Kinetic model for the finite-time thermodynamics of small heat engines

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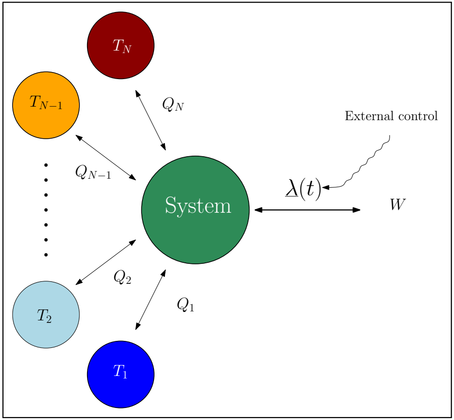
Heat Engines: general considerations



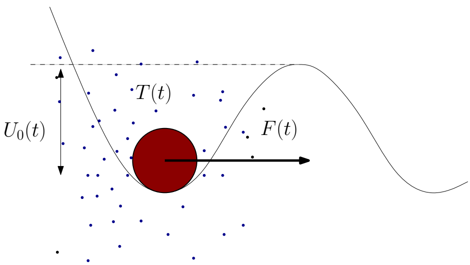
- Engine protocol: cyclical transformation over time τ;
- Adiabatic limit (quasi-static transformation): $\tau \to \infty$;

Desiderata

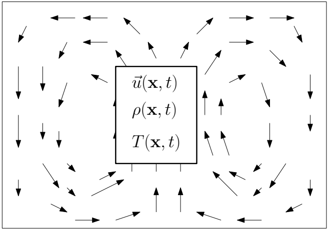
- Predict the dependence of W and Q_i on τ;
- Take into account fluctuations;

Beyond standard thermodynamics: two possible approaches

- $N = 1$ particle
- External time-dependent potential
- Interaction with a reservoir (thermal noise)

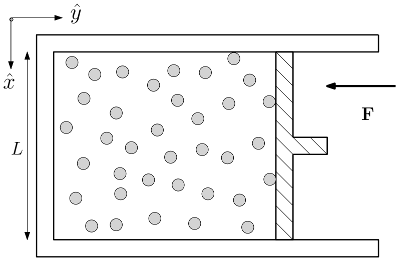


- $N \gg 1$ particles
- Hydrodynamical description (velocity, density, temperature fields)



A paradigmatic small systems

A system composed of $N \sim \mathcal{O}(10^2)$ degrees of freedom



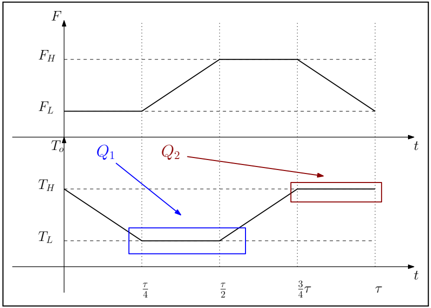
$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{P^2}{2M} + FY$$

(+ elastic collision between particles and piston)

(+ thermal wall on the left side at temperature T_o)

Is it possible to extract mechanical work from this system with a cyclical protocol?

Heat Engine: the Ericsson cycle

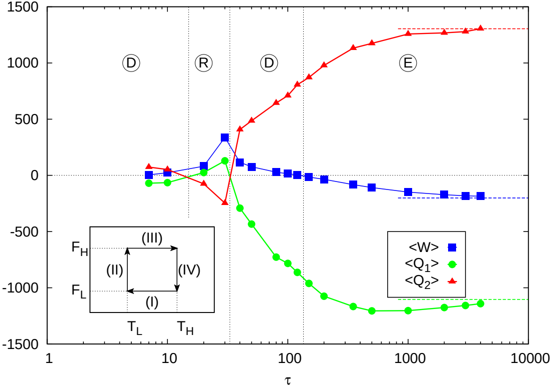


In each segment:

$$W = \int dt \frac{\partial \mathcal{H}}{\partial t} = \int dt \dot{F} X(t)$$

$$Q = \Delta \mathcal{H} - W$$

Results of MD simulations [L.C., A. Puglisi and A. Vulpiani, PRE (2015)]



$$\frac{T_H - T_L}{T} \approx$$

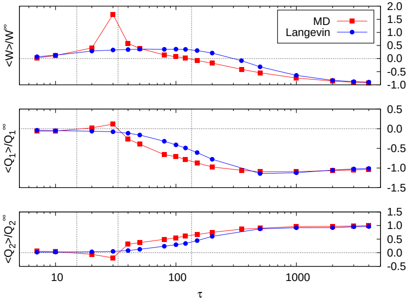
$$\approx \frac{F_H - F_L}{F} \approx 0.1$$

Model with 3 Macroscopic Variables

A coarse grained description is possible:

- X piston position
- V piston velocity
- T gas kin. energy per particle

$$\mathbf{y} = (X, V, T)$$

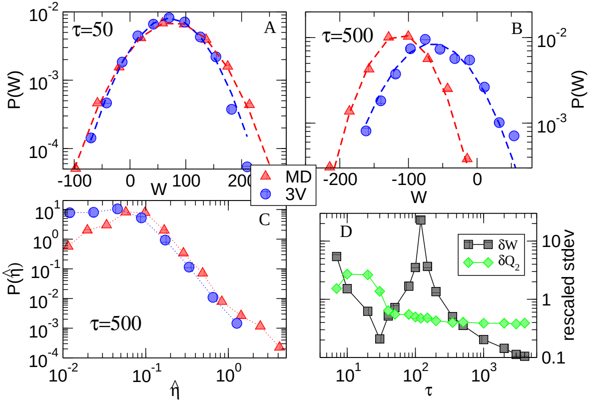


Linear time-dependent Langevin eqn.

$\dot{\mathbf{y}} = \mathbf{A}(\mathbf{t}) \cdot \mathbf{y} + \mathbf{B}(\mathbf{t}) \cdot \boldsymbol{\eta}$ ← white noise

Coefficients of matrix $\mathbf{A}$ are determined via kinetic theory considerations.

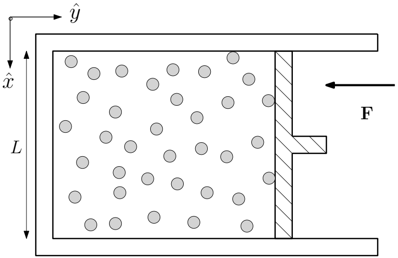
Fluctuations



$$\eta = \frac{-\langle W \rangle}{\langle Q_H \rangle}$$

$$\hat{\eta} = \frac{-W}{Q_H}$$

Summarizing...



- Rich phenomenology (due to $N \neq 1$);
- Fluctuating thermodynamic quantities (due to $N \neq \infty$);
- Non trivial Langevin description.

A good insight into the physics of small systems!

Thank you for the attention!