Fields in TURBULENCE

Part II: Flight-crash events and irreversibility of turbulence

G. Falkovich

based on works with

H Xu, E Bodenschatz, A Pumir, M Shats, H Xia, N Francois, G Boffetta, A Frishman, T Grafke

PRL 2013, 2014, *PNAS* 2014, *PRX* 2014, *PRE* 2015

detailed balance

In systems at thermal equilibrium, the probabilities of forward and backward transitions between any two states are equal,

When a system driven by thermal noise is characterized by a probability current, the fluctuation–dissipation theorem and detailed balance apply in a co-moving reference frame.

Modified detailed balance for NESS

$$\partial_t P(\sigma, t) = \partial_\sigma F$$

$$F = H' P + \mu\gamma^2 P'$$

The detailed balance takes place in the reference frame moving in σ-space with the velocity

$$U(\sigma) = F/P(\sigma)$$

Moral: many NESS have only one degree of freedom deviated from equilibrium; it could be eliminated by passing to a Lagrangian reference frame.

Turbulence is not like that.

Turbulence

$l_F \gg l_D$ direct cascade

$l_F \ll l_D$ inverse cascade

$$\text{Space view } \langle [\mathbf{v}(\mathbf{r},t) - \mathbf{v}(0,t)]_l^3 \rangle = -12\epsilon/d(d+2)$$

Time is greater than space.
Space is an entity, time is in essence a thought of an entity.

J Brodsky

Two-particle manifestations of the cascade and irreversibility

Consider two particles and their velocity difference **u** measured at the distance R

$$\mathbf{u} = \mathbf{v}(\mathbf{r_1}) - \mathbf{v}(\mathbf{r_2})$$

$$\langle du^2/dt \rangle_{t=0} = -4\bar{\epsilon}$$

Hypothetical long-time Richardson law $\langle R^2(t) \rangle \simeq |\epsilon| t^3$

Exact short-time laws for 2d and 3d

$$\left\langle \left[\frac{R_0}{R(t)} \right]^{2/3} \right\rangle - 1 = \frac{2\epsilon t^3}{27 R_0^2}$$

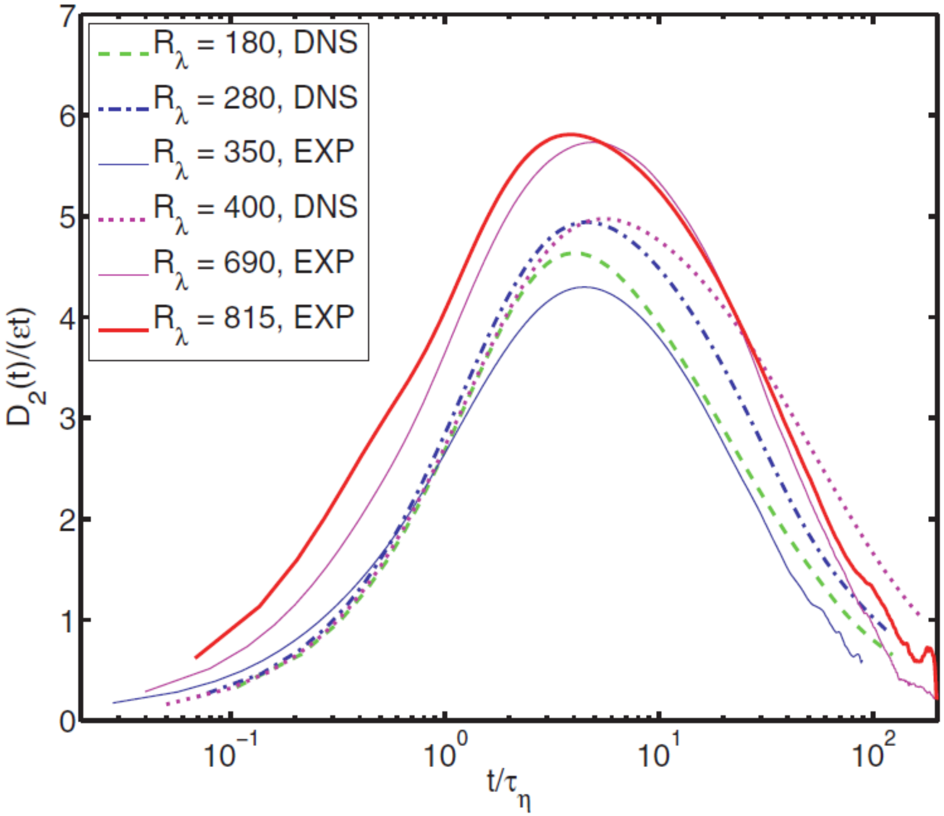
$$\left\langle \left[\frac{R_0}{R(t)} \right]^{5/3} \right\rangle - 1 = \frac{14\epsilon t^3}{81 R_0^2}$$

If we follow a single fluid particle
and the only thing we know is the velocity $\boldsymbol{v}(t)$
how to tell if we are in turbulence?

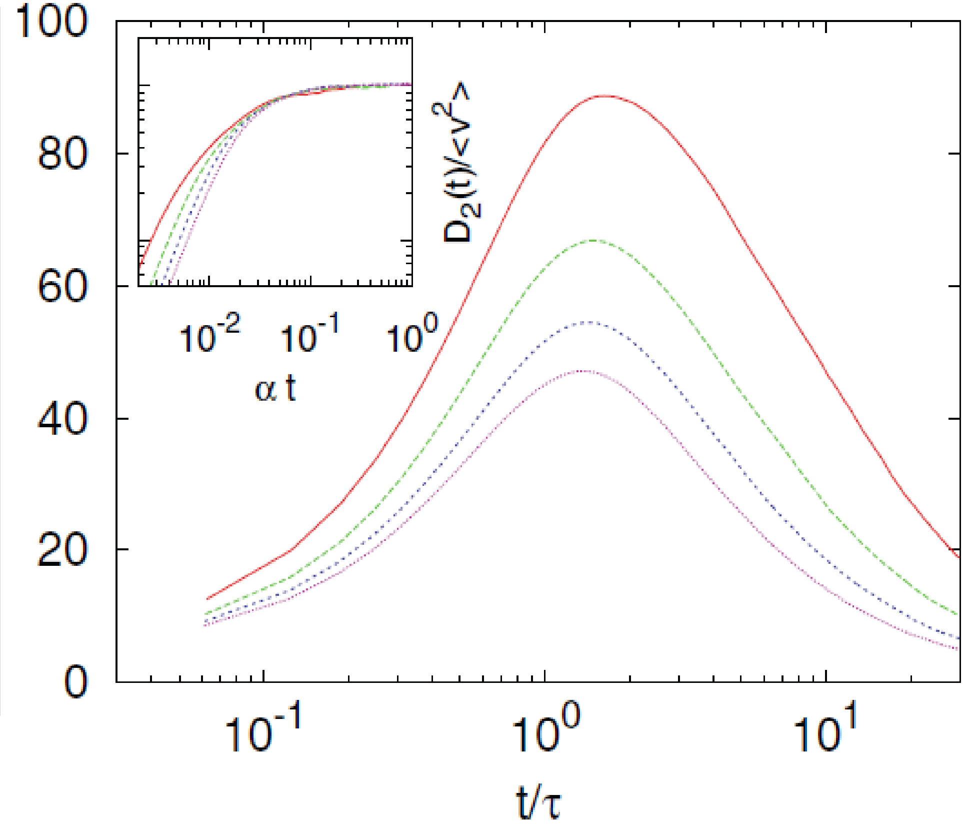
Landau and Obukhov

$$D_2(t) \equiv \langle |\boldsymbol{v}(t) - \boldsymbol{v}(0)|^2 \rangle \simeq \varepsilon t$$

3D



2D



Fluctuation-dissipation theorem?

Lagrangian description of ideal flows is canonical Hamiltonian with Bernoulli invariant, as Hamiltonian $\int (v^2/2 + p) d\boldsymbol{r}$

Since the “coordinate” conjugated to the force ***f*** is **v** then FDT would mean that the time derivative

$$d\langle v_i(0) v_j(t)\rangle / dt \quad = -d < |v(t) - v(0)|^2 > /2dt$$

is equal to the response function

$$R_{ij}(t) = \langle \delta v_i(t) / \delta f_j(0) \rangle = \varepsilon \delta_{ij} H(t)/2$$

for $\quad \langle f_i(0) f_j(t) \rangle = \varepsilon P_{ij} \delta(t)$

If we follow a single fluid particle and
the only thing we know is the velocity $\boldsymbol{v}(t)$
how to tell if we are in turbulence?

Statistics of the velocity difference is time reversible in a stationary incompressible turbulence:

$$\boldsymbol{v}(t) - \boldsymbol{v}(0) \rightarrow -\boldsymbol{v}(-t) + \boldsymbol{v}(0)$$

Then consider the energy $E = v^2/2$

Data from all available energy cascades

Three-Dimensional Turbulence Experiments

A von Kármán swirling water flow between two counter-rotating baffled disks. The flow is confined in a cylindrical tank with an inner diameter of 48.3 cm and a volume of 120 L. The propellers are 20 cm in diameter, each with 12 vertical vanes 4.3 cm in height. The axial distance between the propellers is 33 cm.

Particle Tracking Measurement. The Lagrangian trajectories of the fluid were obtained by optically following tracer particles seeded in the flow. The tracers were polystyrene spheres with diameters of 26 μm. These tracer particles were illuminated by pulsed frequency-doubled neodymium-doped yttrium aluminum garnet lasers, with intensity up to 130 W. Their motion was then recorded by three high-speed complementary metal oxide semiconductor (CMOS) cameras from different viewing angles. Finally, the images were processed to obtain particle trajectories in 3D space and in time.

