

# Superclustering of inertial particles in turbulence

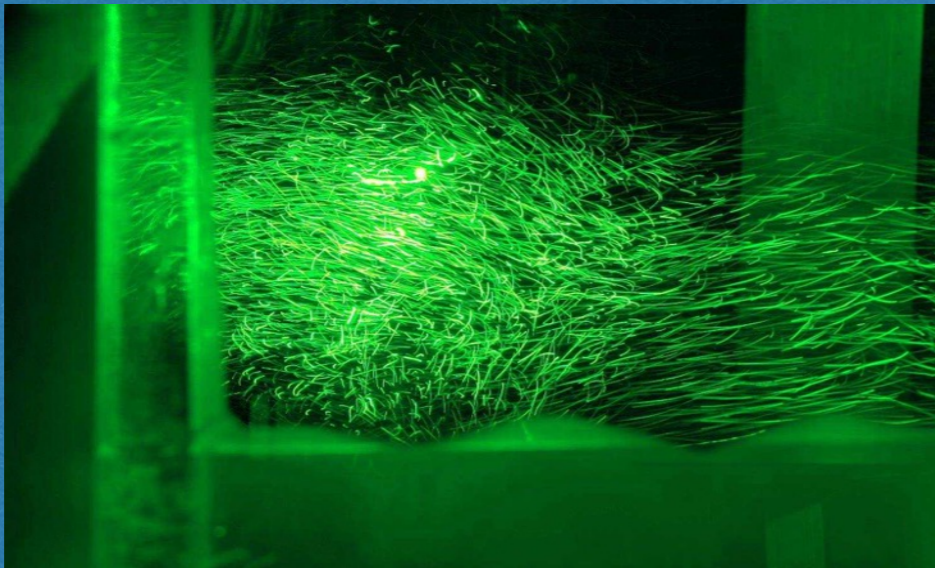
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Mickaël Bourgoïn, Martin Obligado



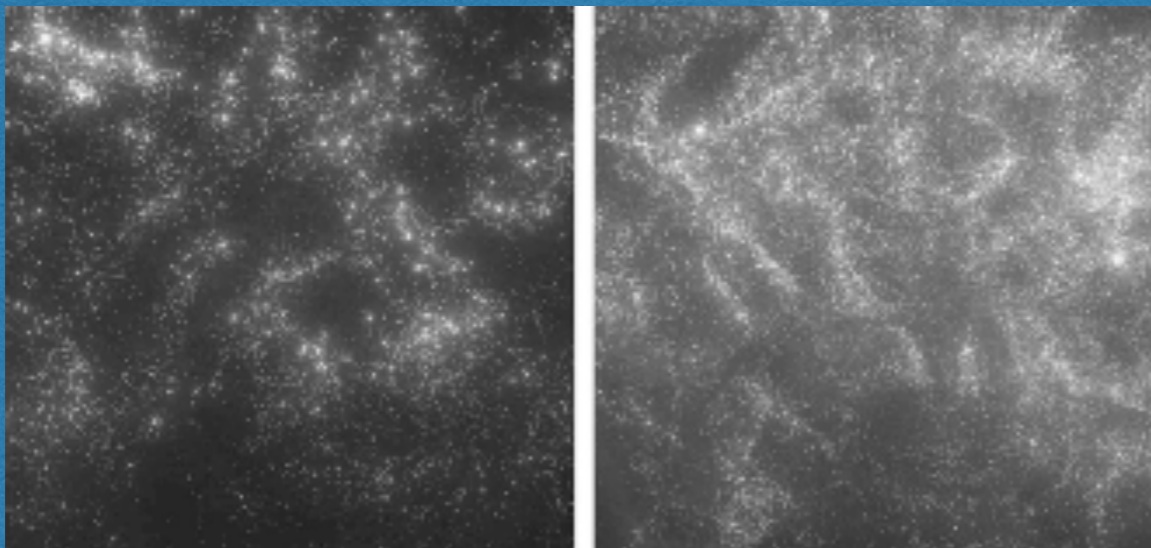
# Tracer particles vs. Inertial particles

**Tracers** behave as fluid particles



PIV, LDV, PTV, etc.

**Inertial particles** do not follow exactly the flow



« material particles »

Ex: water droplets in air

**Stokes number**

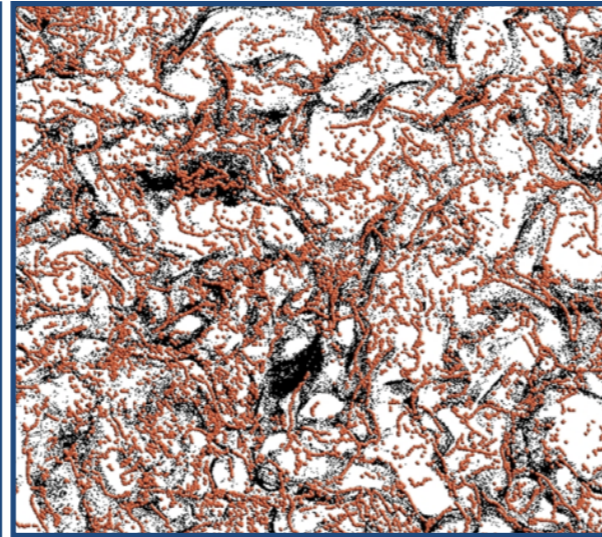
$$St = \tau_p / \tau_k$$
$$\tau_p = \frac{d^2 \rho_p / \rho_f}{18 \nu_f}$$
$$\tau_k = (\nu_f / \epsilon)^{1/2}$$

# Preferential concentration of inertial particles

A striking feature: inertial particles experience preferential concentration



Aliseda, Cartellier, Hainaux & Lasheras. JFM, 468 (2002)



Goto & Vassilicos, PRL 100 (2008)

## Relevant issues:

- impact on collision efficiency
- impact on settling velocity enhancement
- Collective dynamics
- Etc.

## Possible interpretation:

- Dissipative dynamics of inertial particles

Mehlig & Wilkinson, PRL 92 (2004) and others

$$\frac{d\vec{v}}{dt} = \frac{1}{\tau_p} (\vec{u} - \vec{v})$$

- Centrifugal expulsion from eddies  
(role of high strain-low vorticity points)

Wang & Maxey, JFM 1993

- Sweep-stick mechanism  
(role of Zero acceleration points)

Goto & Vassilicos, PRL 100 (2008)

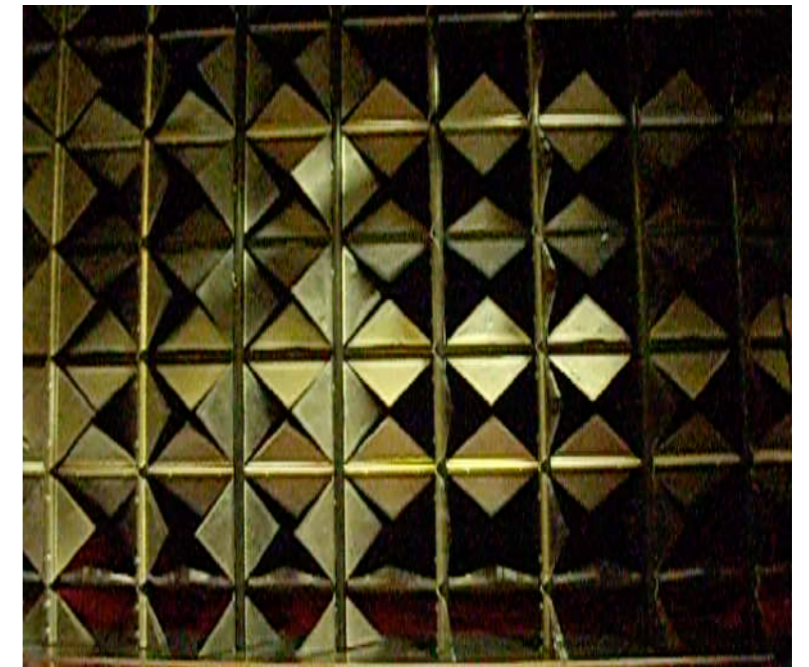
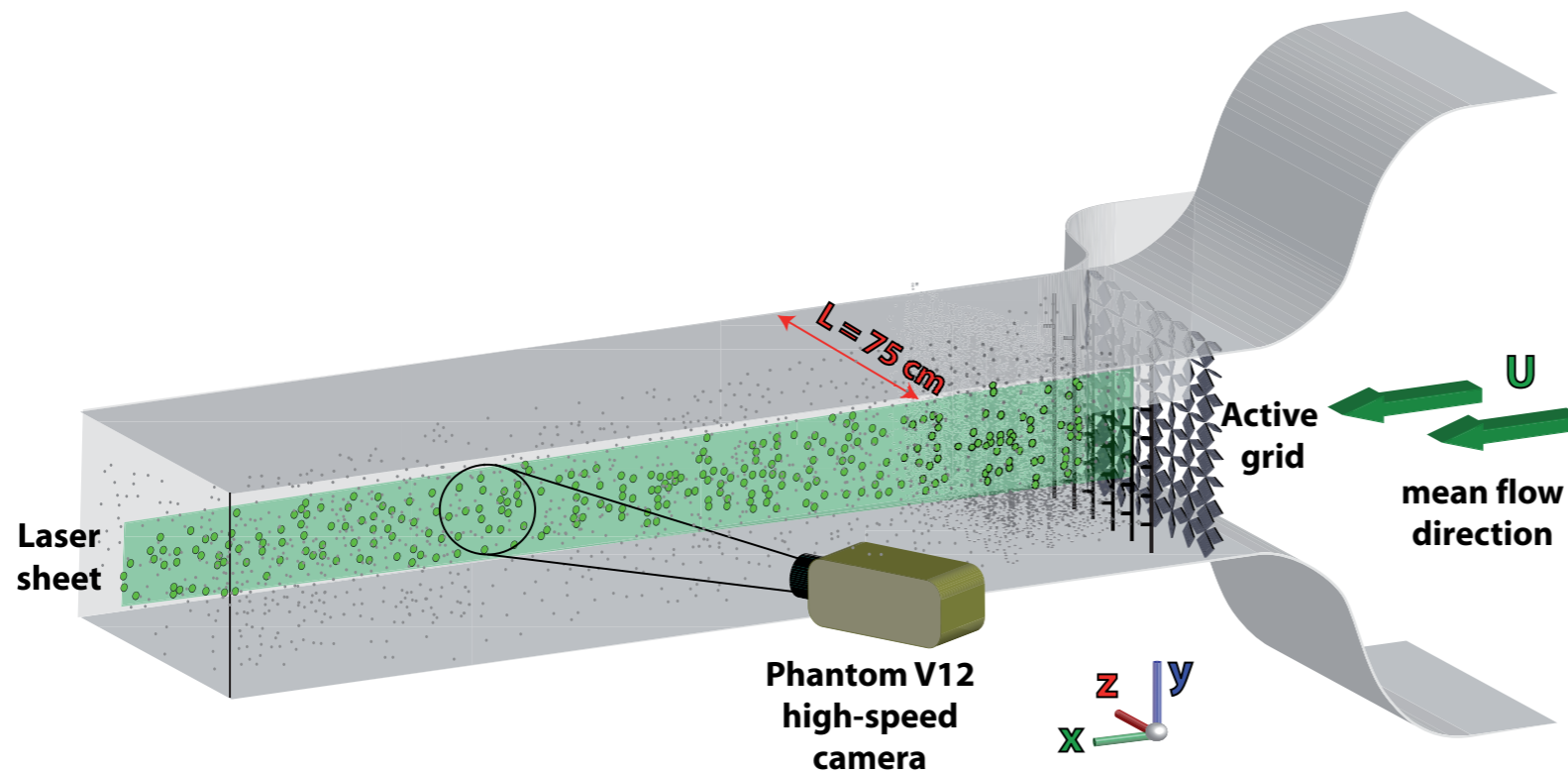
## Some known properties

