PHYSICS AND FLOW OF LIQUID FOAMS

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[Cantat et al., *Les mousses, structure et dynamique*, Belin (2010);
Foams, Structure and Dynamics, Oxford University Press (2013)]

Liquid foam in industry: friend or foe

FRIEND

Ore separation (copper extraction by flotation)

Fire fighting, shock wave attenuation

Food industry, cosmetics, detergents

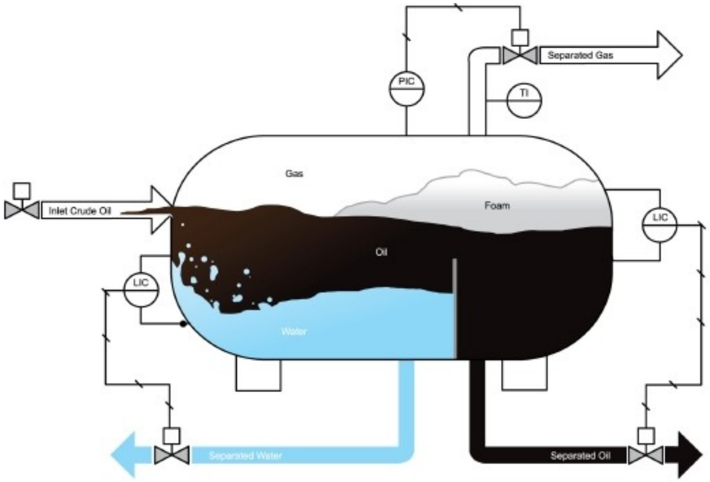
European Alder Spittle Bug larva:
foam nest to remain moist and protected

Liquid foam in industry: friend or foe

FOE

Polluted water: undesirable foam

Catastrophic in lubricants



Oil extraction: oily foam reduces output in well heads and may lead to disruption

OUTLINE

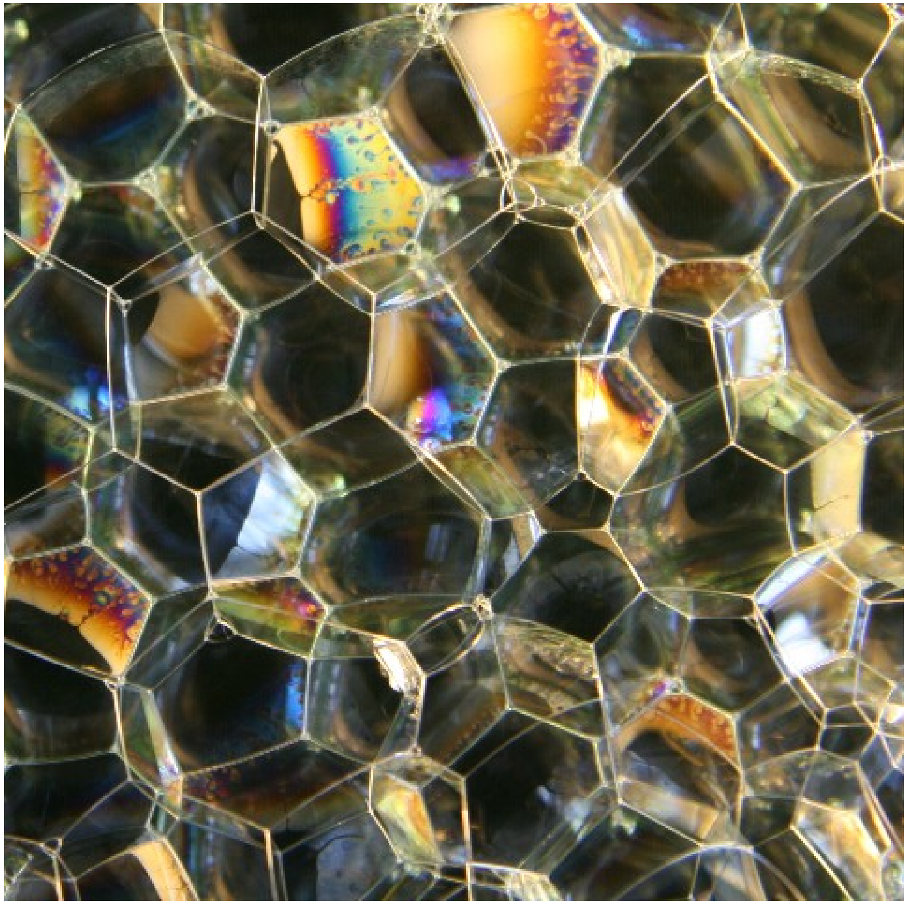
- **structure**

- drainage

- coarsening

- rheology
    - rheometric data
    - micromechanical models
    - flow profiles

- acoustics

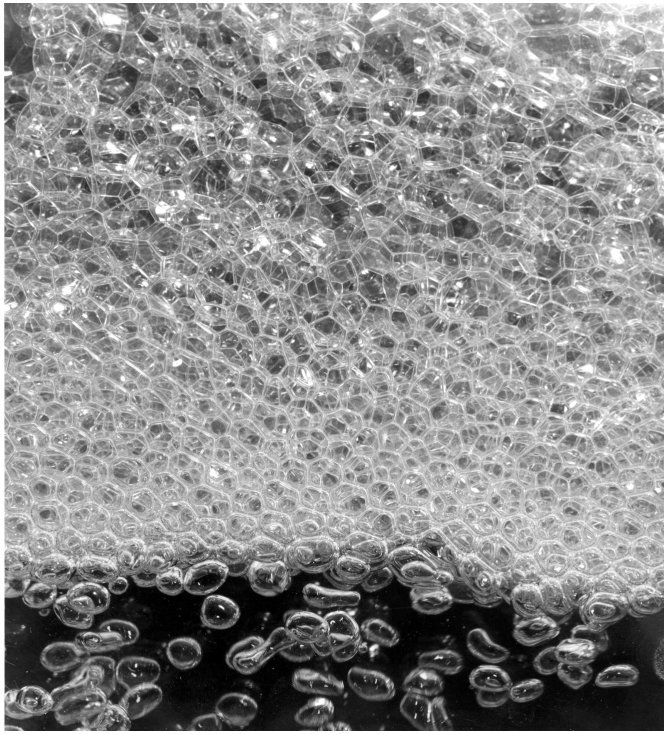


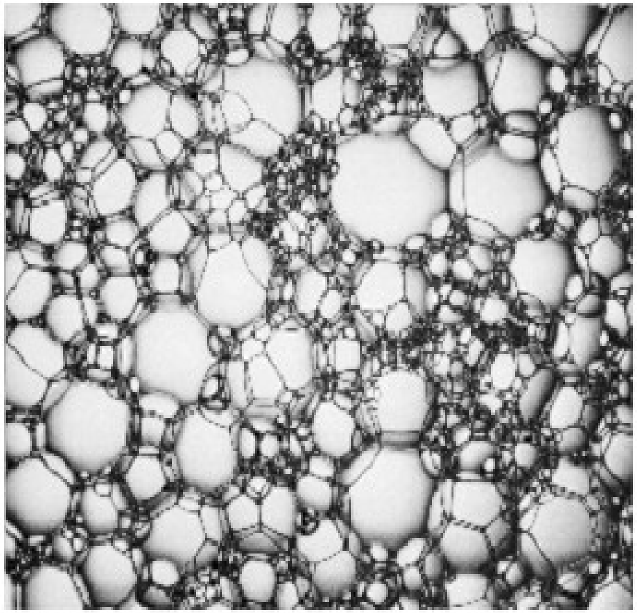
Foams: definition

Foams : dispersions of gas in a liquid

assembly of gas bubbles, deformable, more or less packed…

A COMPLEX SYSTEM…



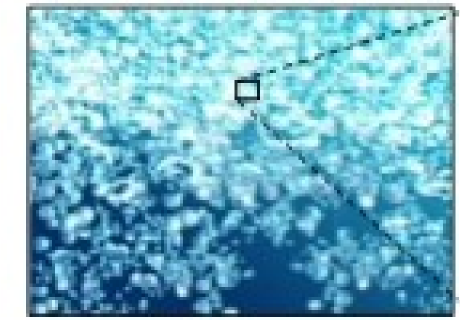


…MULTISCALED
and DISORDERED

- to stabilise a foam: surfactants in solution

Zooming in: hierarchy of scales

Foam **Bubbles** **Liquid channels** **Gas-liquid interface**



cm

Bubble average radius will be denoted *R* (or *a*)

Liquid films

(10-100 nm)

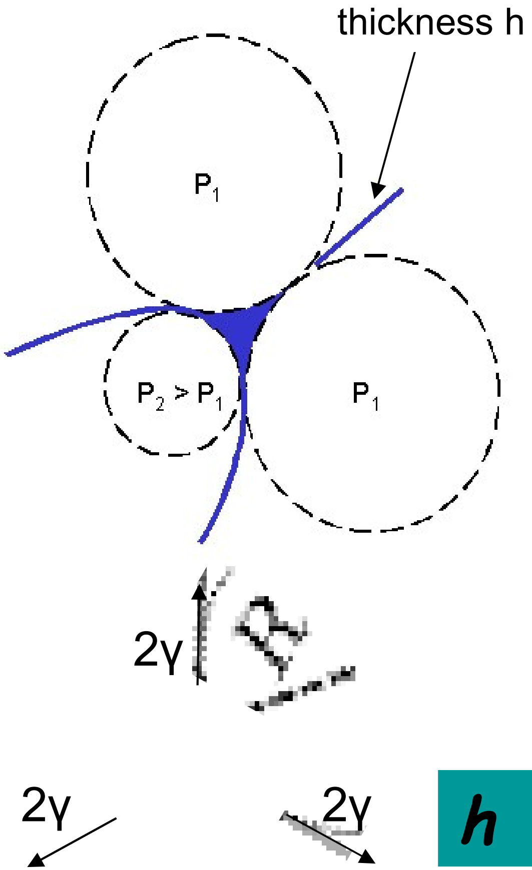
→ A hierarchical material, organised at mesoscopic scales

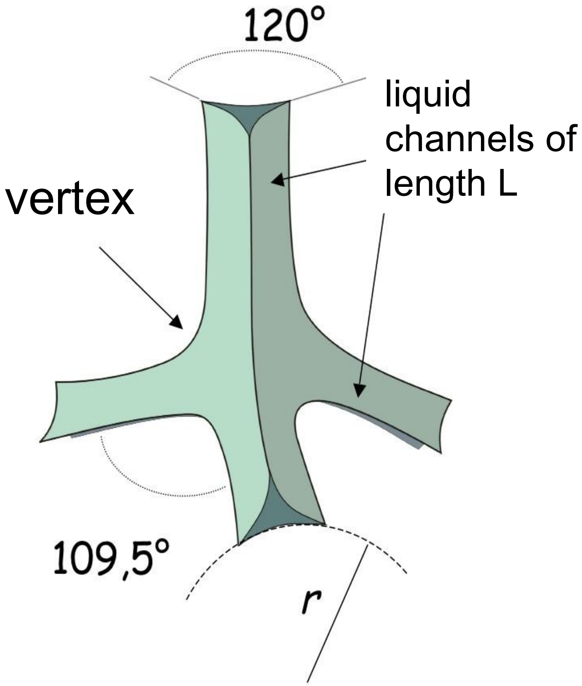
→ Couplings between properties at each scales

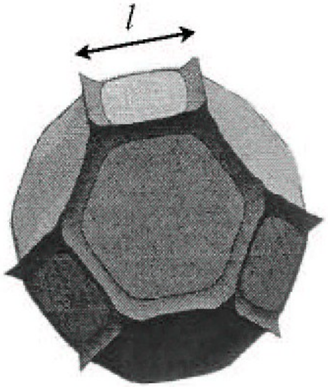
Characteristic size and liquid distribution

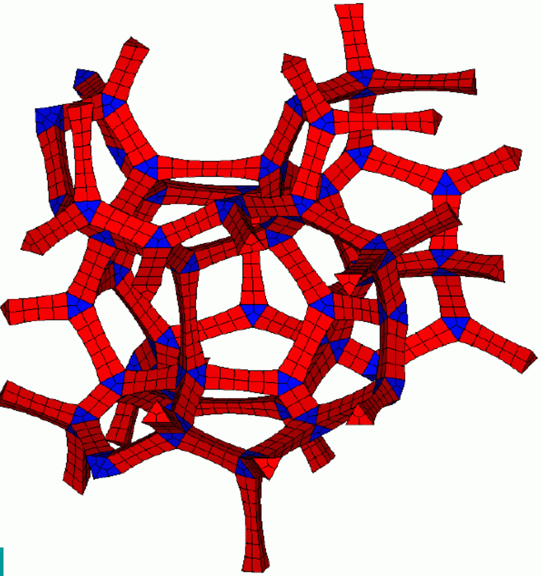
The films which separate bubbles
always meet three-fold,
at an angle = 120°

At junction between films,
a channel (Plateau border)









$h \ll r \ll L \sim$ bubble size

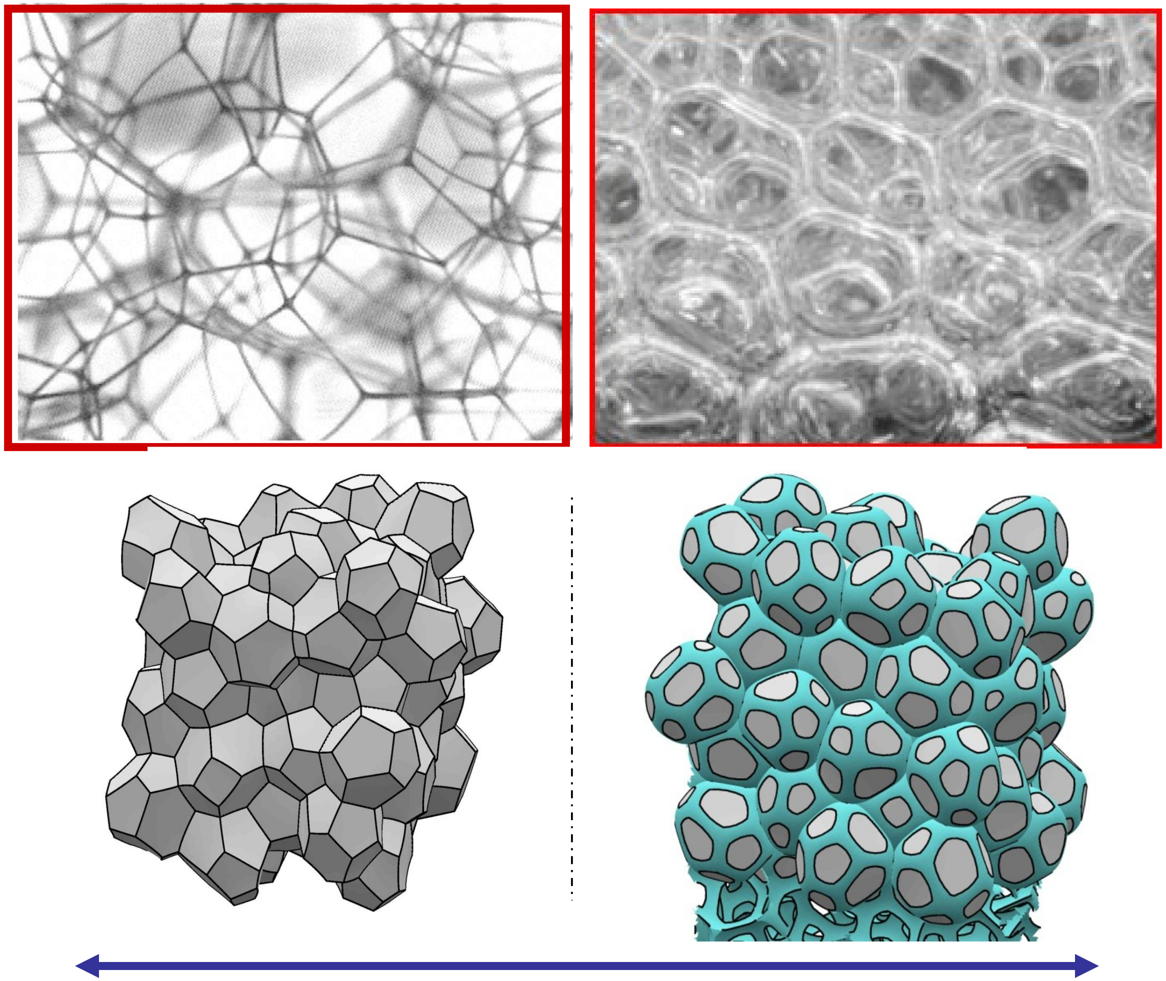
The liquid within a foam: contained in channels, linked by vertices

Liquid volume fraction

liquid fraction:

$\varepsilon = V_{liquid}/V_{foam}$

or ϕ_l



Dry foam
$\varepsilon < 0.01$

Wet foam
$\varepsilon > 0.10$

The liquid fraction dictates the degree of packing of the bubbles within the foam

Foam stability

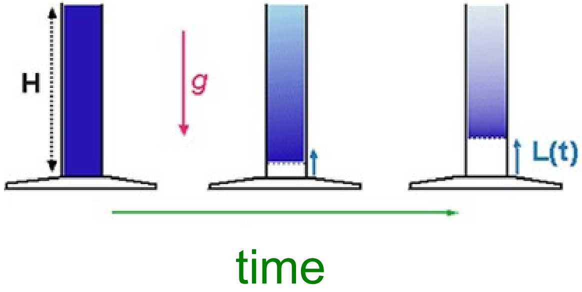
Foam = metastable system

Once formed, it ages irreversibly

DRAINAGE

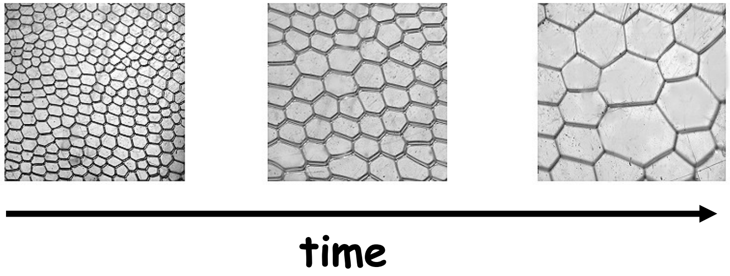
Effect of gravity

The foam dries up with time



COARSENING

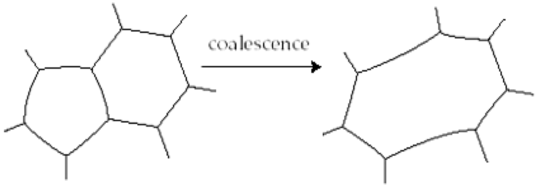
The bubbles grow with time



FILM RUPTURE

Instabilities in the soap films and coalescence

Eventual destruction of the foam



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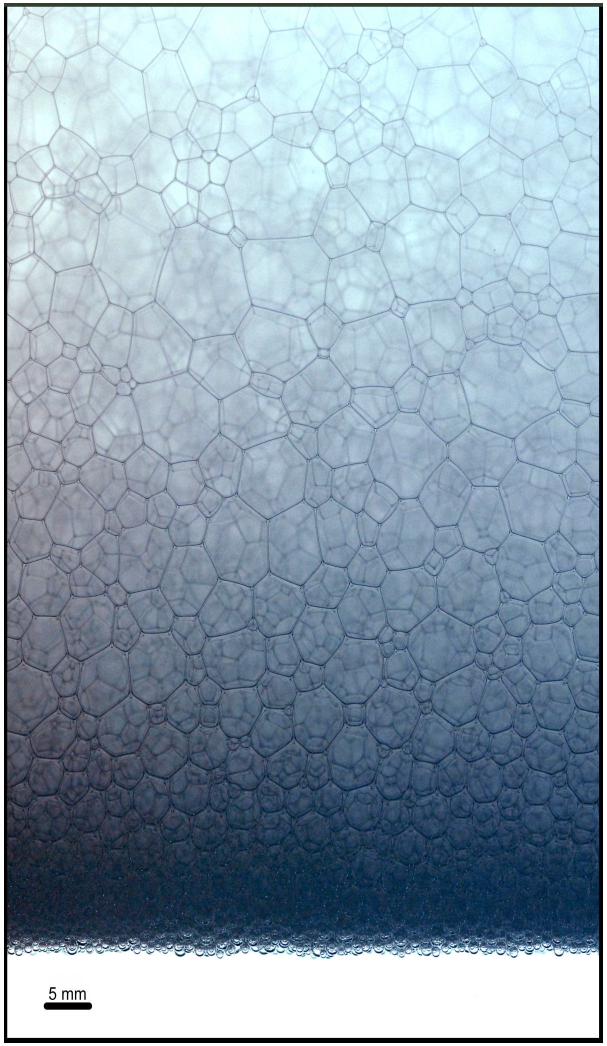
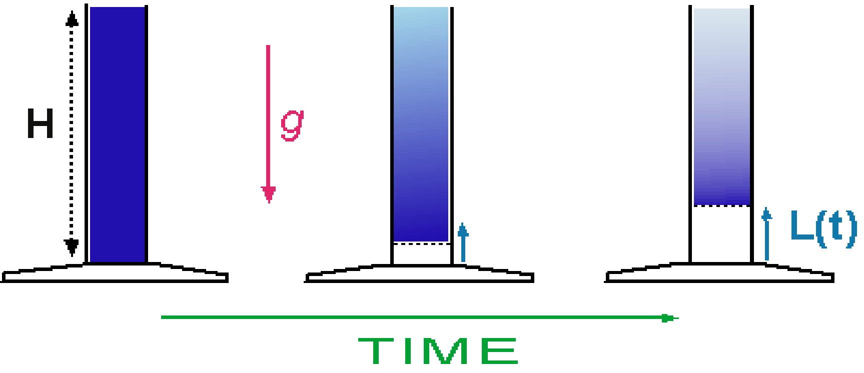


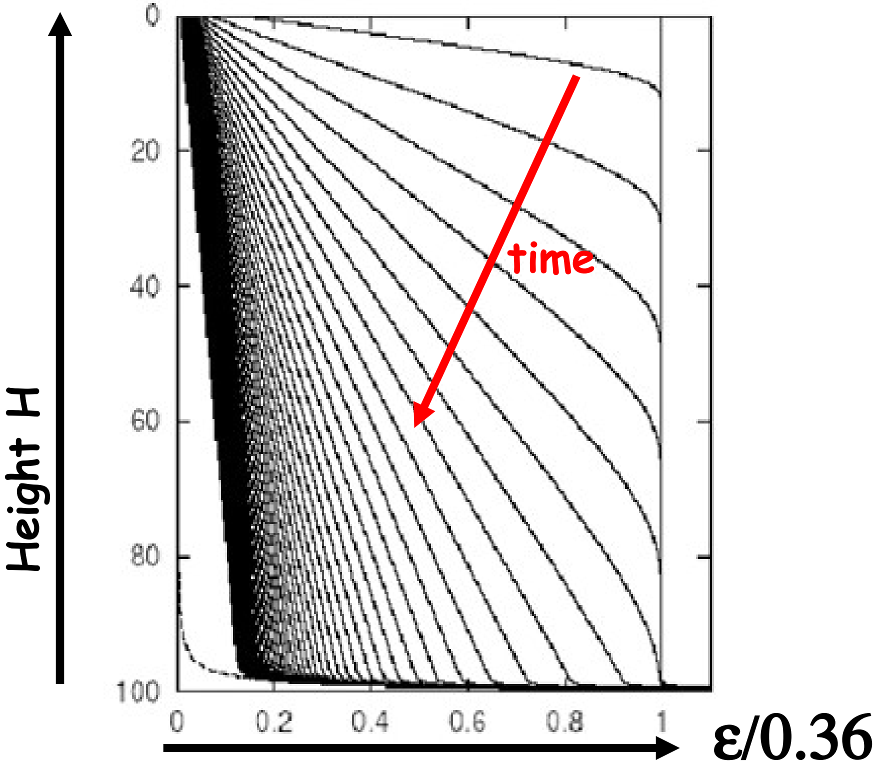
Drainage?



