Lagrangian time irreversibility at a glance.

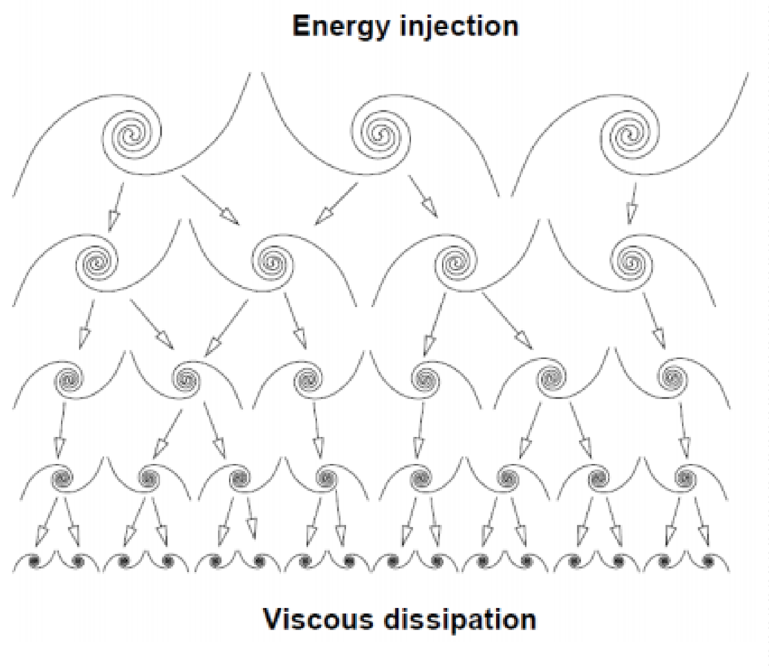
ANNA FRISHMAN
GREGORY FALKOVICH

Weizmann Institute of Science, Israel

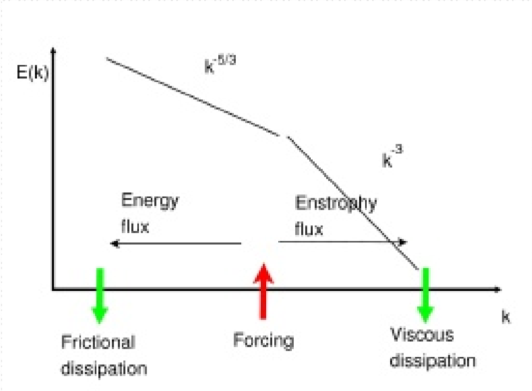
Time irreversibility in turbulence

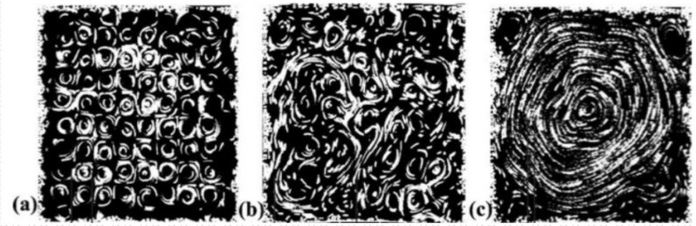
Eulerian Framework

3d: Kolmogorov



2d: Kraichnan





Shats, Xia, Punzmann, Phys. Rev. E, 71, 046409, (2005)

Time irreversibility in turbulence

In the inertial range of scales:

3d: $\langle u_l{}^3 \rangle = -\frac{4}{5}|\epsilon| R_0.$ 2d: $\langle u_l{}^3 \rangle = \frac{3}{2}|\epsilon| R_0.$

Where ϵ is the kinetic energy flux, positive in 3D and negative in 2D.

$\vec{R}_0$ - The separation between two points.