- Maximum for Stokes number around 1
- Clusters typical size of order  $10 \eta$
- Fractal cluster geometry

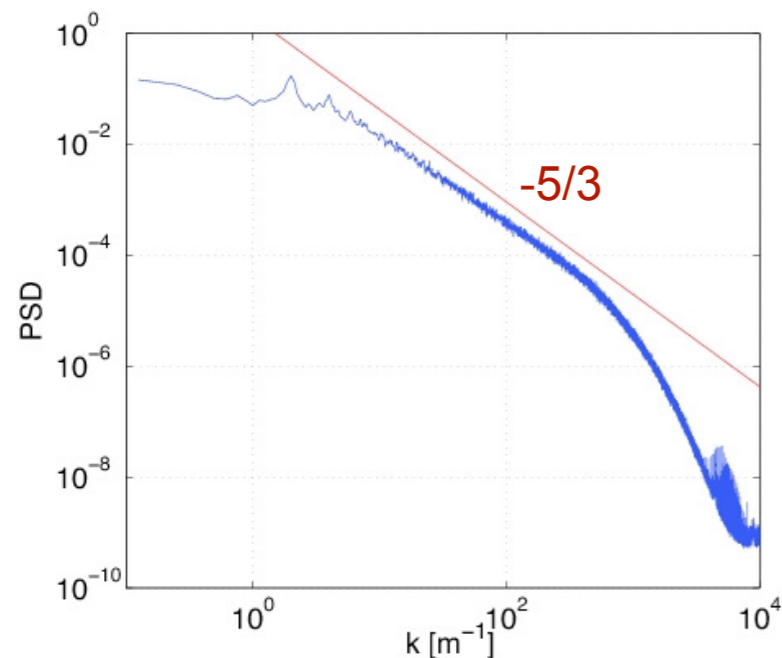
Do clusters clusterize ? (Super-clustering)

# Experimental setup

Obligado et al., JoT 2014



## Turbulent velocity spectrum

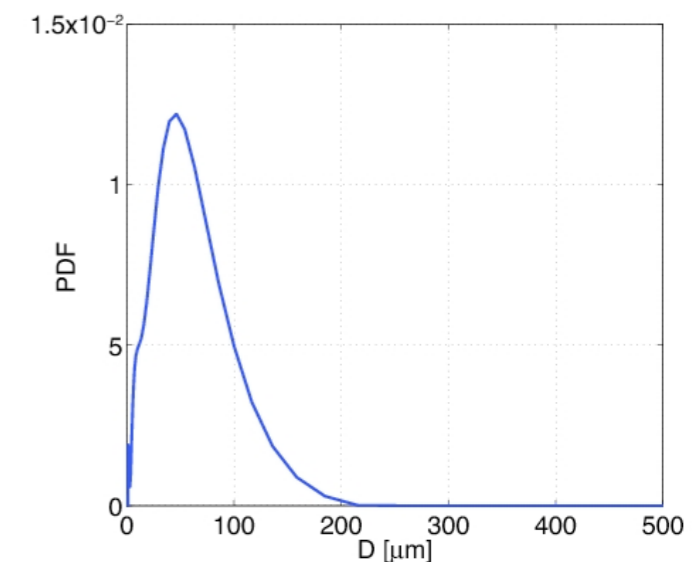


$R_\lambda$	$U$ (m/s)	$L$ (cm)	$\eta$ ( $\mu\text{m}$ )	$\epsilon$ ( $\text{m}^3\text{s}^{-3}$ )	$St$
234	3.4	13.0	280	.69	2.1
264	4.0	13.2	240	1.2	3.3
295	4.8	13.5	208	2.0	4.3
331	5.7	13.8	178	3.4	5.8
357	6.4	14.0	160	4.7	6.6
400	7.6	14.3	140	7.7	9.9

Note : here  $R_\lambda$  and  $St$  are equivalent

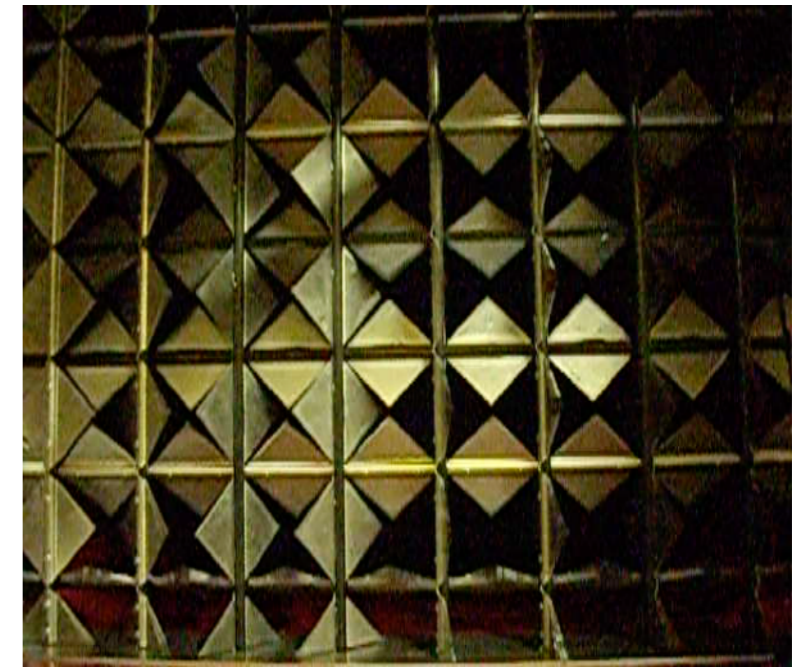
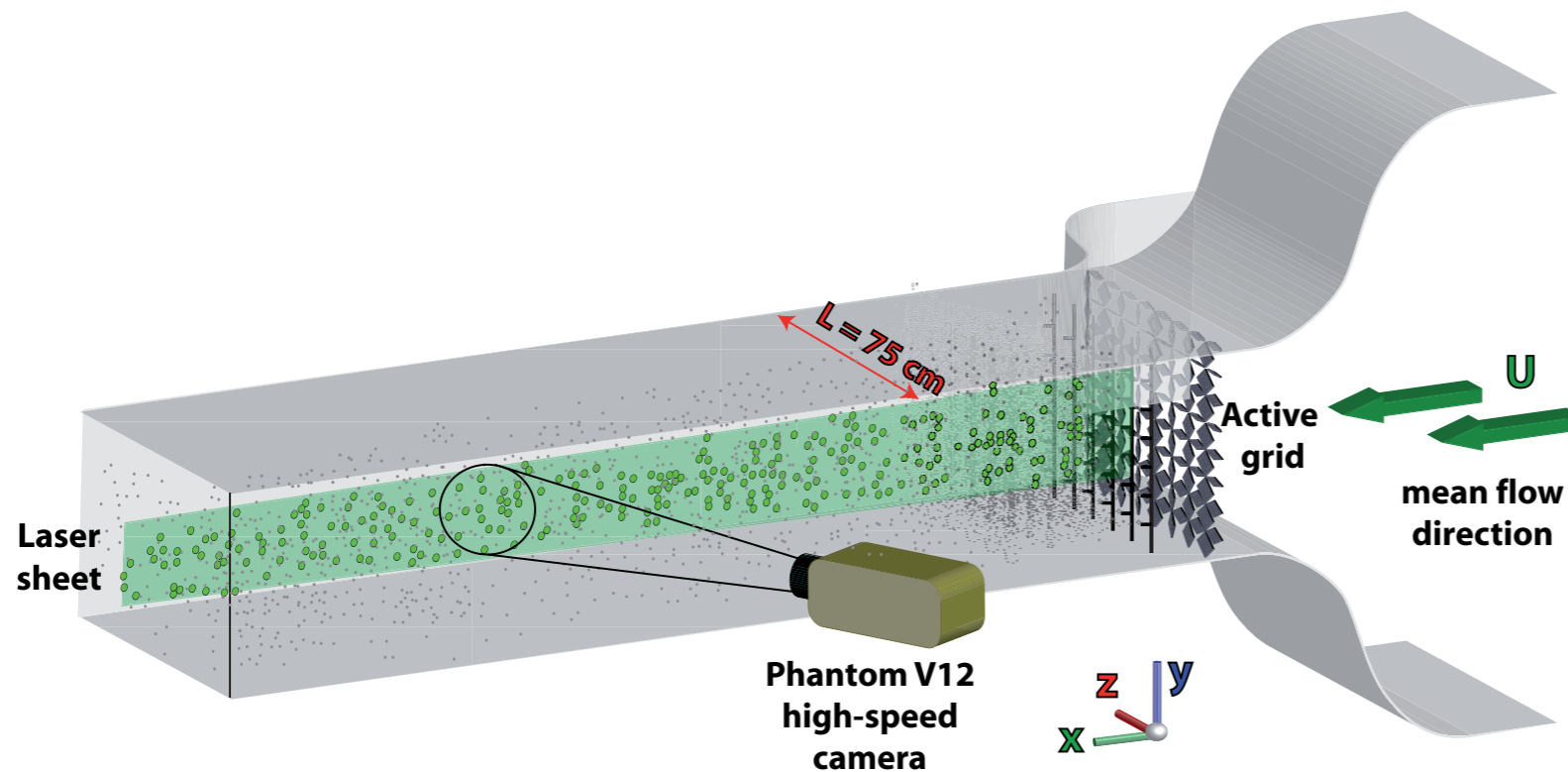
$$St = (d/\eta)^2 (1 + 2\rho_p/\rho_f) / 36$$