Effect due to gravity: gas and liquid demix

What does control the speed / the macroscopic characteristics of drainage?

Liquid fraction profile:



Drainage: theory

resembles gravity-induced flow in porous media, but:

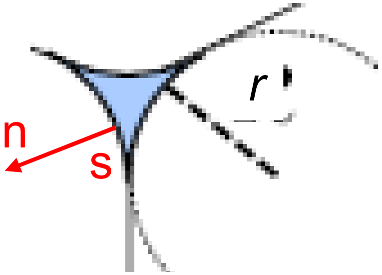
- pores are Plateau borders which section (or radius *r*) adapt to the liquid content;

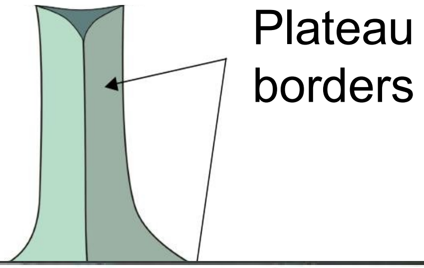
- capillary contribution *γ*/*r* to the pressure, in addition to hydrostatic pressure;

- boundaries are not solid walls.

Permeability *K* of a single Plateau border: defined as $\vec{v} \equiv \langle \vec{u} \rangle = -\frac{s}{\eta} K \vec{\nabla} p$

cross section $s = \left(\sqrt{3} - \frac{\pi}{2}\right) r^2$





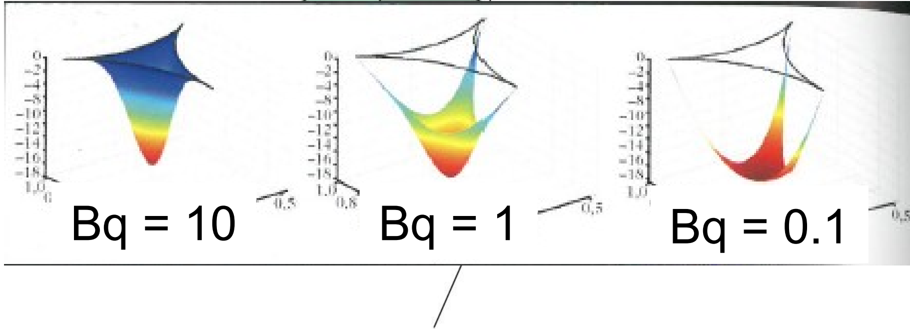
Velocity field in the Plateau border:

Stokes $\vec{\nabla} p = \eta \Delta \vec{u}$, boundary condition

$$\eta_s \frac{\partial^2 u_s}{\partial s^2} = \eta \left. \frac{\partial u}{\partial n} \right|_s$$

$\downarrow$

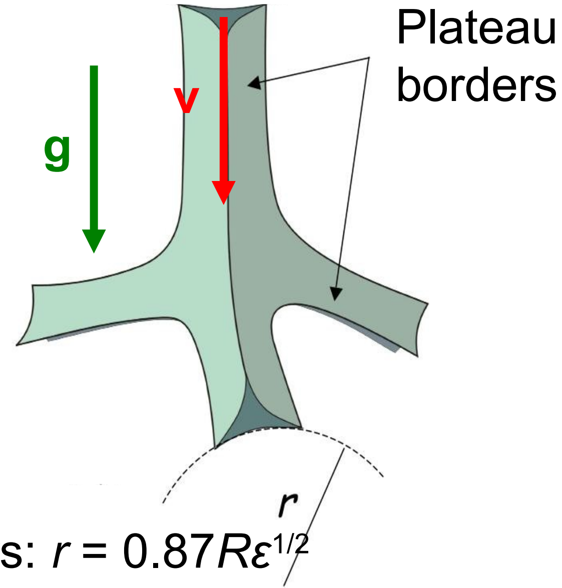
surface shear viscosity



Flow profile (hence *K*) depends on Bq = $\eta_s/\eta r$

Drainage: theory

Assumptions: constant bubble size, dry foam (viscous resistance in the Plateau borders only)



Continuity equation: $\frac{\partial \varepsilon}{\partial t} + \vec{\nabla} \cdot (\varepsilon \vec{v}) = 0$

Darcy's law: $\vec{v} = \frac{\alpha}{\eta}(\rho \vec{g} - \vec{\nabla} p_L)$

Permeability $\alpha = sK/\boxed{3}$

comes from an average over Plateau border orientations

Laplace pressure: p_L = cste - γ/r

If most of the liquid is in the Plateau borders: $r = 0.87R\varepsilon^{1/2}$

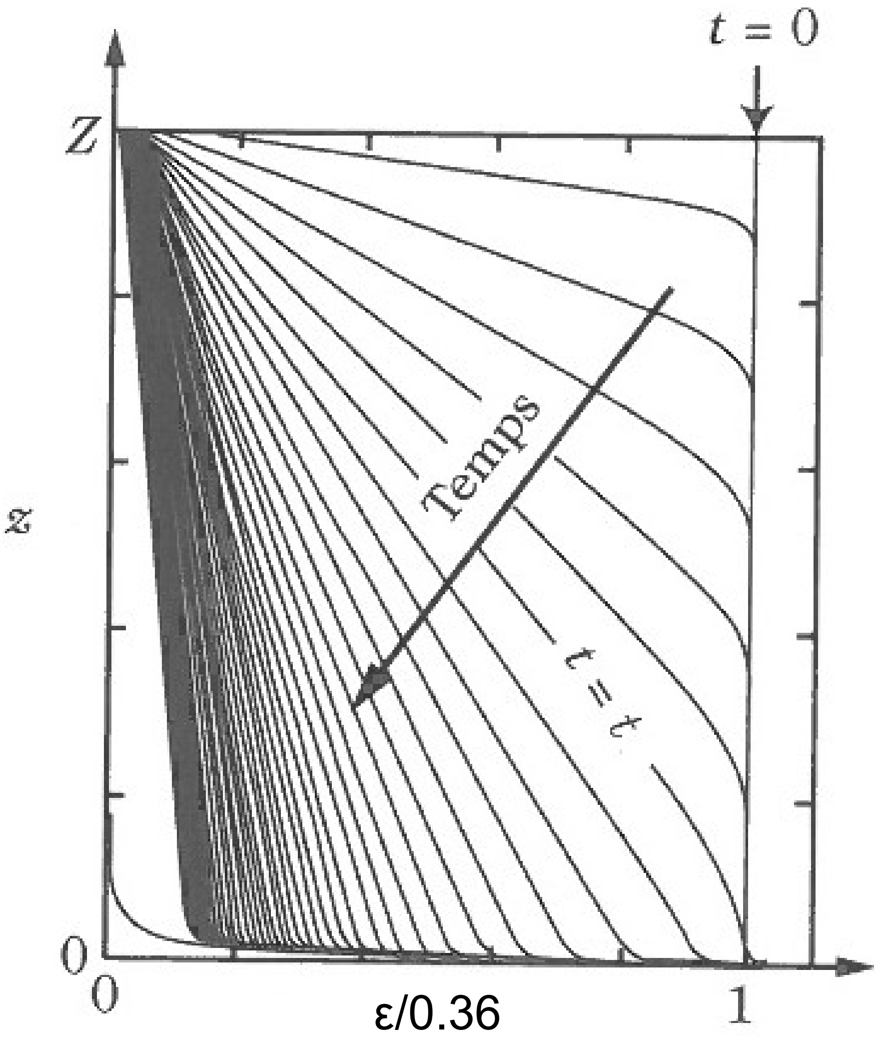
Drainage equation: $\frac{\partial \tilde{\varepsilon}}{\partial \tilde{t}} - \frac{\partial}{\partial \tilde{z}}\left(\underbrace{\boxed{\tilde{\varepsilon}^2}}_{\text{gravity}} + \underbrace{\boxed{\frac{1}{2}\sqrt{\tilde{\varepsilon}}\frac{\partial \tilde{\varepsilon}}{\partial \tilde{z}}}}_{\text{capillarity}}\right) = 0$

variables rescaled by: $z^* = (\gamma/\rho g)^{1/2}$, $t^* = 8.3\eta/KR(\rho g\gamma)^{1/2}$, $\varepsilon^* = 0.083\gamma/\rho g R$

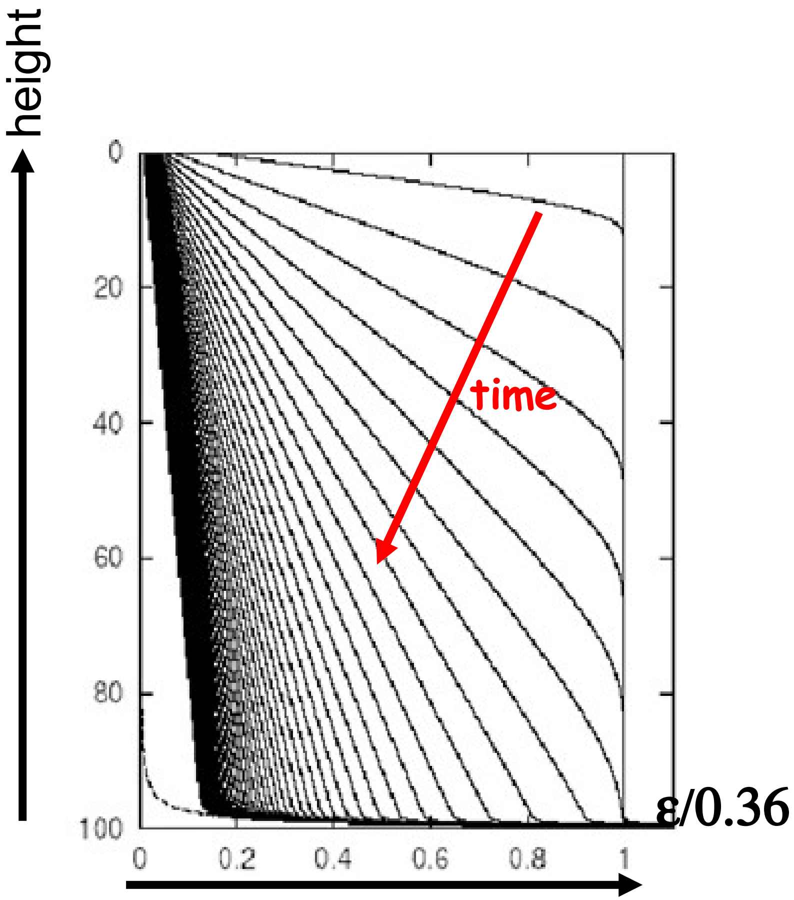
Asymptotics: see [Koehler, Hilgenfeldt & Stone, *Langmuir* (2000)]

Drainage: theory

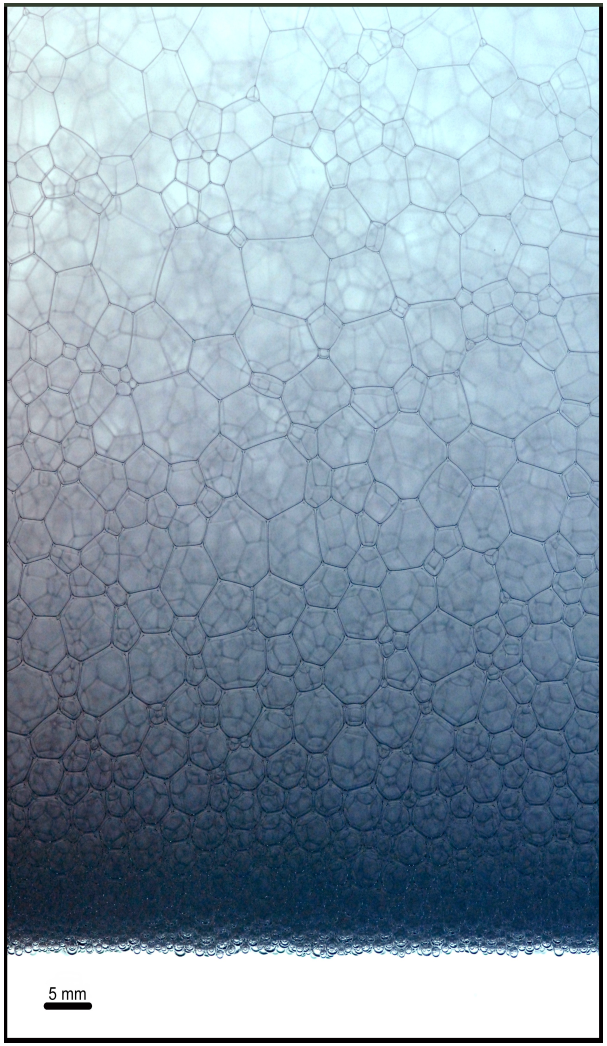
simulation from drainage equation



experiment



Capillary rise



Final state after drainage:

A slab of foam
remains wet close to the
interface with the drained out solution

capillary hold-up over a height ~ $\gamma/\rho g R$

OUTLINE

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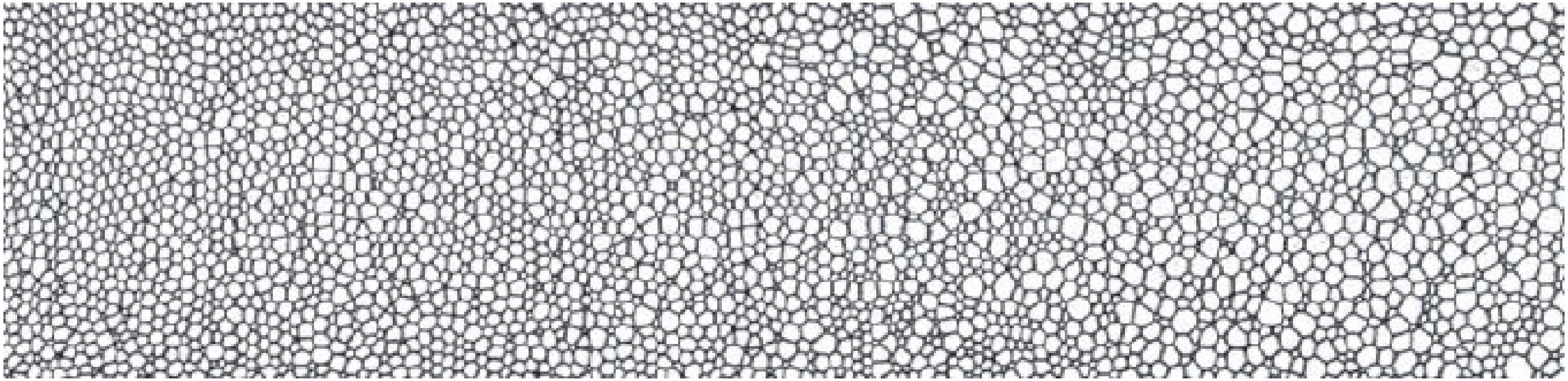
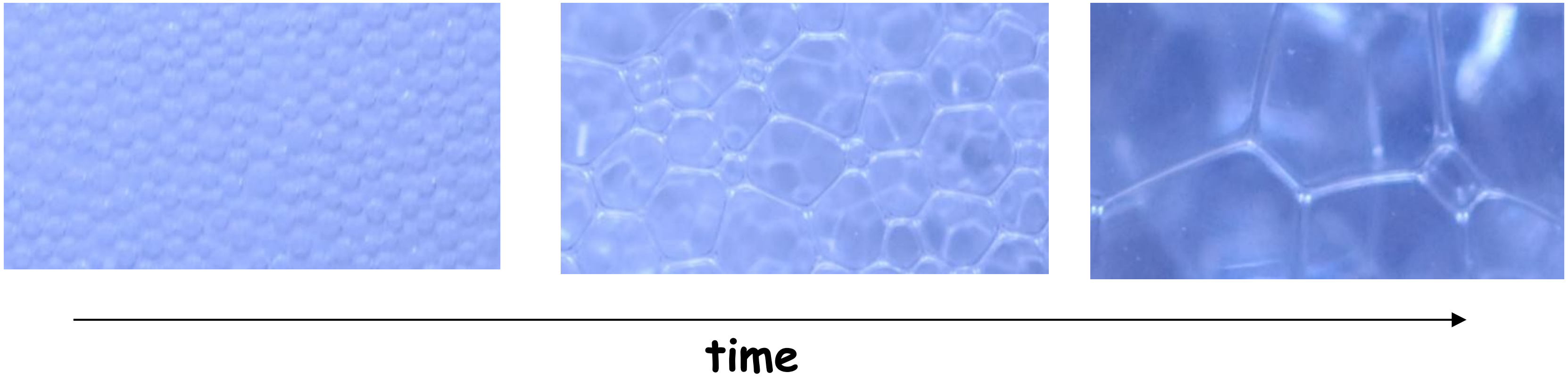
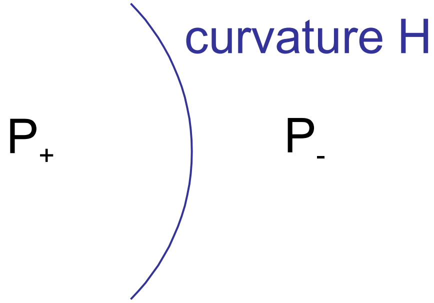


Coarsening?



Increase of the average bubble size, decrease of the total number of bubbles

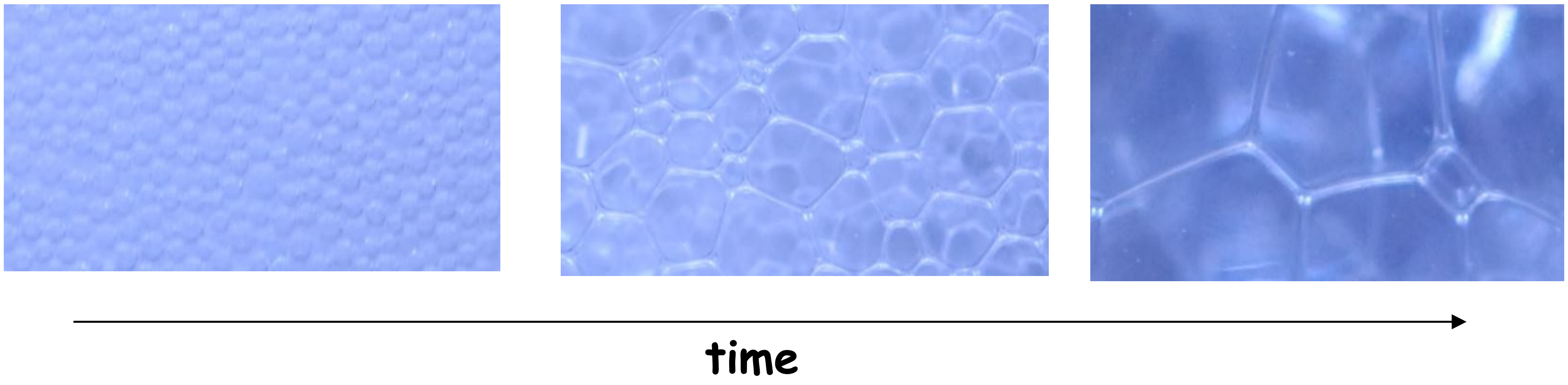
Laplace pressure: higher pressure at the convex side of a curved interface



$$P_+ - P_- = 2\gamma H$$

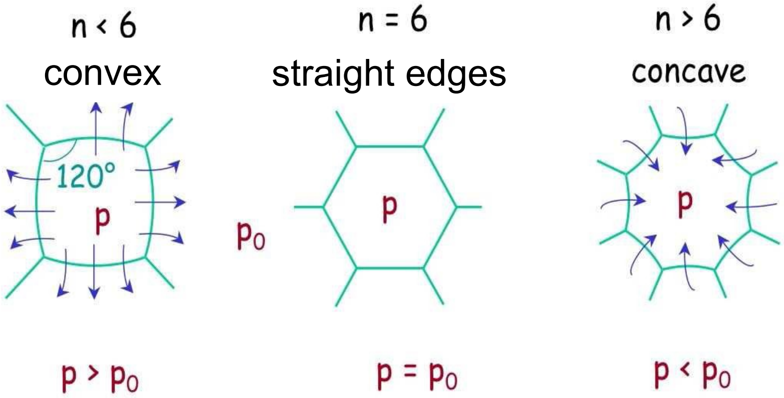
Gas thus tends to permeate from convex bubbles to concave ones

Coarsening?