Three-Dimensional Numerical Simulations

The results of direct numerical simulation (DNS) of 3D turbulent flows presented in this work came either from simulations in Lyon at a relatively low Reynolds number, obtained directly using a pseudospectral code developed at Johns Hopkins University

4. Two-Dimensional Numerical Simulations

We performed numerical simulations of the Navier–Stokes equation for a 2D incompressible velocity field, $\mathbf{u} = (u, v)$,

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} - \alpha \mathbf{u} + \mathbf{f}, \qquad \text{[S6]}$$

$$p(\mathbf{x}, t) = \mathbf{V} \cdot \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \mathbf{u} \cdot \mathbf{a} = \mathbf{u} \cdot \left(-\nabla P + \nu \nabla^2 \mathbf{u} - \alpha \mathbf{u} + \mathbf{f} \right)$$

$$R_\alpha \equiv (l_\alpha / l_F)^{2/3} = (\varepsilon_\alpha / \varepsilon_I)^{1/2} \eta_I^{1/3} \Big/ \alpha.$$

5. Two-Dimensional Turbulence Experiments

2D turbulence was studied experimentally using two methods of turbulence generation: electromagnetically driven turbulence in layers of electrolyte and Faraday-wave-driven turbulence in vertically vibrated containers.

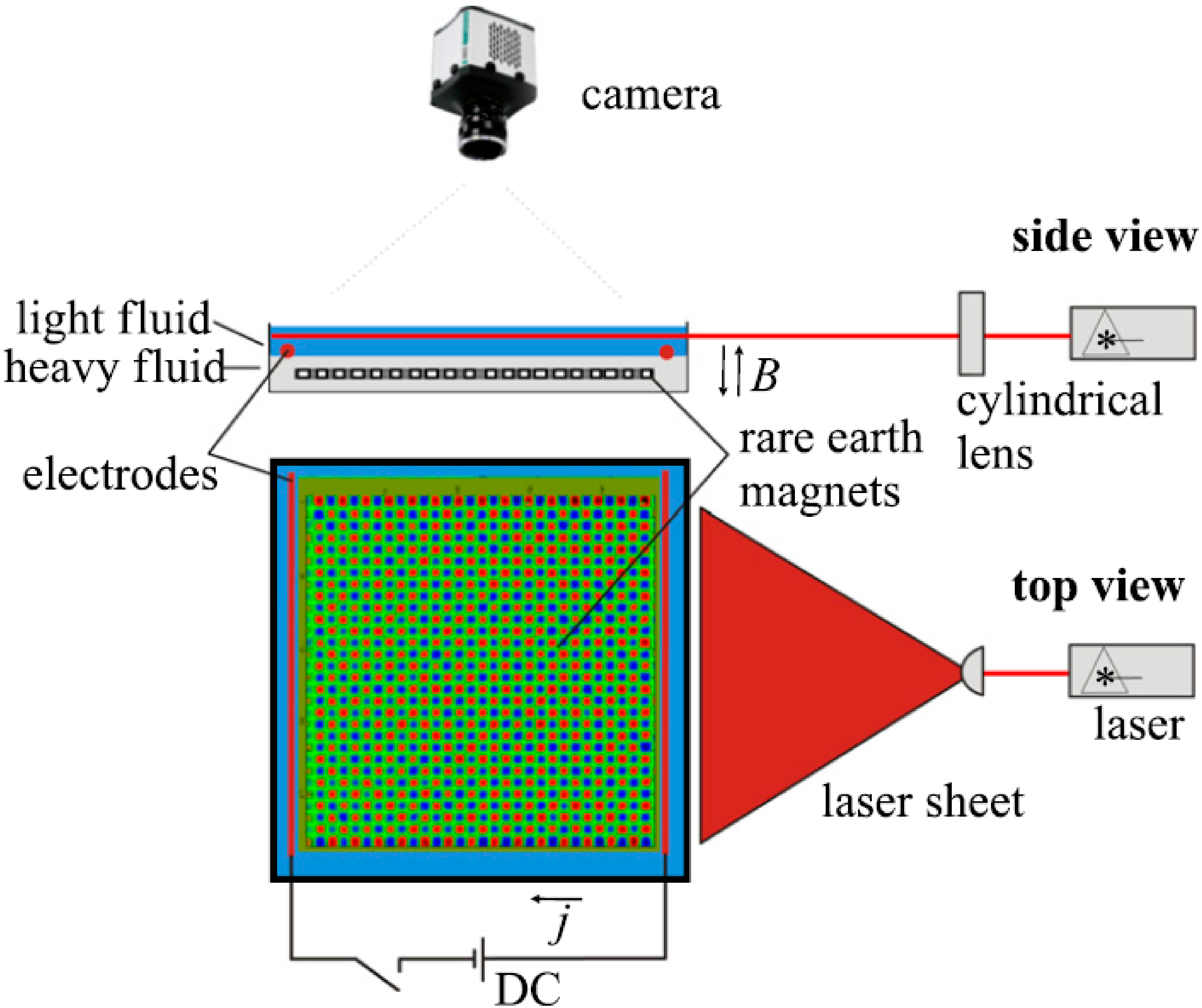


Fig. S2. Experimental setup for the electromagnetically driven turbulence.

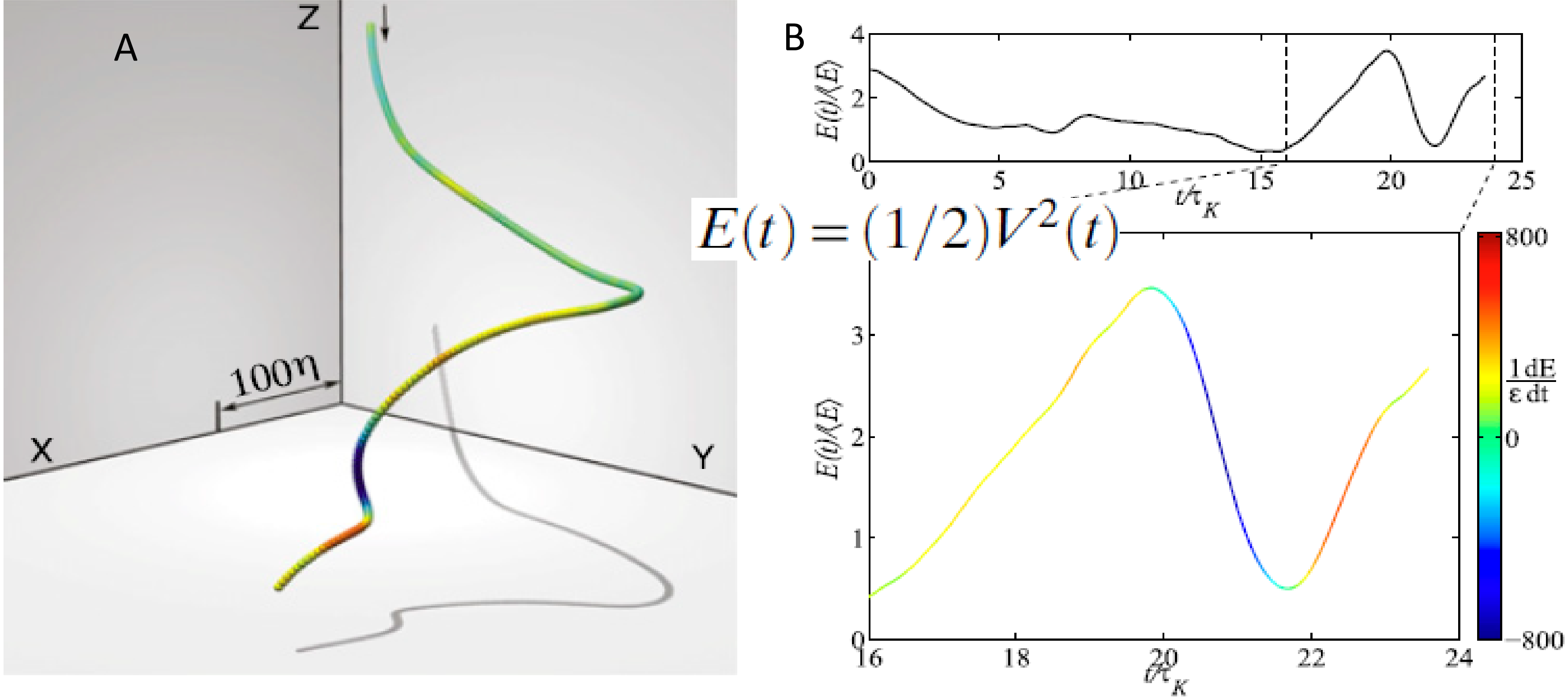
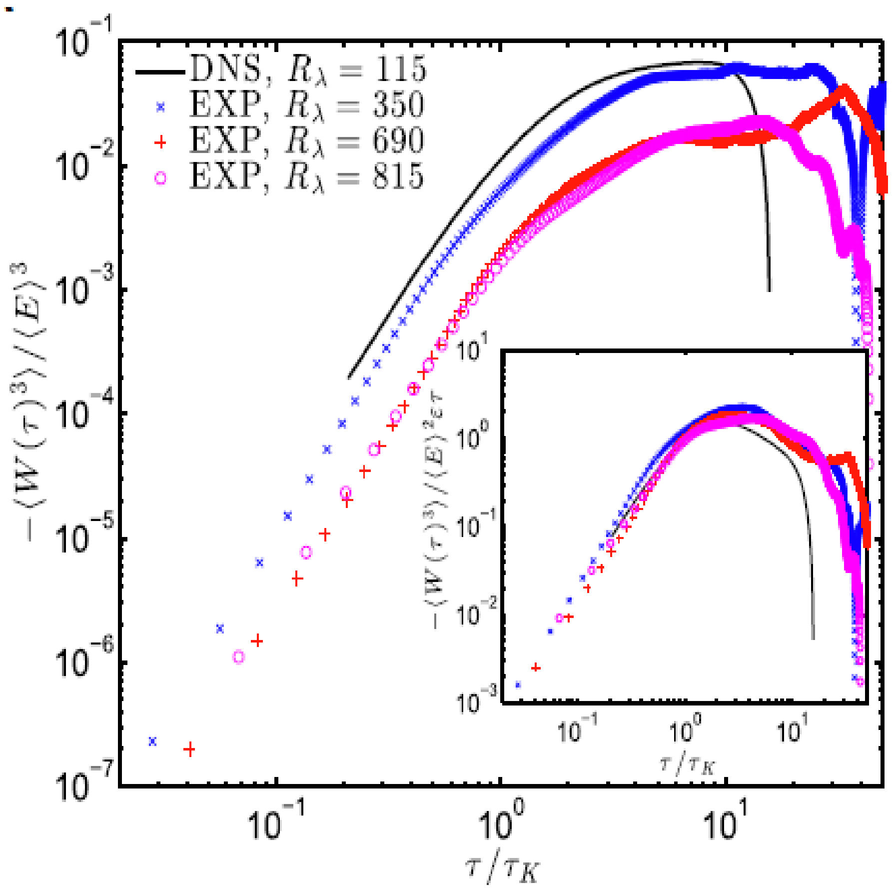
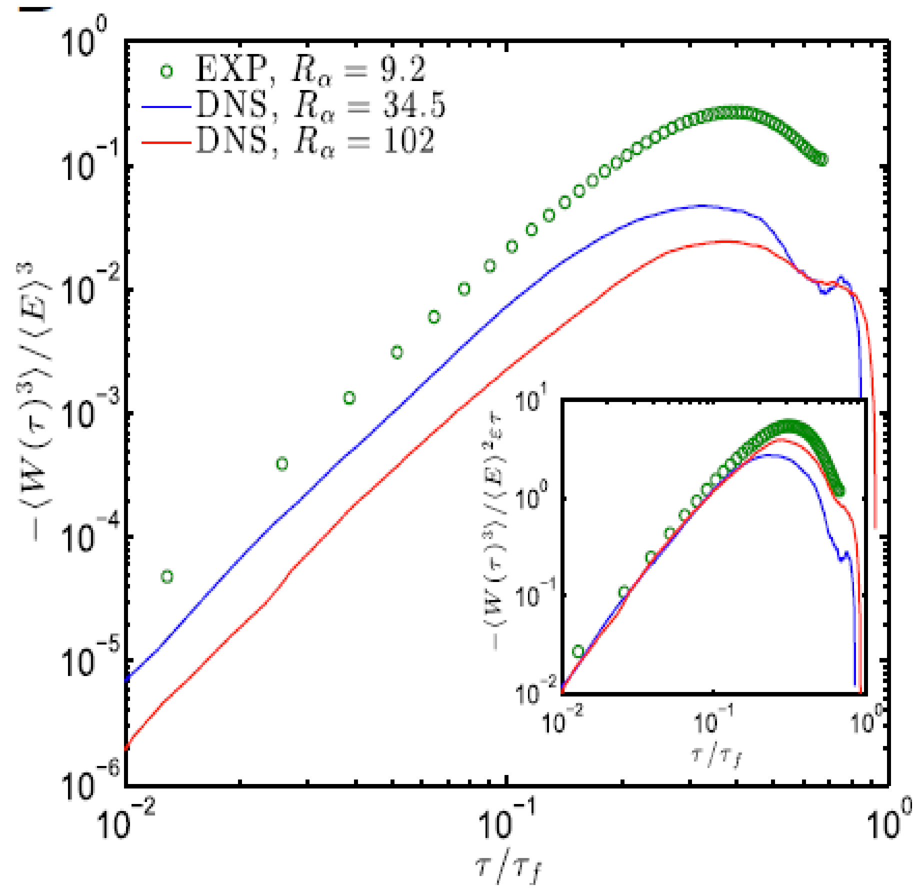