## Size distribution

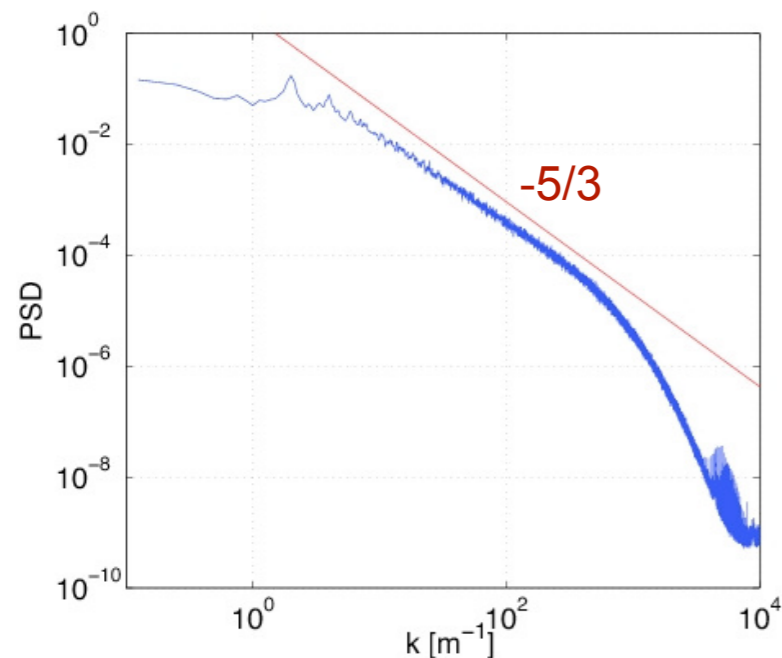


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Obligado et al., JoT 2014



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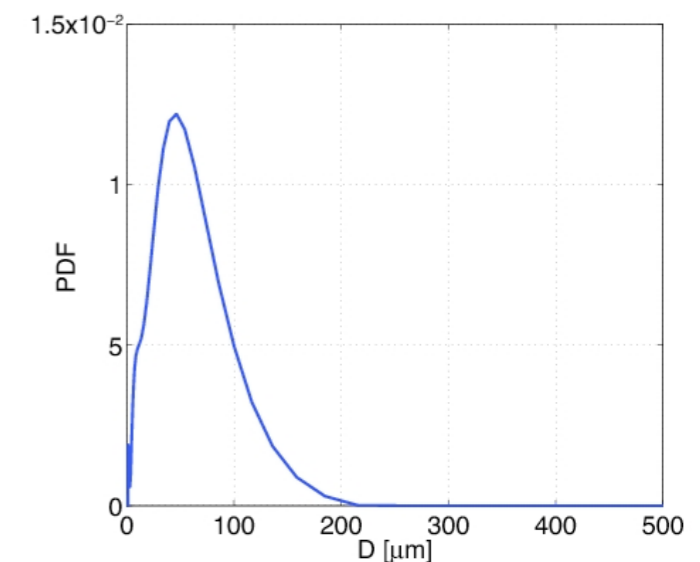


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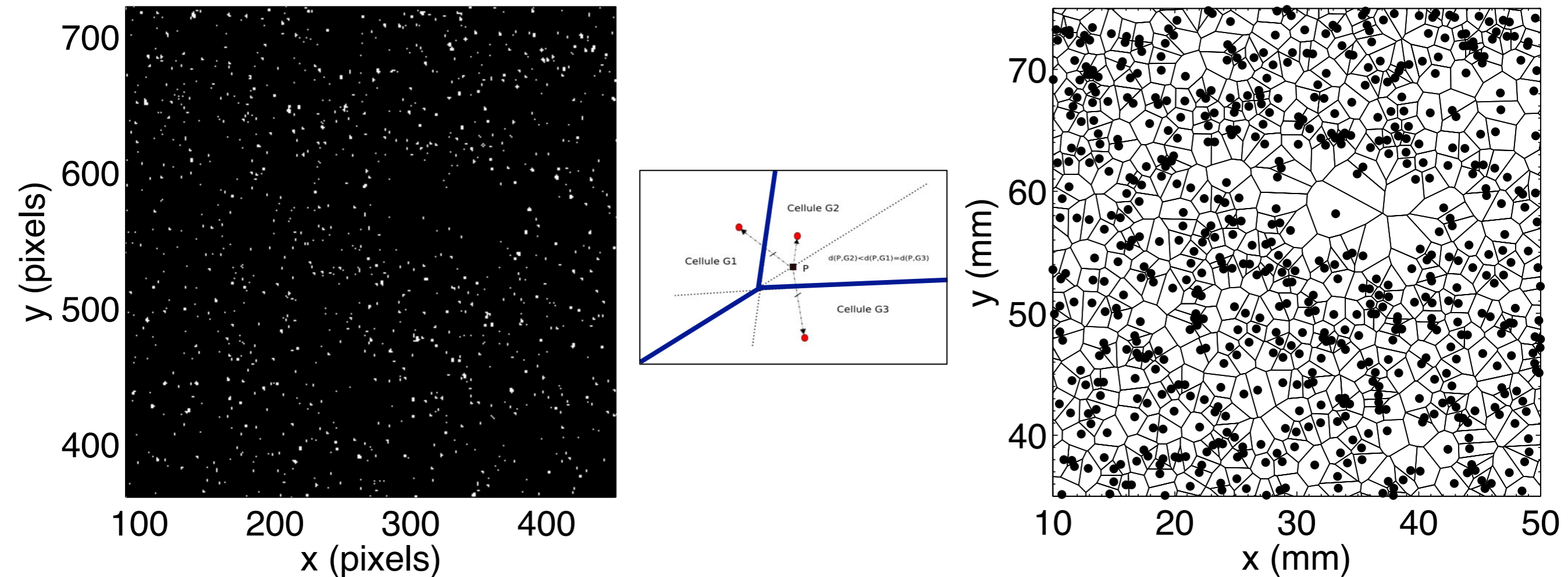
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## Size distribution



# Quantifying clustering with Voronoï tessellation

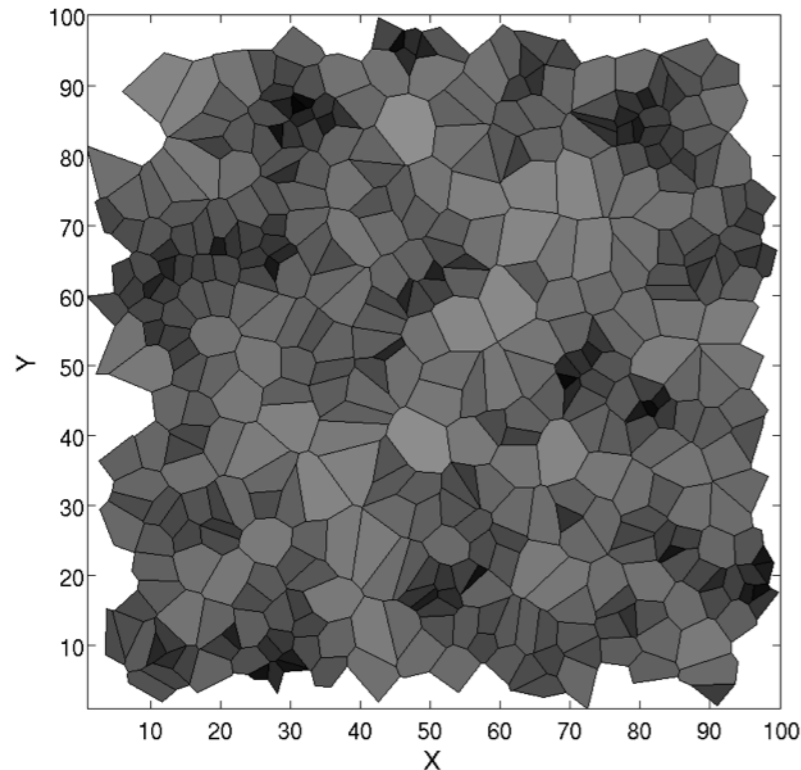
## Voronoi Tessellation



- Local concentration is estimated at an intrinsic length scale  $C_{loc} = A^{-1}$
- Both global and local (multi-scale) information.
- Allow cluster identification and characterization.
- Easily ported to Lagrangian framework (tracking of Voronoi cells).

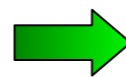
# Quantifying clustering with Voronoï tessellation

Voronoï Tessellation - some known properties



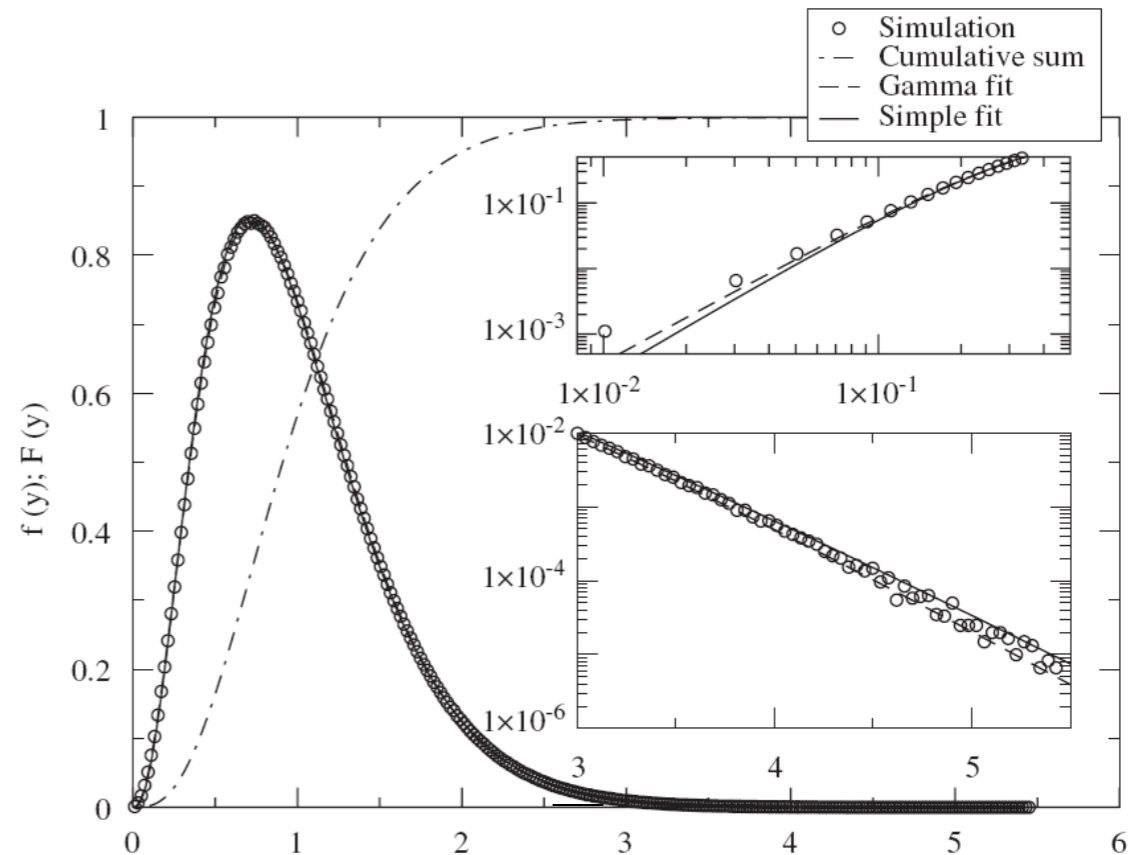
$$\bar{A} = C_0^{-1}$$

Average Voronoï area does not carry any structural information



$$\mathcal{V} = A / \bar{A}$$

Normalized Voronoï area



## Random Poisson Process (RPP)

- No known analytical form of the PDF of  $A$  for a RPP, but it is well approximated by Gamma function.