Increase of the average bubble size, decrease of the total number of bubbles

Relation number of faces F/curvature/pressure difference (illustrated in 2D):



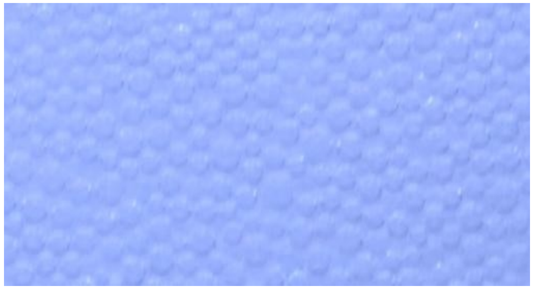
+ direct correlation between number of neighbours and bubble size

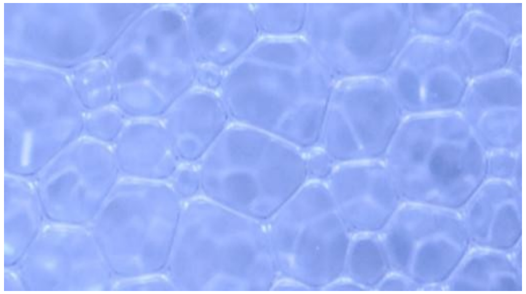
3D foam:

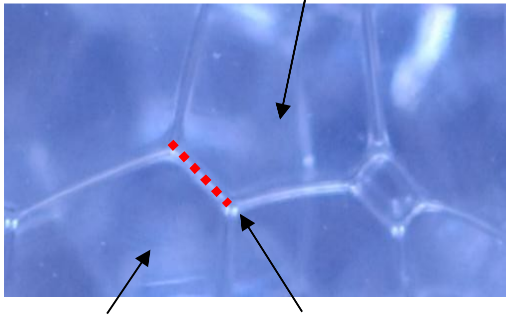
-small bubbles, F < 13: shrink and vanish

-large bubbles, F > 16: grow

Coarsening: theory







bubble *i*
pressure P_i, volume V_i

bubble *j*
pressure P_j

face *ij*
surface S_{ij}
curvature H_{ij}

Growth rate: $$\frac{\mathrm{d}V_i}{\mathrm{d}t} = -\boxed{a_1}\sum_{\langle ij\rangle}(P_i - P_j)S_{ij}$$

depends o

hence $$\frac{1}{V_i^{1/3}}\frac{\mathrm{d}V_i}{\mathrm{d}t} = -\boxed{2\gamma a_1}\boxed{\sum_{\langle ij\rangle}\frac{S_{ij}H_{ij}}{V_i^{1/3}}}$$

diffusivity

characteristic life time
$\tau(F) \sim V^{2/3}/D_{\text{eff}}q(F)$

effective diffusion constant D geometric charge q

Evolution of the number of bubbles N over a foam volume V_{tot}:

n the number of faces F
, positive for $F \leq 13$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\sum_{F=4}^{13}\frac{N(F)}{\tau(F)} = -D_{\text{eff}}\sum_{F=4}^{13}\left(Nq\langle V^{-2/3}\rangle\right)(F)$$

$$= -D_{\text{eff}}\frac{N^{5/3}}{V_{\text{tot}}^{2/3}}\underbrace{\sum_{F=4}^{13}\frac{N(F)}{N}q(F)\frac{\langle V(F)^{-2/3}\rangle}{\langle V^{-2/3}\rangle}}_{}$$

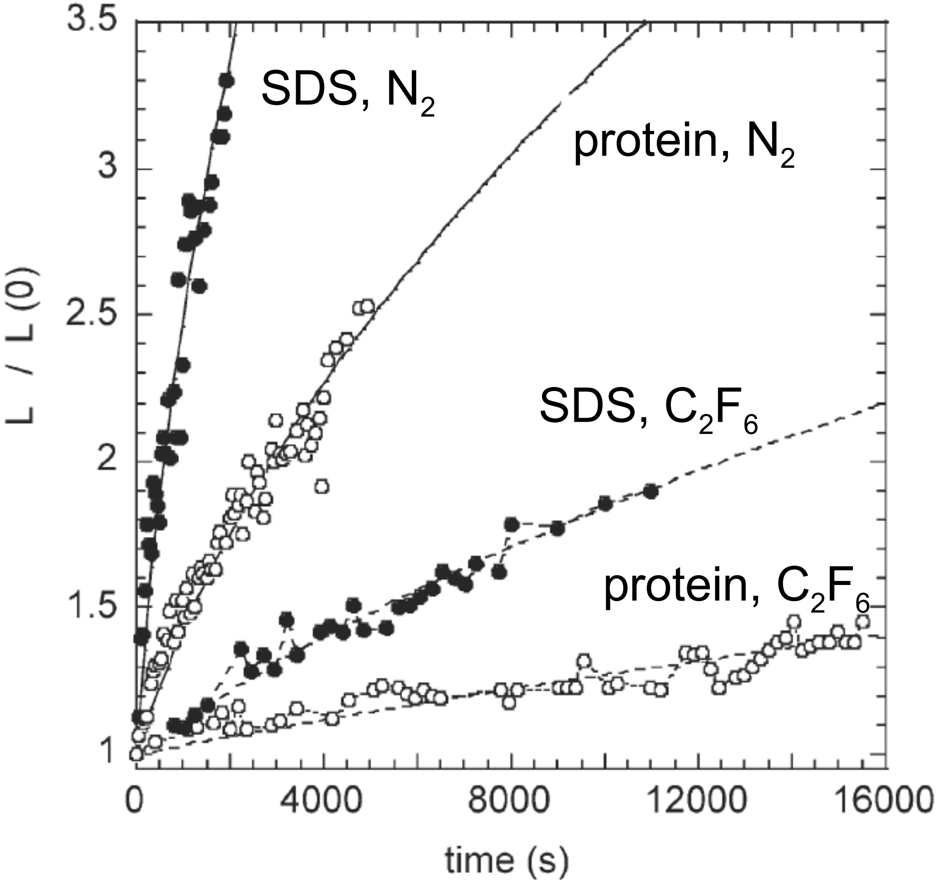
$$\Longrightarrow N(t) = N_0\left(1+\frac{t}{\boxed{\tau}}\right)^{-3/2}$$

dimensionless constant O(1),
steady at long time (self-similar coarsening)

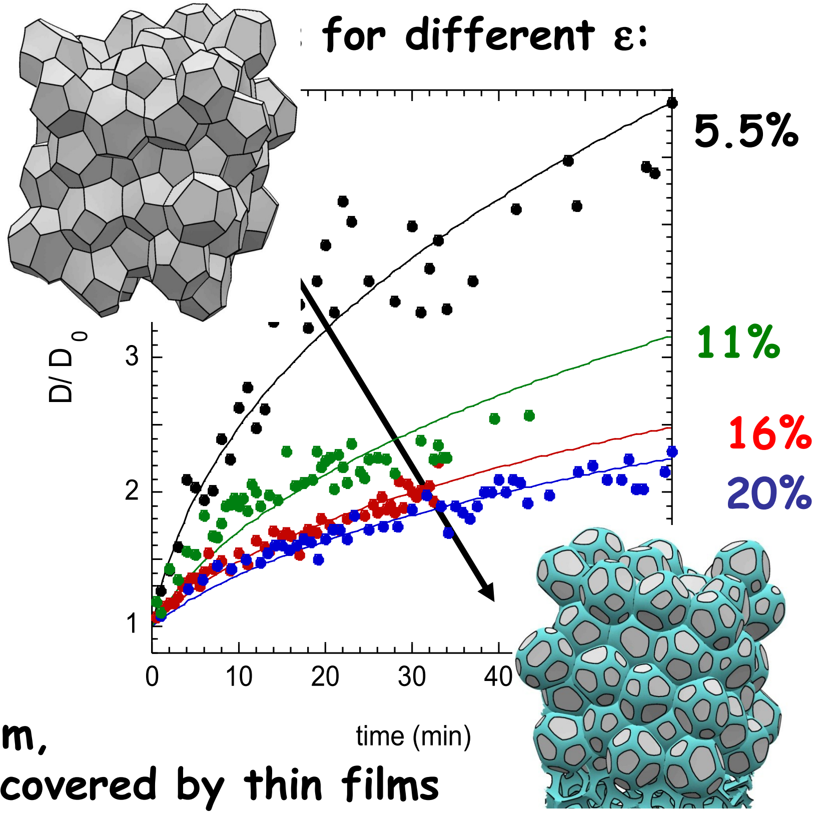
characteristic coarsening
time $\tau \sim V_0^{2/3}/D_{\text{eff}}$

Coarsening: time scales

Increase of the average diameter D: a diffusive-like process $D \sim \sqrt{1+t/\tau}$



Influence of the gas and of the surfactant (modifies film thickness)



The wetter the foam,
the less surface is covered by thin films

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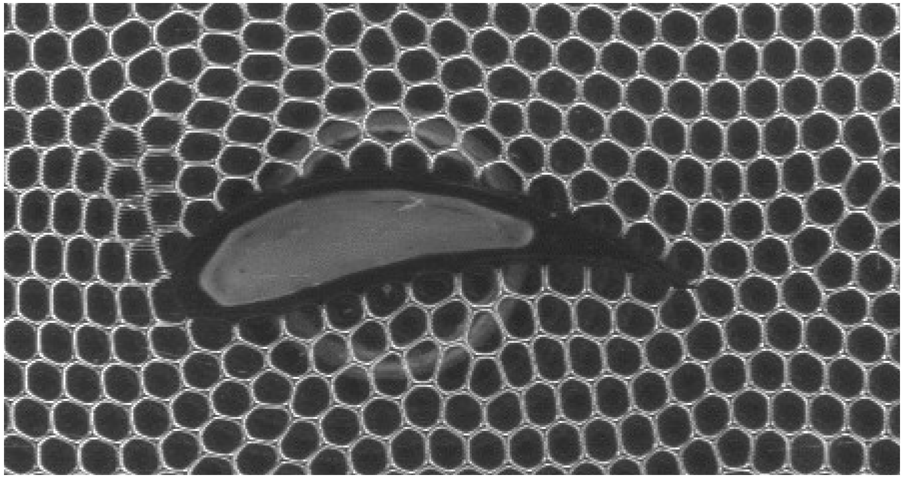
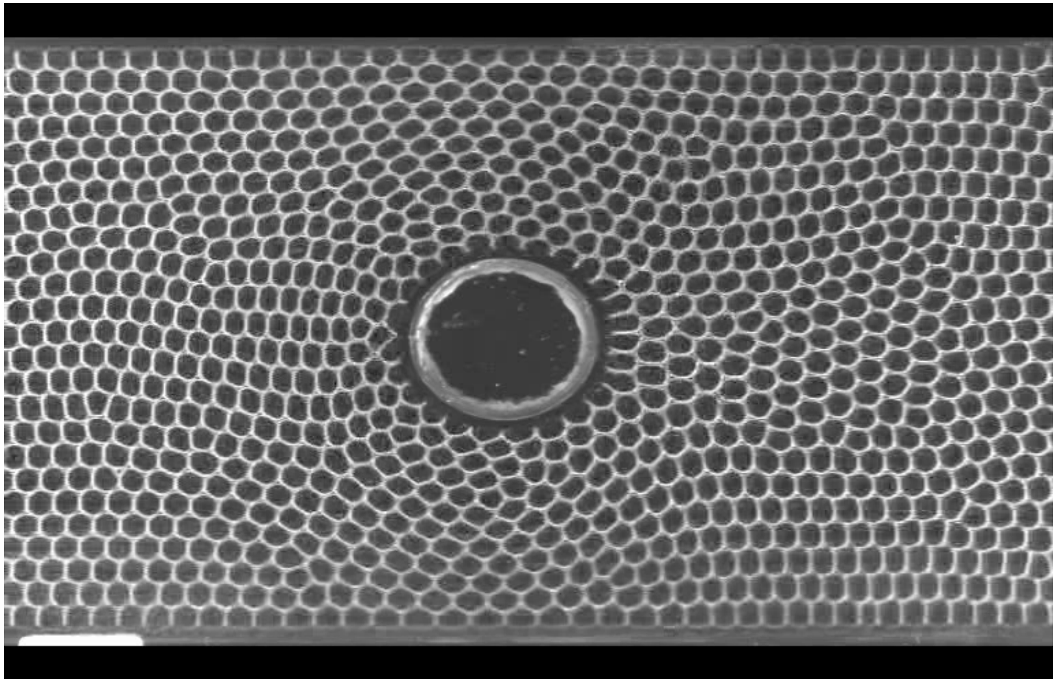


[Cohen-Addad, Höhler & Pitois, *Annu. Rev. Fluid Mech.* (2013);
Dollet & Raufaste, *C. R. Physique* (2014)]

- acoustics

RHEOLOGY

A 2D example:

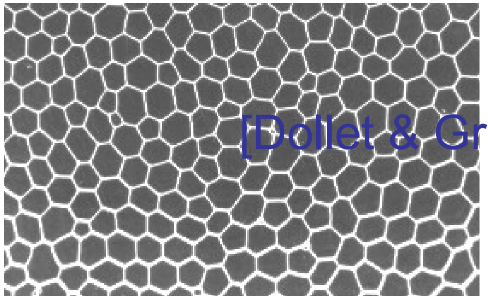


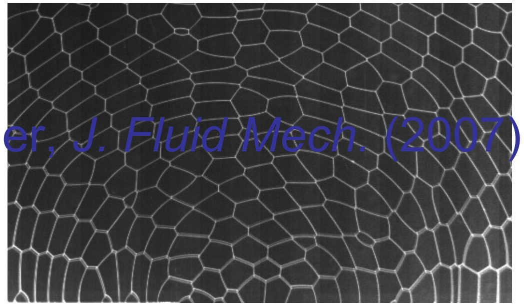
Elasticity: bubbles deform → surface energy ↑ → shear modulus ~ γ/a

~ $10\text{-}10^3$ Pa

>> bulk modulus ~ $1/P$ ~ 10^5 Pa

[Dollet & Graner, *J. Fluid Mech.* (2007); Dollet, *J. Rheol.* (2010)]





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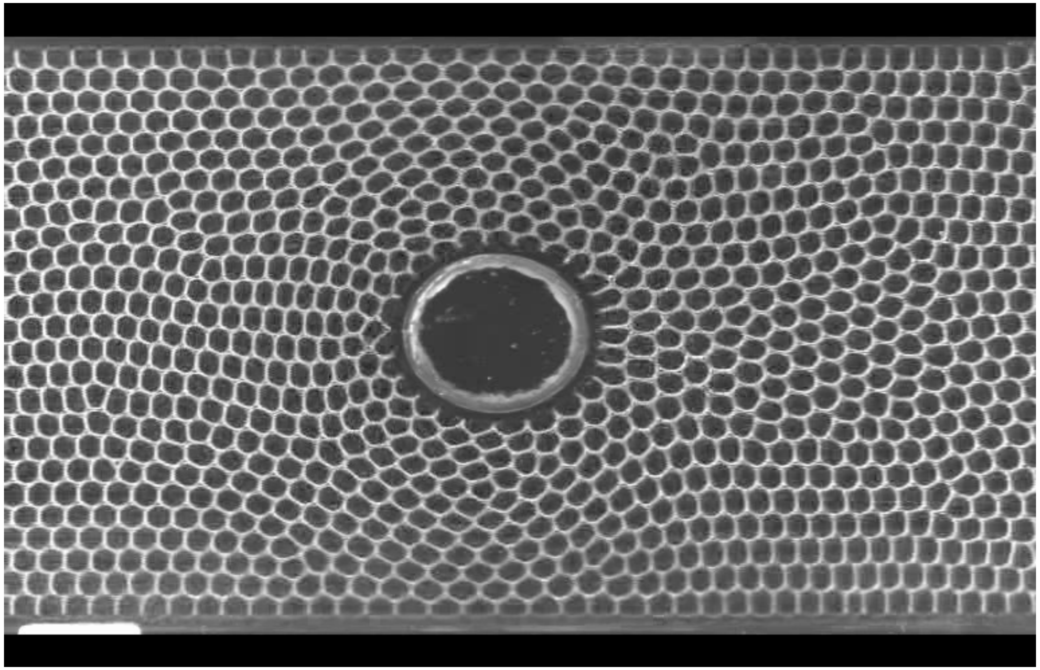
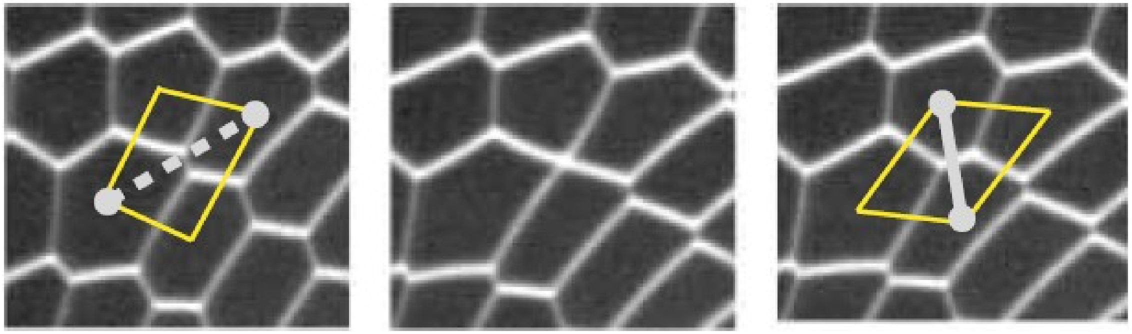


Plasticity: bubbles rearrange (T1s) → saturation of elastic stress → yield stress ~ γ/a
→ plastic energy dissipation per T1 ~ γa^2



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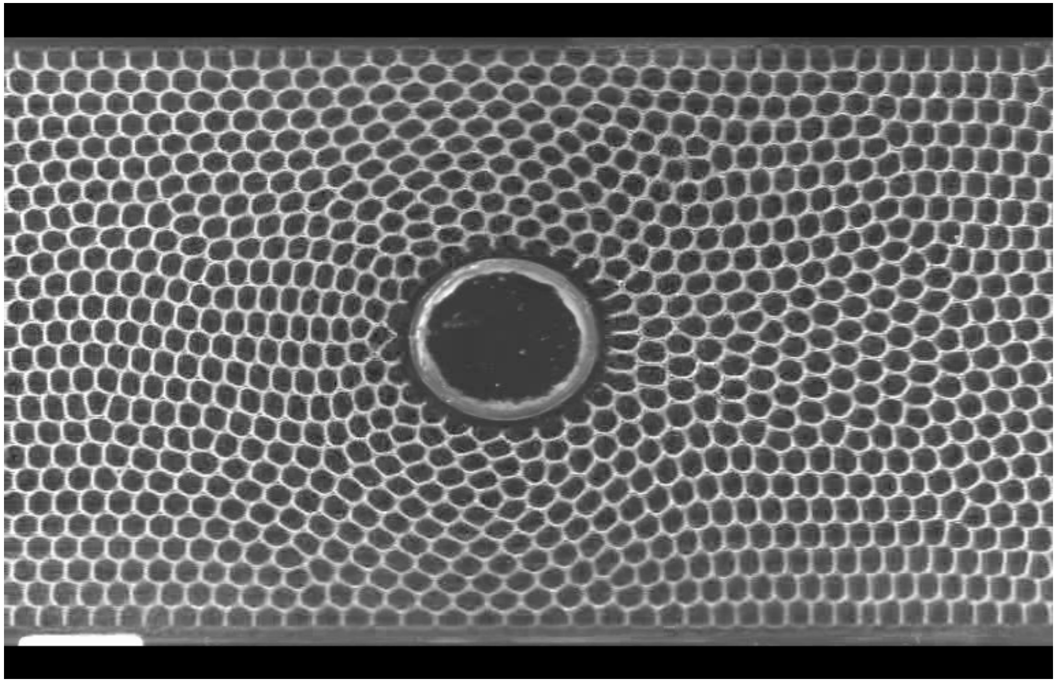

Dissipation: viscous flows in the films/Plateau borders
depends on surfactant dynamics via boundary conditions
very difficult! many pending issues

[Denkov et al., *Soft Matter* (2009);
Cohen-Addad, Höhler & Pitois, *Annu. Rev. Fluid Mech.* (2013);
Seiwert el al., *Phys. Rev. Lett*. (2013)]

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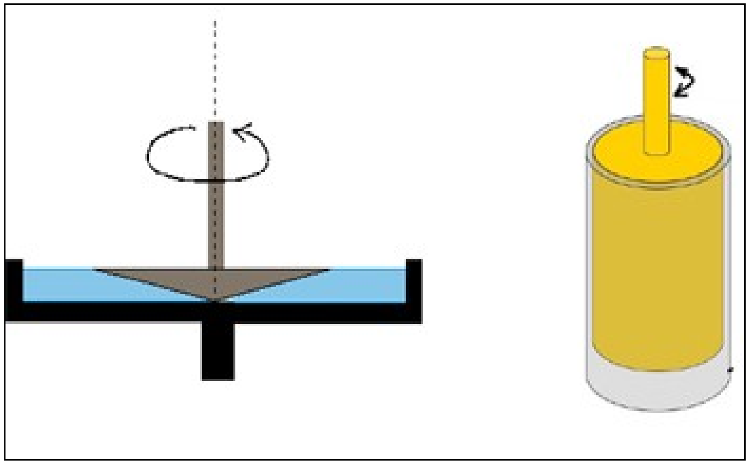
Rheometry: methods

Rheometry



Oscillatory shear: an elastic modulus *G'* and a viscous one *G''*, a yield point…

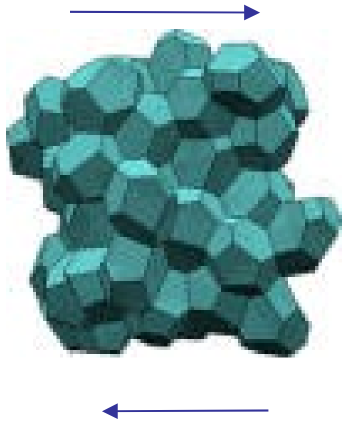
$\sigma = G^*(\omega)\,\gamma$ and $G^* = G' + iG''$

Continuous shear: shear rate, viscosity, stress

$$\sigma = f(\dot{\gamma})$$

+ creep + relaxation + …

Mechanics of foams: specificities



Foam mechanics

GAS + LIQUID = SOLID!

