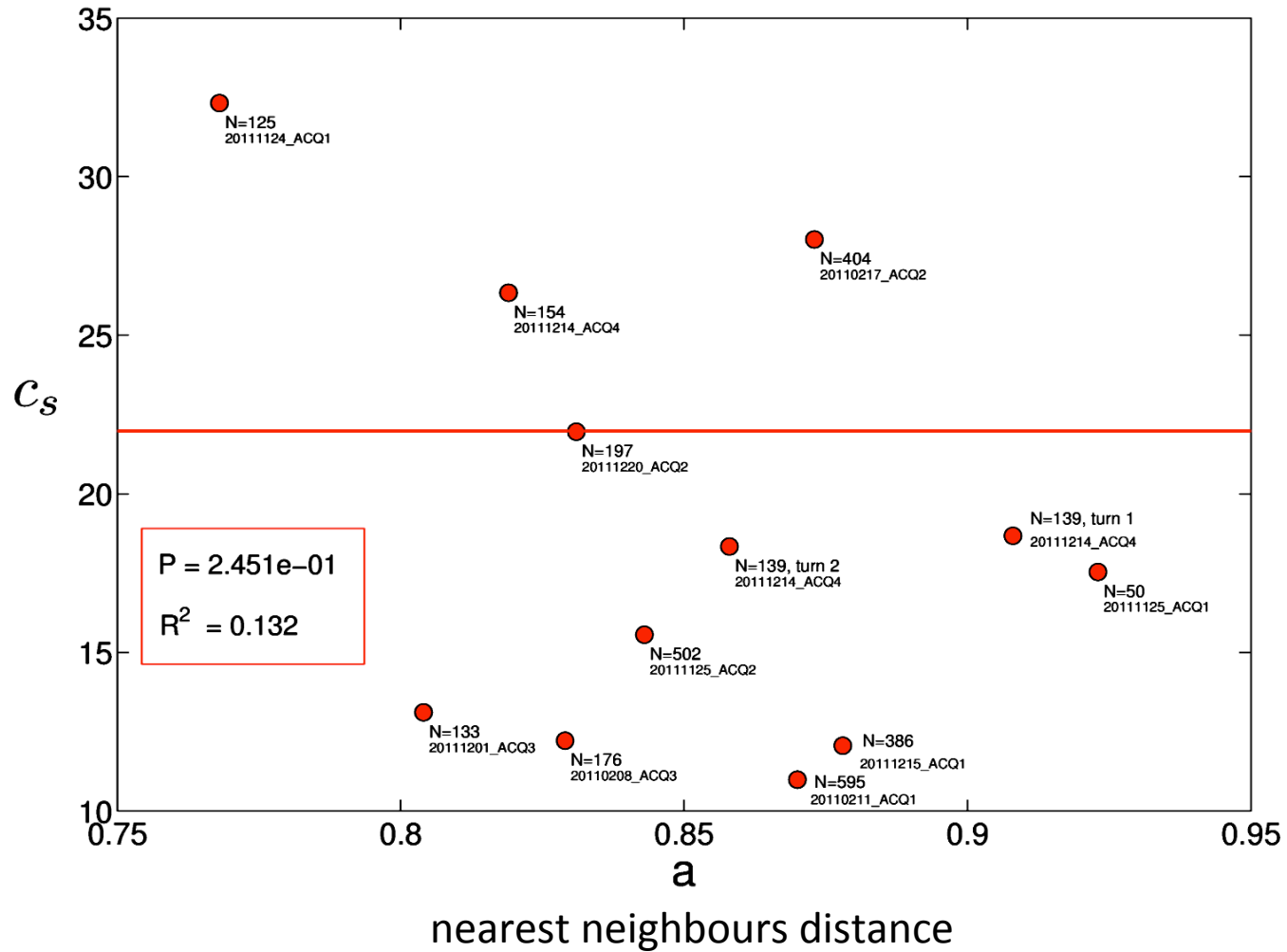
$c_s$  is  $\sim 4$  time larger than the birds speed

$c_s$  does NOT depend on density, nor on system size



# making sense of the variability of $c_s$

## attempt #1



# questions

*propagation*

- why a linear propagation law?