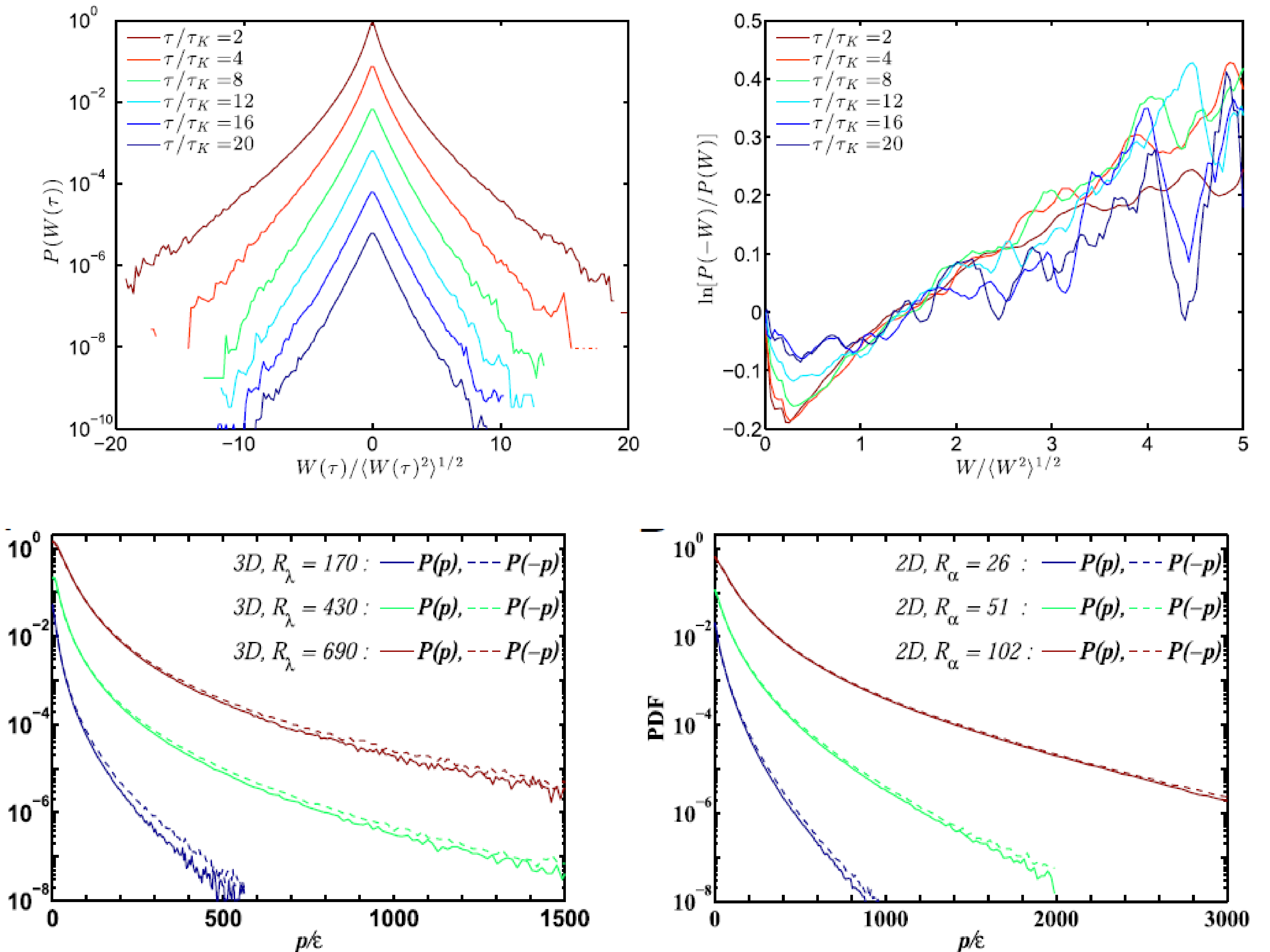
Fig. 1. Asymmetry of the statistics of energy differences. (*A*) The trajectory of a fluid particle in a 3D laboratory flow at $R_\lambda = 690$. The color coding refers to the instantaneous power $p(t) = \mathrm{d}E/\mathrm{d}t = \mathbf{a}(t) \cdot \mathbf{V}(t)$ acting on the fluid particle, showing that energy builds up slowly and dissipates quickly. The particle enters the observation volume from above and leaves from below. The scale bar is expressed in terms of the Kolmogorov scale η, which is the dissipation scale of this flow, $l_D = \eta = 30\ \mu\mathrm{m}$. (*B*) The evolution of the kinetic energy $E(t)$ of the same particle as a function of time, in units of the Kolmogorov time τ_K, the fastest time scale of the flow, characterizing the dynamics at scale l_D. *B, Upper* is for the entire trajectory, while *Lower* magnifies the period with strong energy change,