A visco-elasto-plastic, ageing material:

the worst case… (?)

Experimentally, care should be taken of:

wall slip

transients

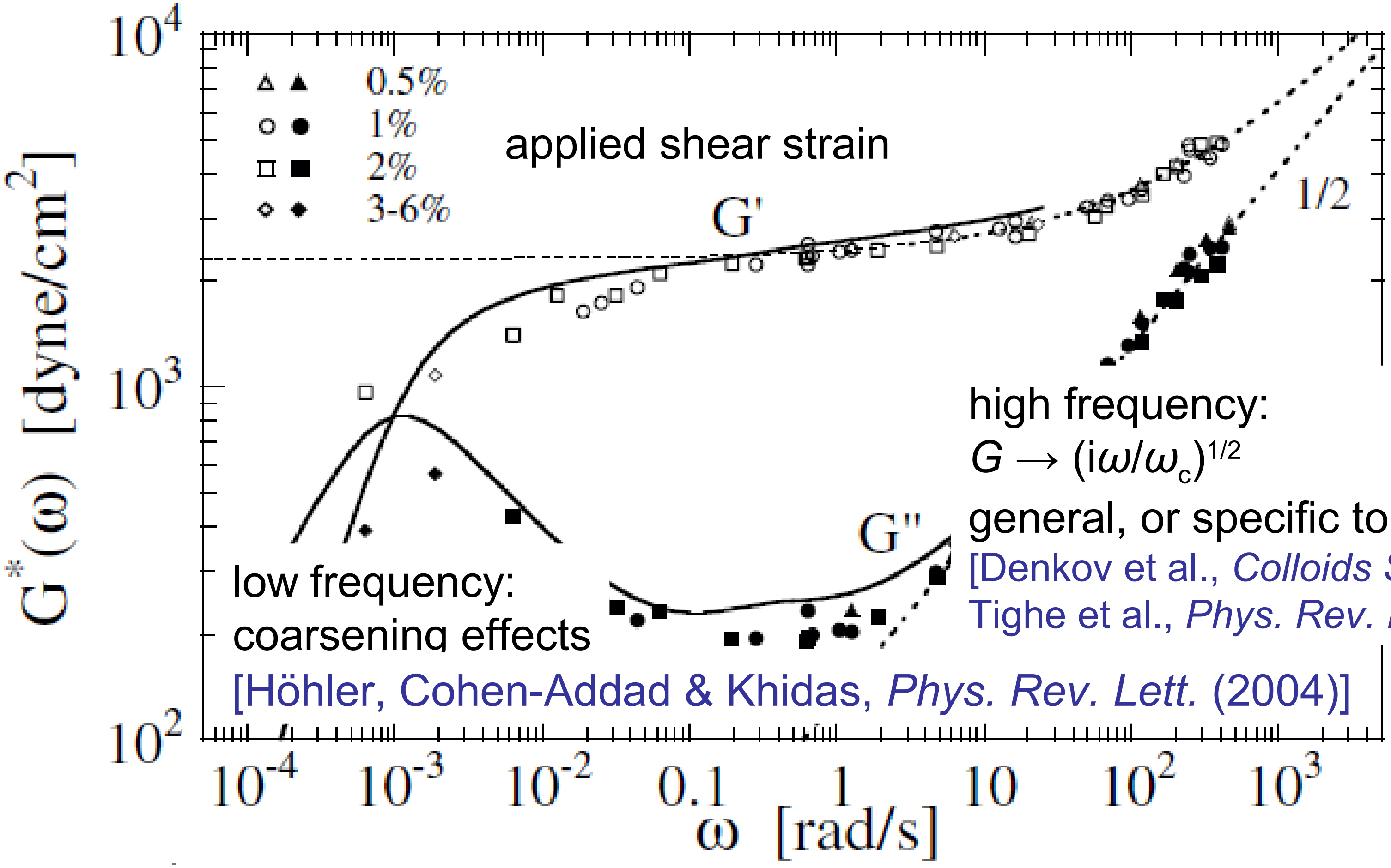
nonlinearities

measurement time

memory effects

shear localisation

Rheometry: measurements



applied shear strain

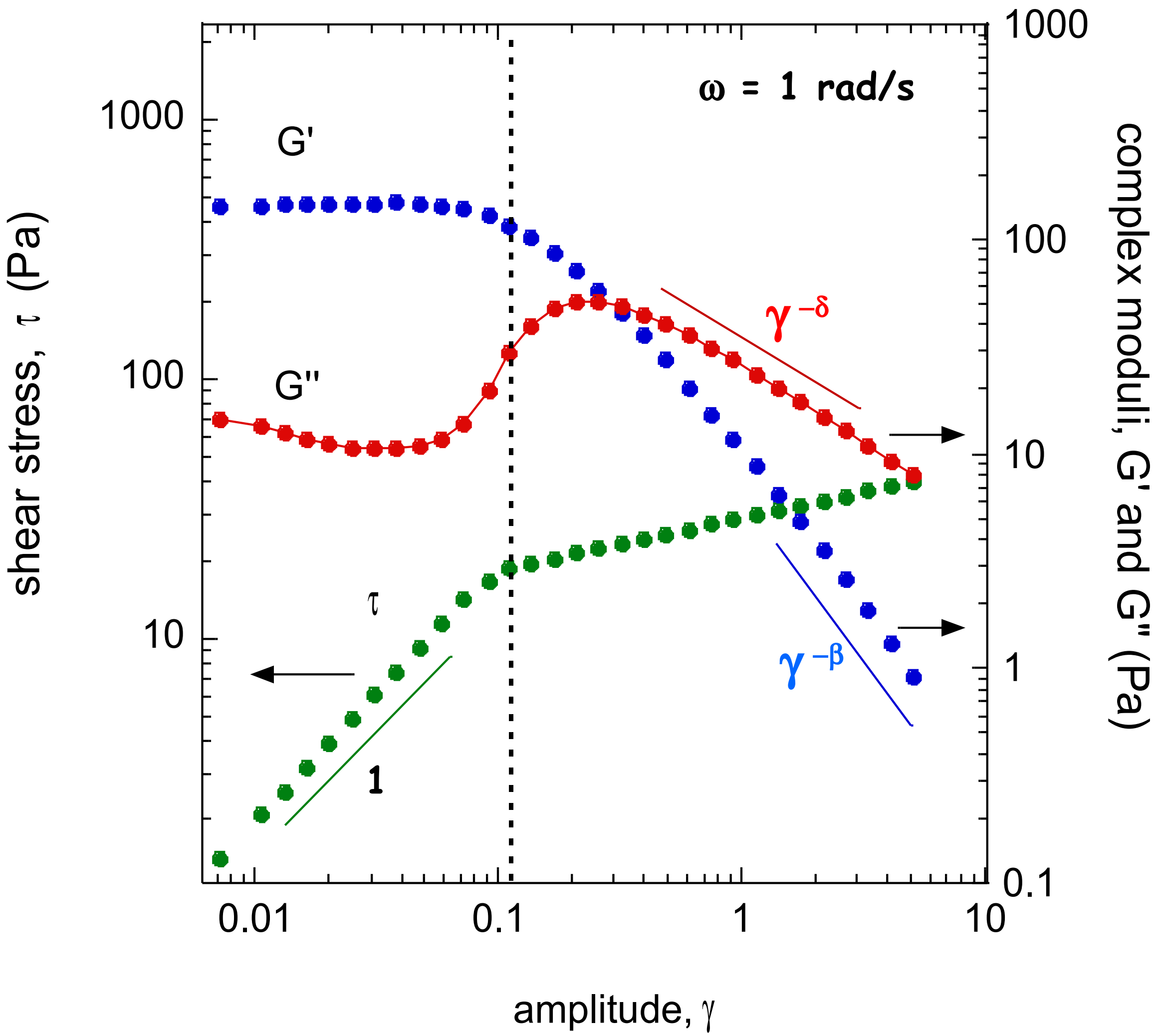
low frequency:
coarsening effects
[Höhler, Cohen-Addad & Khidas, *Phys. Rev. Lett.* (2004)]

high frequency:
$G \rightarrow (i\omega/\omega_c)^{1/2}$
general, or specific to foams?
[Denkov et al., *Colloids Surf. A* (2006);
Tighe et al., *Phys. Rev. Lett.* (2010)]

[Saint-Jalmes & Durian, *J. Rheol.* (1999);
Gopal & Durian, *Phys. Rev. Lett.* (2003);
Höhler & Cohen-Addad, *J. Phys. Condens. Matter* (2005);
Marze, Guillermic & Saint-Jalmes, *Soft Matter* (2009)]

Rheometry: measurements

Strain sweep: usual response of soft, disordered media

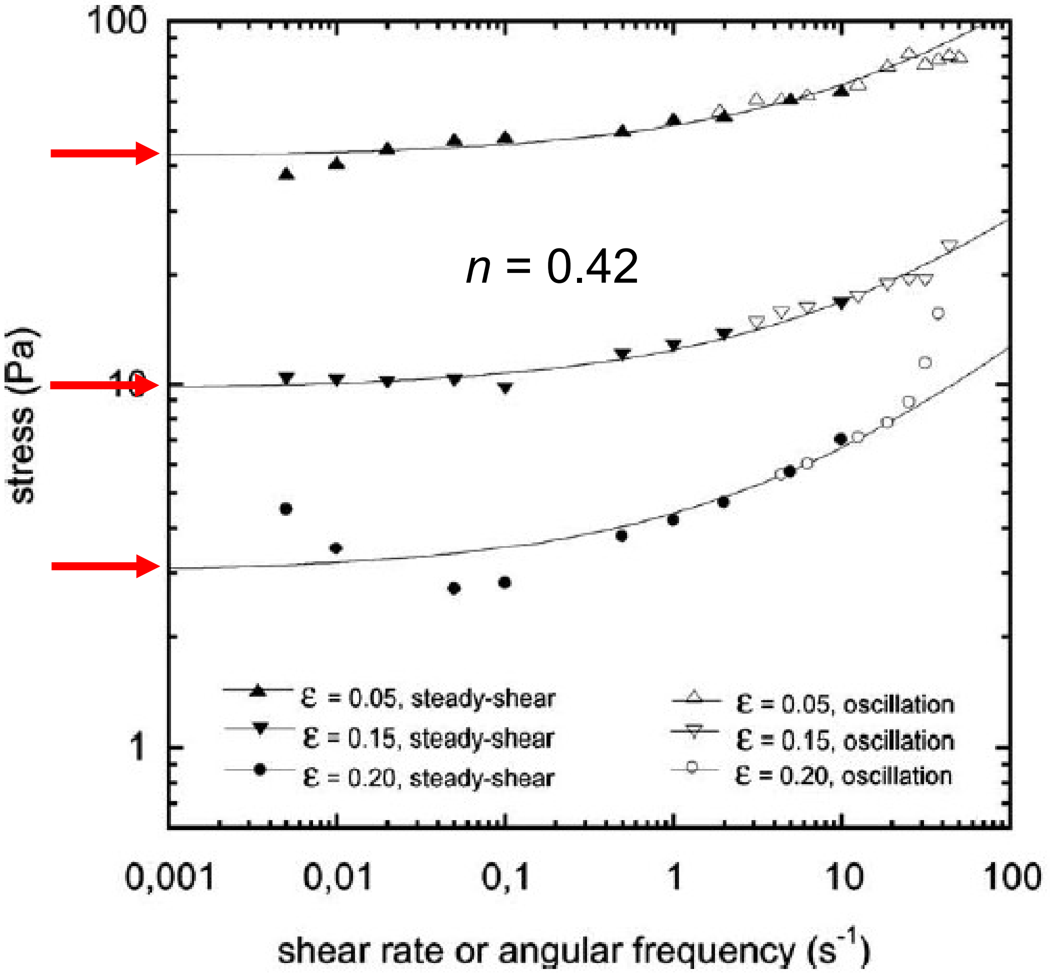


- G' > G'' at low γ
- linear stress at low γ
- yield stress and strain
- same trends for all ε

D ~ 100μm ; ε = 0.15

[Marze, Guillermic & Saint-Jalmes, *Soft Matter* (2009)]

Rheometry: measurements



- yield stress, decreasing function of the fluid fraction ε

- stress increasing with shear rate: Herschel-Bulkley fit $\sigma = \sigma_Y + (\dot{\gamma}\tau_P)^n, n \approx 1/2$

[Marze, Langevin & Saint-Jalmes, *J. Rheol.* (2008)]

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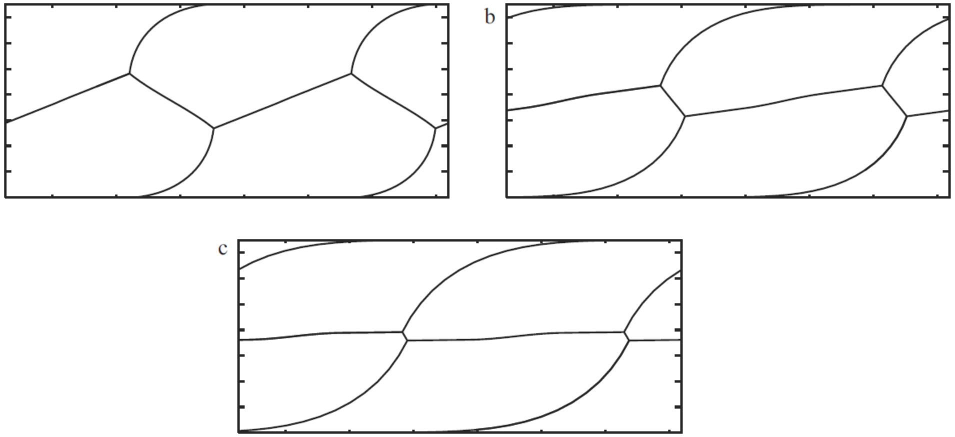
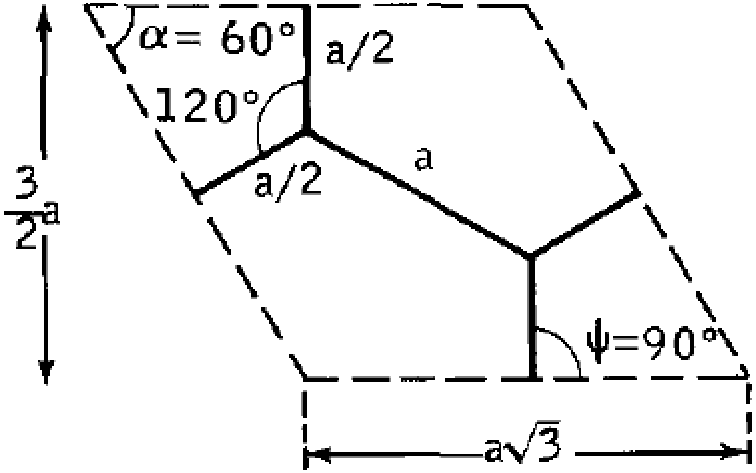


RHEOLOGY: elastoplastic micromechanics

- prediction of the shear modulus and yield stress vs. bubble size and liquid fraction
- monodisperse hexagonal foam under shear [Princen, *J. Colloid Interface Sci.* (1983)]
- unit cell:



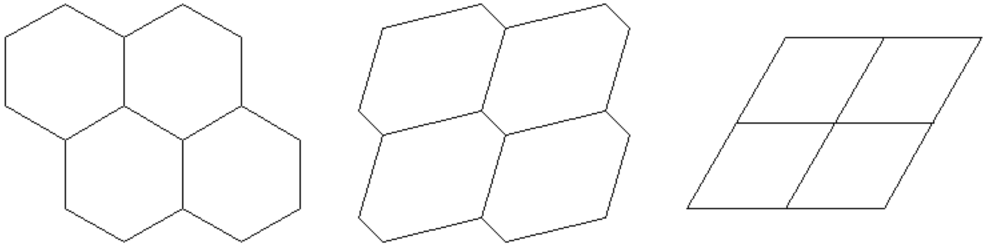
strain = $2\Delta x/3a$

- horizontal force exerted on the top boundary $F = 2\gamma \cos(\psi)$
- ψ computed from strained geometry + Plateau's 120° rule
- shear stress $\tau = F/(a\sqrt{3})$
- shear modulus = $\tau/\Delta x$ as $\Delta x \to 0$

  $= \gamma\sqrt{3}/6a$

RHEOLOGY: elastoplastic micromechanics

- yield strain and T1:

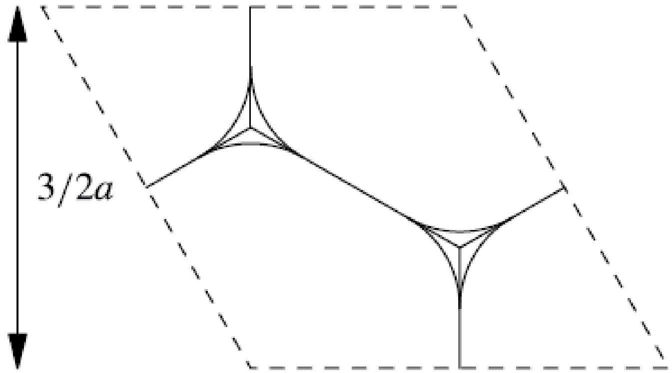


yield strain = $2/\sqrt{3}$

yield stress = $\gamma/a\sqrt{3}$

- T1: change of topology
- elastic stress relaxes
- elastic energy is dissipated (rate-independent, plastic dissipation)

- effect of liquid fraction:



- shear modulus is unaffected
- but yield stress ↓ as liquid fraction ↑
- vanishes for ϕ_l = 1 – π/2√3 = 9.3%

RHEOLOGY: origins of dissipation

- tough, and open question!

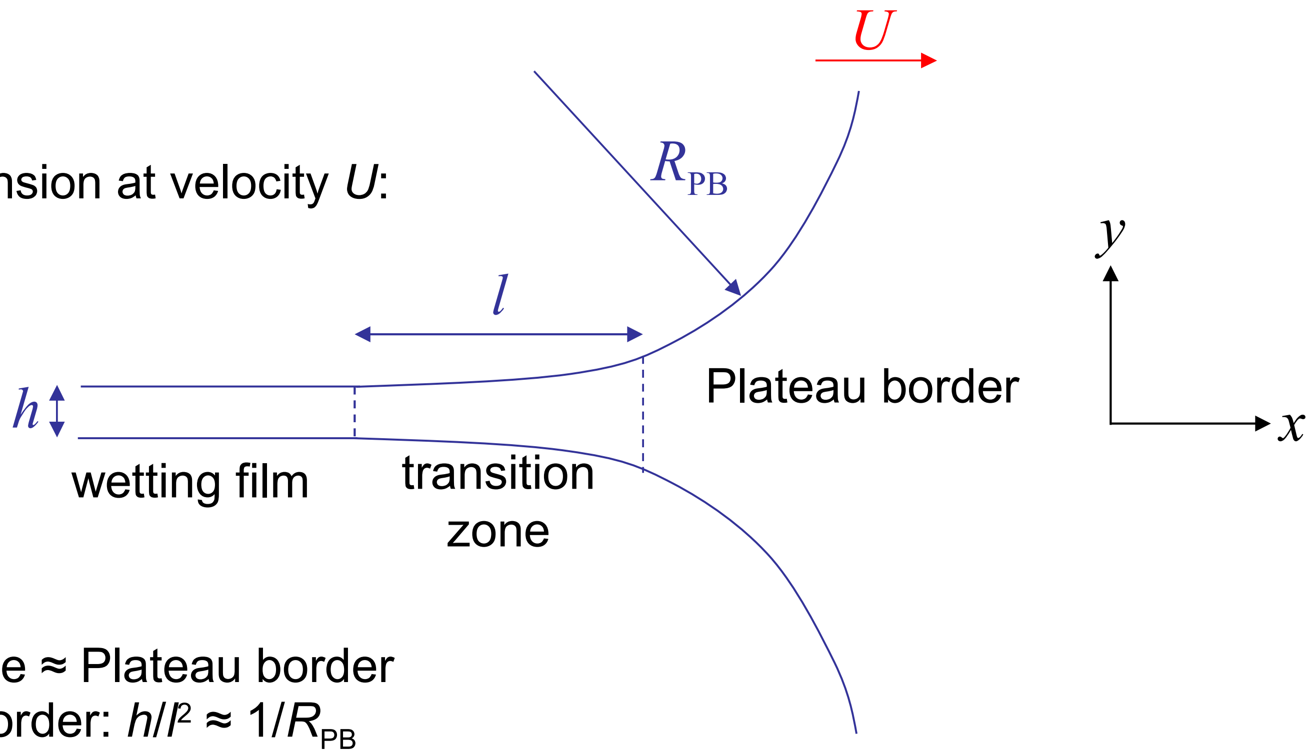
- depends on surfactant dynamics [Langevin, *Annu. Rev. Fluid Mech.* (2014)]

- two extreme cases: free shear vs. no slip boundary condition

- what happens to an extending film? (a key question, but not the only one, in sheared foams) [Seiwert et al., *Phys. Rev. Lett.* (2013); Seiwert, Dollet & Cantat, *J. Fluid Mech.* (2014)]

RHEOLOGY: origins of dissipation

Film in extension at velocity U:



- pressure: transition zone ≈ Plateau border
  curvatures of same order: $h/l^2 \approx 1/R_{\mathrm{PB}}$