*orientation waves, not density waves*

- why a very weak attenuation?
- how to make sense of the variability of  $c_s$  ?

*start*

- why turns occur spontaneously ?
- what triggers the start of the turn ?
- why initiators are on the edges ?

# standard theory of flocking

$$\vec{v}_i(t+1) = \vec{v}_i(t) + J \sum_{k \in \mathcal{E}_i} \vec{v}_k(t) + \vec{\xi}_i$$

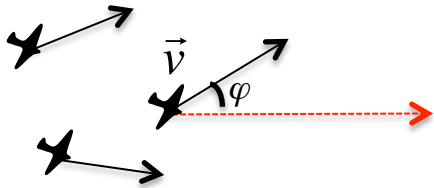
alignment force

typical flocking model (Vicsek model)

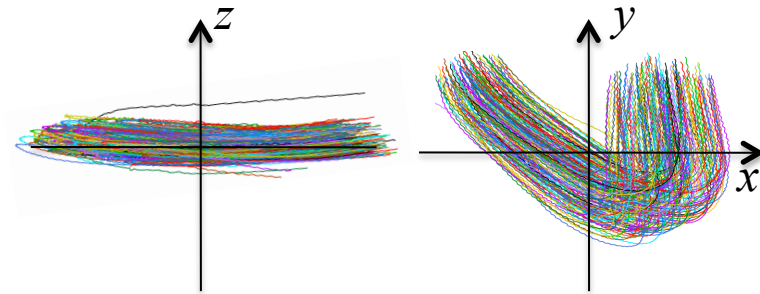
$$|\vec{v}_i| = v = \text{const}$$

active nature

- planar order parameter:



$$v_i^x + i v_i^y = v e^{i\varphi_i}$$



- high polarization:  $\varphi \sim 0$

$$\frac{\partial \varphi}{\partial t} = J \nabla^2 \varphi + \xi$$

alignment



$$\omega = iJk^2$$


$$x \sim \sqrt{t}$$

• damping **×**

• diffusive propagation **×**

# What is missing ?

$$\frac{\partial \varphi}{\partial t} = J \nabla^2 \varphi + \xi = -\frac{\delta H}{\delta \varphi} + \xi$$

  
force

The force acts on the velocity  
NO rotational inertia - overdamping

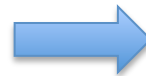
*but turns are smooth*

There is a global continuous symmetry (rotation of velocities),  
it has strong consequences on correlations *Cavagna et al. Pnas 107 (2010)*

Implications on the dispersion law ?

*direction AND curvature propagate in turns – turns occur on the short scales*

Hamiltonian structure of equations  
Inertia + global continuous symmetry  
CONSERVATION LAW



$$\left\{ \begin{array}{l} \frac{\partial \varphi}{\partial t} = \frac{1}{\chi} s \\ \frac{\partial s}{\partial t} = Ja^2 \nabla^2 \varphi \end{array} \right.$$

# Hamiltonian description ?

Active nature of individuals  $|\vec{v}_i| = v = \text{const}$

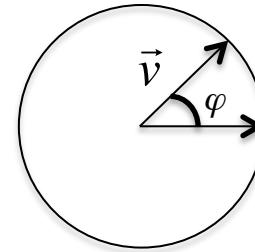
*not Hamiltonian in  $(\vec{r}_i, \vec{v}_i)$  !!!!*

$\vec{v}_i = v e^{i\varphi_i}$  + rotational symmetry  $\rightarrow$

*Hamiltonian in  $(\varphi_i, s_i)$*

$$H = \int \frac{d^3x}{a^3} \left[ \frac{1}{2} J a^2 (\vec{\nabla} \varphi)^2 + \frac{s^2}{2\chi} \right]$$