- $\sigma_{\mathcal{V}}^{\text{RPP}} = \sqrt{\langle \mathcal{V}^2 \rangle_{\text{RPP}} - 1} \simeq 0.53$

# Clusters definition

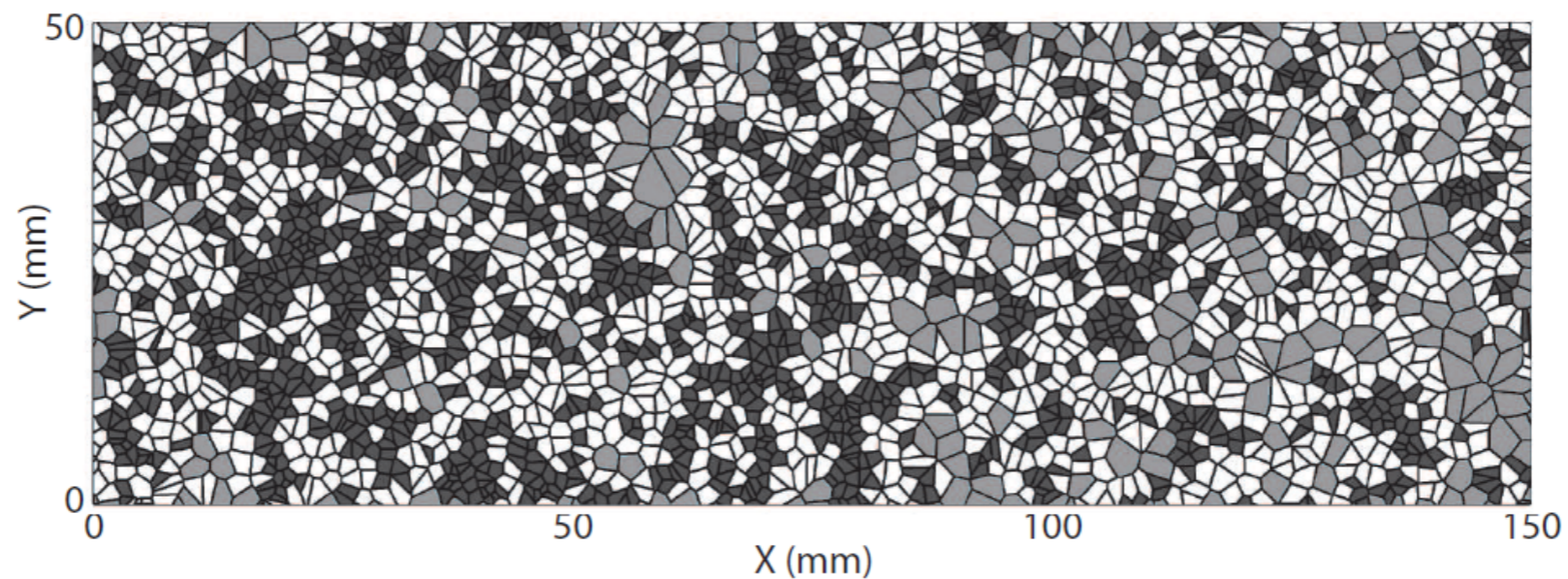
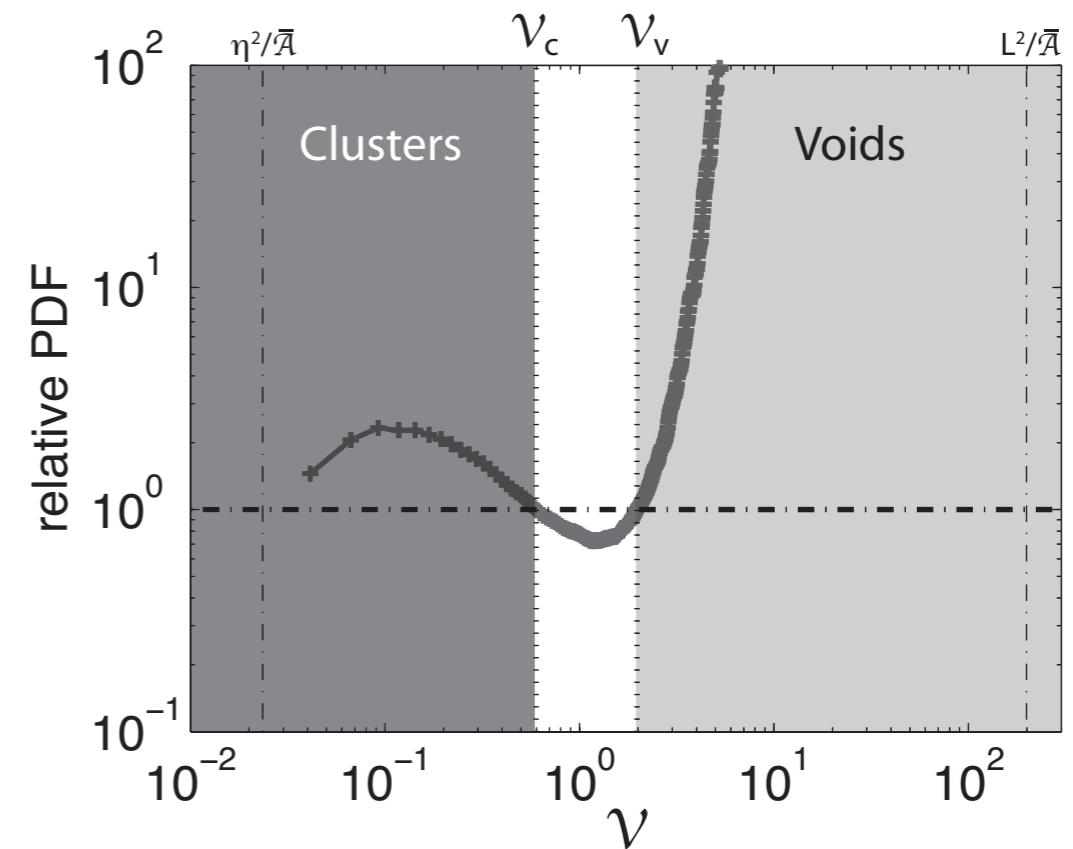
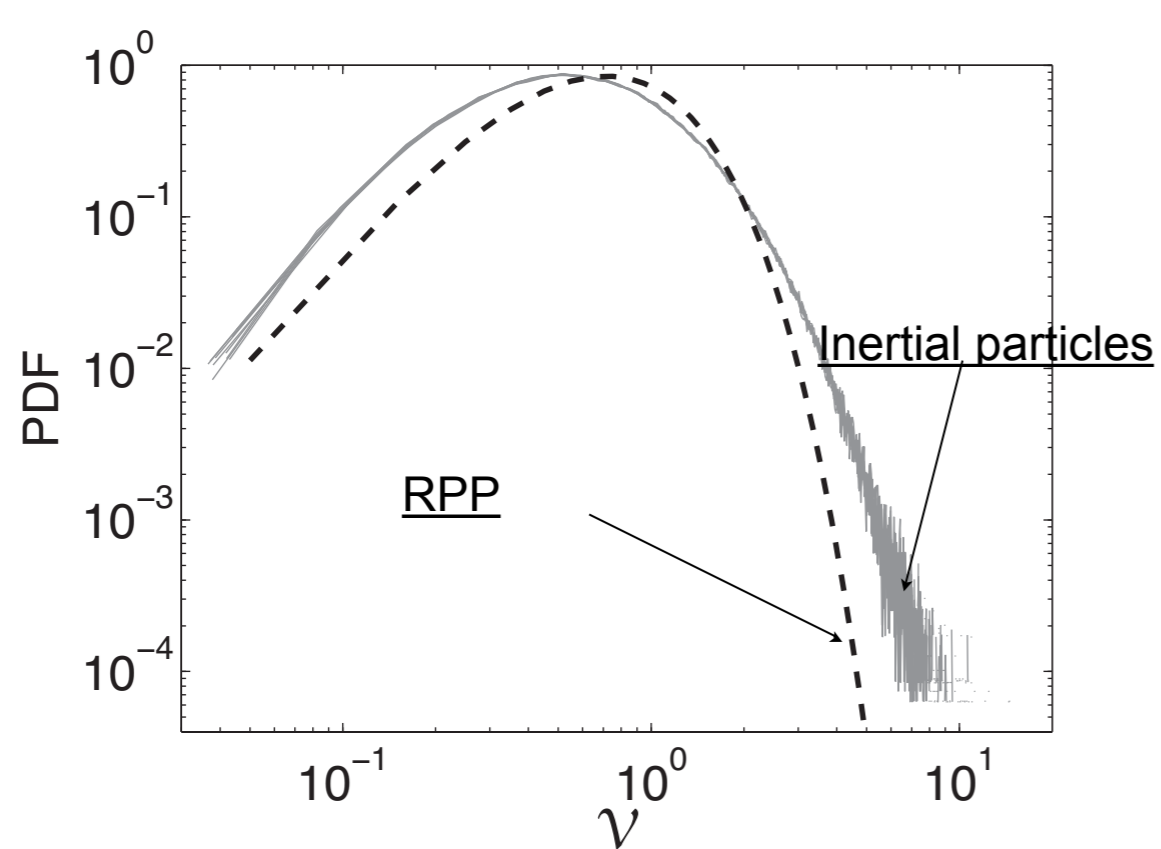
## Voronoi Analysis