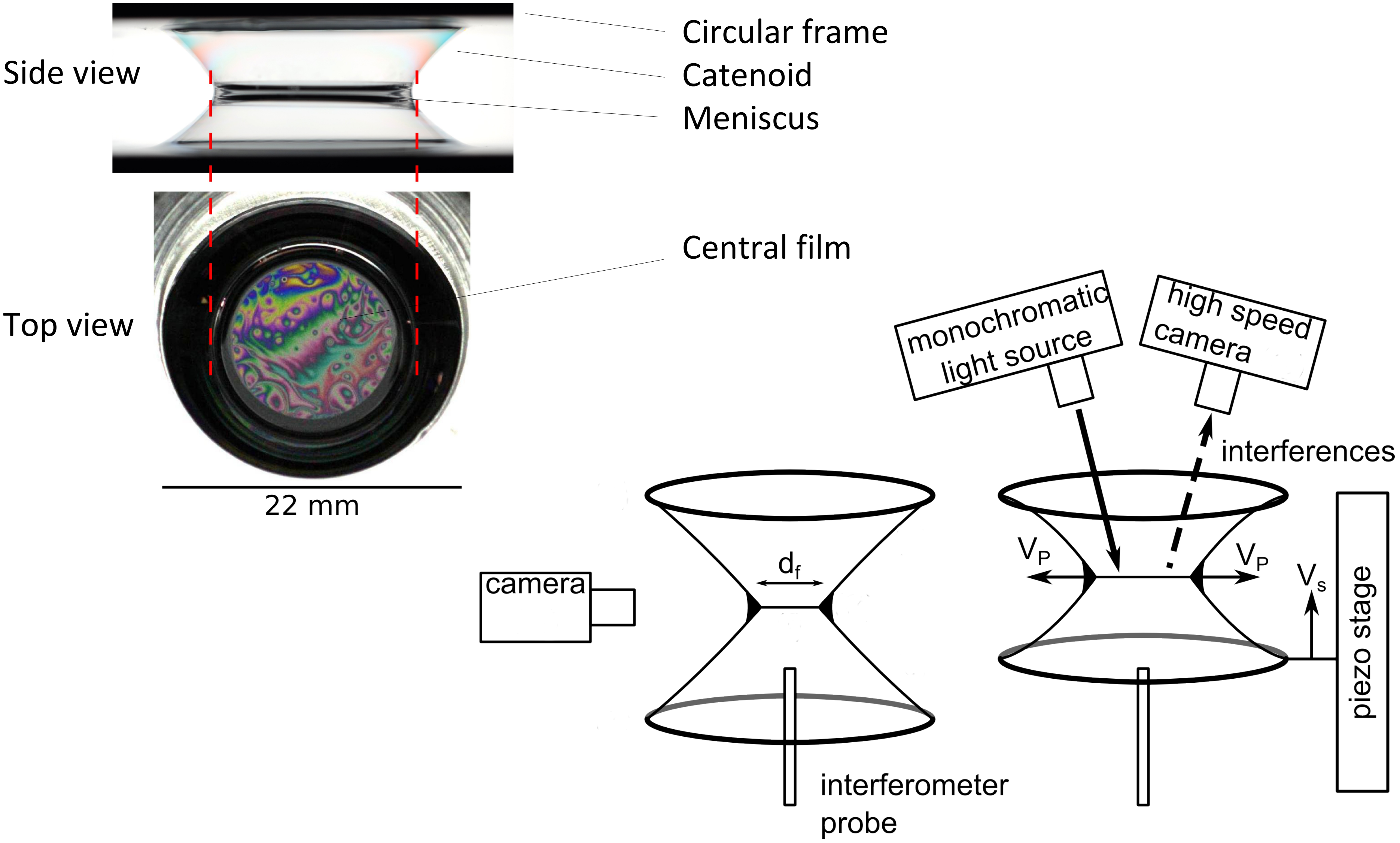
- transition zone: dictated by surface tension and viscosity
  viscous stress $\eta\Delta\mathbf{v} \approx$ Laplace pressure gradient $\mathrm{d}p_L/\mathrm{d}x$ with $p_L \approx \gamma h/l^2$
  $\rightarrow \eta U/h^2 \approx \gamma h/l^3$

- scalings: $h \approx R_{\mathrm{PB}}\mathrm{Ca}^{2/3}$ (Frankel law), $l \approx R_{\mathrm{PB}}\mathrm{Ca}^{1/3}$ with $\mathrm{Ca} = \eta U/\gamma$ the capillary number

$$\sigma_v(\dot{\gamma}) \approx (\gamma/a)^{1/3}(\eta\dot{\gamma})^{2/3}$$

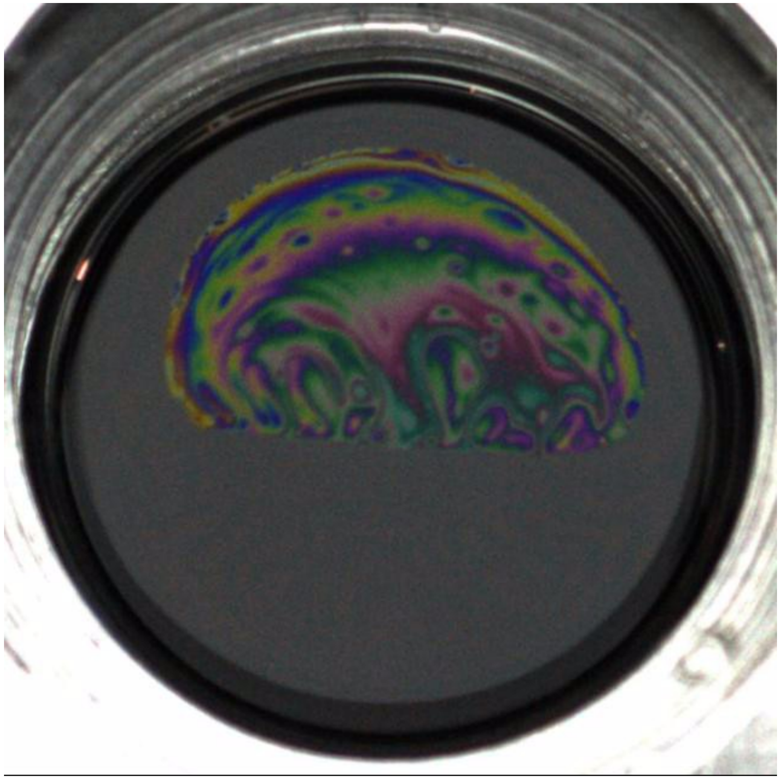
- leads to a prediction for the viscous stress

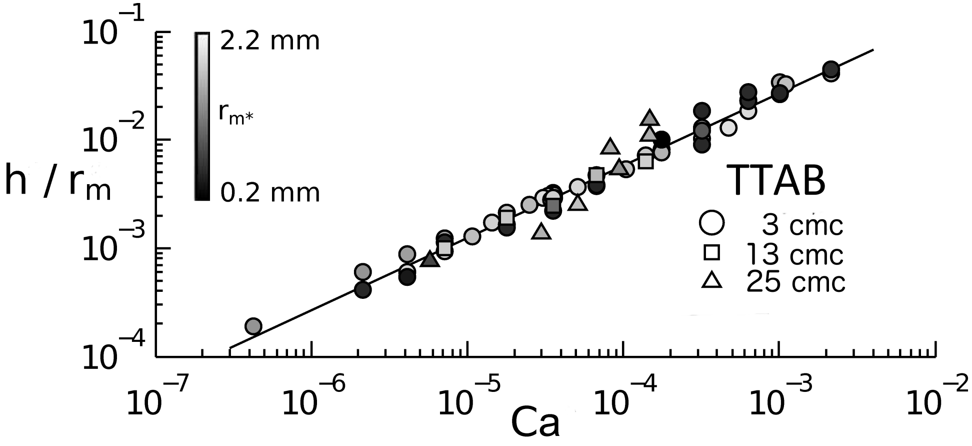
Film extension



RHEOLOGY: origins of dissipation

- what happens on a single film in extension [Seiwert et al., *Phys. Rev. Lett.* (2013); Seiwert, Dollet & Cantat, *J. Fluid Mech*. (2014)]:



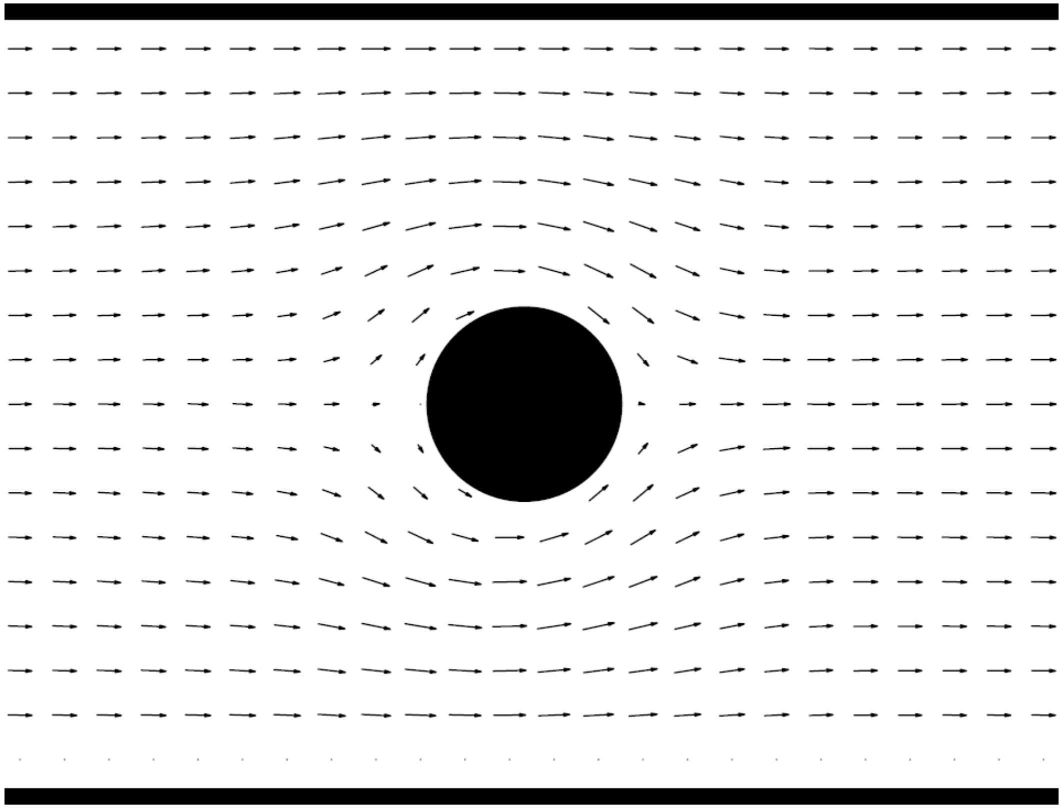


Frankel law perfectly recovered

- but there are MANY other complications and sources of dissipation in a sheared foam, still a lot of pending issues…

OUTLINE

- structure
- drainage
- coarsening
- **rheology**
    - rheometric data
    - micromechanical models
    - **flow profiles**
- acoustics



RHEOLOGY: local aspects

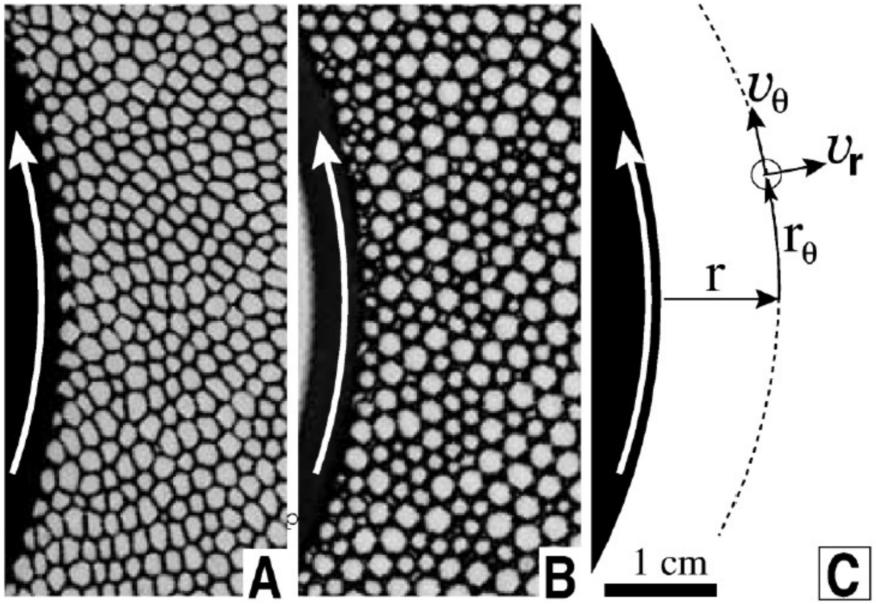
Limitations of macroscopic rheometry: is the shear rate uniform? Is the shear stress vs. shear rate law representative of the mechanical response?

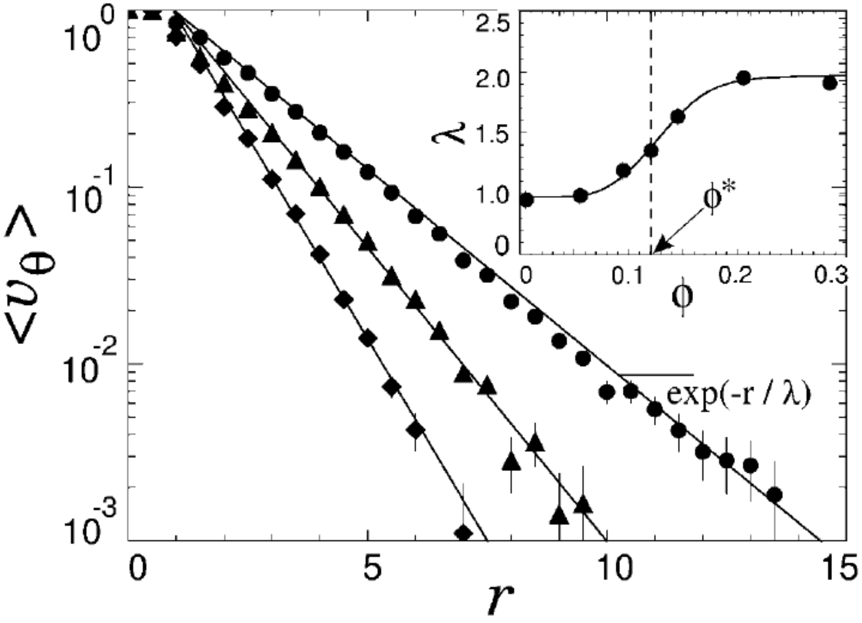
Problematics of shear localisation [Schall & van Hecke, *Annu. Rev. Fluid Mech.* (2010)], confinement/nonlocality [Goyon et al., *Nature* (2008)]…

Necessary to measure local information: advantage of 2D foams

RHEOLOGY: local aspects

- shear localisation [Debrégeas, Tabuteau & di Meglio, *Phys. Rev. Lett*. (2001)]





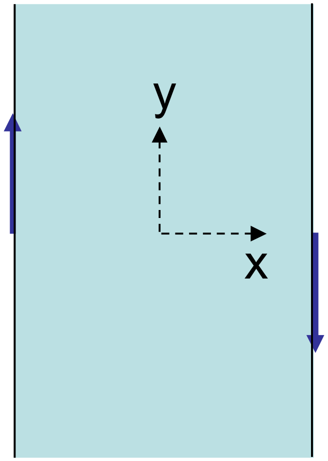
- velocity profile decays exponentially close to the inner rotating cylinder

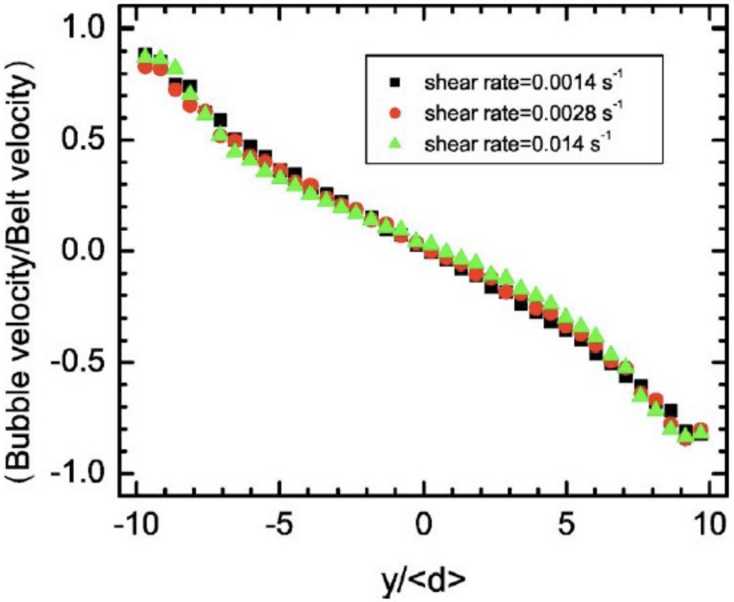
- stress in non-homogeneous → go to plane Couette geometry

$\vec{u} = u(y)\vec{e}_x$: shear stress constant across the Couette cell (if no wall friction)

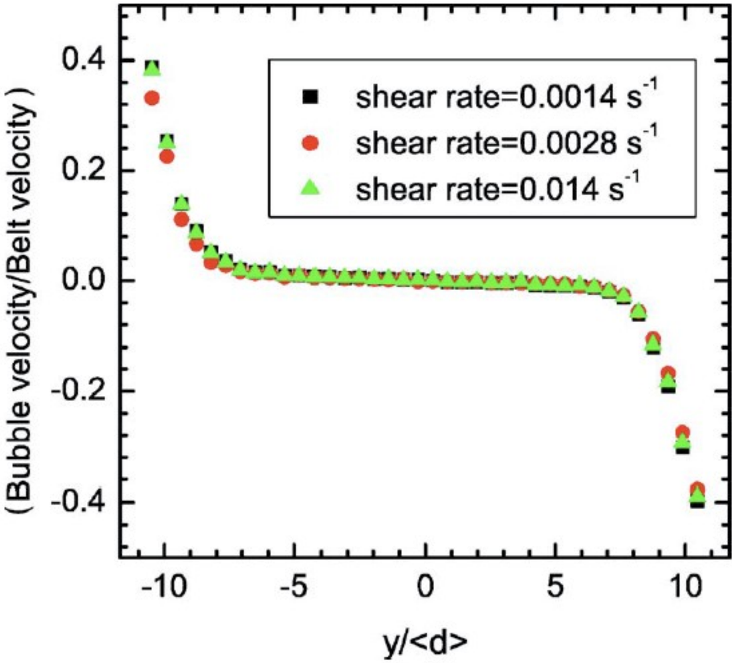
RHEOLOGY: local aspects

- shear localisation depends on foam/wall friction [Wang, Krishan & Dennin, *Phys. Rev. E* (2006)]





without top/bottom walls (bubble raft): no shear localisation

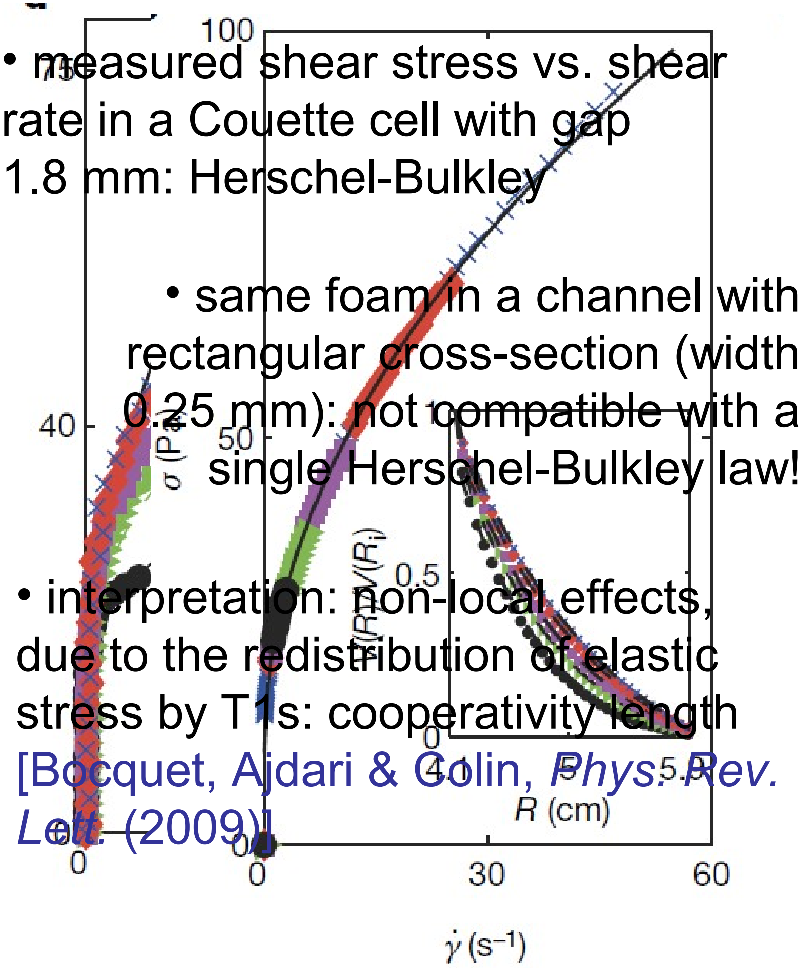


with top/bottom walls: shear localisation, rate dependence [Katgert, Möbius & van Hecke, *Phys. Rev. Lett.* (2008)]

- so unconfined foams seem well-behaved. But…

RHEOLOGY: local aspects

- breakdown of the relationship $\sigma(\dot{\gamma})$ in a Poiseuille channel flow of an emulsion [Goyon et al., *Nature* (2008); *Soft Matter* (2010)]