*Rotational inertia*



$\varphi$  Parameterizes rotations of  $\vec{v}$

$s$  Generates rotations (**SPIN**)

*Like circular motion BUT in internal space*

canonical equations:

$$\begin{cases} \frac{\partial \varphi}{\partial t} = \frac{\delta H}{\delta s} = \frac{1}{\chi} s \\ \frac{\partial s}{\partial t} = -\frac{\delta H}{\delta \varphi} = J a^2 \nabla^2 \varphi \end{cases}$$

conservation law:

$$\frac{\partial s}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0 \quad \text{with: } \vec{j} = -J a^2 \vec{\nabla} \varphi$$

$s \sim$  curvature

*Excess curvature cannot be dissipated but propagate !*

# Predictions

Model F (Hohenberg-Halperin 1977)

planar ferromagnet

lattice model for superfluid He2

$$\left\{ \begin{array}{l} \frac{\partial \varphi}{\partial t} = \frac{1}{\chi} s \\ \frac{\partial s}{\partial t} = Ja^2 \nabla^2 \varphi \end{array} \right. \quad \rightarrow \quad \frac{\partial^2 \varphi}{\partial t^2} - \frac{Ja^2}{\chi} \nabla^2 \varphi = 0$$



linear dispersion law:

$$\omega = c_s k$$

$$x = c_s t$$



speed of propagation:  $c_s = \sqrt{\frac{Ja^2}{\chi}}$

the coupling  $J$  can be measured through the polarization  $\Phi$  :