$\vec{u}$ - The velocity difference between the points, $u_l = \vec{u} \cdot \vec{R} / R.$

Lagrangian framework: Tracers in a flow

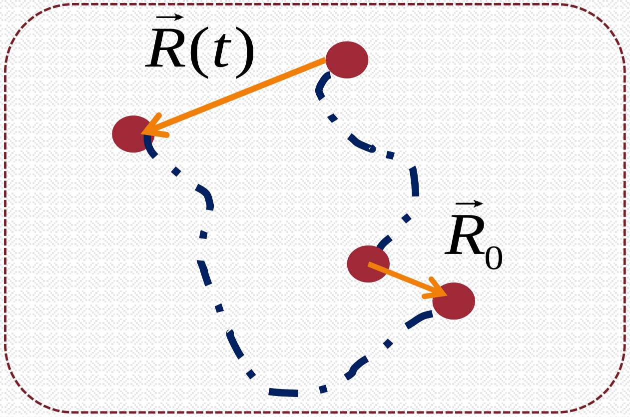
- Small fluid parcels that can be treated as point particles.
- They move in space according to the velocity field of the flow.
- An example is small, neutrally buoyant particles.

Lagrangian time irreversibility

What is the footprint of time irreversibility of turbulence on the dynamics of pairs of tracers in the flow?

A pair of tracers -notations

The separation between the tracers $\vec{R}(t) = \vec{r}_2(t) - \vec{r}_1(t)$ is initially fixed to $\vec{R}_0$.



Denote their velocity difference $\vec{u}(t) = \vec{v}(t, r_1 + R) - \vec{v}(t, r_1)$.

Time irreversibility for pairs at short times

In the inertial range:

The 4/5 and 3/2 laws can be written as $\left\langle \frac{du^2}{dt} \right\rangle_{t=0} = -4\epsilon$

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Pair dispersion: Ballisitics

$$\vec{R}(t) = \vec{R}_0 + \vec{u}(\vec{R}_0, 0)t.$$

Does ballistics completely determine short time evolution, hiding time irreversibility?

Irreversibility in the inertial range

For an incompressible, statistically isotropic, translational invariant and stationary flow.

The Lagrangian manifestation of the Kolmogorov 4/5 law,

3d: $$\left\langle \left[\frac{R_0}{R(t)} \right]^{5/3} \right\rangle - 1 = \frac{14\epsilon t^3}{81 R_0^2} + O(t^4)$$

2d: $$\left\langle \left[\frac{R_0}{R(t)} \right]^{2/3} \right\rangle - 1 = \frac{2\epsilon t^3}{27 R_0^2} + O(t^4)$$

Valid for short times-accessible to experiments.

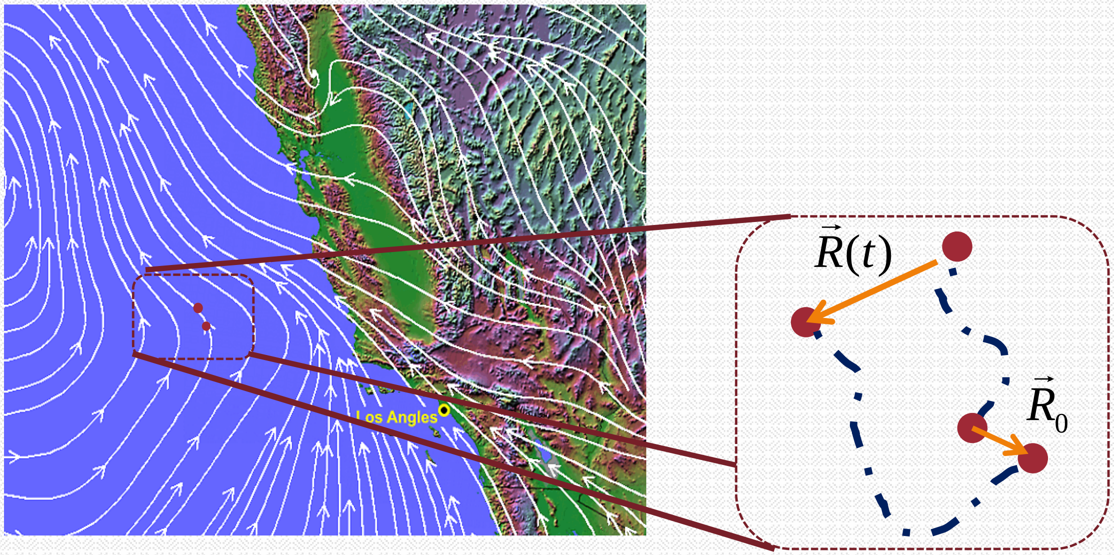
Smooth flow

For separations smaller than the typical scale of spatial velocity change

$$\vec{u}(t) = \sigma(t)\vec{R}(t)$$

$$\sigma_{ij}(t) = \nabla_j v_i(t)$$

A random matrix.