The statistics of the energy increments

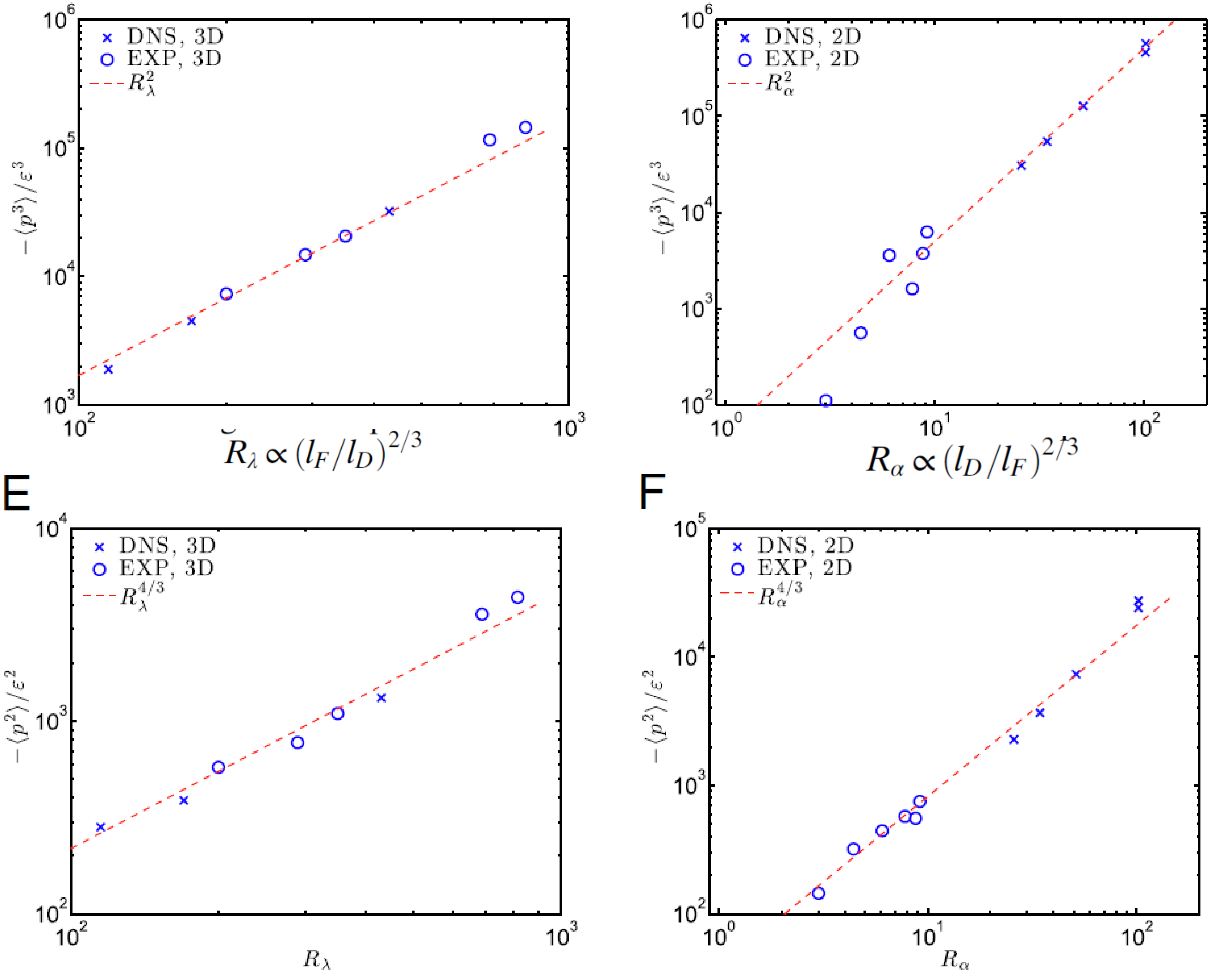
$$W(\tau) = E(t+\tau) - E(t)$$







$\tau/\tau_K = 2$
$\tau/\tau_K = 4$
$\tau/\tau_K = 8$
$\tau/\tau_K = 12$
$\tau/\tau_K = 16$
$\tau/\tau_K = 20$
$P(W(\tau))$
$W(\tau)/\langle W(\tau)^2\rangle^{1/2}$
$\tau/\tau_K = 2$
$\tau/\tau_K = 4$
$\tau/\tau_K = 8$
$\tau/\tau_K = 12$
$\tau/\tau_K = 16$
$\tau/\tau_K = 20$
$\ln[P(-W)/P(W)]$
$W/\langle W^2\rangle^{1/2}$
3D, $R_\lambda = 170$: P(p), P(−p)
3D, $R_\lambda = 430$: P(p), P(−p)
3D, $R_\lambda = 690$: P(p), P(−p)
p/ε
2D, $R_\alpha = 26$: P(p), P(−p)
2D, $R_\alpha = 51$: P(p), P(−p)
2D, $R_\alpha = 102$: P(p), P(−p)
PDF
p/ε



DNS, 3D
EXP, 3D
R_λ^2
$-\langle p^3\rangle/\varepsilon^3$
$R_\lambda \propto (l_F/l_D)^{2/3}$
DNS, 2D
EXP, 2D
R_α^2
$-\langle p^3\rangle/\varepsilon^3$
$R_\alpha \propto (l_D/l_F)^{2/3}$
E
DNS, 3D
EXP, 3D
$R_\lambda^{4/3}$
$-\langle p^2\rangle/\varepsilon^2$
R_λ
F
DNS, 2D
EXP, 2D
$R_\alpha^{4/3}$
$-\langle p^2\rangle/\varepsilon^2$
R_α

A fast particle takes flight and then sharply decelerates

$$\text{distance} \sim V\tau$$

velocity difference

$$\delta V(\tau) \simeq (\varepsilon V \tau)^{1/3}$$

$$(\varepsilon V \tau)^{1/3} / (\varepsilon\tau)^{1/2} = (T/\tau)^{1/6}$$

$$W(\tau) \approx V \cdot \delta V \sim V^{4/3} (\varepsilon\tau)^{1/3}$$

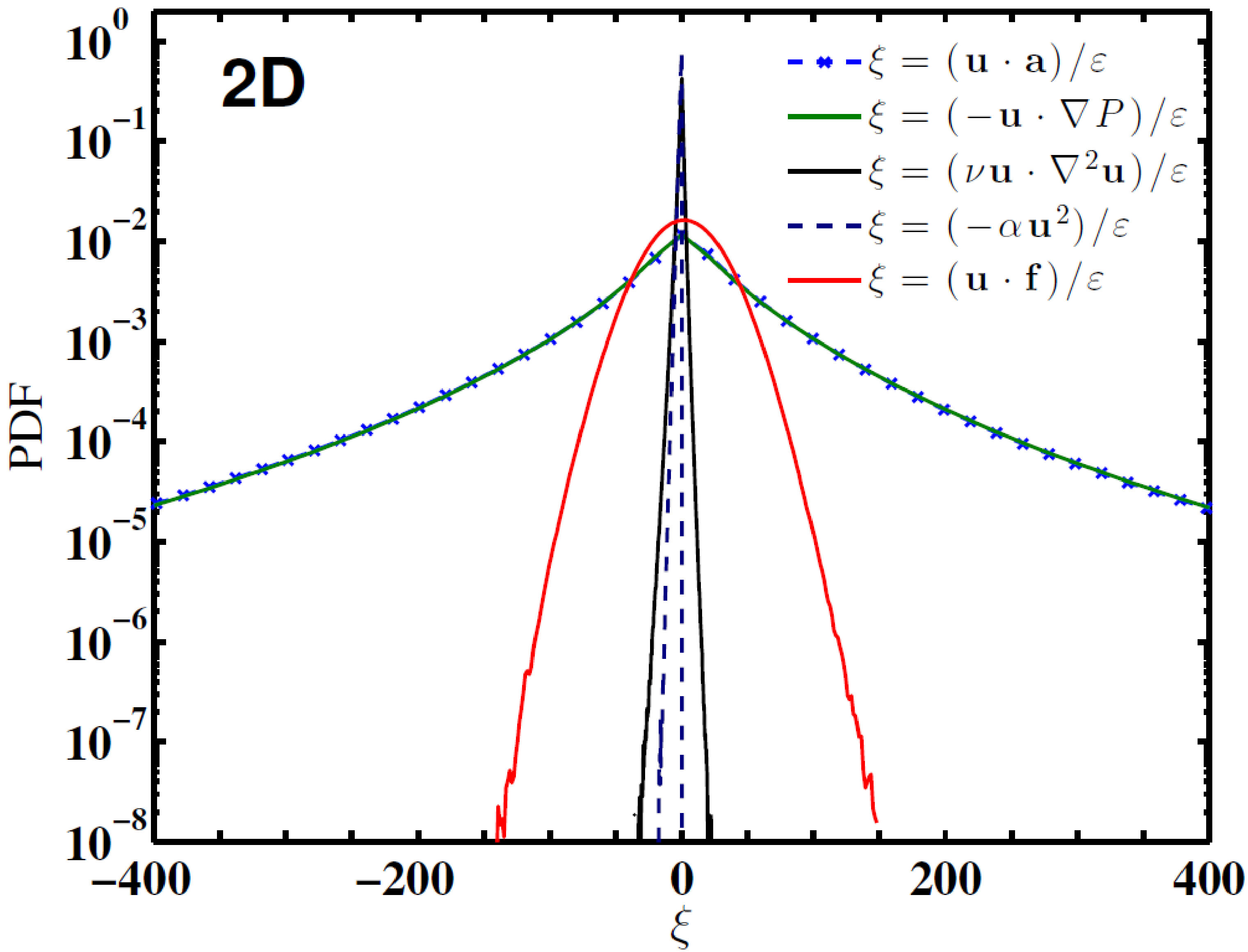
$$\langle W^3(\tau) \rangle \propto U_{rms}^3 (\varepsilon U_{rms} \tau)$$