$$J = \frac{\varepsilon}{1 - \Phi}$$

$$\Phi = \left\| \frac{1}{N} \sum_i \frac{\vec{v}_i}{\|\vec{v}_i\|} \right\|$$

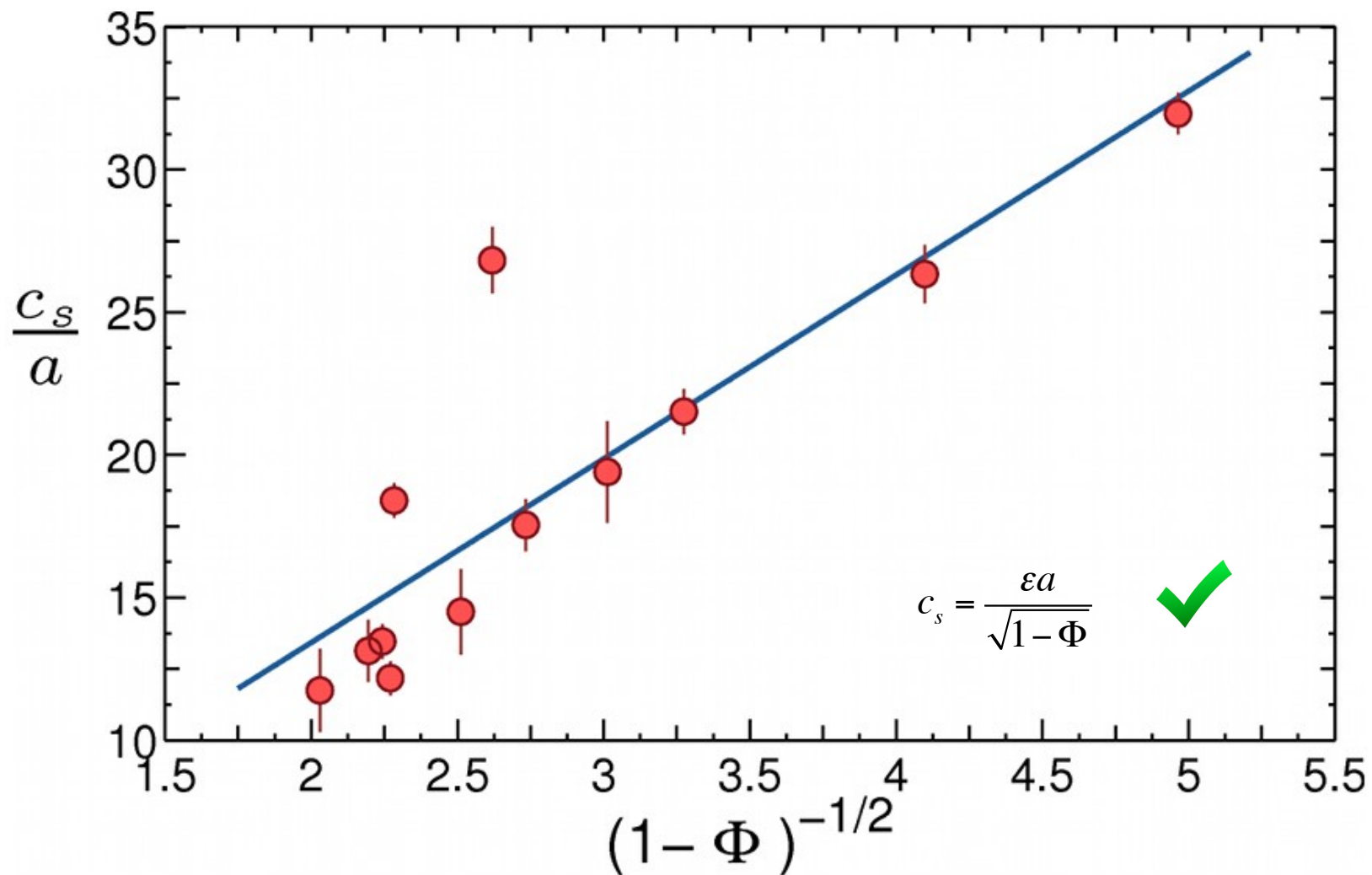
$\Phi$  is experimentally accessible



$$c_s = \frac{a\varepsilon}{\sqrt{1 - \Phi}}$$

*the speed of propagation of the turn across the flock must be larger in more ordered flocks*

# experimental test of the theory



# why turns occur spontaneously

standard Heisenberg  
on a lattice

$$\tau^{rel} \sim L^{d-2}$$

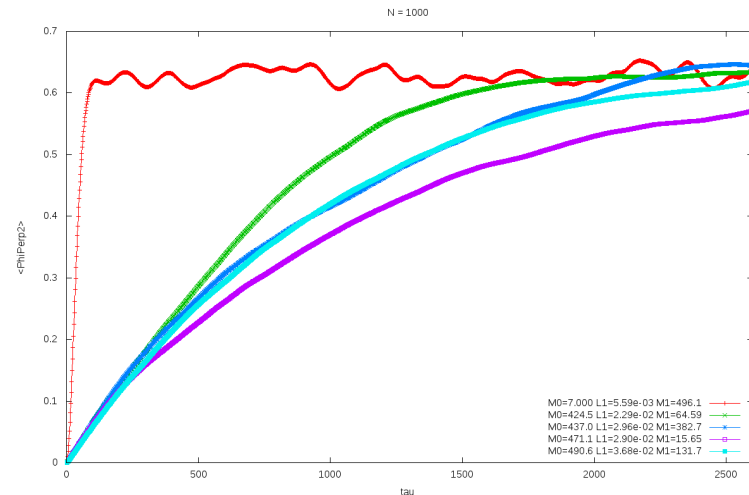
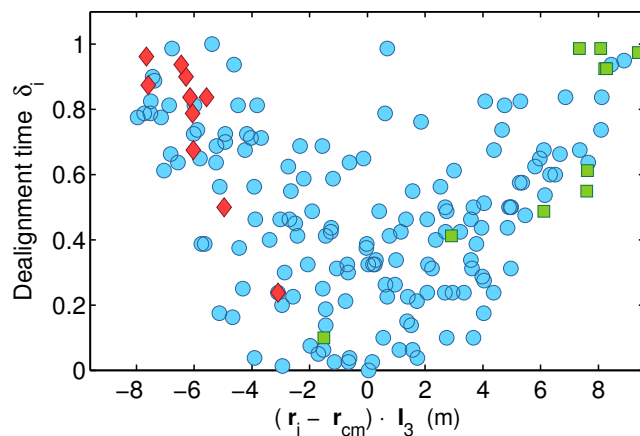
the system changes  
global state on large scales

what is different  
in flocks ?