- measured shear stress vs. shear rate in a Couette cell with gap 1.8 mm: Herschel-Bulkley

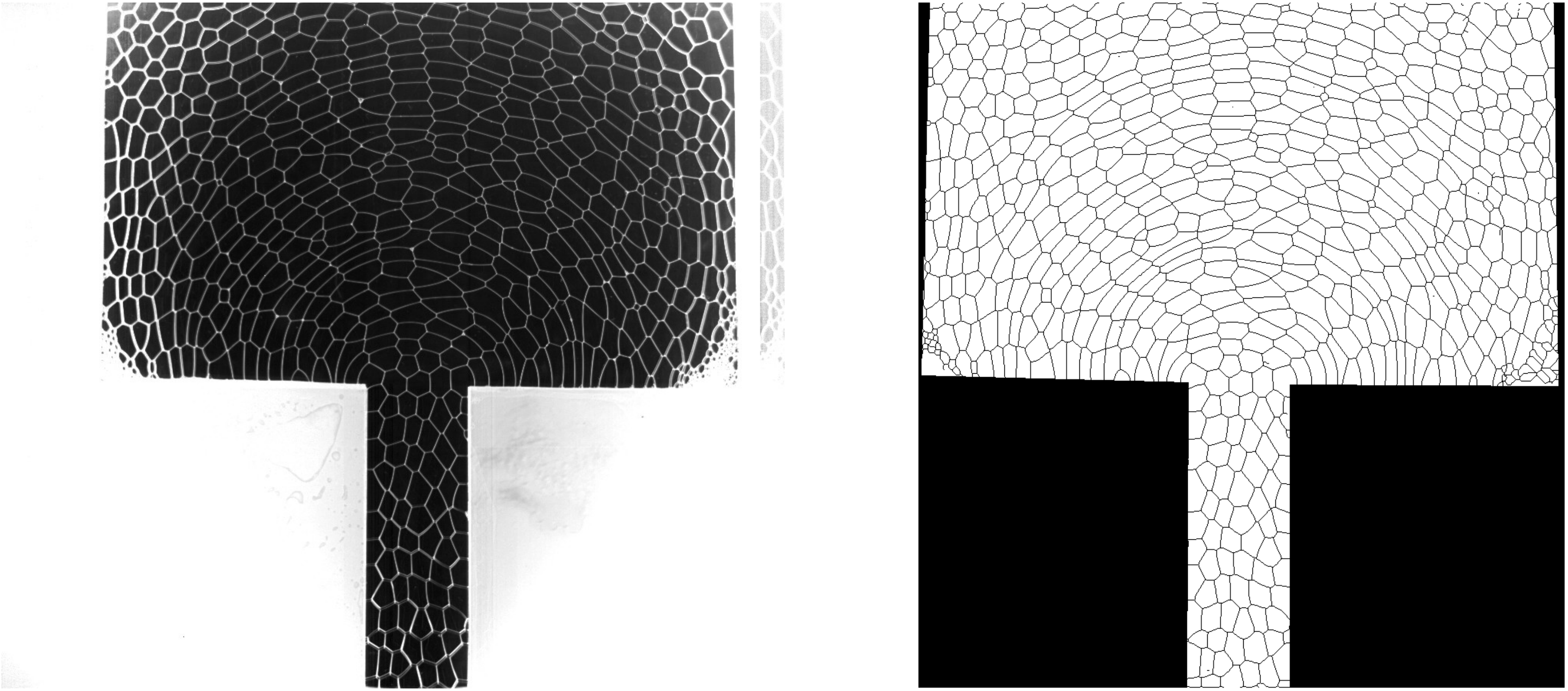
- same foam in a channel with rectangular cross-section (width 0.25 mm): not compatible with a single Herschel-Bulkley law!

- interpretation: non-local effects, due to the redistribution of elastic stress by T1s: cooperativity length [Bocquet, Ajdari & Colin, *Phys. Rev. Lett.* (2009)]

- for foams: ask Andrea [Dollet, Scagliarini & Sbragaglia, *J. Fluid Mech*. (2015)]

LOCAL ANALYSIS: principle

Image analysis: threshold and skeletonisation



- individual tracking of bubbles: velocity and T1 fields
- tracking of bubble edges: elastic stress field

LOCAL ANALYSIS: velocity

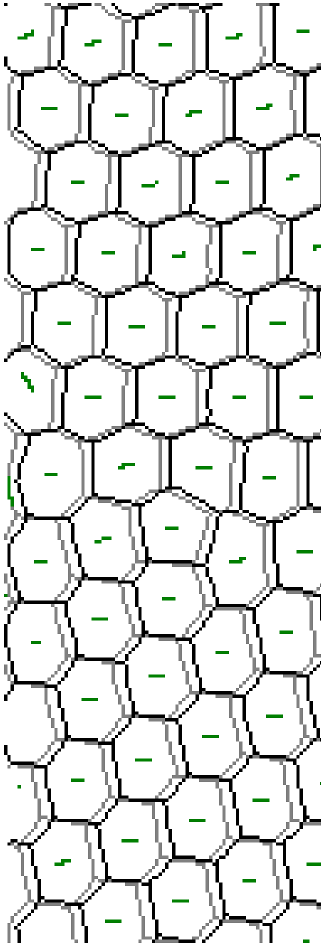


image *n*

displacement of each bubble centre

image *n*+1

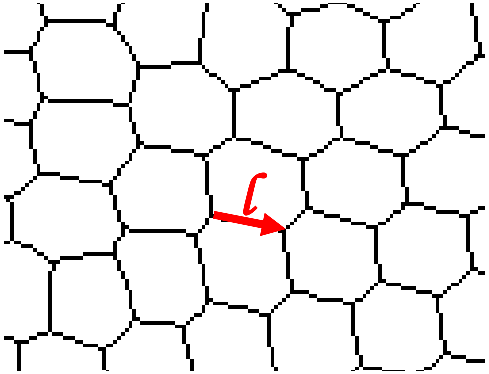
averaged in time and per box:

velocity field

+ neighbour swapping: plastic event, T1

LOCAL ANALYSIS: elastic stress

2D elastic stress tensor



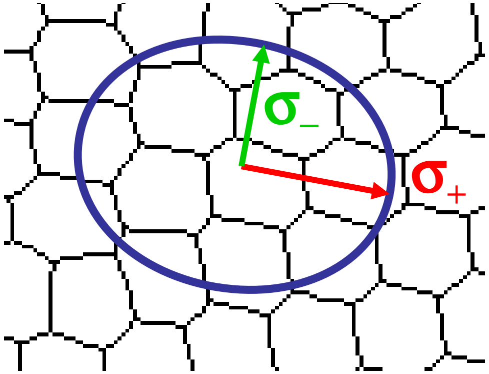
$$\bar{\bar{\sigma}}_{\text{él}} = \lambda \rho \left\langle \frac{\vec{\ell} \otimes \vec{\ell}}{\ell} \right\rangle$$

line tension ≈ $2\gamma h$

edge areal density

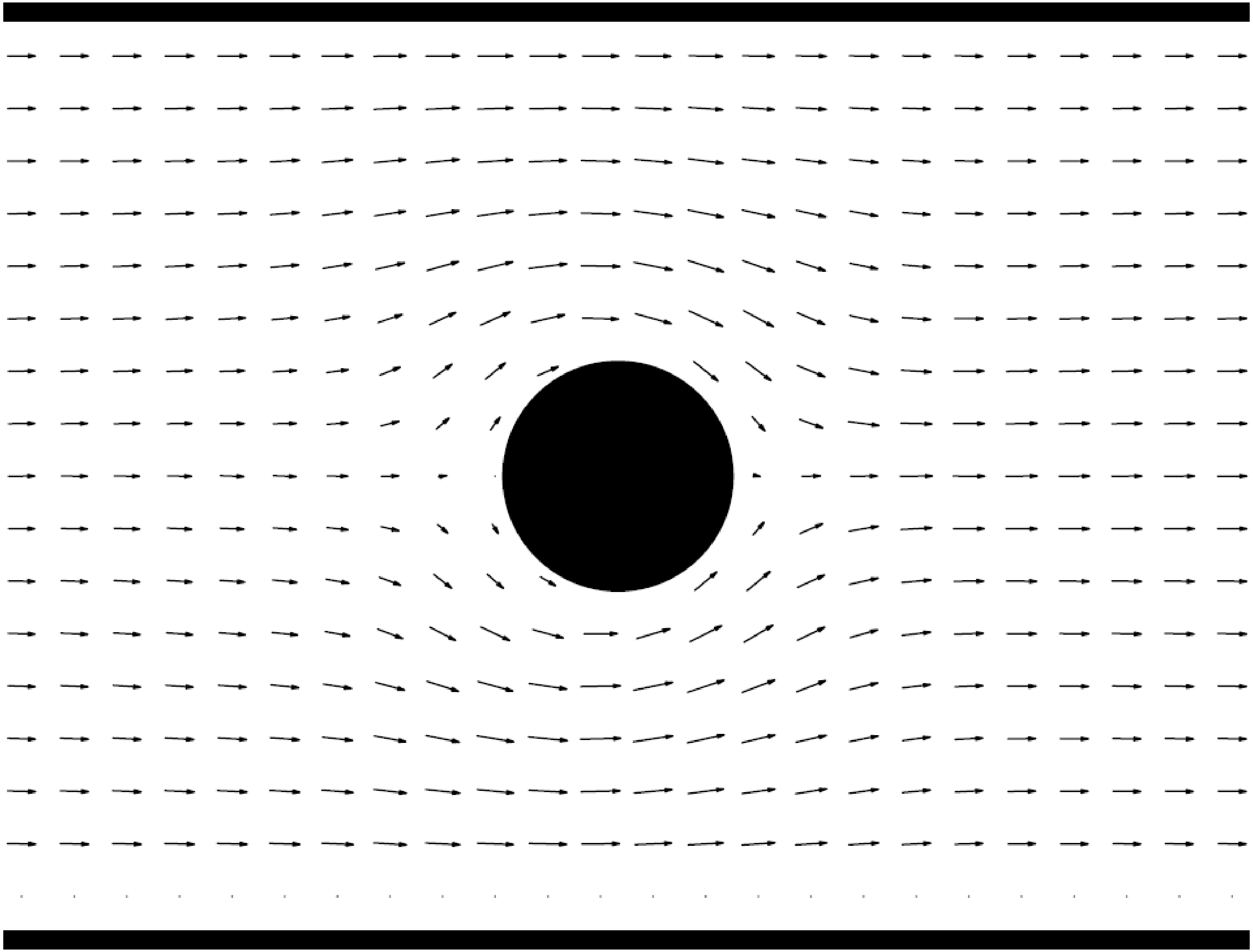
average in time and per box

elliptical representation of the elastic stress tensor

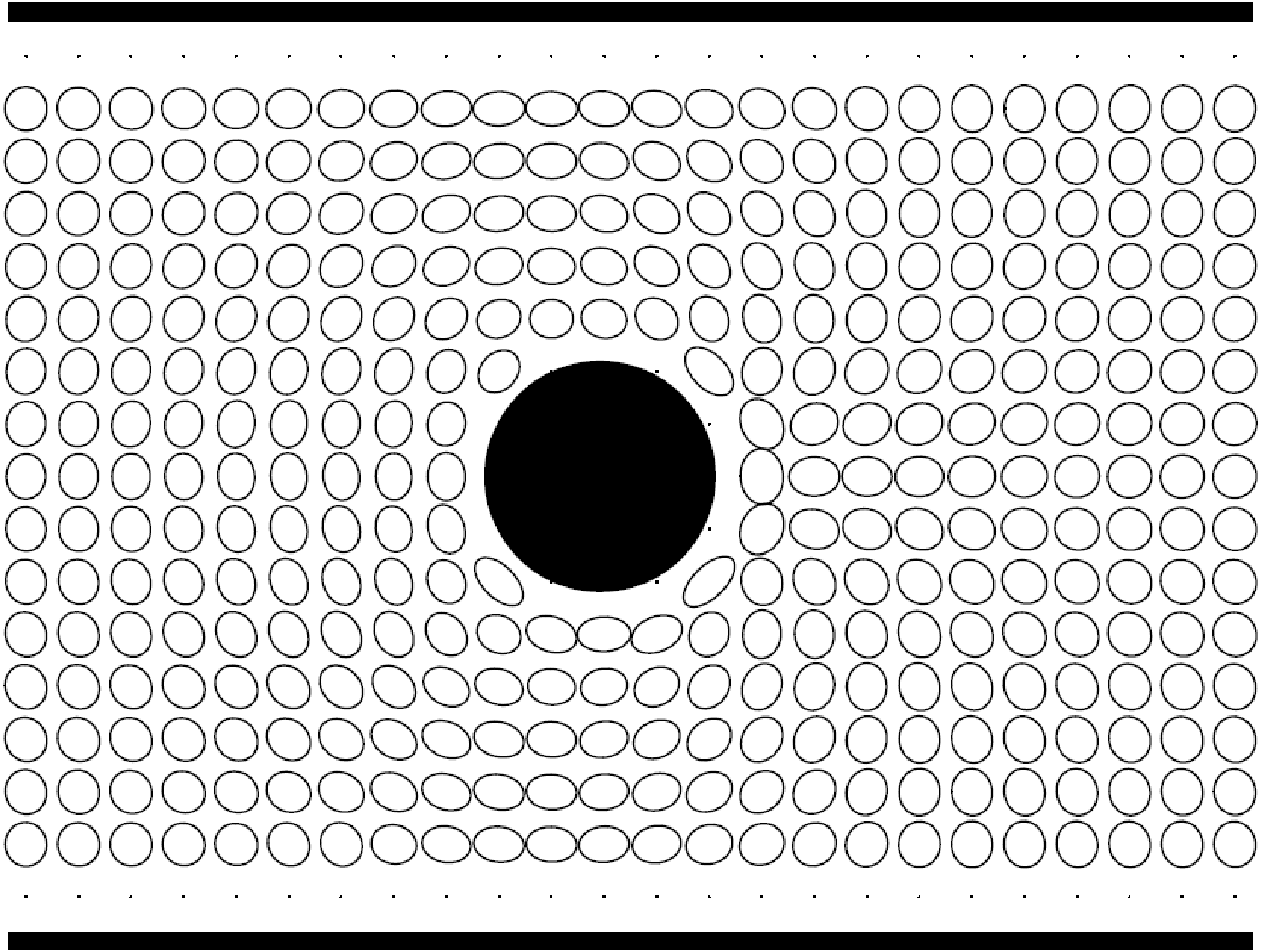


[Dollet & Graner, J. Fluid Mech. 2007]

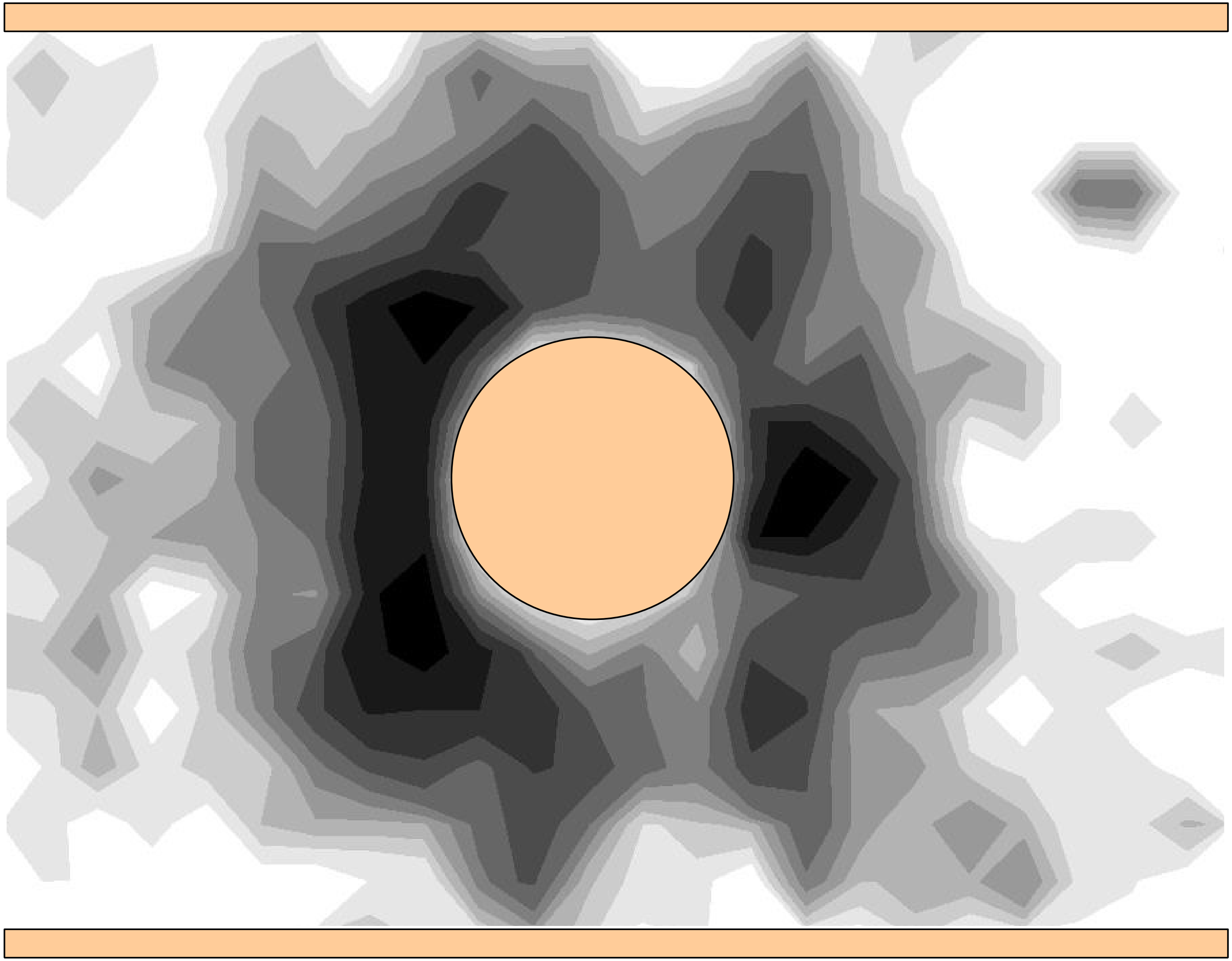
LOCAL ANALYSIS: maps of the flow



LOCAL ANALYSIS: maps of the flow



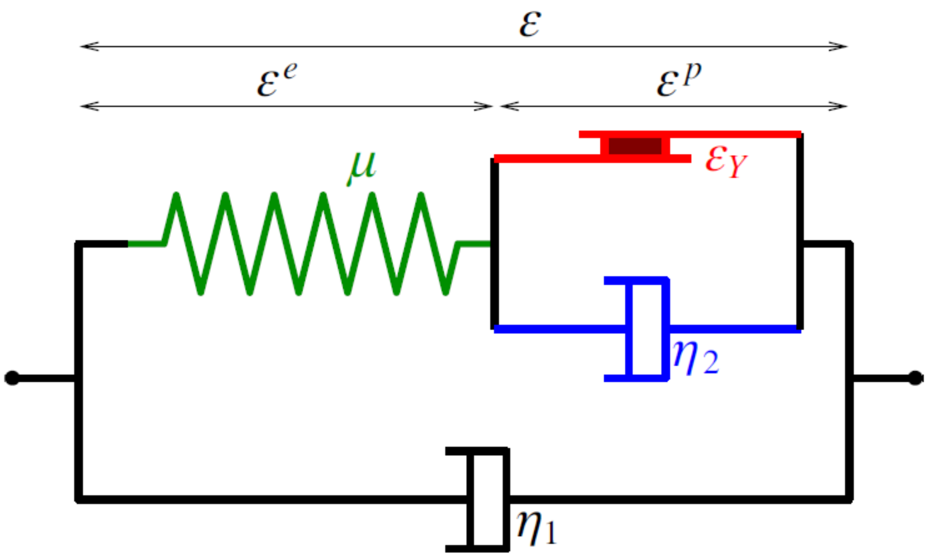
LOCAL ANALYSIS: maps of the flow



LOCAL ANALYSIS: comparison with models

- need for a tensorial viscoelastoplastic model:

$$
\begin{aligned}
\sigma_{\text{tot}} &= -pI + 2\eta_1\dot{\varepsilon} + 2\mu\varepsilon^e, \\
\dot{\varepsilon} &= \dot{\varepsilon}^e + \dot{\varepsilon}^p, \\
\dot{\varepsilon}^p &= \begin{cases} \dfrac{1}{\lambda}\dfrac{|\varepsilon^e| - \varepsilon_Y}{|\varepsilon^e|}\,\varepsilon^e, & \text{when } |\varepsilon^e| > \varepsilon_Y, \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
$$