$$p \approx W(\tau_K)/\tau_K \sim \varepsilon (T/\tau_K)^{2/3}$$

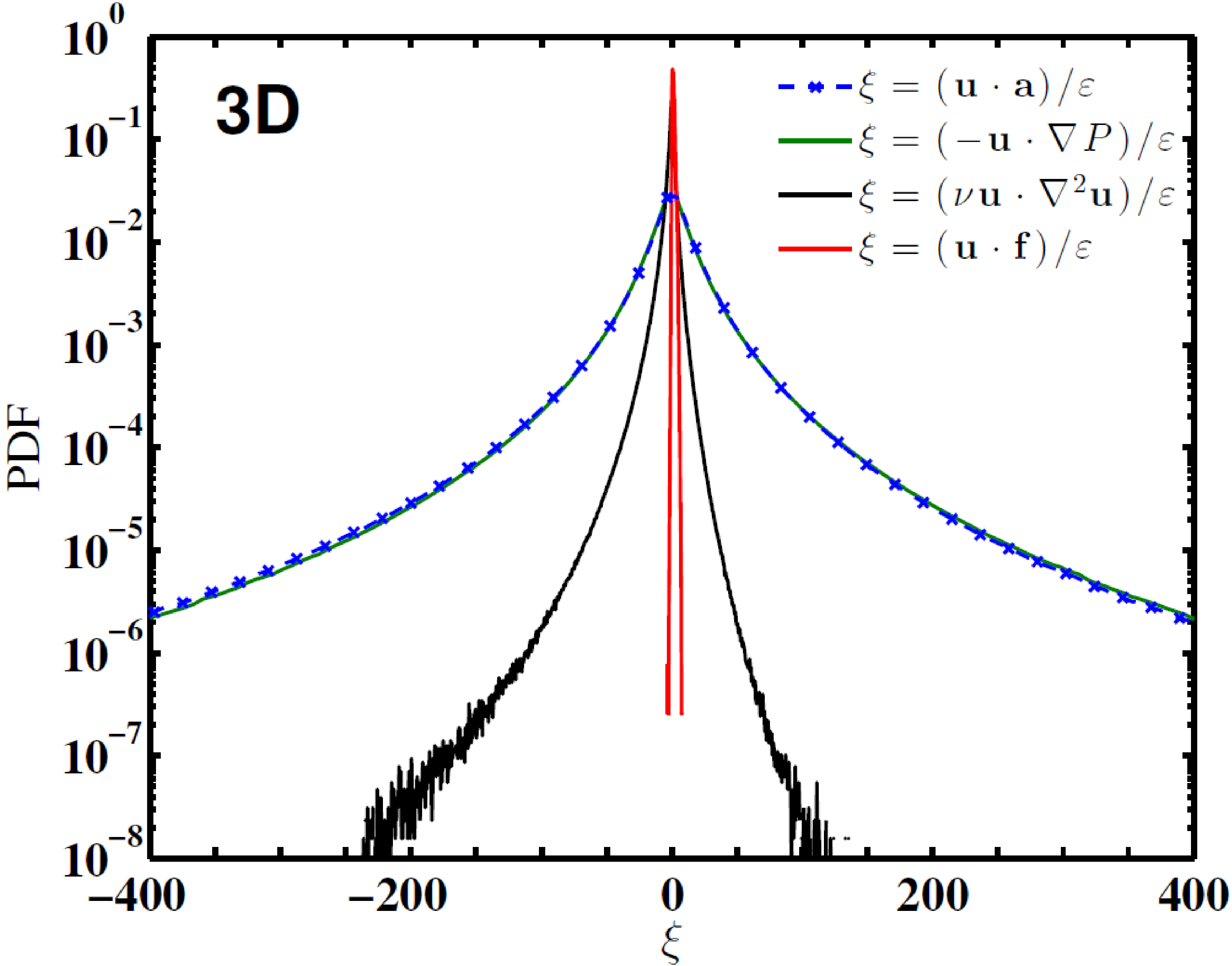
$$\langle p^3 \rangle \propto \varepsilon^3 (T/\tau_K)^2 \propto \varepsilon^3 R_\lambda^2$$

$$\langle p^2 \rangle \propto \varepsilon^2 R_\lambda^{4/3}$$

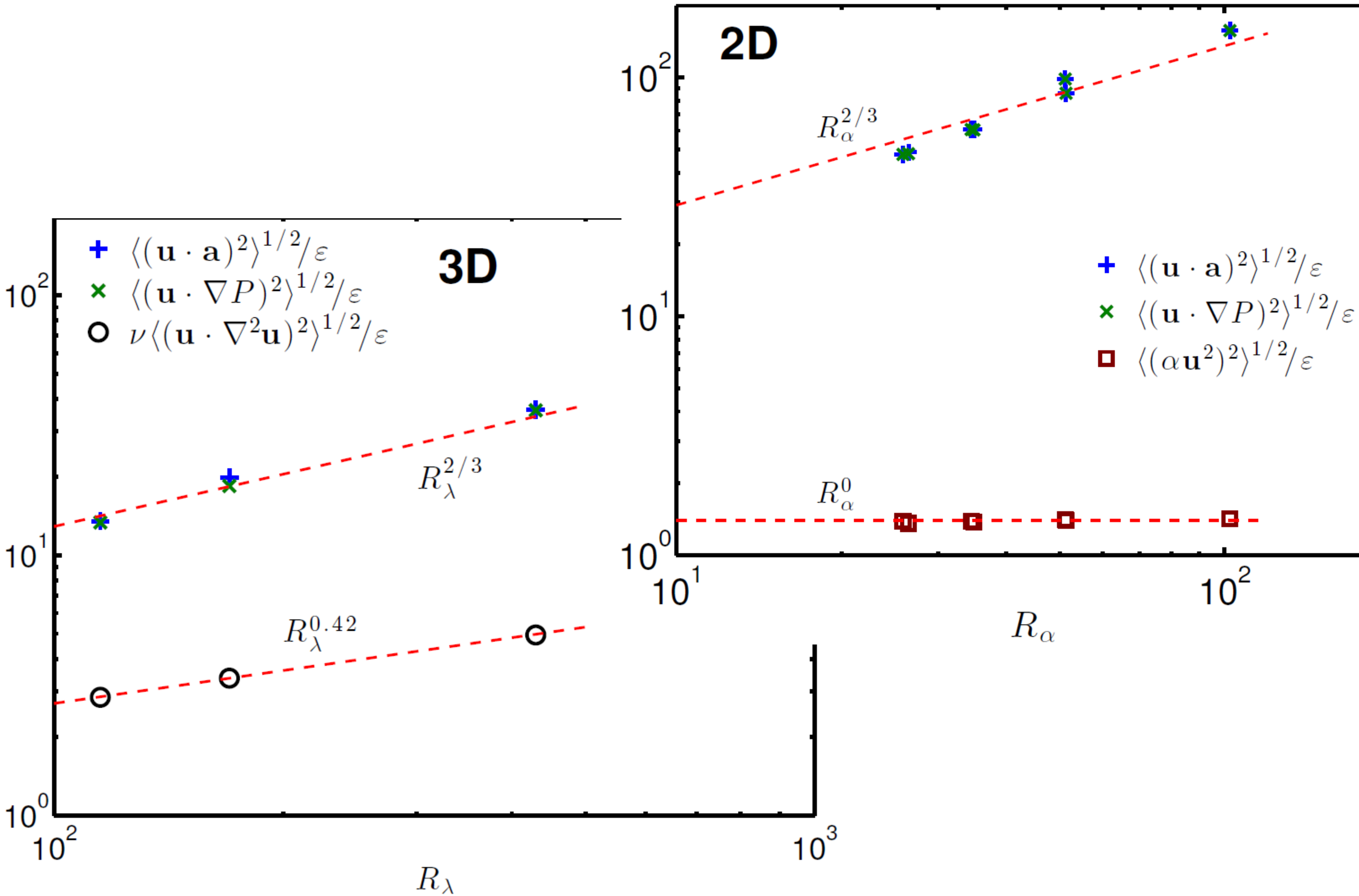
Role of different forces in power fluctuations



Role of different forces in power fluctuations



Standard deviations as functions of Reynolds numbers



CONTRIBUTIONS OF PRESSURE, DISSIPATION AND FORCING TO THE THIRD MOMENT OF POWER in 2d

R_α	26	51	102
$\langle p^3\rangle/\langle p^2\rangle^{3/2}$	-0.28	-0.20	-0.12
$\langle(-\mathbf{u}\cdot\nabla P)^3\rangle/\langle(-\mathbf{u}\cdot\nabla P)^2\rangle^{3/2}$	-0.13	-0.11	-0.068
$\langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^3\rangle/\langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^2\rangle^{3/2}$	-1.99	-1.60	-1.23
$\langle(-\alpha\mathbf{u}^2)^3\rangle/\langle(-\alpha\mathbf{u}^2)^2\rangle^{3/2}$	-2.04	-2.06	-2.14
$\langle(-\mathbf{u}\cdot\nabla P)^3\rangle/\langle p^3\rangle$	0.47	0.56	0.56
$\langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^3\rangle/\langle p^3\rangle$	1.27×10^{-4}	6.78×10^{-5}	4.08×10^{-5}
$\langle(-\alpha\mathbf{u}^2)^3\rangle/\langle p^3\rangle$	1.79×10^{-4}	4.39×10^{-5}	1.34×10^{-5}
$3\langle(-\mathbf{u}\cdot\nabla P)^2(-\alpha\mathbf{u}^2)\rangle/\langle p^3\rangle$	0.55	0.42	0.39
$3\langle(-\mathbf{u}\cdot\nabla P)^2(\mathbf{u}\cdot\mathbf{f})\rangle/\langle p^3\rangle$	-0.31	-0.25	-0.23
$3\langle(-\mathbf{u}\cdot\nabla P)^2(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})\rangle/\langle p^3\rangle$	0.29	0.27	0.27

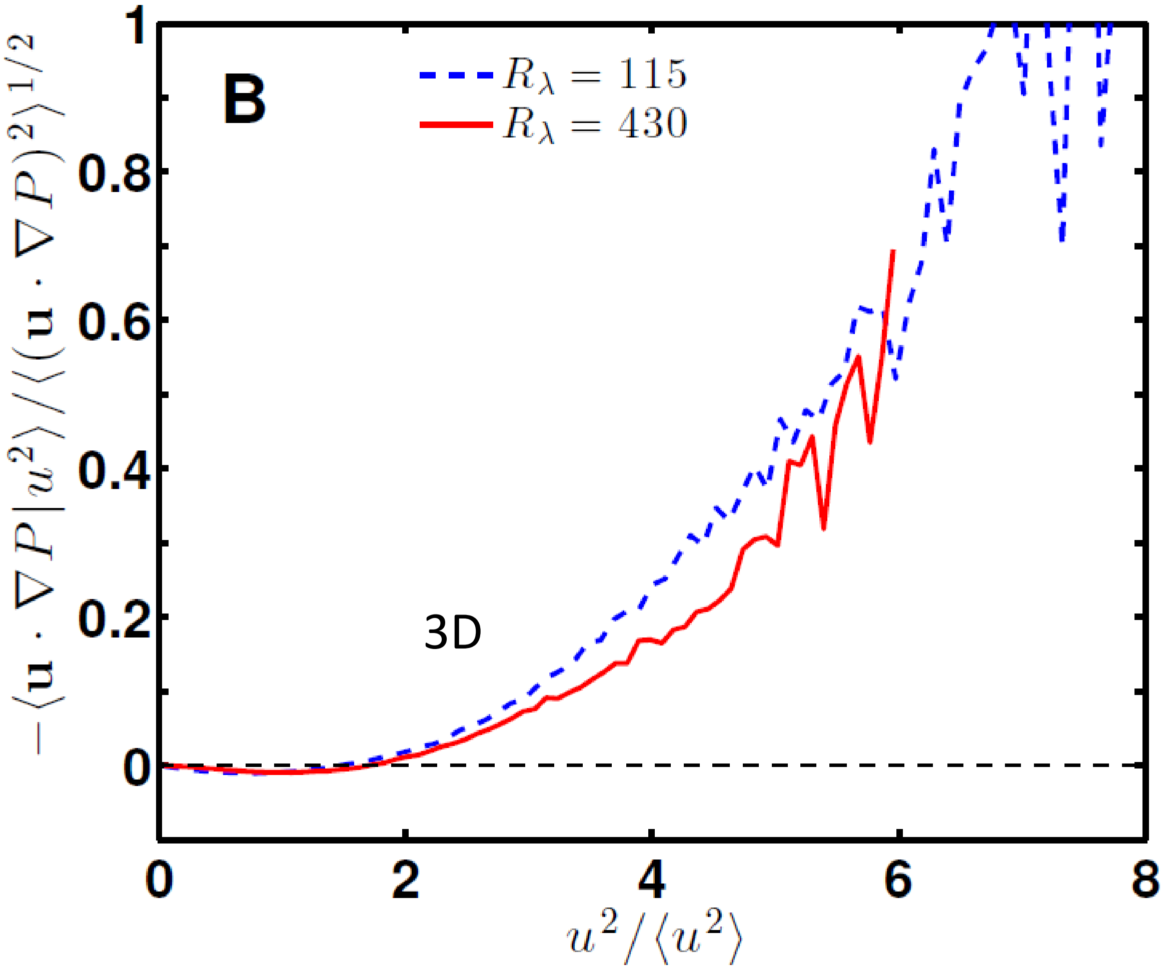
The main contribution to the third moment of the power fluctuations comes from pressure.