the network is random  
interactions are NOT symmetric



peripheral clusters  
more sensitive  
to noise





## conclusions

- ◆ high order in the group (large stiffness) grants a more efficient propagation of information

*impact on collective dynamical response*

- ◆ current dynamical models of flocking lack an essential term
- ◆ a new model including inertial terms and conservation laws is able to explain all known features of flocking

*new dynamical properties of the ordered phase*

- ◆ non symmetric random interaction network + inertial dynamics can produce spontaneous changes of collective state on short scales

# COBBS group



*Massimiliano Viale  
Stefania Melillo  
Alessandro Attanasi  
Ed Shen  
Lorenzo Del Castello  
Asja Jelic  
Oliver Pohl  
Edmondo Silvestri  
Leonardo Parisi  
Agnese D'Orazio*

*and... the Red Van*



Andrea Cavagna



Irene Giardina



Dasyhelea flavirons



# The inertial spin model

from phases to velocities:

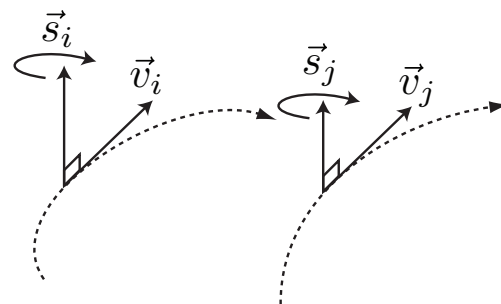
$$\begin{aligned} \frac{d\vec{v}_i}{dt} &= \frac{1}{\chi} \vec{s}_i \times \vec{v}_i \\ \frac{d\vec{s}_i}{dt} &= \vec{v}_i \times \left[ J \sum_{j \sim i} \vec{v}_j - \eta \frac{d\vec{v}_i}{dt} + \vec{\xi}_i \right] \\ \frac{d\vec{r}_i}{dt} &= \vec{v}_i \end{aligned}$$

rotational  
dissipation

noise

$$\langle \vec{\xi}_i(t) \cdot \vec{\xi}_j(t') \rangle = 2d\eta T \delta_{ij} \delta(t-t')$$

model G



$$\chi \frac{d^2 \vec{v}_i}{dt^2} + \chi \left( \frac{d\vec{v}_i}{dt} \right)^2 + \eta \frac{d\vec{v}_i}{dt} = J \left( \sum_{j \sim i} \vec{v}_j \right)^\perp + \vec{\xi}_i^\perp$$

inertial term

centripetal  
term

dissipation

social force

noise

the Vicsek model is  
recovered in the  
overdamped limit

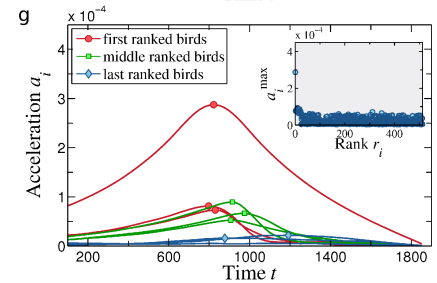
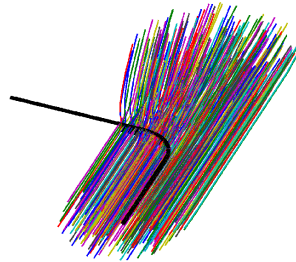
# General dispersion relations