Monchaux et al., PoF 2010

Monchaux et al., JMF 2012

Tagawa et al., JFM 2012

Obligado et al., JoT 2014

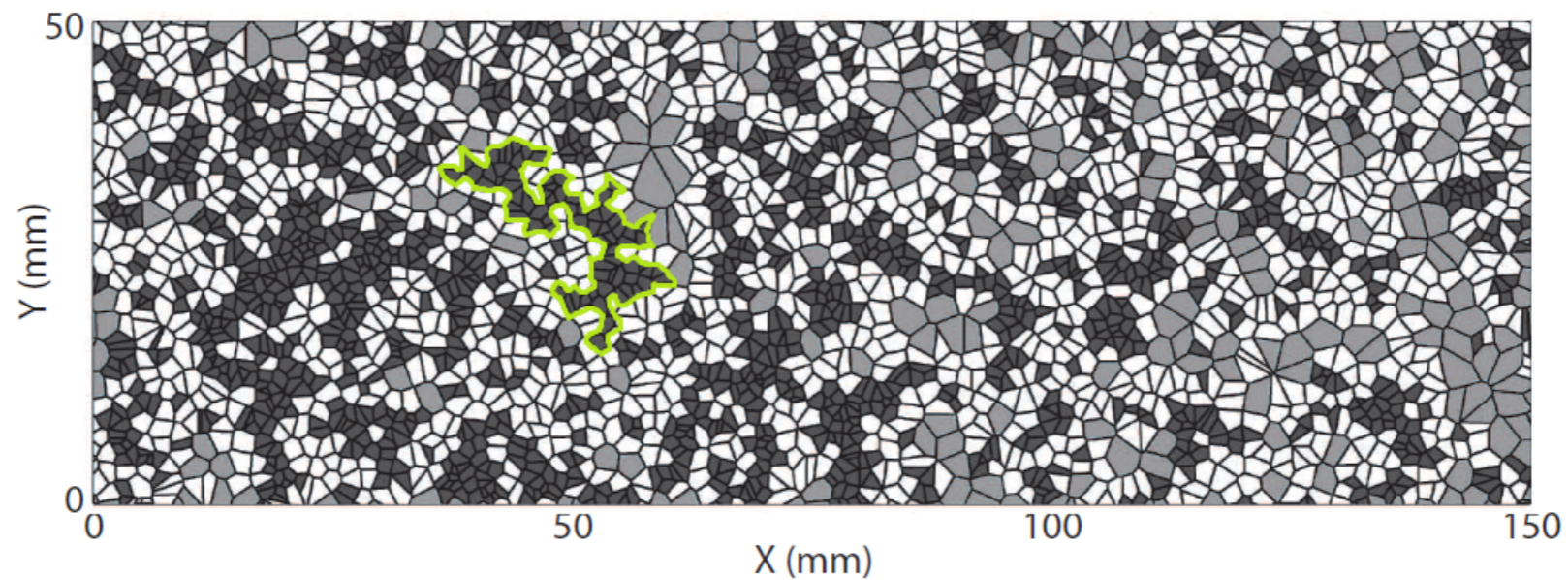
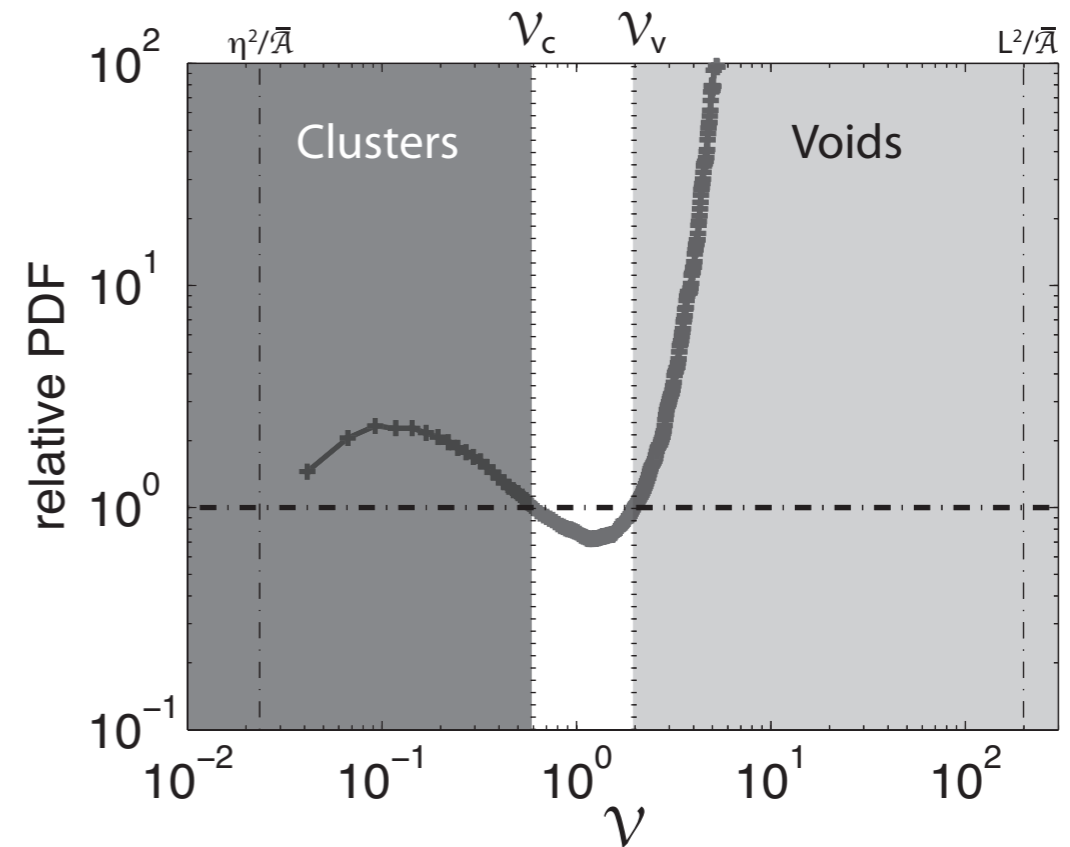
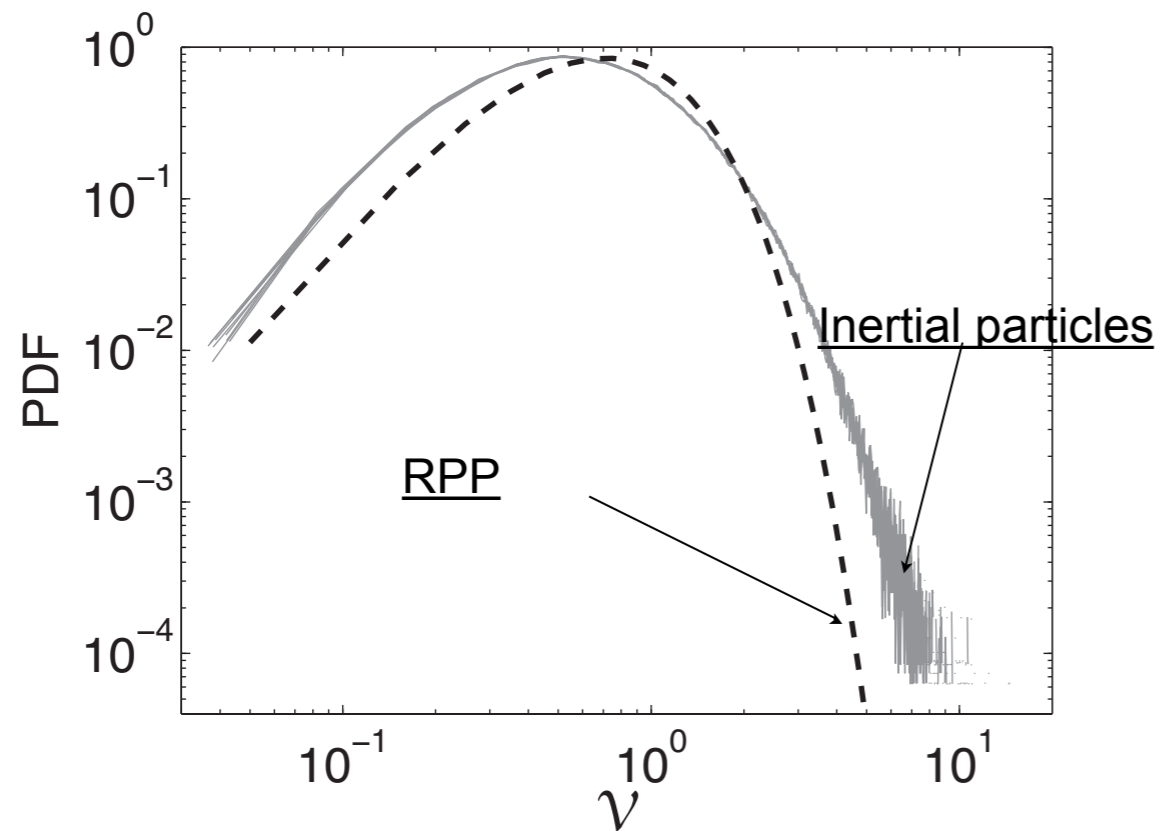




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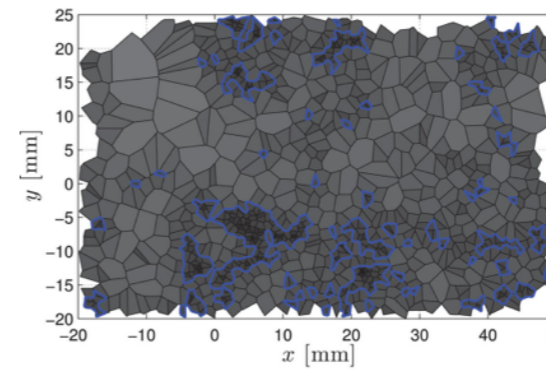
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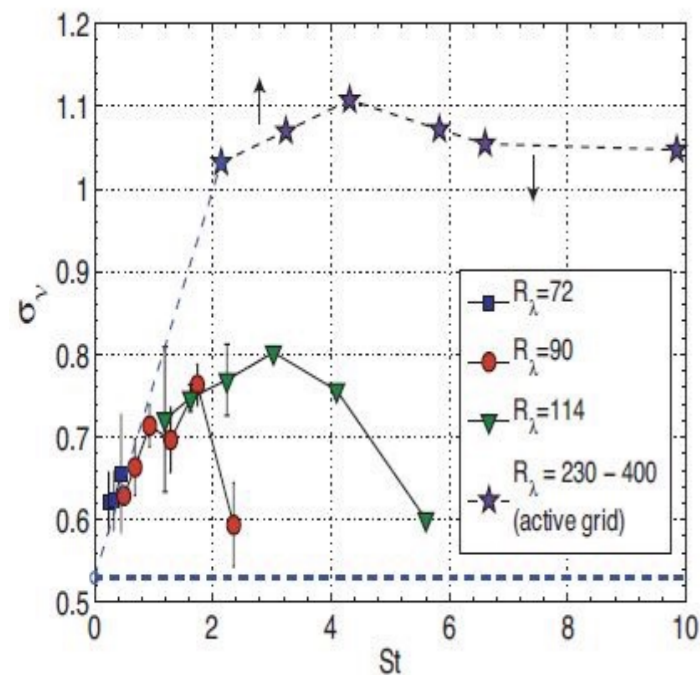
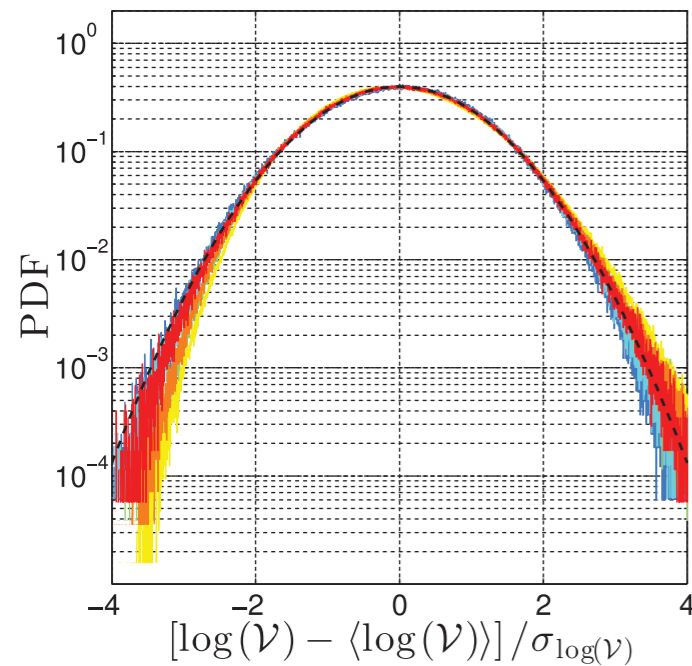
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## Voronoi Analysis