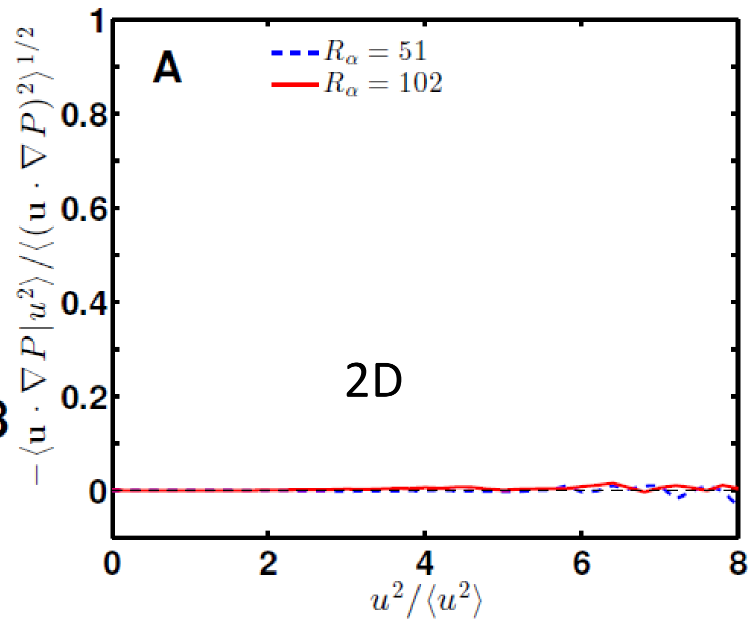
CONTRIBUTIONS OF PRESSURE, DISSIPATION AND FORCING TO THE THIRD MOMENT OF POWER in 3d

R_λ	115	170	430
$\langle p^3\rangle/\langle p^2\rangle^{3/2}$	-0.53	-0.65	-0.67
$\langle(-\mathbf{u}\cdot\nabla P)^3\rangle/\langle(\mathbf{u}\cdot\nabla P)^2\rangle^{3/2}$	0.13	0.15	0.023
$\langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^3\rangle/\langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^2\rangle^{3/2}$	-3.7	-4.3	-4.1
$\langle(-\mathbf{u}\cdot\nabla P)^3\rangle/\langle p^3\rangle$	-0.24	-0.19	-0.034
$\langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^3\rangle/\langle p^3\rangle$	6.47×10^{-2}	3.18×10^{-2}	1.55×10^{-2}
$3\langle(-\mathbf{u}\cdot\nabla P)^2(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})\rangle/\langle p^3\rangle$	1.98	1.30	1.23
$3\langle(-\mathbf{u}\cdot\nabla P)^2(\mathbf{u}\cdot\mathbf{f})\rangle/\langle p^3\rangle$	$*$	$*$	-0.19

The main contribution to the third moment of the power fluctuations comes from the cross-correlation of the pressure with dissipation.

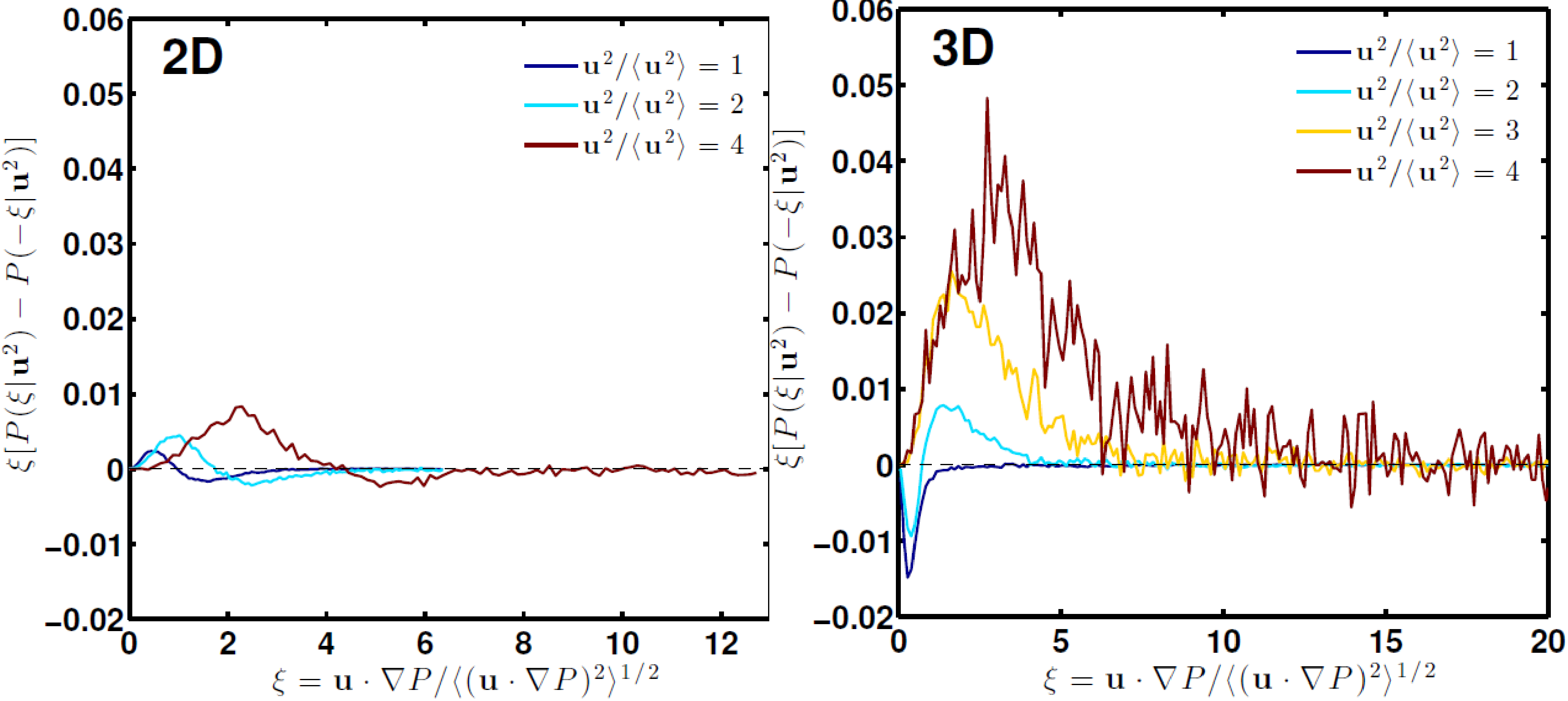
The role of pressure in redistributing kinetic energy





Redistribution of kinetic energy in turbulent flows

Alain Pumir,[1,2,3,*] Haitao Xu,[1,3] Guido Boffetta,[1,4] Gregory Falkovich,[1,5,6] and Eberhard Bodenschatz