Such a linear scaling is present also in 2d direct cascade.

Smooth flow-a conservation law

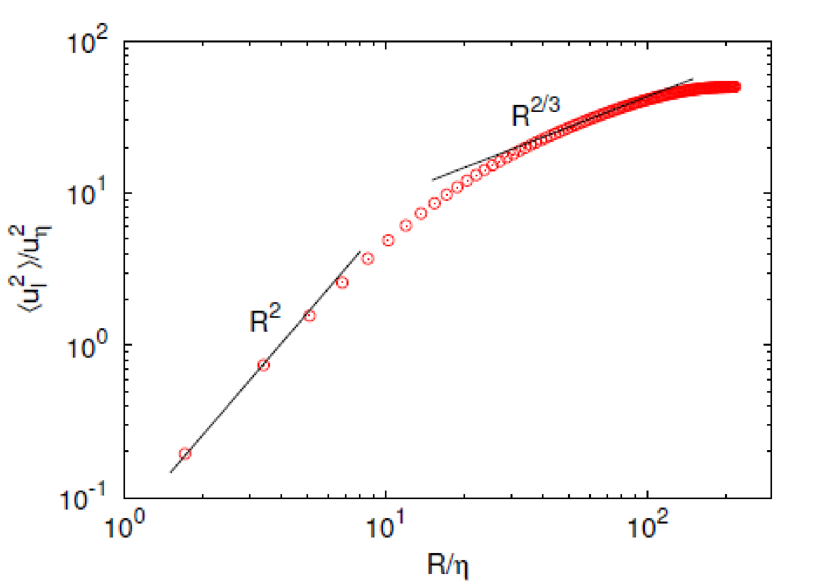
Instead of time irreversible Lagrangian relations, a conservation law:

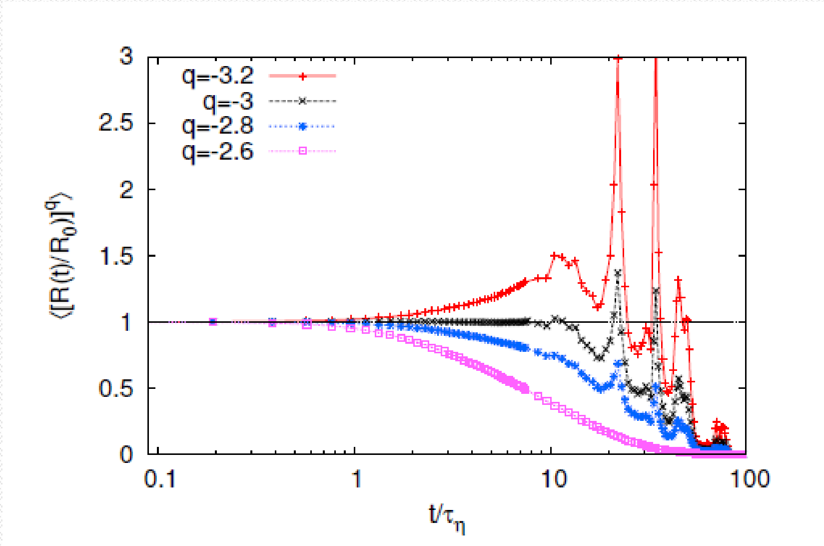
$$\left\langle R^{-d}(t) \right\rangle = R_0^{\ -d}$$

Conserved even for a non stationary flow!