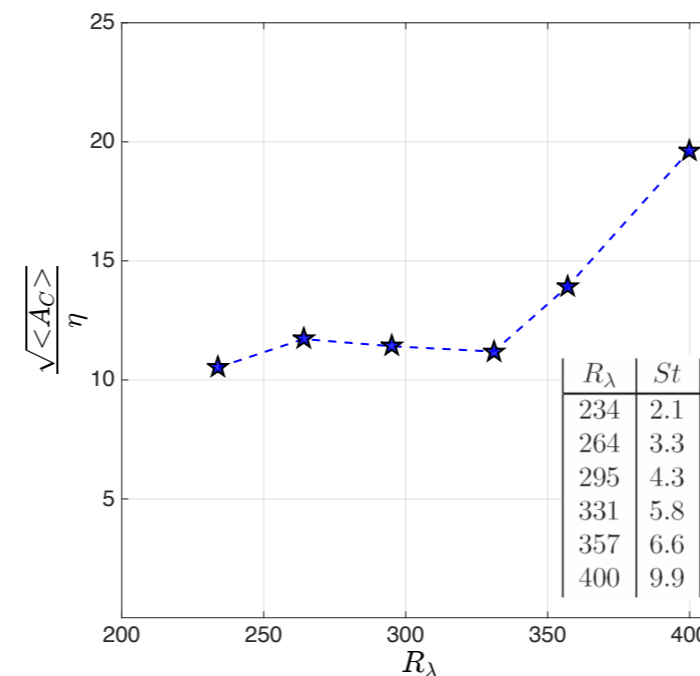
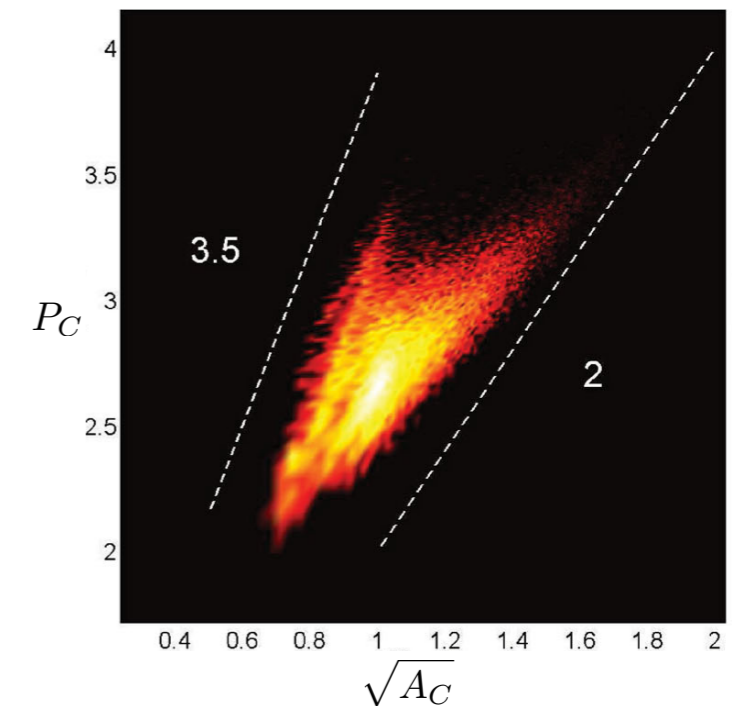
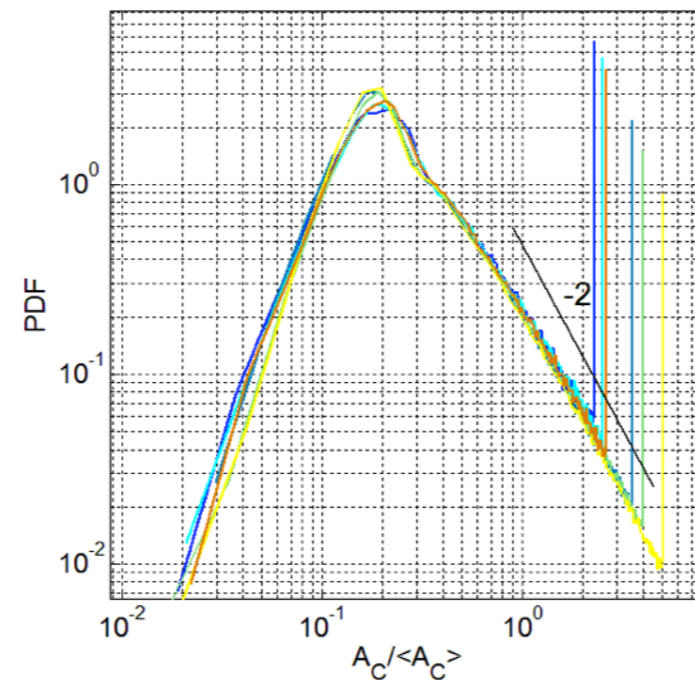


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## Statistics of Voronoi areas



## Clusters geometry



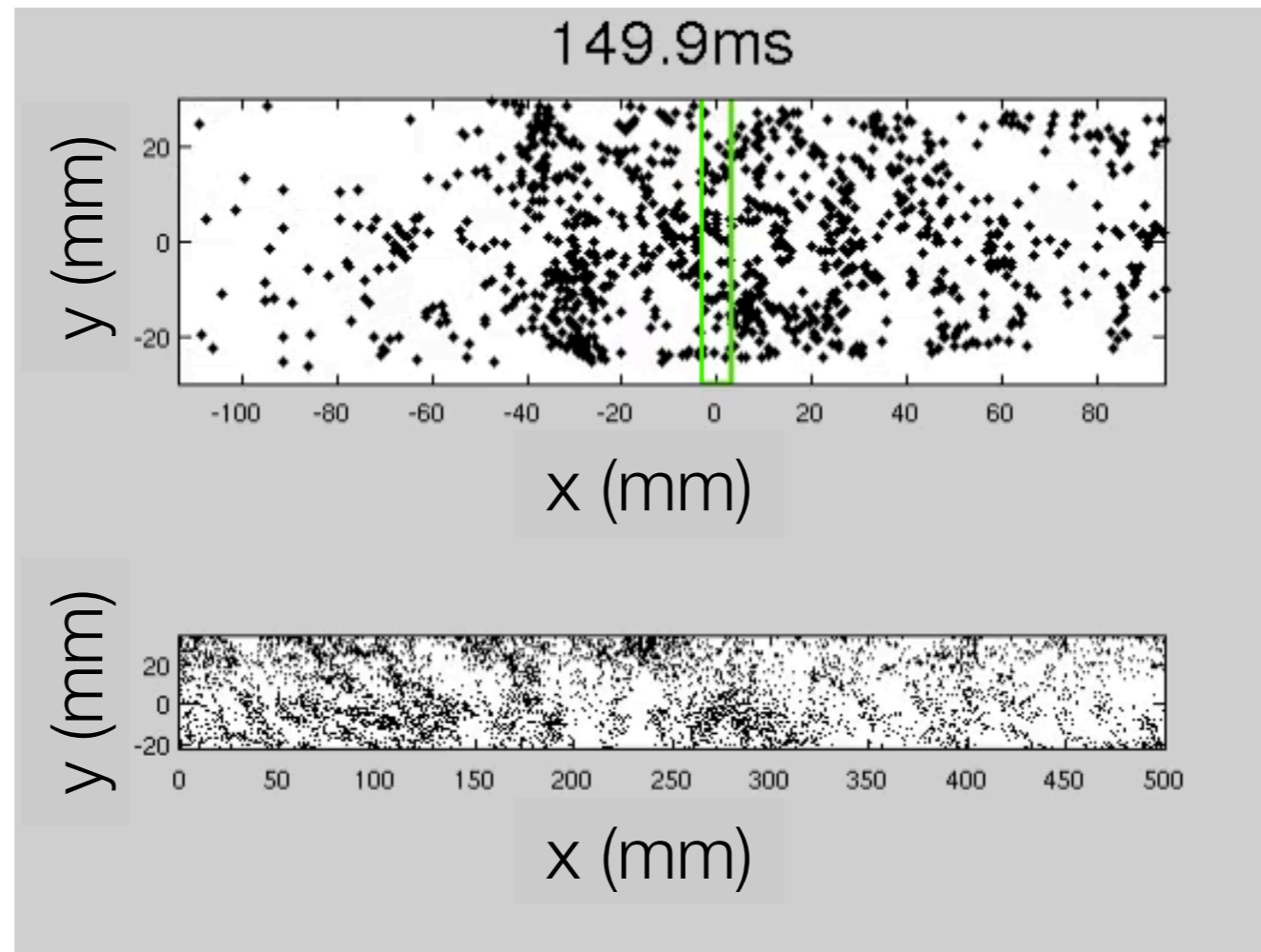
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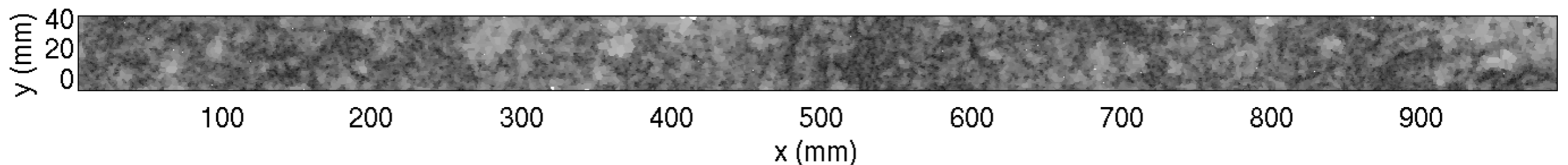
# Taylor hypothesis applied to high speed imaging