$$\omega = i \frac{\eta}{\chi} + c_s k \sqrt{1 - \frac{k_0^2}{k^2}}$$

$$k_0 = \frac{\eta}{2\sqrt{J a^2 \chi}}$$

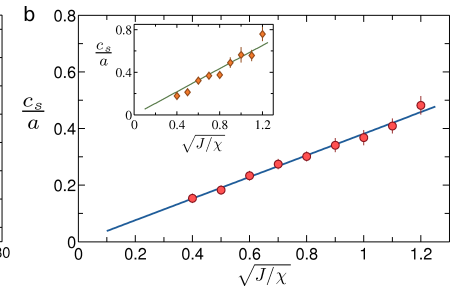
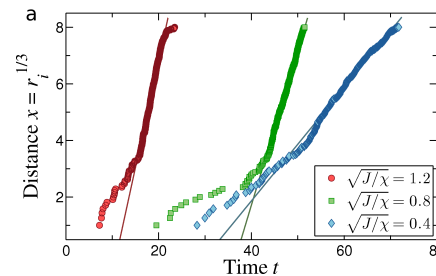
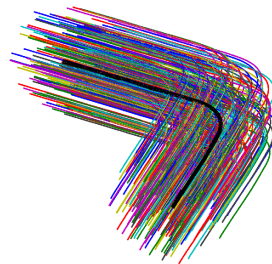
- overdamped regime

$$k_0 = \frac{\eta}{2\sqrt{J a^2 \chi}} \gg \frac{1}{L}$$



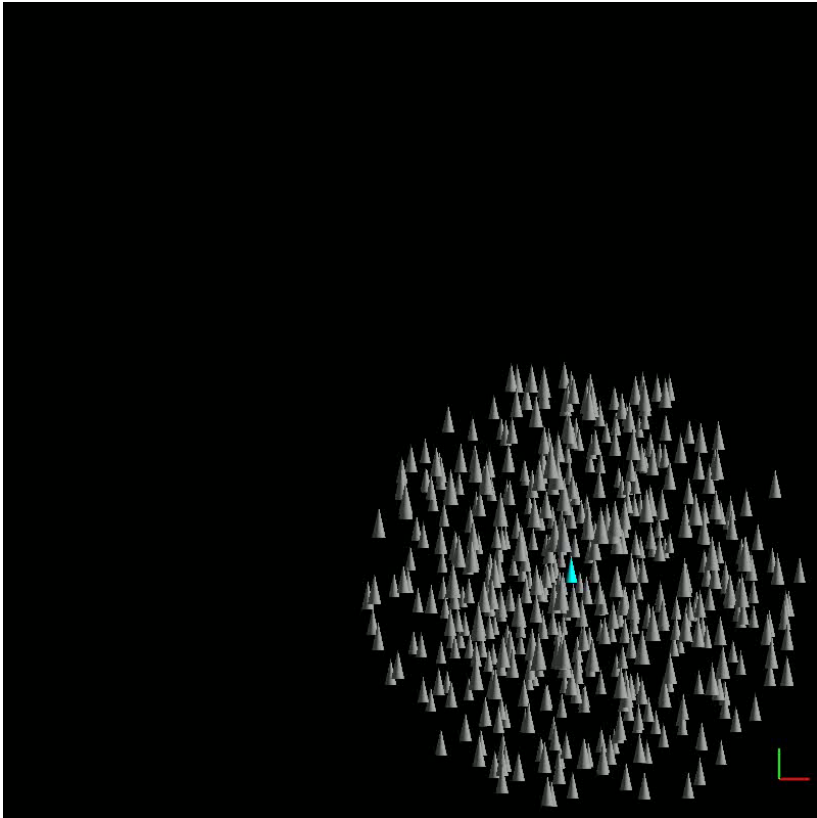
- underdamped regime

$$k_0 = \frac{\eta}{2\sqrt{J a^2 \chi}} \ll \frac{1}{L}$$



some small dissipation does NOT affect the linear propagation  
Birds are in the underdamped regime

overdamped



underdamped