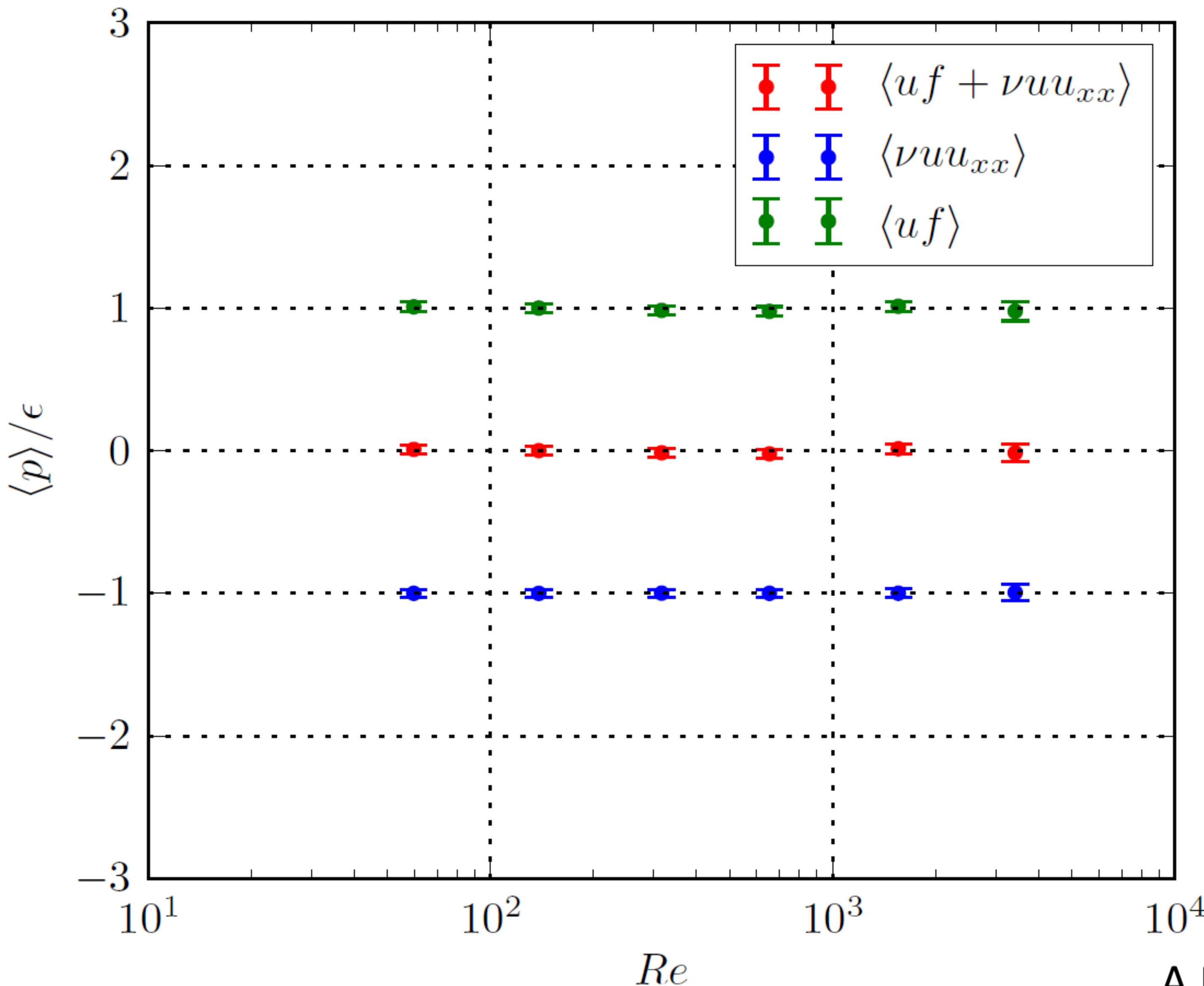
The contribution of different values of $\xi \equiv -\mathbf{u}\cdot\nabla P/\langle(\mathbf{u}\cdot\nabla P)^2\rangle^{1/2}$ to the conditional average $\langle -\mathbf{u}\cdot\nabla P|\mathbf{u}^2\rangle$

In 3D *all* values of ξ contribute to transfer energy to fast particles.

Our 3D world is unfair: From rich to poor, from slow to fast.