Reconstruction of large scale fields

## “Linear Camera” reconstruction

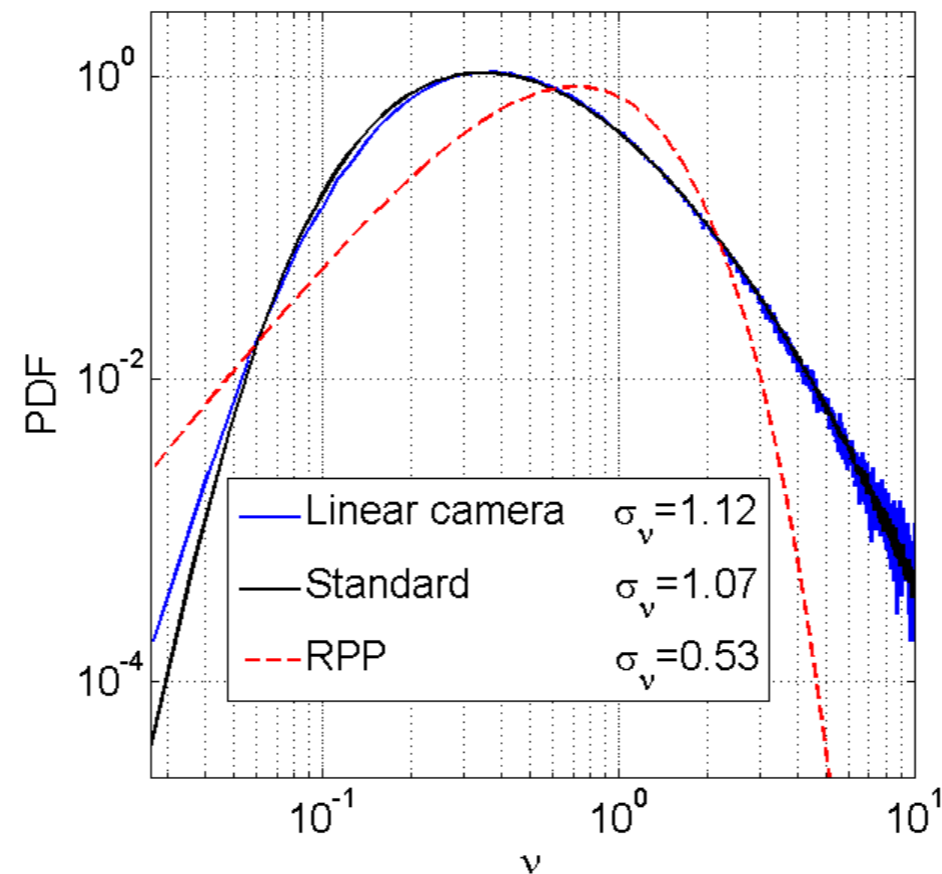
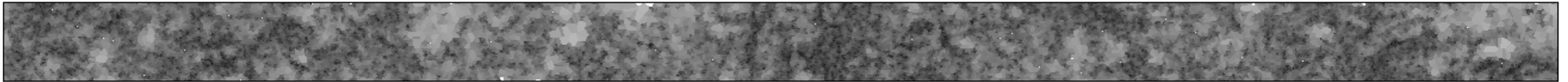


We can reconstruct a large scale field and apply Voronoï Tessellation :



# Validation of Taylor hypothesis

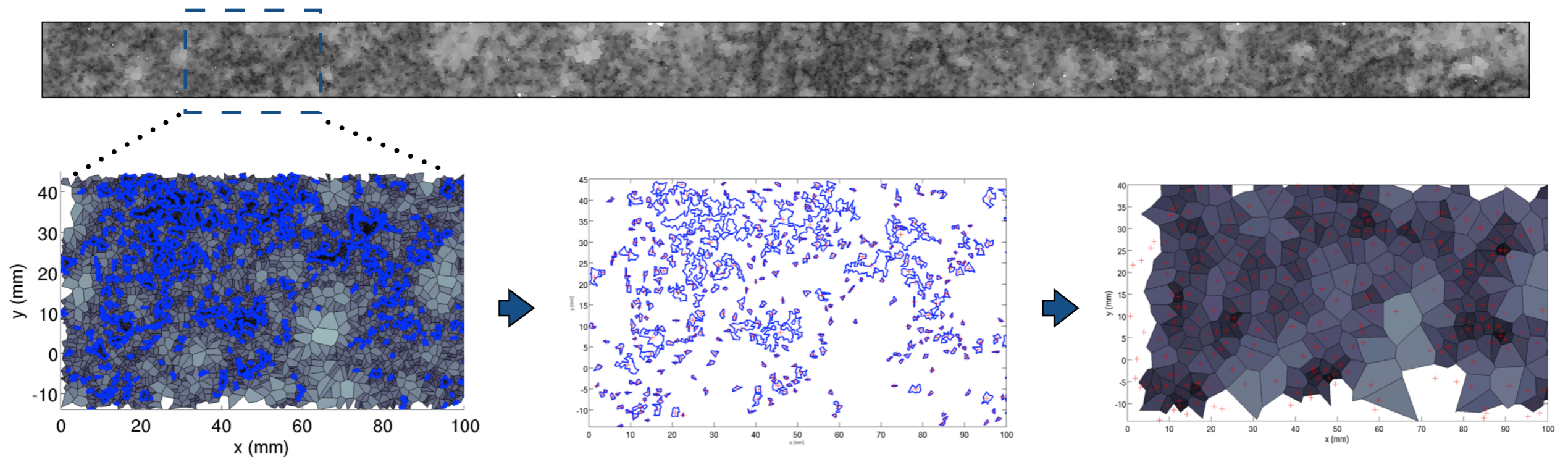
Voronoi analysis of particles centers



Statistics of Voronoi areas reconstructed using the Taylor hypothesis are identical than the statistics obtained with the classical image per image analysis

# Evidence of Super-Clustering

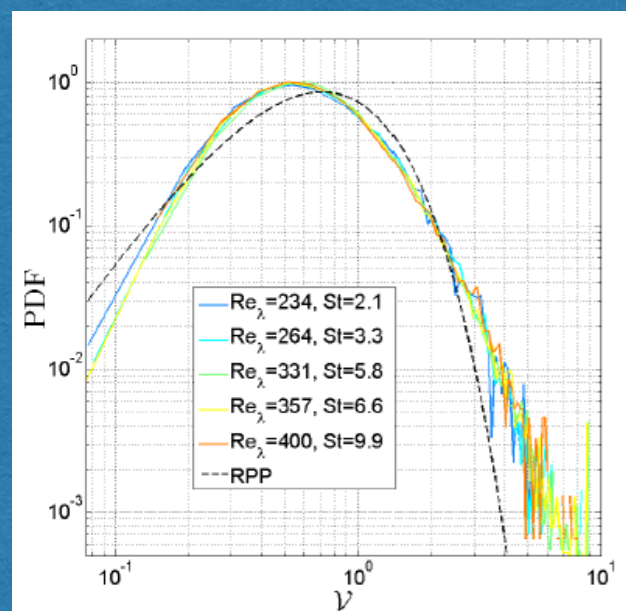
Voronoi analysis of cluster centers



1. Identify clusters of particles from Voronoi diagram

2. Find clusters of mass

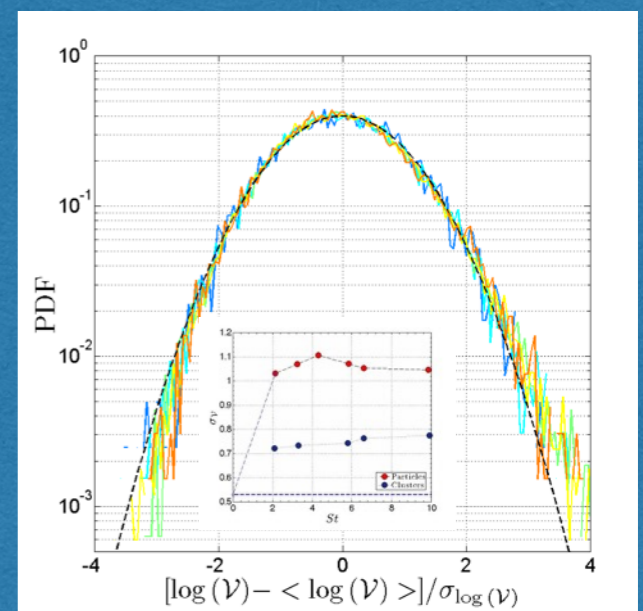
3. Voronoi analysis of clusters center of mass



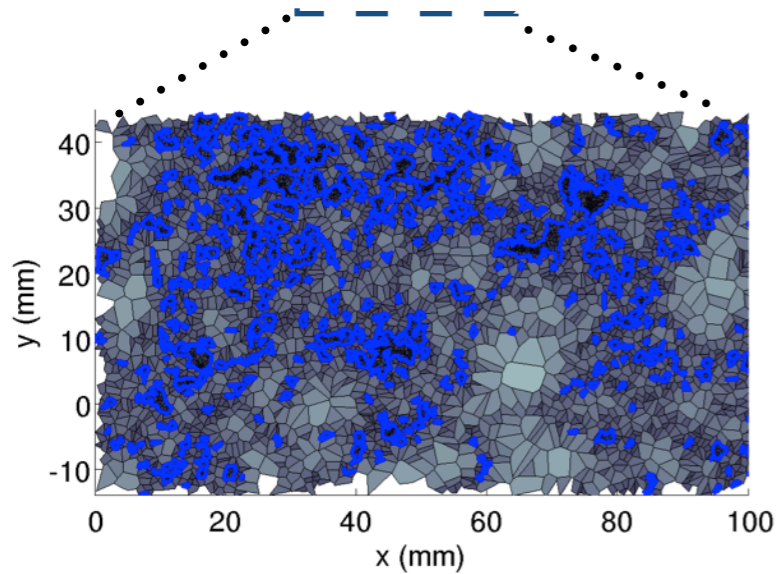
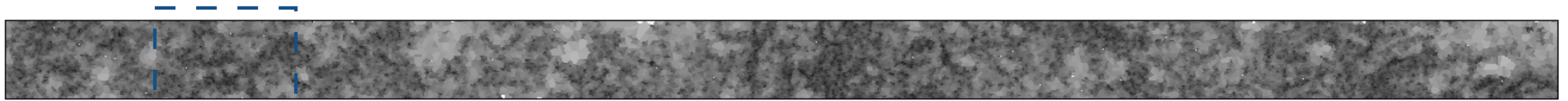
## Super-clustering !

Log-normal Super-clustering  
(Similar behavior as particle clusters)

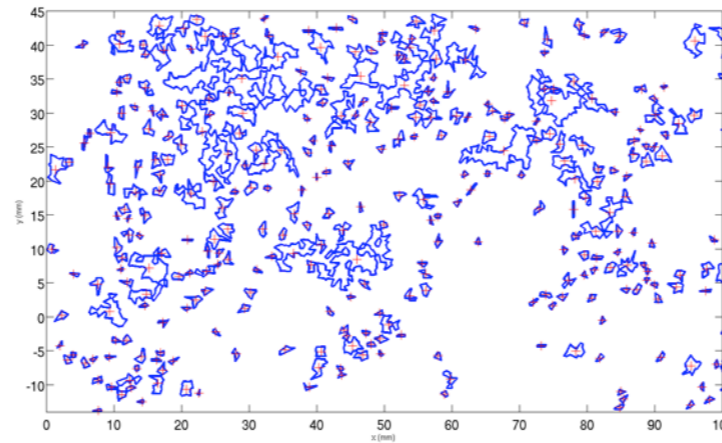
Super-Clustering is less pronounced than clustering