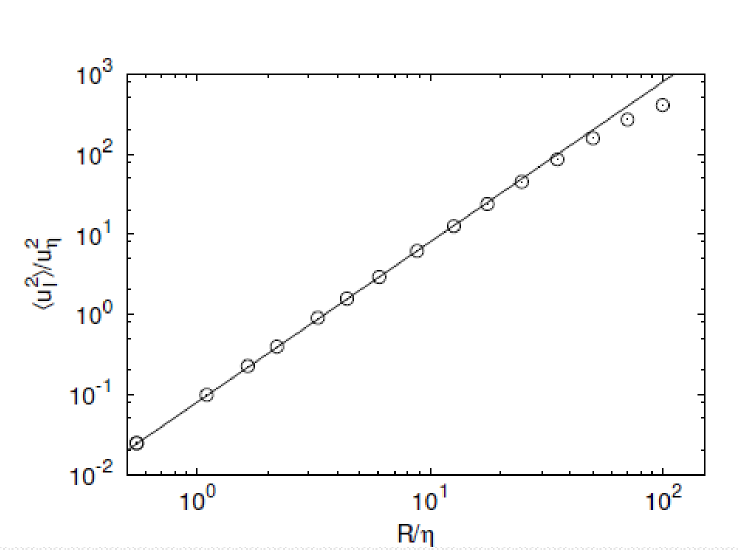
Useful as a check of isotropy/dimensionality/compressibility of the flow.