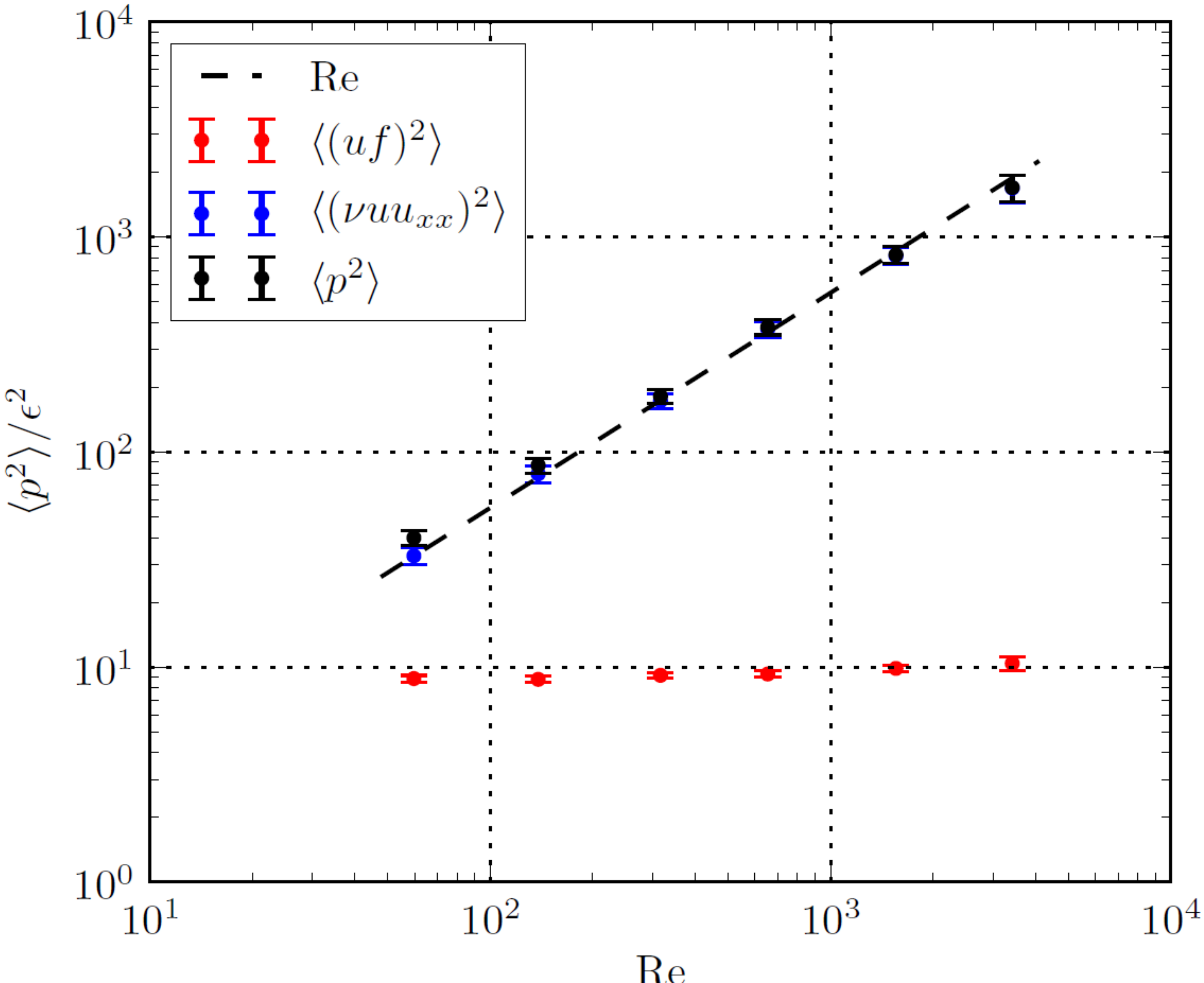
Burgers turbulence

$$u_t + uu_x = f + \nu u_{xx}$$

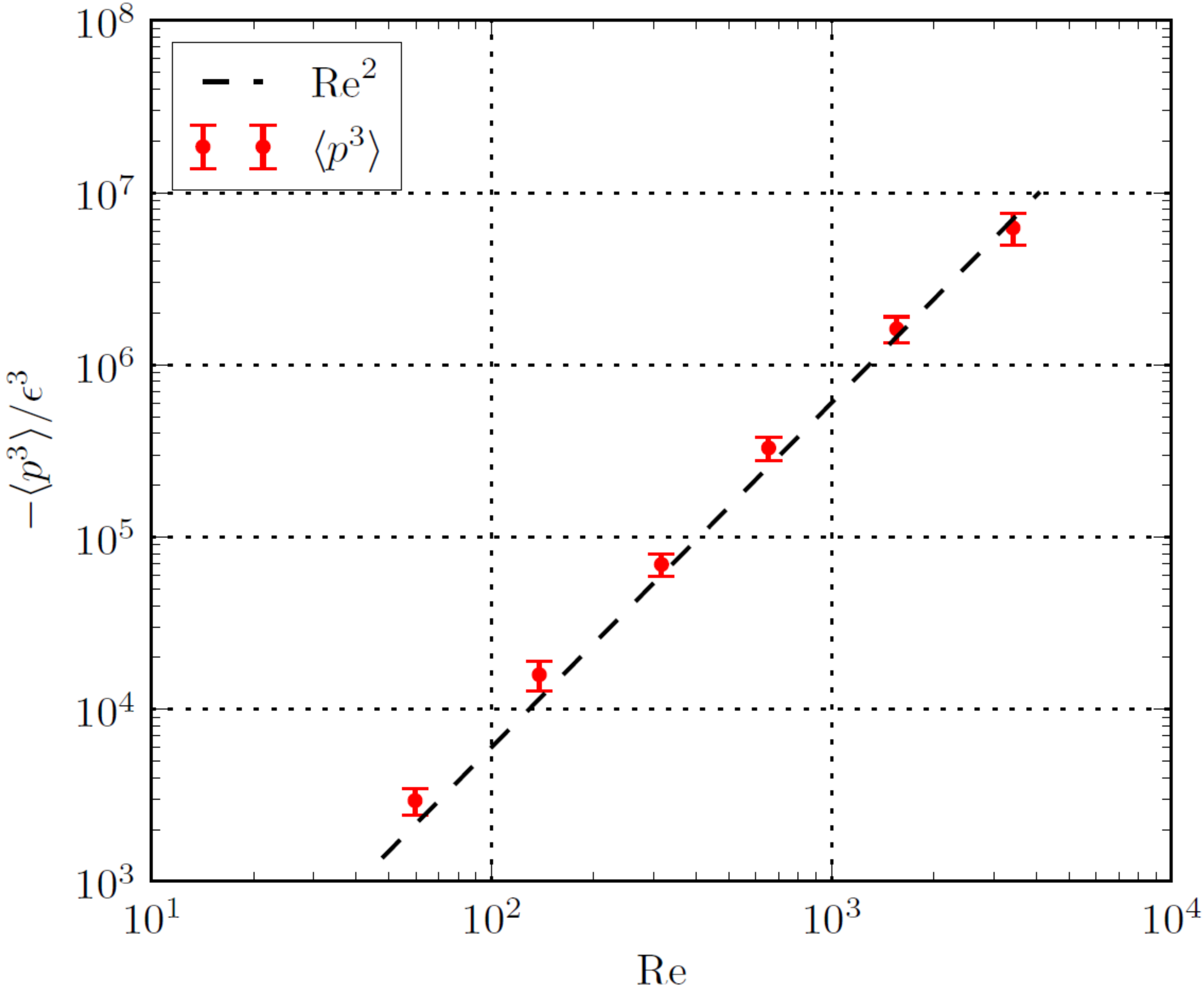


A Frishman, T Grafke

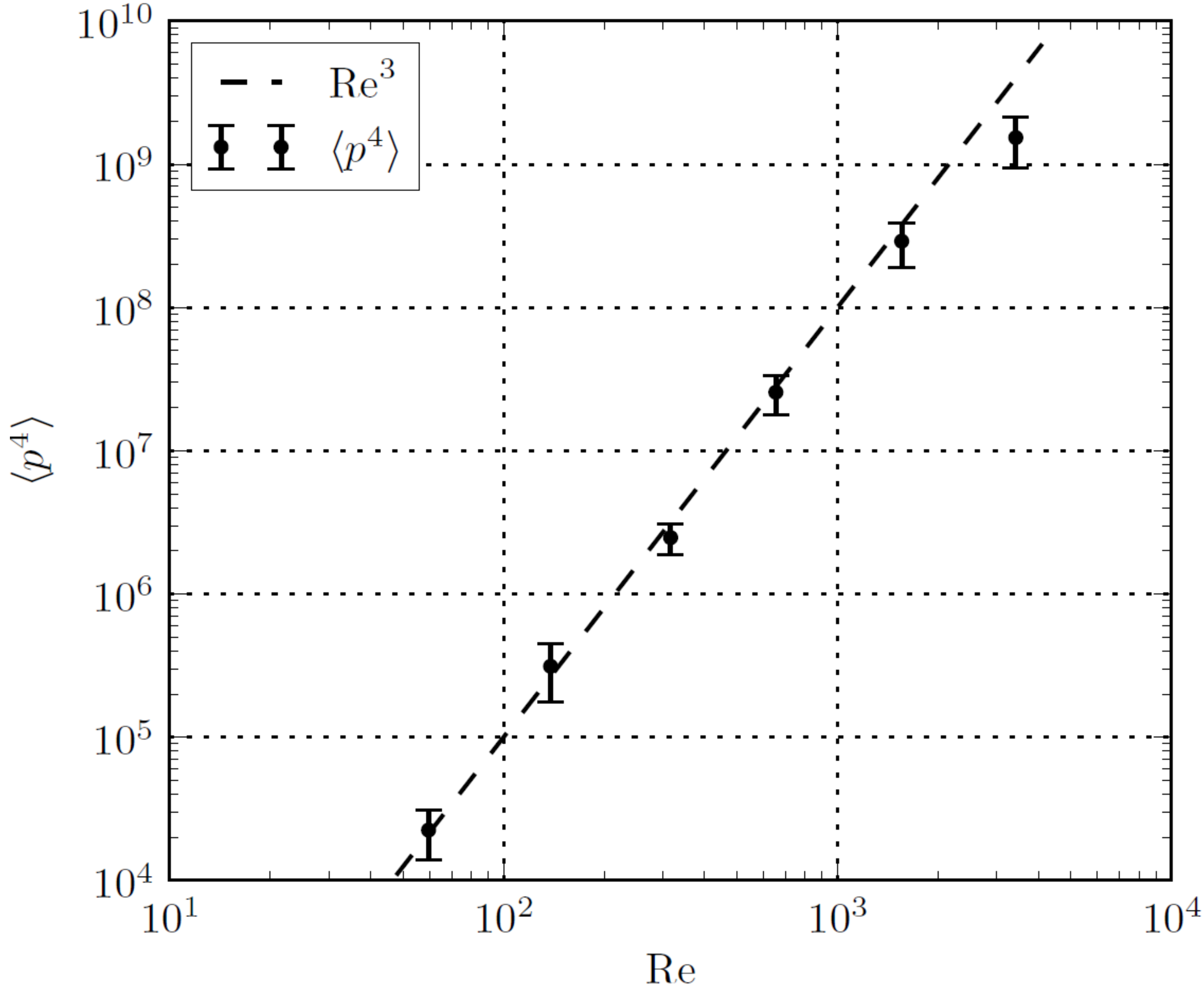
Burgers turbulence



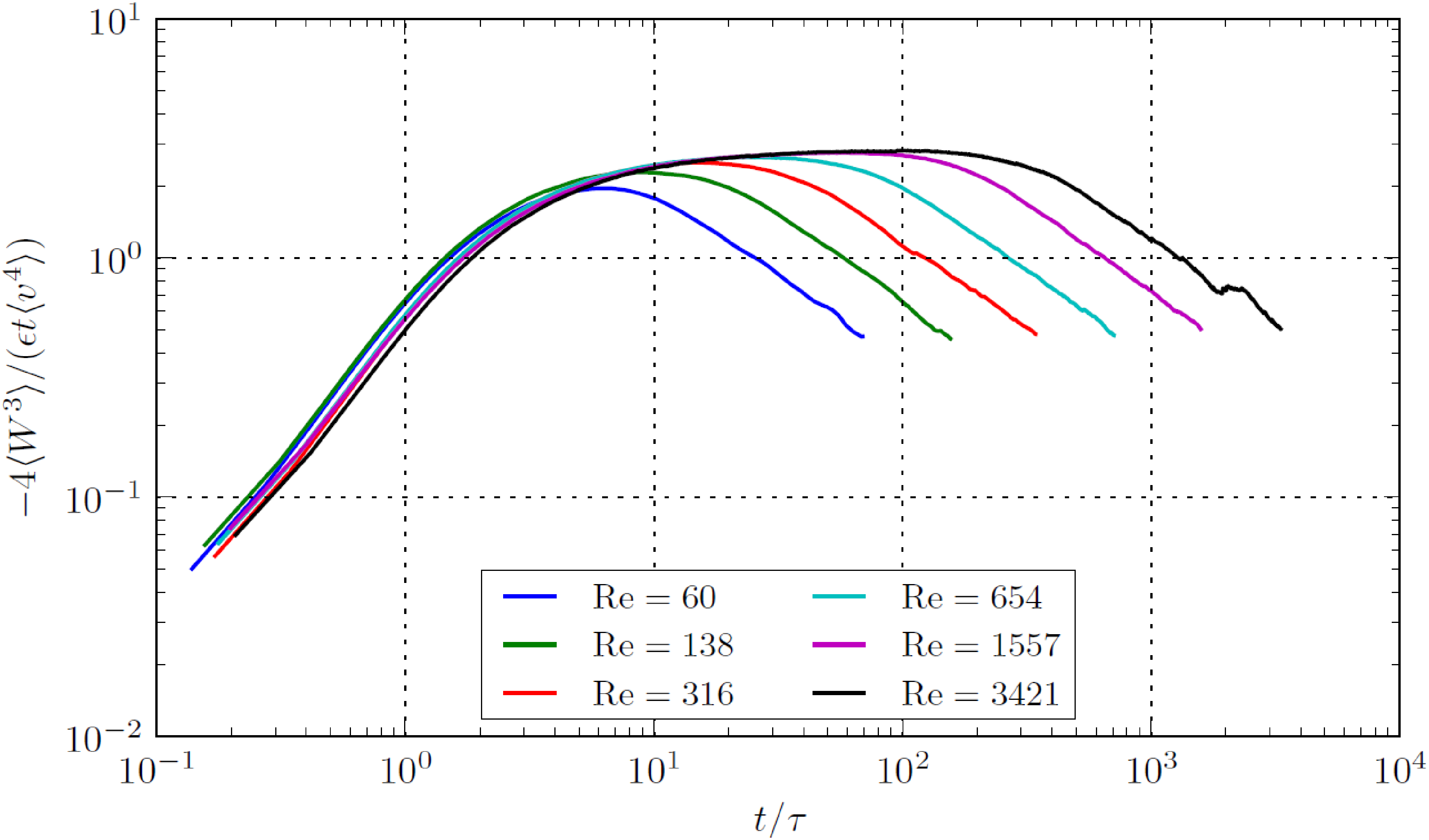
Burgers turbulence



Burgers turbulence



Burgers turbulence



Take-home lesson

One can quantitatively determine the degree of non-equilibrium of large system by measuring irreversibility of time evolution of a single degree of freedom.

Summary

Irreversibility is a fundamental aspect of the evolution of natural systems, and quantifying its manifestations is a challenge in any attempt to describe nonequilibrium systems. In the case of fluid turbulence, an emblematic example of a system very far from equilibrium, we show that the motion of a single fluid particle provides a clear manifestation of time irreversibility. Namely, we observe that fluid particles tend to lose kinetic energy faster than they gain it. This is best seen by the presence of rare "flight–crash" events, where fast moving particles suddenly decelerate into a region where fluid motion is slow. Remarkably, the statistical signature of these events establishes a quantitative relation between the degree of irreversibility and turbulence intensity.

Fluid Mechanics

The multi-disciplinary field of fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering. In this book, the fundamental ideas of fluid mechanics are presented from a physics perspective.

Using examples taken from everyday life, from hydraulic jumps in a kitchen sink to Kelvin–Helmholtz instabilities in clouds, the book provides readers with a better understanding of the world around them. It teaches the art of fluid-mechanical estimates and shows how the ideas and methods developed to study the mechanics of fluids are used to analyse other systems with many degrees of freedom in statistical physics and field theory.

Aimed at undergraduate and graduate students, the book assumes no prior knowledge of the subject and only a basic understanding of vector calculus and analysis. It contains 32 exercises of varying difficulties, from simple estimates to elaborate calculations, with detailed solutions to help readers understand fluid mechanics.

Gregory Falkovich is a Professor in the Department of Physics of Complex Systems, Weizmann Institute of Science. He has researched in plasma, condensed matter, fluid mechanics, statistical and mathematical physics and cloud physics and meteorology, and has won several awards for his work.

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Cover designed by Hart McLeod Ltd

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-1-10700-575-4

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Fluid Mechanics

A Short Course for Physicists

GREGORY FALKOVICH

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