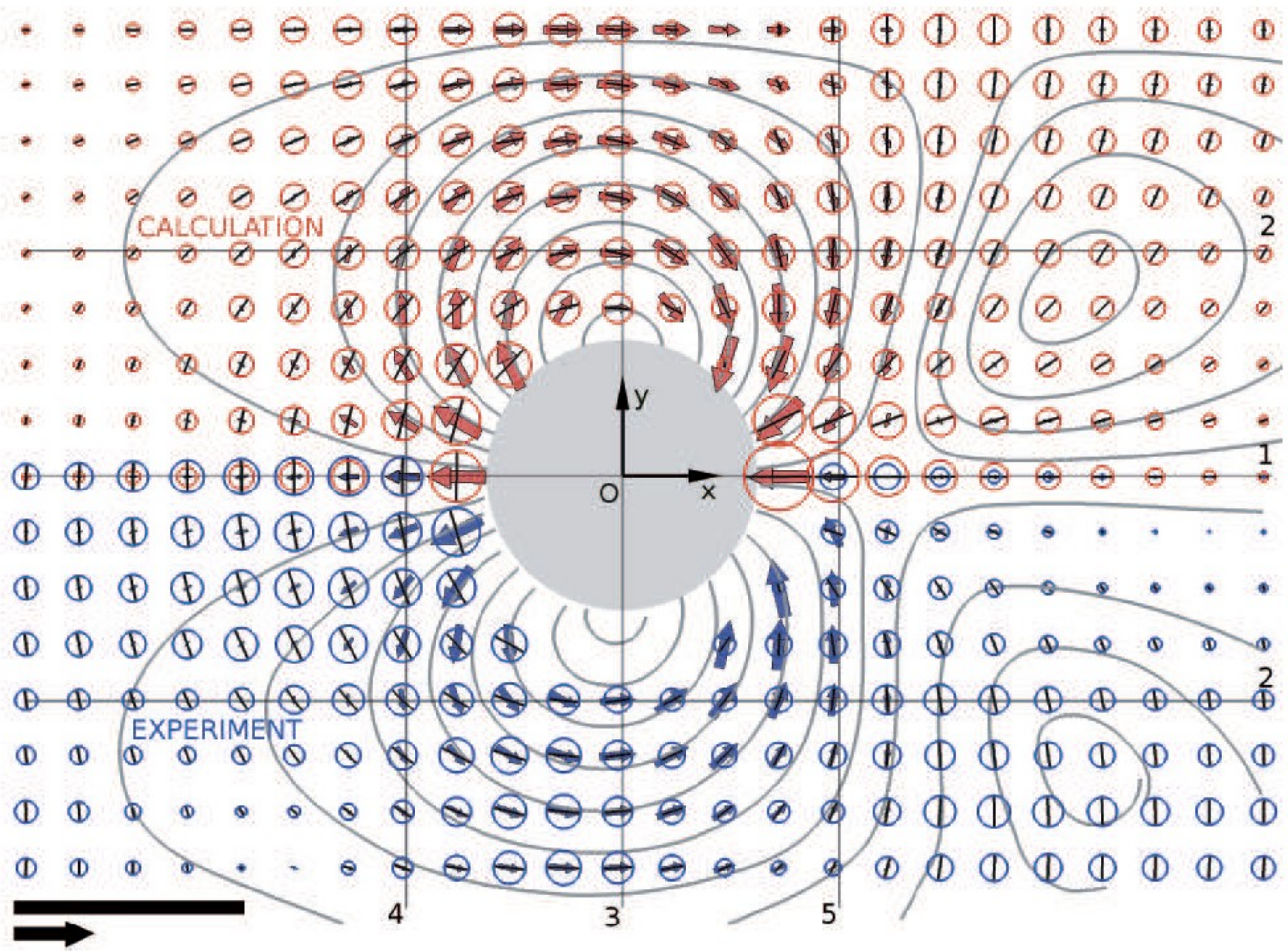
$$
\begin{aligned}
\rho\dot{\mathbf{v}} &= \operatorname{div}\sigma_{\text{tot}} + \mathbf{f}_{\text{ext}}, \\
\operatorname{div}\mathbf{v} &= 0.
\end{aligned}
$$

LOCAL ANALYSIS: comparison with models

- comparison experiments/model:

numerical resolution of the model

experimental measurements



[Cheddadi et al., *Eur. Phys. J. E* (2011)]

OUTLINE

- structure
- drainage
- coarsening
- rheology
    - rheometric data
    - micromechanical models
    - flow profiles
- **acoustics**

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[Pierre, Dollet & Leroy, *Phys. Rev. Lett.* (2014)]

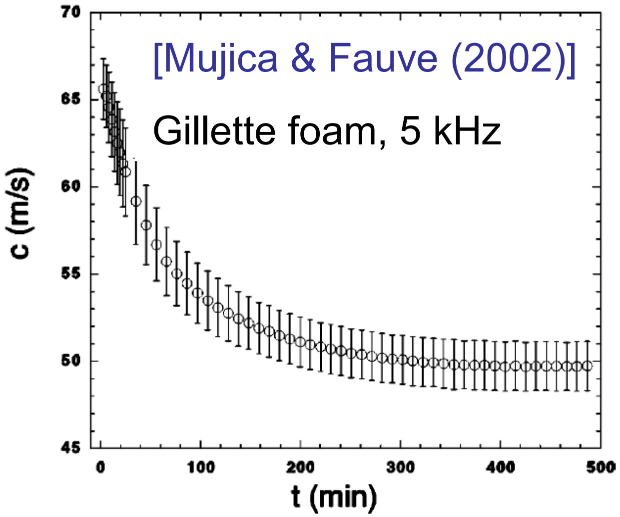
Acoustics of liquid foams: motivations

- liquid foams: known as good acoustical insulators

- use in practice: blast mitigation [Britan et al., *Shock Waves* (2013); Del Prete et al., *Shock Waves* (2013)]

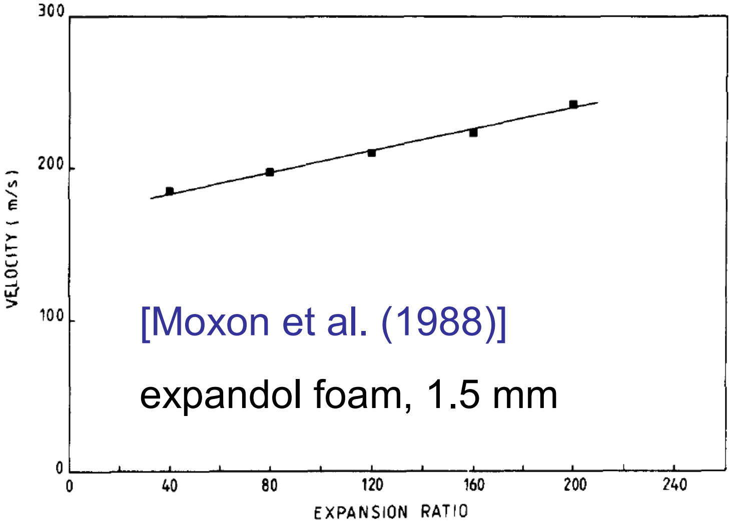
- but it is not much known why [Goldfarb et al., *Shock Waves* (1997); Mujica & Fauve, *Phys. Rev. E* (2002)]

- controversy: some results report small speed of sound, some much larger [Moxon, Torrance & Richardson, *Appl. Acoust.* (1988)]



[Mujica & Fauve (2002)]

Gillette foam, 5 kHz



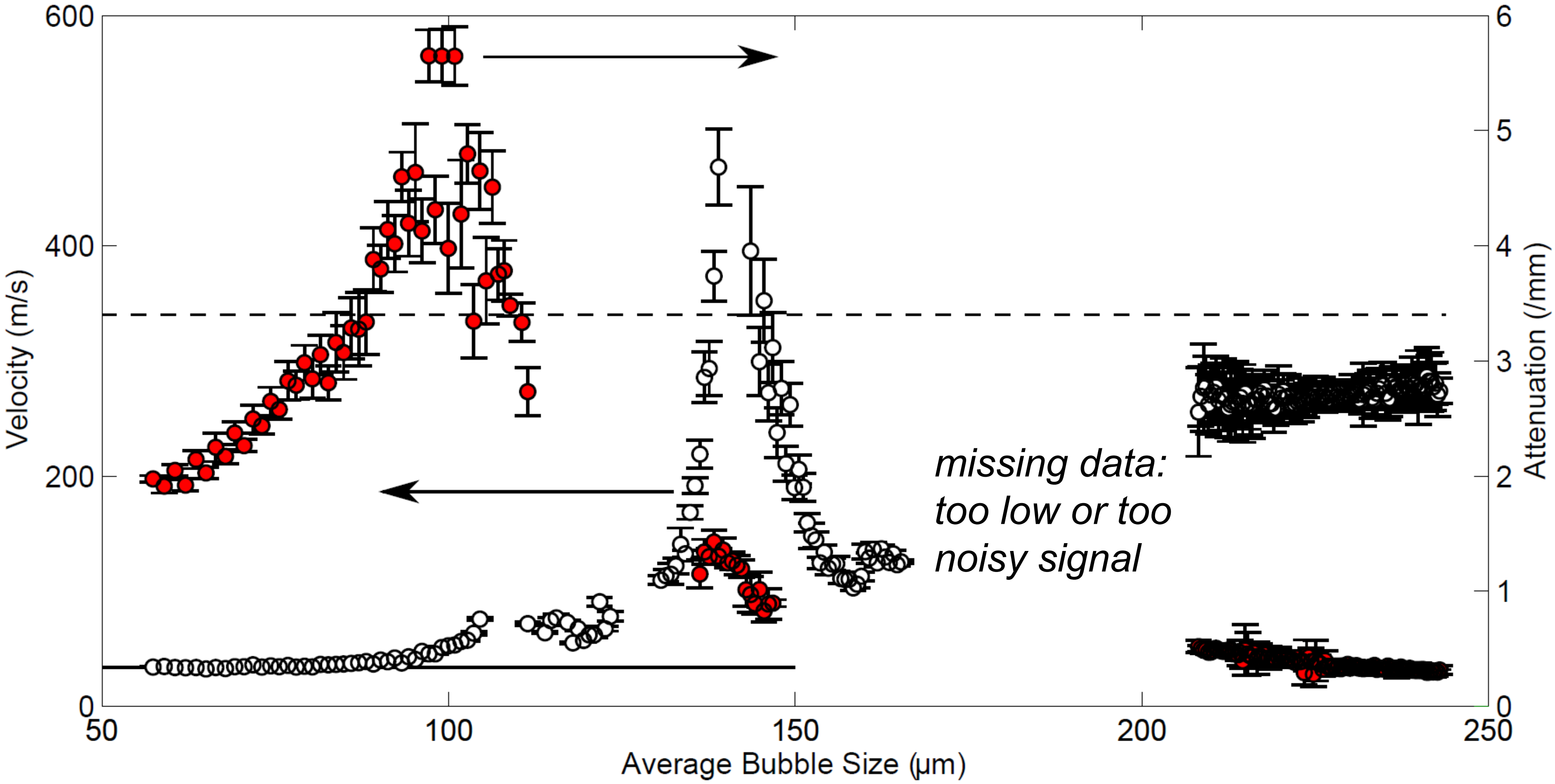
[Moxon et al. (1988)]

expandol foam, 1.5 mm

- our aim: more experimental measurements of transmission of sound in liquid foams → shaving foams, and home-made foams

EXPERIMENTS: results 40 kHz

- controlled foams: SDS solution (10 g/L), C_2F_6 gas, liquid fraction = 10%



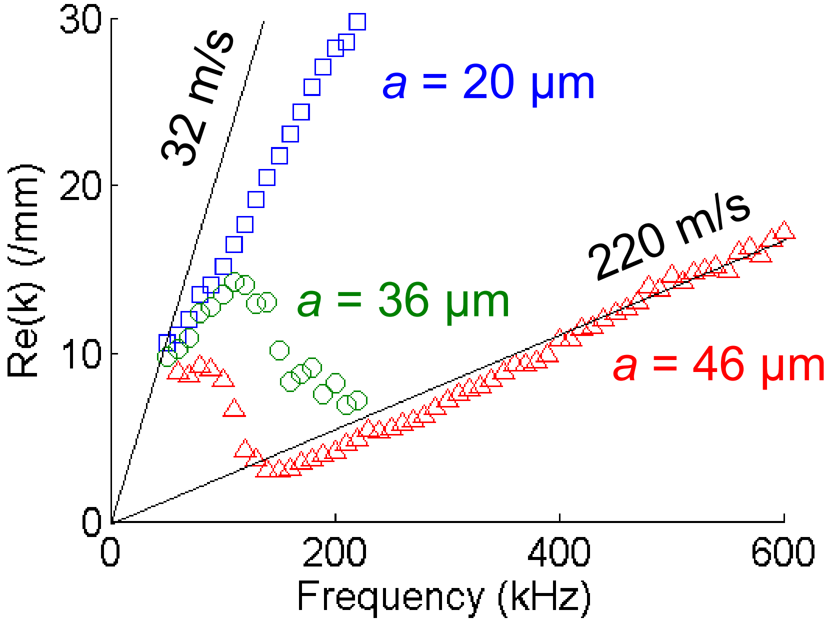
missing data: too low or too noisy signal

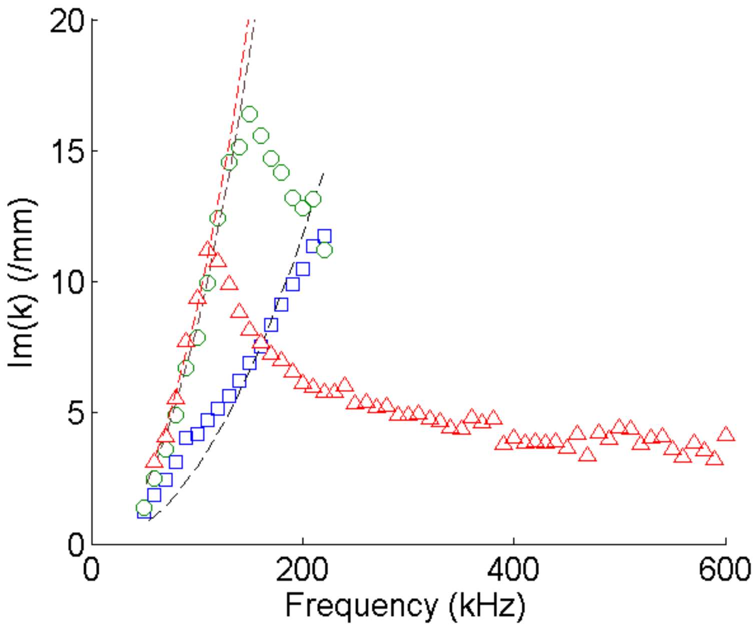
- materials & methods [Pierre, Elias & Leroy, *Ultrasonics* (2013); Pierre et al., submitted to *Phys. Rev. E*]

EXPERIMENTS: results 60-600 kHz

- controlled foams: SDS solution (10 g/L) + xanthane, air saturated in C_6F_{14}, $\phi_l = 11\%$

Re(k) = ω/c



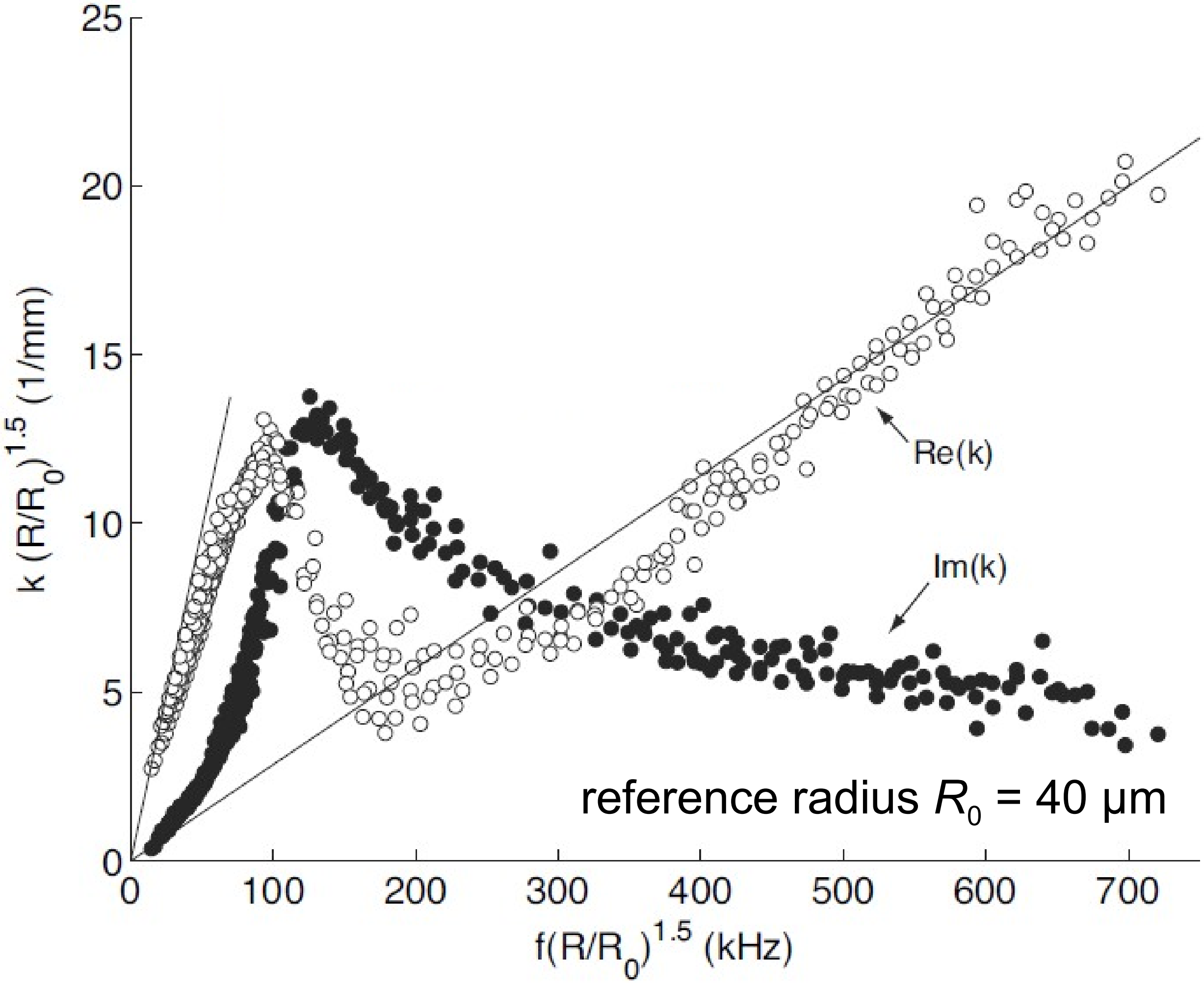


Qualitative trends:

- low frequency, small bubbles: low speed of sound (30-40 m/s)
- high frequency, large bubbles: high speed of sound (220-250 m/s)
- in between: resonant behaviour, maximum of attenuation

EXPERIMENTS: results 60-600 kHz

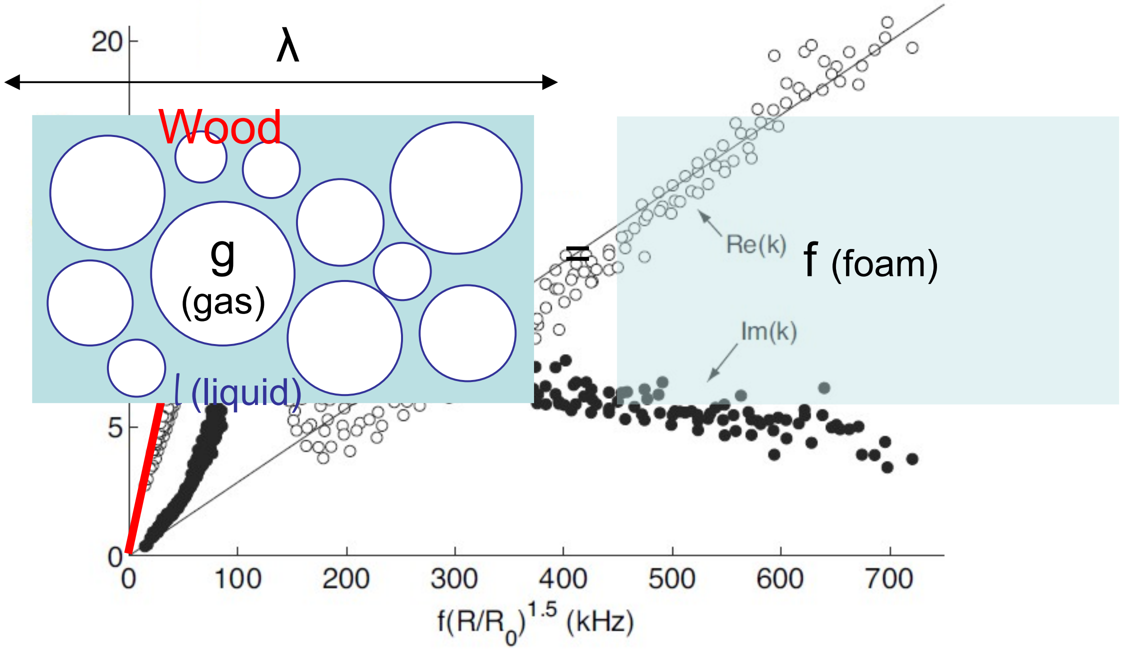
- rescaling of the data:



reference radius R_0 = 40 μm

EFFECTIVE MEDIUM THEORY: WOOD’S MODEL

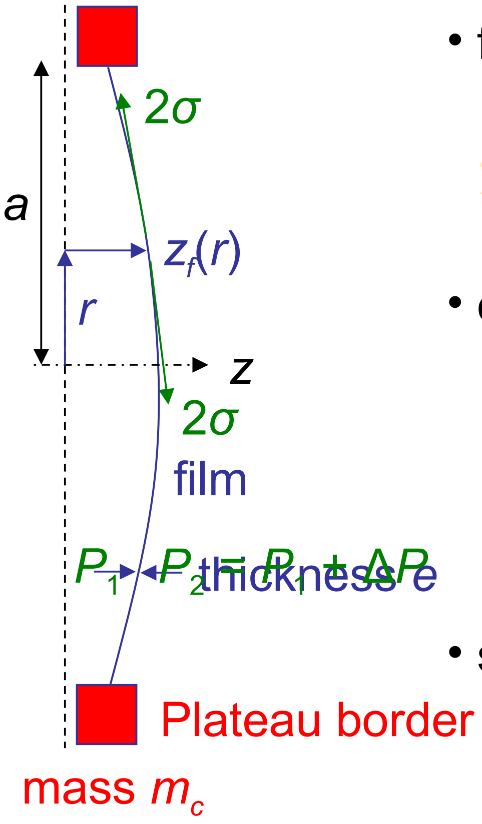
- simplest model: *λ*>>*a*, foam = effective medium
  - effective density $\rho_f = \phi_\ell \rho_\ell + (1-\phi_\ell)\rho_g \simeq \phi_\ell \rho_\ell$
  - effective compressibility $\chi_f = \phi_\ell \chi_\ell + (1-\phi_\ell)\chi_g \simeq (1-\phi_\ell)\chi_g$
  - speed of sound $c_{\mathrm{Wood}} = \dfrac{1}{\sqrt{\rho_f \chi_f}} \simeq \dfrac{1}{\sqrt{\phi_\ell (1-\phi_\ell)\rho_\ell \chi_g}}$



- works only for low frequencies and/or small bubbles

TOY MODEL

- assumption of Wood’s model: the acoustic-induced motion of all water material elements is the same everywhere
- but foam = thin films (~ 100 nm) + large Plateau borders (~ 10 µm)
  - very different inertia: do they vibrate similarly?