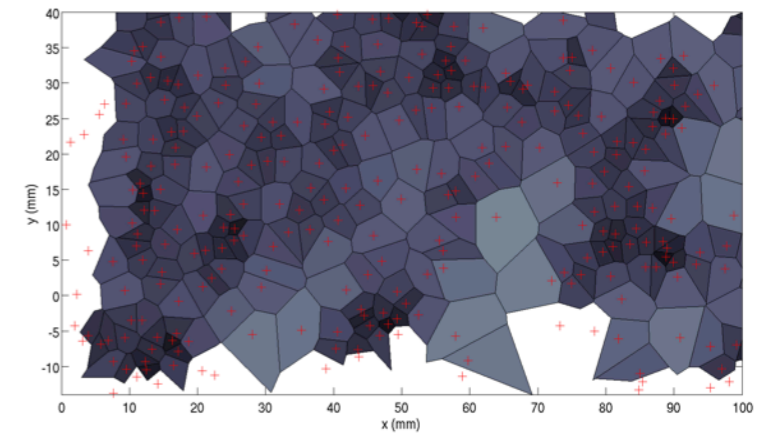
# Super-clusters identification



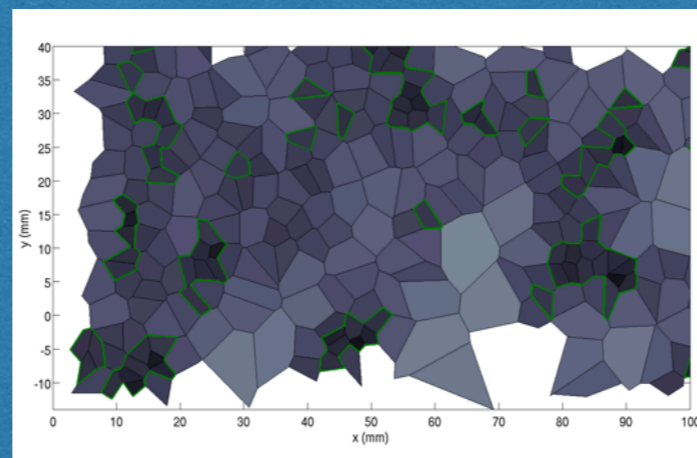
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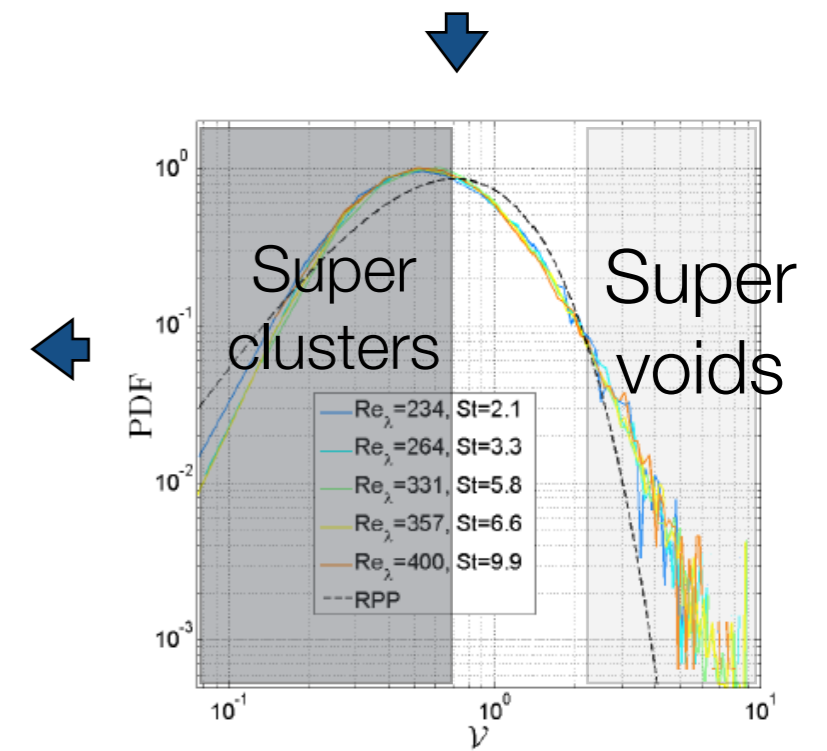
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3. Voronoï analysis of clusters center of mass

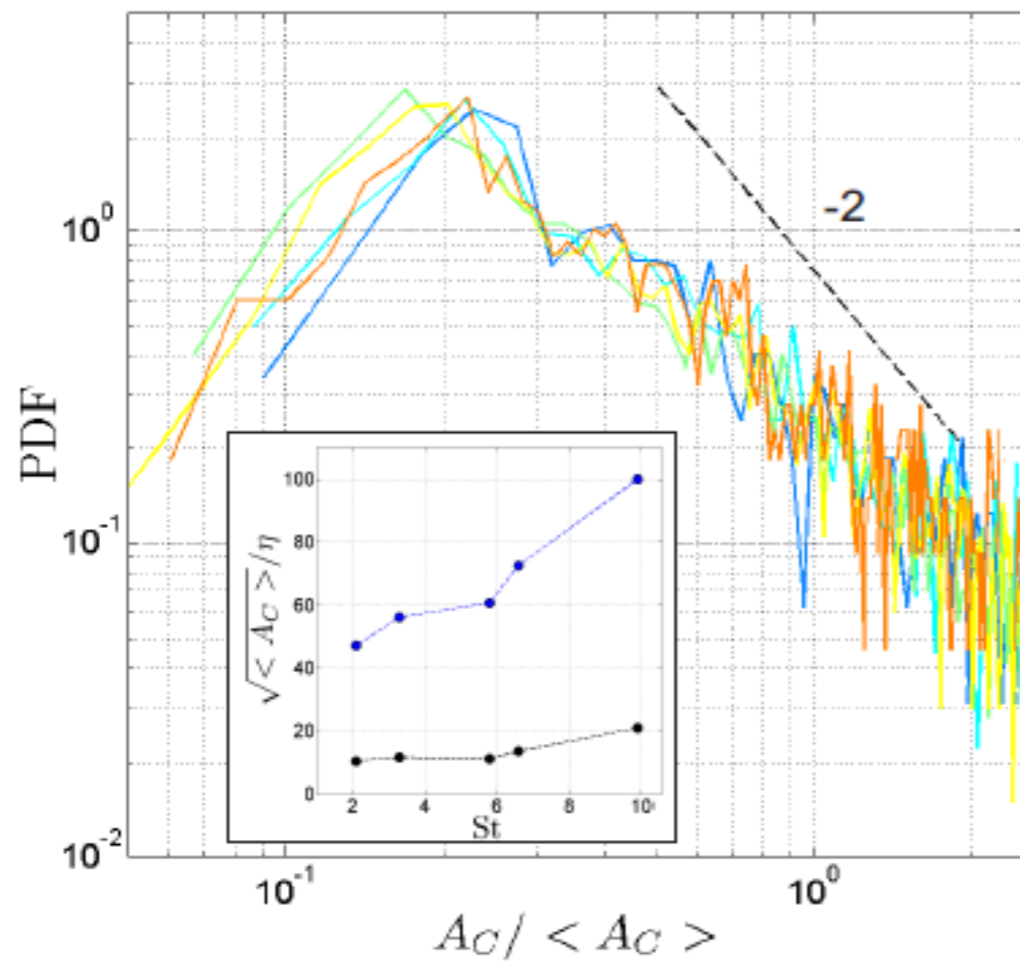


Super-clusters

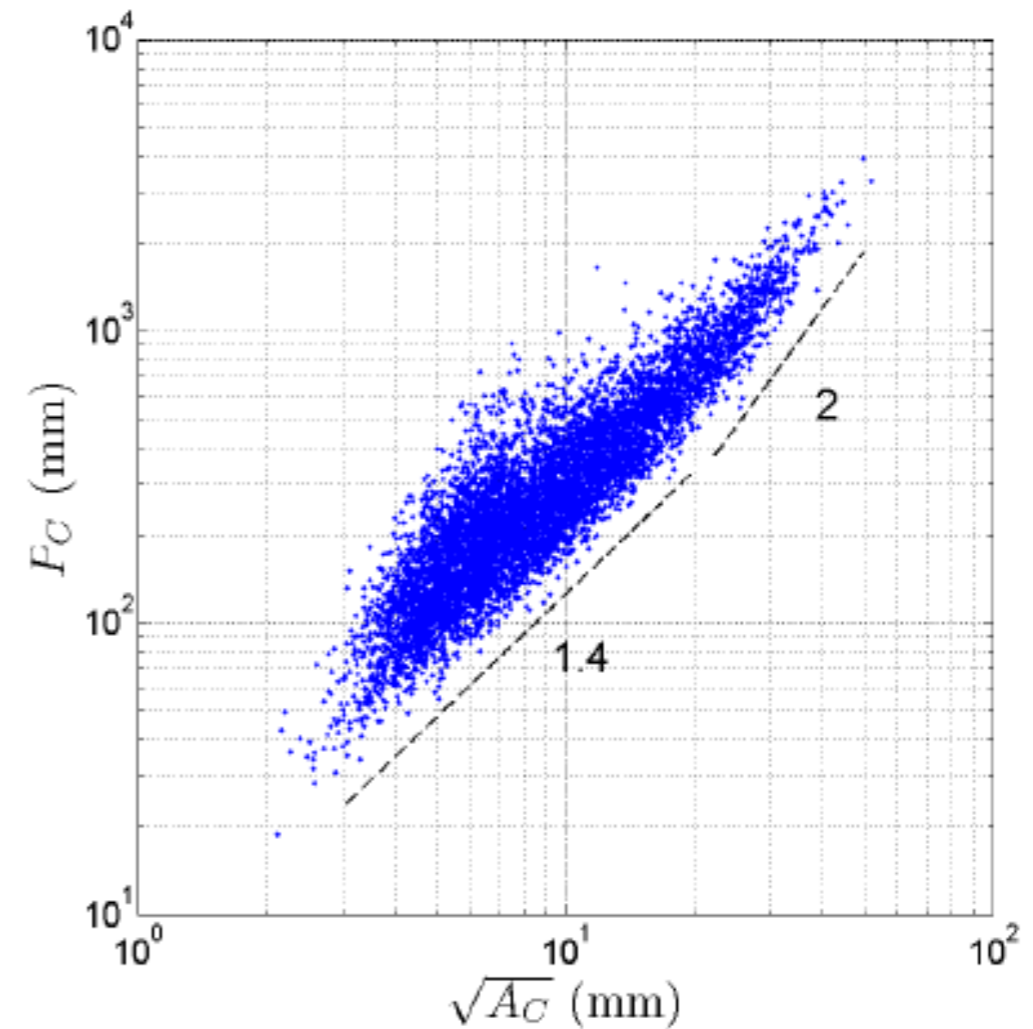


# Super-clusters geometry

Super-clusters have a typical size  
 $\sim 50 - 100 \eta$



Super-clusters have  
fractal geometry



# Take Home messages

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- Inertial particles in turbulence tend to segregate in clusters
  - Lognormal distribution of Voronoï areas
  - Maximum of clustering for  $St \sim 1$
  - Fractal geometry
  - Typical size  $\sim 10 \eta$
- Clusters of inertial particles tend to segregate in super-clusters
  - Lognormal distribution of Voronoï areas
  - Clustering less pronounced than for particles themselves
  - Fractal geometry
  - Typical size  $\sim 50 - 100 \eta$ , increases with  $St$  (or Reynolds)
- Perspectives
  - Disentangle trends with  $St$  and Reynolds ?
  - Dynamical aspects of clusters and super-clusters ?
  - Hyper-clusters ?

**Thank you !**