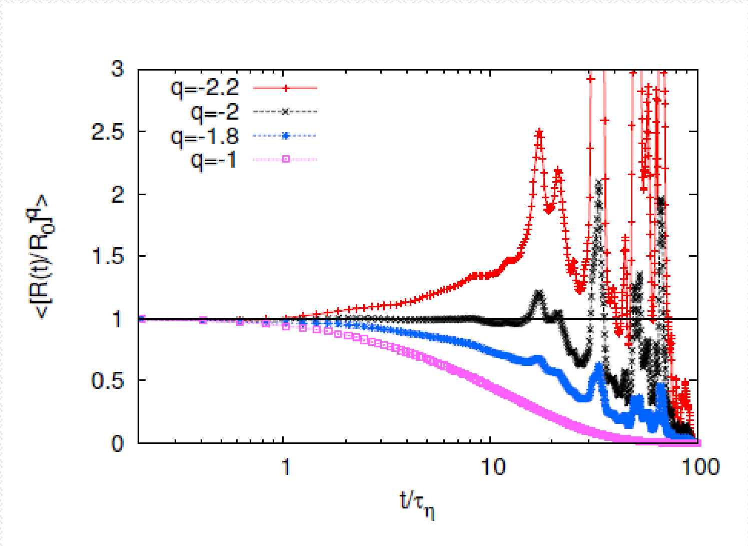
3D





2D



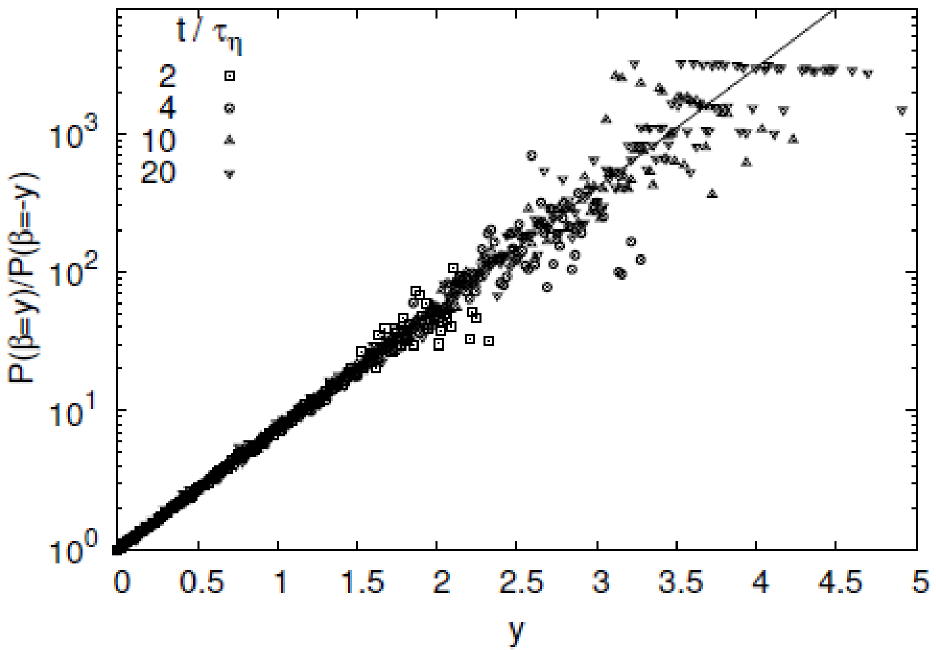


AF ., Boffetta, G., De Lillo, F., & Liberzon, A. (2015) PRE (*arXiv:1501.02570*)

Insensitivity to time irreversibility in 2d

$$\beta = \ln\left(\frac{R(t)}{R_0}\right)$$

$$\frac{P(\beta = y)}{P(\beta = -y)} = e^{2y}$$



At long times looks like an Evans-Cohen-Morriss/ Gallavotti-Cohen type fluctuation relation

Time irreversibility in compressible flows

- 1d Burgers equation as a model.
- Consider pairs of tracers, chosen from an initial homogeneous distribution in space.
- There is a 4/5 law for Burgers, and its Lagrangian version is again

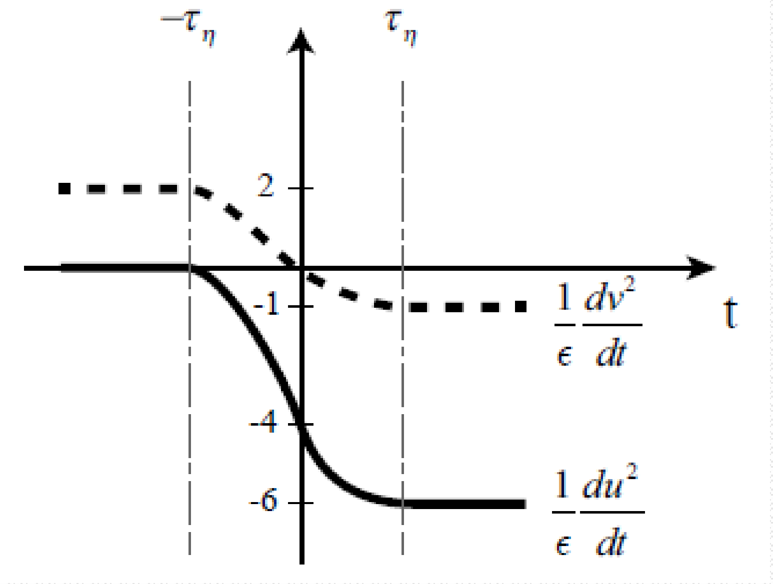
$$\left\langle \frac{du^2}{dt} \right\rangle_{t=0} = -4\epsilon$$

Time irreversibility in compressible flows

- In Burgers turbulence the mechanism for energy dissipation are shocks.
- The distribution of tracers doesn't remain homogeneous, they cluster in shocks.
- The Lagrangian flux law changes during the viscous time scale τ_η.

Time irreversibility in compressible flows

The inviscid dynamics contains a jump at t=0.



This does not occur in incompressible flows.

AF, GF. *Phys. Rev. lett*. 113.2 (2014): 024501.

Summary

- In the inertial range the time irreversibility of pair dispersion, related to 4/5 law, can be seen at short times.
- For smooth flows a conservation law exists instead. In 2d it is extended to a symmetry relation for the pdf, insensitive to time reversal.
- For Burgers turbulence, a redistribution of tracers during a short time causes a new kind of time asymmetry.

Thank you for your attention!

More details:

Incompressible flow

GF, AF (2013) *Phys. Rev. Lett*. 110:214502

AF ., Boffetta, G., De Lillo, F., & Liberzon, A. (2015) *arXiv:1501.02570*.

Burgers

AF, GF. *Phys. Rev. lett*. 113.2 (2014): 024501.