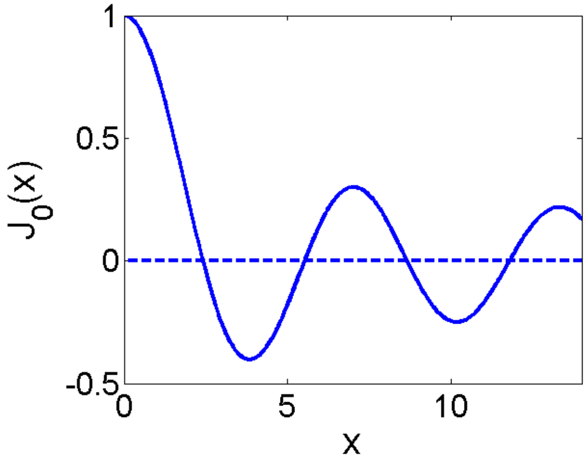
- force balance on a ring between *r* and *r* + d*r*

$$2\pi\rho e r \frac{\partial^2 z_f}{\partial t^2} = -2\pi r \Delta P(t) + 4\pi\sigma \frac{\partial}{\partial r}\left(r \frac{\partial z_f}{\partial r}\right)$$

- oscillations at frequency ω: $z(r,t) \rightarrow e^{i\omega t} z(r)$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}z_f}{\mathrm{d}r}\right) + q^2 z = \frac{\Delta P}{2\sigma}$$

$$q^2 = \frac{\rho e}{2\sigma}\omega^2$$

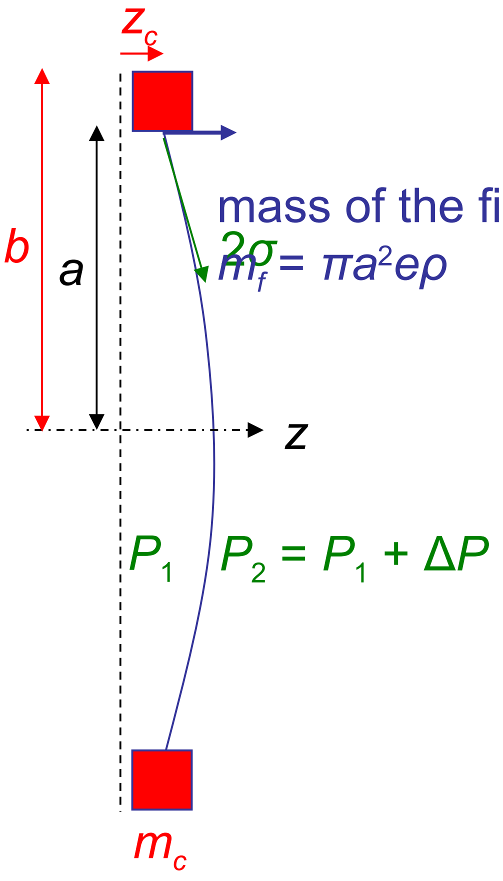


- solution

$$z_f(r) = A J_0(qr) + \frac{\Delta P}{2\sigma q^2}$$

TOY MODEL

- assumption of Wood's model: the acoustic-induced motion of all water material elements is the same everywhere
- but foam = thin films (~ 100 nm) + large Plateau borders (~ 10 µm)
  - very different inertia: do they vibrate similarly?



mass of the film

$m_f = \pi a^2 e\rho$

- force balance on the Plateau border

$$m_c \frac{\mathrm{d}^2 z_c}{\mathrm{d}t^2} = -\pi(b^2 - a^2)\Delta P(t) - 4\pi\sigma a \left(\frac{\partial z_f}{\partial r}\right)_{r=a} \boxed{- 2\pi\xi a \left(\frac{\partial^2 z_f}{\partial t\, \partial r}\right)_{r=a}}$$

- hence

phenomenological friction force

$$-m_c\omega^2 z_c = \pi(b^2 - a^2)\Delta P + 4\pi\sigma(1 - \mathrm{i}\omega\tau)aqAJ_1(qa)$$

- matching: $z_c = z_f(r = a)$; amplitude of motion:

$$z_f(r) = \frac{\Delta P}{\rho e\omega^2}\left[1 - \frac{m_c - m_f(b^2 - a^2)/a^2}{m_c + m_f(1 - \mathrm{i}\omega\tau)\mathcal{H}(qa)}\frac{}{J_0(qa)}\right]$$

$\tau = \xi/\sigma$

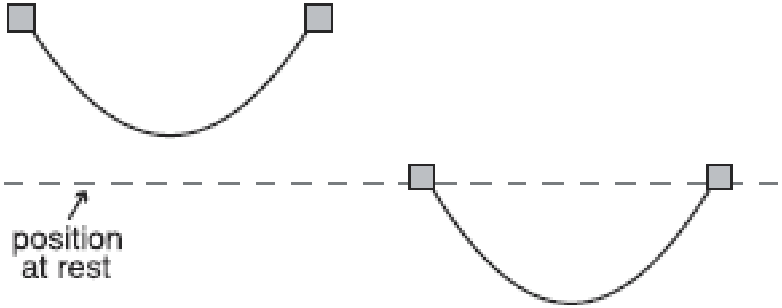
$$\mathcal{H}(qa) = 2J_1(qa)/[qaJ_0(qa)]$$

TOY MODEL

- small frequency/size: if $qa << 1$, $z_f(r) \rightarrow b^2\Delta P/a^2\rho e\omega^2$ independent of r

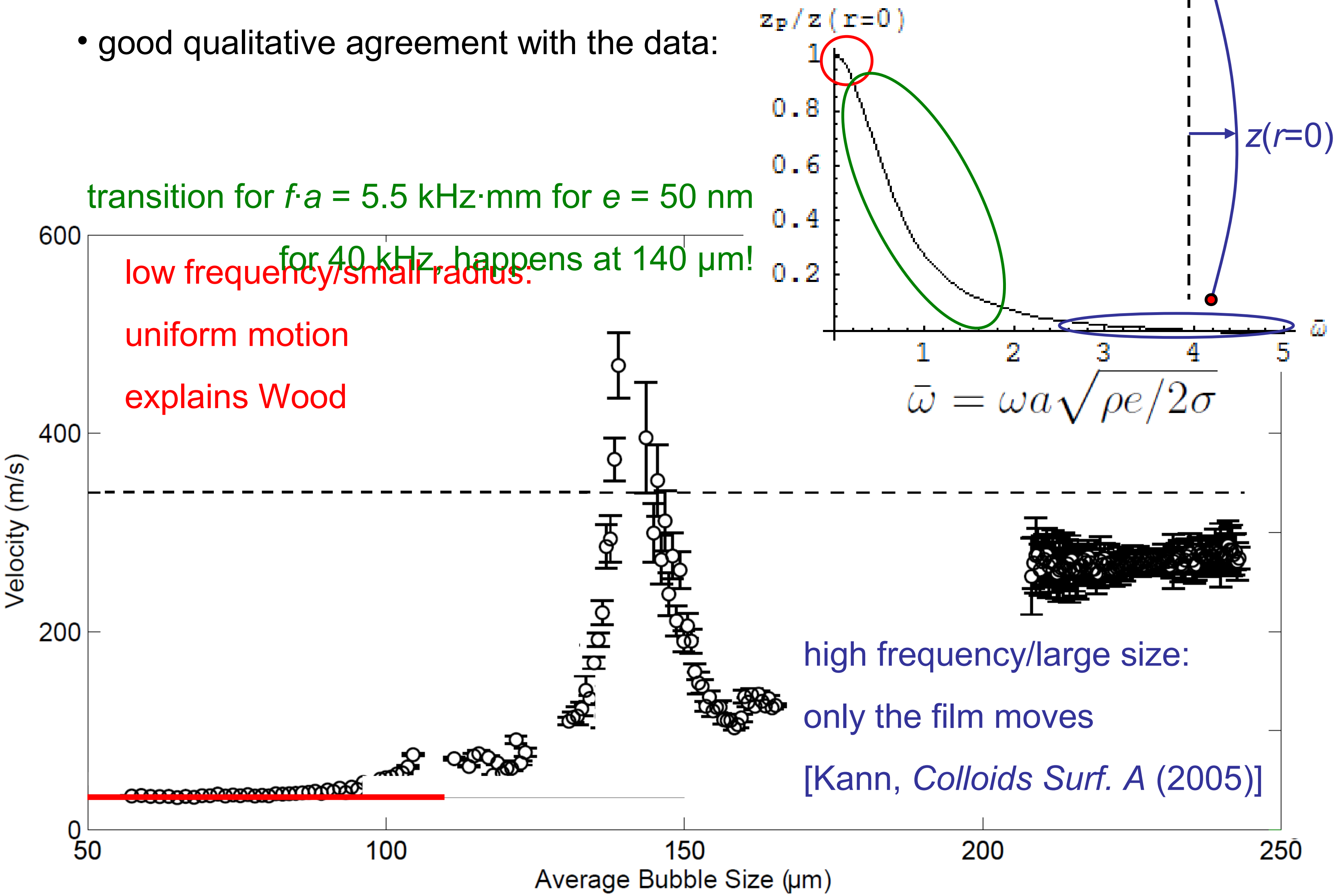
The Plateau border and the film move in phase, with a comparable amplitude → justifies Wood's model

- large frequency/size: only the film moves → compatible with Kann's model [Kann, *Colloids Surf. A* (2005)]



MODELLING: rough comparison

- good qualitative agreement with the data:



transition for $f \cdot a$ = 5.5 kHz·mm for e = 50 nm

for 40 kHz, happens at 140 µm!

low frequency/small radius:

uniform motion

explains Wood

$$\bar{\omega} = \omega a \sqrt{\rho e / 2\sigma}$$

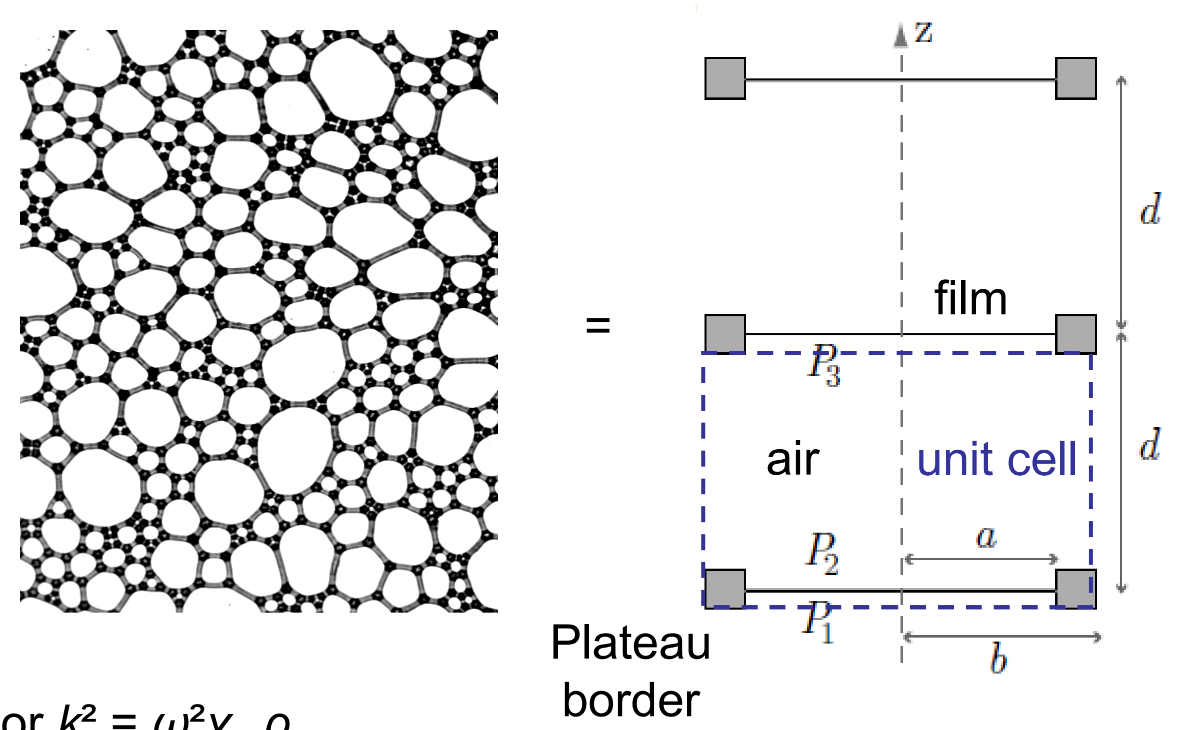
high frequency/large size:

only the film moves

[Kann, *Colloids Surf. A* (2005)]

FULL MODEL

- prediction of the wavevector: assume the following foam structure



- wavevector $k^2 = \omega^2 \chi_{\text{eff}} \rho_{\text{eff}}$

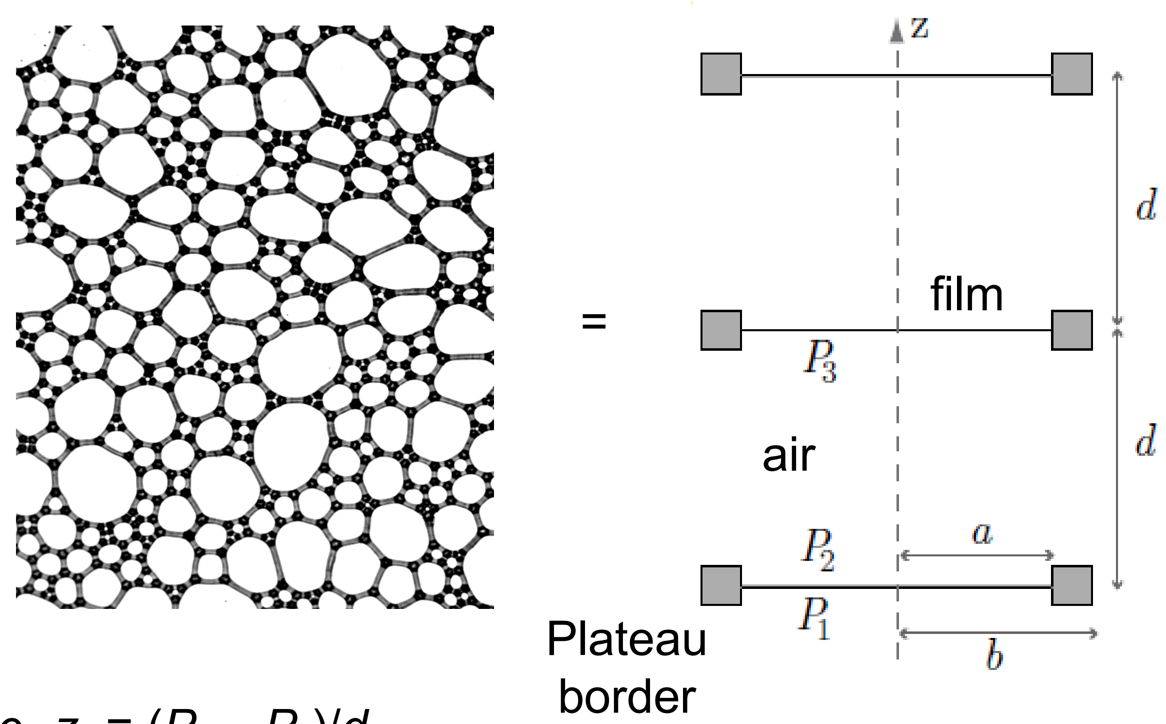
with an effective compressibility = relative variation of volume per unit pressure

$\chi_{\text{eff}} = (1 - \phi_l)\chi_l + \phi_l \chi_g$ like in Wood's model

and an effective density = inverse of acceleration of the unit cell per unit volumetric force

FULL MODEL

- effective density: $d \ll \lambda \rightarrow$ uniform displacement amplitude z_a in the unit cell



- hence $\omega^2 \rho_{\text{eff}} z_a = (P_3 - P_1)/d$
- air displacement: continuity with film + Plateau border displacement

$z_a = x\langle z\rangle + (1 - x)z_c$ with $x = a^2/b^2$ (decreasing function of ϕ_l)

+ Euler equation for the air: $-m_a \omega^2 z_a = (P_2 - P_3)\pi b^2$

FULL MODEL

- prediction of the wavevector: $k^2 = \omega^2 \chi_{\text{eff}} \rho_{\text{eff}}$

with $\chi_{\text{eff}} = (1 - \phi_l)\chi_l + \phi_l \chi_g$

and an effective density: $\rho_{\text{eff}} = (1 - \Phi)\rho_a + \Phi' \rho$

with an effective liquid fraction:

$$\Phi' = \frac{\Phi_c + \Phi_f(1 - i\omega\tau)\mathcal{H}(qa)}{1 + \left(x^2 \frac{\Phi_f + \Phi_c}{\Phi_f} - 2x\right)[1 - \mathcal{H}(qa)] - i\omega\tau x \mathcal{H}(qa)}$$

where $\phi_l = \phi_c + \phi_f$

ϕ_f → liquid fraction in the films only

ϕ_c → liquid fraction in the Plateau borders only

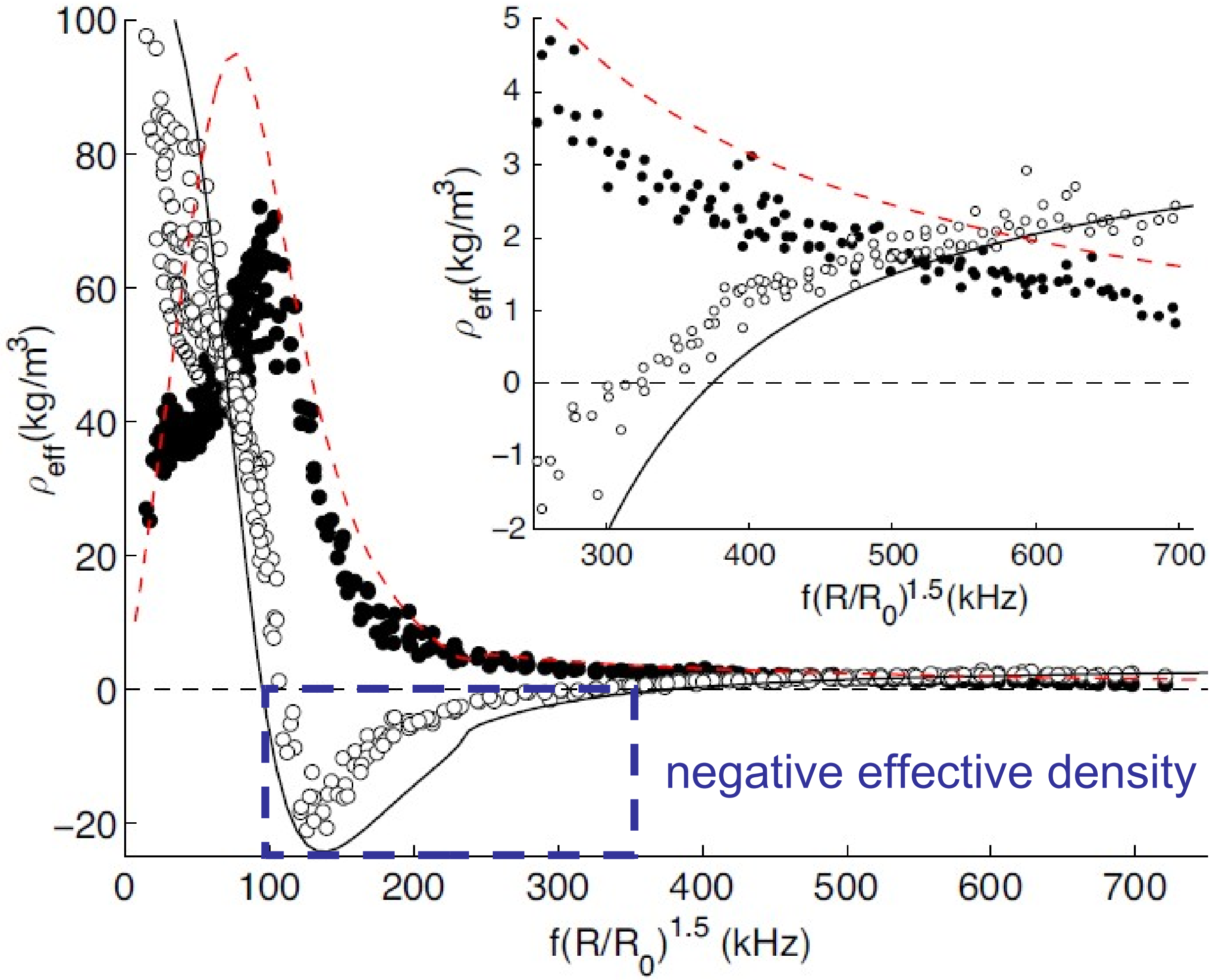
$$1/\Phi' = (1-x)^2/\Phi_p + x^2/\Phi_f$$

and $\mathcal{H}(qa) = 2J_0(qa)/[qaJ_1(qa)]$

- $qa << 1$: $\mathcal{H}(qa) \to 1$, $\phi' \to \phi$, Wood model recovered

COMPARISON TO EXPERIMENTS

- dissipation time $\tau = 10^{-5}$ s and thickness e = 70 nm fitting parameters



- intermediate regime: acoustic forcing and acceleration of film out of phase
- $k^2 \sim \rho_{eff}$: evanescent waves, barrier to acoustic propagation